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Tokovi snaga u kompleksnom domenu primjenom Newton-Raphson metoda

Seminarski rad

- Strukture i režimi rada elektroenergetskih sistema -

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Sarajevo, juli 2024.

Sadržaj

Poglavlje 1

Tokovi snaga

Proračun tokova snaga je jedan od baznih proračuna koji se vrše u svrhu planiranja, projektovanja te eksploatacije elektroenergetskog sistema. Tokovi snaga su proračuni koji se vrše za slučaj kada se elektroenergetski sistem nalazi u kvazistacionarnom stanju.

Pretpostavlja se da su svi elementi sistema simetrični te da je potrošnja balansirana, te se samim tim proračun svede na proračun pozitivne sekvence u sistemu simetričnih komponenti (jednofazni krug).

Prvi zadatak kod proračuna tokova snaga je određivanje kompleksnih napona svih sabirnica, dok je drugi zadatak da se na osnovu proračunatih napona čvorova i modela sistema odrede struje i snage kroz sve elemente sistema.

1.1 Newton-Raphson metoda

Newton-Raphson metod (NR) je najrašireniji metod za proračunavanje tokova snaga, pružajući bolje performanse u prenosnim mrežama u poređenju sa svim ostalim algoritmima. Koristi se za pronalaženje nultačaka (korijena) nelinearnih realnih funkcija.

Pri rješavanju sistema od n nelinearnih jednačina sa n nepoznatih, svaku i-tu nelinearnu funkciju možemo zapisati u obliku:

$$f_i(\mathbf{x}) = 0,$$

gdje je $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$ vektor nepoznatih.

Za najčešću aproksimaciju (linearizaciju) nelinearne funkcije koristi se Taylorov polinom prvog reda. Da bi funkcija mogla biti linearizovana Taylorovim polinomom, potrebno je da funkcija ima derivacije svakog reda.

Kada se svaka od n jednačina zapiše u ovom obliku, te se jednačbe konvertuju u matrično-vektorski format:

$$\begin{bmatrix} f_1([\mathbf{x}^{(0)}]) \\ f_2([\mathbf{x}^{(0)}]) \\ \vdots \\ f_n([\mathbf{x}^{(0)}]) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 - x_1^{(0)} \\ x_2 - x_2^{(0)} \\ \vdots \\ x_n - x_n^{(0)} \end{bmatrix} = 0,$$

gdje su:

- $[\mathbf{f}([\mathbf{x}^{(0)}])]$ vektor vrijednosti funkcija za trenutne vrijednosti nepoznatih,
- $[\mathbf{J}]$ matrica Jacobijana,
- $[\Delta\mathbf{x}^{(0)}]$ pomjeraj vektora nepoznatih.

Te se vektor nepoznatih za narednu iteraciju odredi kao:

$$[\mathbf{x}^{(1)}] = [\mathbf{x}^{(0)}] + [\Delta\mathbf{x}^{(0)}],$$

te se proces nastavlja iterativno dok se ne ispuni uvjet konvergencije:

$$\max|\mathbf{f}([\mathbf{x}^{(k)}])| < \varepsilon,$$

ili dok se ne dosegne maksimalni broj iteracija za slučaj divergencije.

Za svaki od čvorova su poznata dva od četiri parametra koja ga opisuju. Potrebno je napisati jednačine u zavisnosti od modela sistema i poznatih vrijednosti, kako bismo odredili kompleksne napone čvorova i ostale nepoznate vrijednosti (struje, snage, gubitke). Neki od tih odnosa su nelinearni, pa se sistem jednačina rješava koristeći Newton-Raphson metod koji je prethodno opisan.

Za proračun tokova snaga potrebno je kreirati jednačine koje opisuju model. Kako za kompleksnu snagu čvora vrijedi:

$$s_i = v_i \cdot i_i^*$$

potrebno je formirati jednačine koje će povezati snage čvorova i poznate varijable posmatranog čvora. Neka Y označava matricu admitanse mreže. Struja čvora i je data kao:

$$i_i = \sum_{j=1}^{n} y_{ij} v_j$$

pa je tada snaga čvora i:

$$s_i = v_i \sum_{j=1}^{n} y_{ij}^* v_j^*$$

Kako su za različite čvorove poznati različiti parametri, u tu svrhu se razlikuju tri tipa čvorova (PQ, PV i balansni (*slack*)) za koje se postavljaju različiti tipovi jednačina po snagama koje zajedno čine matematički model cjelokupne razmatrane mreže.

1. Za čvorove potrošačkog tipa (PQ):
Za PQ čvorove su poznate aktivna P_i i reaktivna Q_i snaga potrošača, odnosno čitava kompleksna snaga $s_i = P_i + jQ_i = s_{i_n}(k_Z v_i^2 + k_I v_i + k_p)$. Potrebno je odrediti kompleksnu vrijednost napona v_i za taj čvor. Za određivanje polazi se od relacije:

$$s_i = s_{i_n}(k_Z v_i^2 + k_I v_i + k_p)$$

$$\Rightarrow s_i - s_{i_n}(k_Z v_i^2 + k_I v_i + k_p) = 0$$

$$\Rightarrow v_i \sum_{j=1}^{n} y_{ij}^* v_j^* - s_{i_n}(k_Z v_i^2 + k_I v_i + k_p) = 0$$

pa se formiraju funkcije u kompleksnom domenu:

$$f_{i_S} = v_i \sum_{j=1}^{n} y_{ij}^* v_j^* - s_{i_n}(k_Z v_i^2 + k_I v_i + k_p) = 0 \tag{1.1}$$

$$f_{i_S}^* = v_i^* \sum_{j=1}^{n} y_{ij} v_j - s_{i_n}^*(k_Z v_i^2 + k_I v_i + k_p) = 0 \tag{1.2}$$

2. Za čvorove s regulacijom napona (PV):
Za PV čvorove su poznate aktivna (mehanička) snaga P_i i modul napona čvora $|v_i|$. Potrebno je odrediti ugao kompleksnog napona čvora Φ_i, te reakivnu snagu Q_i za taj čvor. Za određivanje polazi se od relacija:

$$P_i = P_i^{inj}$$

$$\Rightarrow P_i - P_i^{inj} = 0$$

a kako vrijedi $P_i = \frac{1}{2}(s_i + s_i^*)$, tada je i:

$$\frac{1}{2}(s_i + s_i^*) - P_i^{inj} = 0$$

$$\Rightarrow \frac{1}{2}(v_i \sum_{j=1}^{n} y_{ij}^* v_j^* + v_i^* \sum_{j=1}^{n} y_{ij} v_j) - p_i^{inj}$$

S druge strane za napon čvora vrijedi:

$$v_i v_i^* = |v_i|^2$$

$$\Rightarrow v_i v_i^* - |v_i|^2 = 0$$

pa se formiraju funkcije:

$$f_{i_P} = \frac{1}{2}(v_i \sum_{j=1}^{n} y_{ij}^* v_j^* + v_i^* \sum_{j=1}^{n} y_{ij} v_j) - p_i^{inj} = 0, \tag{1.3}$$

$$f_{|v_i|} = v_i v_i^* - |v_i|^2 = 0. \tag{1.4}$$

3. Za balansne čvorove:
Za balansni čvor je poznata kompleksna vrijednost napona čvora v_i. Potrebno je odrediti aktivnu P_i i reakivnu snagu Q_i za taj čvor, odnosno kompleksnu snagu s_i. Za određivanje polazi se od relacija:

$$Re(v_i) = v_i^{(sl)}$$

$$Im(v_i) = 0$$

ukoliko je ugao balansnog čvora Φ_i jednak nuli.
Kako vrijedi:

$$Re(v_i) = \frac{1}{2}(v_i + v_i^*)$$

$$Im(v_i) = \frac{i}{2}(v_i^* - v_i)$$

tada je:

$$\frac{1}{2}(v_i + v_i^*) = v_i^{(sl)}$$

odnosno:

$$\frac{1}{2}(v_i + v_i^*) - v_i^{(sl)} = 0$$

i

$$\frac{i}{2}(v_i^* - v_i) = 0$$

pa se formiraju funkcije:

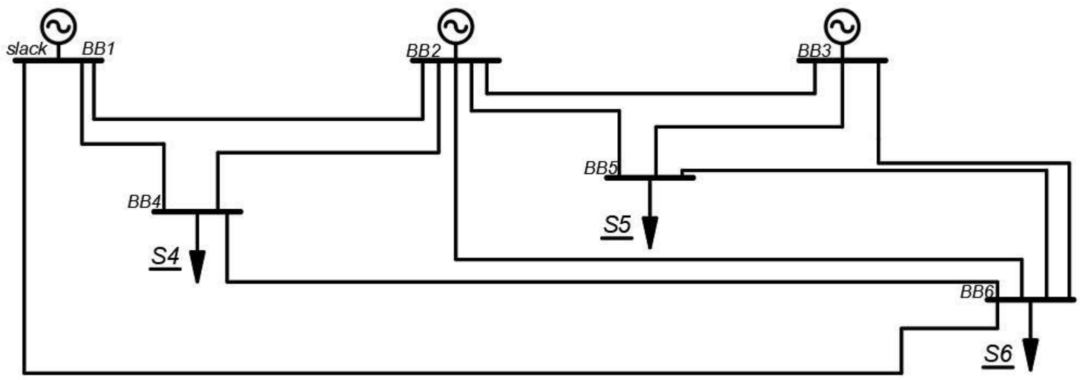
$$f_{v_i^{Re}} = \frac{1}{2}(v_i + v_i^*) - v_i^{(sl)} = 0, \tag{1.5}$$

$$f_{v_i^{Im}} = \frac{i}{2}(v_i^* - v_i) = 0. \tag{1.6}$$

Ove jednačine se rješavaju iterativno koristeći Newton-Raphson metod sve dok se ne ispuni kriterij konvergencije ili se ne dosegne maksimalni broj iteracija kako je ranije navedeno.

1.2 Postavka problema

Odrediti nepoznate kompleksne napone na sabirnicama elektroenergetskog sistema prikazanog na slici 1.1. Sabirnica BB1 je balansna (slack) sa naponom $v_1 = 1.05e^{j0^\circ}$ [p.u.]. Sabirnica BB2 je generatorska sa regulacijom napona (PV), gdje je aktivna injektirana snaga $p_2 = 0.5$ [p.u.] ,a modul napona $v_2 = 1.05$ [p.u.]. Sabirnica BB3 je također generatorska (PV), gdje je aktivna injektirana snaga $p_3 = 0.6$ [p.u.], a modul napona $v_3 = 1.07$ [p.u.]. Na sabirnice BB4, BB5 i BB6 spojeni su potrošači (PQ čvorovi) čija je snaga $s_4 = s_5 = s_6 = 0.7 + j0.7$ [p.u.]. Impedanse prenosnih linija su date u tabeli 1.1.



Slika 1.1: Prikaz elektroenergetskog sistema sa šest sabirnica

Linija	Od sabirnice	Do sabirnice	r [p.u.]	x [p.u.]	b [p.u.]
1	BB1	BB2	0.1	0.2	0.04
2	BB1	BB4	0.05	0.2	0.04
3	BB1	BB5	0.08	0.3	0.06
4	BB2	BB3	0.05	0.25	0.06
5	BB2	BB4	0.05	0.1	0.02
6	BB2	BB5	0.1	0.3	0.04
7	BB2	BB6	0.07	0.2	0.05
8	BB3	BB5	0.12	0.26	0.05
9	BB3	BB6	0.02	0.1	0.02
10	BB4	BB5	0.2	0.4	0.08
11	BB5	BB6	0.1	0.3	0.06

Tabela 1.1: Parametri linija

U tabeli 1.1 su dati parametri linija, na osnovu kojih se određuju podužna impedansa i poprečna provodnost sistema (elementi π ekvivalenta linije):

$$z_{ij} = r_{ij} + jx_{ij},$$

$$y_{sh,ij} = jb_{ij},$$

gdje su i i j sabirnice između kojih se nalazi prenosna linija.

1.2.1 Rješenje problema

Matrica admitanse sistema je:

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & 0 & Y_{14} & Y_{15} & 0 \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} \\ 0 & Y_{32} & Y_{33} & 0 & Y_{35} & Y_{36} \\ Y_{41} & Y_{42} & 0 & Y_{44} & Y_{45} & 0 \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} \\ 0 & Y_{62} & Y_{63} & 0 & Y_{65} & Y_{66} \end{bmatrix}$$

Prvi red matrice [Y] je:

$$Y_{11} = y_{21} + y_{41} + y_{51} + \frac{y_{\mathrm{sh}12}}{2} + \frac{y_{\mathrm{sh}14}}{2} + \frac{y_{\mathrm{sh}15}}{2}$$

$$Y_{21} = Y_{12} = \frac{1}{z_{12}}, \quad Y_{41} = Y_{14} = \frac{1}{z_{14}}, \quad Y_{51} = Y_{15} = \frac{1}{z_{15}}$$

Drugi red matrice [Y] je:

$$Y_{22} = y_{21} + y_{23} + y_{24} + y_{25} + y_{26} + \frac{y_{\mathrm{sh}12}}{2} + \frac{y_{\mathrm{sh}23}}{2} + \frac{y_{\mathrm{sh}24}}{2} + \frac{y_{\mathrm{sh}25}}{2} + \frac{y_{\mathrm{sh}26}}{2}$$

$$Y_{23} = Y_{32} = \frac{1}{z_{23}}, \quad Y_{24} = Y_{42} = \frac{1}{z_{24}}, \quad Y_{25} = Y_{52} = \frac{1}{z_{25}}, \quad Y_{26} = Y_{62} = \frac{1}{z_{26}}$$

Treći red matrice [Y] je:

$$Y_{33} = y_{23} + y_{35} + y_{36} + \frac{y_{\mathrm{sh}23}}{2} + \frac{y_{\mathrm{sh}35}}{2} + \frac{y_{\mathrm{sh}36}}{2}$$

$$Y_{35} = Y_{53} = \frac{1}{z_{35}}, \quad Y_{36} = Y_{63} = \frac{1}{z_{36}}$$

Četvrti red matrice [Y] je:

$$Y_{44} = y_{41} + y_{24} + y_{45} + \frac{y_{\mathrm{sh}14}}{2} + \frac{y_{\mathrm{sh}24}}{2} + \frac{y_{\mathrm{sh}45}}{2}$$

$$Y_{45} = Y_{54} = \frac{1}{z_{45}}, \quad Y_{56} = Y_{65} = \frac{1}{z_{56}}$$

Peti red matrice [Y] je:

$$Y_{55} = y_{51} + y_{25} + y_{35} + y_{45} + y_{56} + \frac{y_{\mathrm{sh}15}}{2} + \frac{y_{\mathrm{sh}25}}{2} + \frac{y_{\mathrm{sh}35}}{2} + \frac{y_{\mathrm{sh}45}}{2} + \frac{y_{\mathrm{sh}56}}{2}$$

Šesti red matrice [Y] je:

$$Y_{66} = y_{26} + y_{36} + y_{56} + \frac{y_{\mathrm{sh}26}}{2} + \frac{y_{\mathrm{sh}36}}{2} + \frac{y_{\mathrm{sh}56}}{2}$$

Nepoznate koje je potrebno odrediti, odnosno varijable sistema su kompleksni naponi čvorova i konjugovano kompleksni naponi čvorova koji se posmatraju kao odvojene varijable:

$$[x^{(0)}] = \begin{bmatrix} v_1^{(0)} \\ v_2^{(0)} \\ v_3^{(0)} \\ v_4^{(0)} \\ v_5^{(0)} \\ v_6^{(0)} \\ v_1^{*(0)} \\ v_2^{*(0)} \\ v_3^{*(0)} \\ v_4^{*(0)} \\ v_5^{*(0)} \\ v_6^{*(0)} \end{bmatrix} = \begin{bmatrix} 1.05 \\ 1.05 \\ 1.07 \\ 1.05 \\ 1.05 \\ 1.05 \\ 1.05 \\ 1.05 \\ 1.07 \\ 1.05 \\ 1.05 \\ 1.05 \end{bmatrix}$$

Magnitudu napona svih čvorova izjednačavamo s magnitudom napona na balansnom čvoru. Analogno važi i za uglove. Ukoliko se u mreži nalazi generator sa regulacijom napona, napon tog čvora inicijaliziramo sa vrijednošću koja se regulira.

Naredni korak je formiranje vektora funkcija ovisno o vrstama čvorova u mreži. Čvor 1 je balansni, pa vrijedi sljedeće:

$$f_{v_1^{Re}} = \frac{1}{2}(v_1 + v_1^*) - v_1^{(sl)} = 0,$$

$$f_{v_1^{Im}} = \frac{i}{2}(v_1^* - v_1) = 0.$$

Za čvor 2 koji je PV čvor, vrijede sljedeće jednadžbe:

$$f_{2_P} = \frac{1}{2}(v_2 \sum_{j=1}^{6} y_{2j}^* v_j^* + v_2^* \sum_{j=1}^{6} y_{2j} v_j) - p_2^{inj} = 0,$$

$$f_{|v_2|} = v_2 v_2^* - v_2^2 = 0.$$

Za čvor 3 koji je PV čvor, vrijede sljedeće jednadžbe:

$$f_{3_P} = \frac{1}{2}(v_3 \sum_{j=1}^{6} y_{3j}^* v_j^* + v_3^* \sum_{j=1}^{6} y_{3j} v_j) - p_3^{inj} = 0,$$

$$f_{|v_3|} = v_3 v_3^* - v_3^2 = 0.$$

Za čvor 4 koji je PQ čvor, vrijede sljedeće jednadžbe:

$$f_{4_S} = v_4 \sum_{j=1}^{6} y_{4j}^* v_j^* - s_{4_n}(k_Z v_4^2 + k_I v_4 + k_p) = 0$$

$$f_{4_S}^* = v_4^* \sum_{j=1}^{6} y_{4j} v_j - s_{4_n}^*(k_Z v_4^2 + k_I v_4 + k_p) = 0$$

Za čvor 5 koji je PQ čvor, vrijede sljedeće jednadžbe:

$$f_{5_S} = v_5 \sum_{j=1}^{6} y_{5j}^* v_j^* - s_{5_n}(k_Z v_5^2 + k_I v_5 + k_p) = 0$$

$$f_{5_S}^* = v_5^* \sum_{j=1}^{6} y_{5j} v_j - s_{5_n}^*(k_Z v_5^2 + k_I v_5 + k_p) = 0$$

Za čvor 6 koji je PQ čvor, vrijede sljedeće jednadžbe:

$$f_{6_S} = v_6 \sum_{j=1}^{6} y_{6j}^* v_j^* - s_{6_n}(k_Z v_6^2 + k_I v_6 + k_p) = 0$$

$$f_{6_S}^* = v_6^* \sum_{j=1}^{6} y_{6j} v_j - s_{6_n}^*(k_Z v_6^2 + k_I v_6 + k_p) = 0$$

Konačni vektor funkcija je:

$$[f([x])] = \begin{bmatrix} f_{1_S} \\ f_{2_S} \\ f_{3_S} \\ f_{4_S} \\ f_{5_S} \\ f_{6_S} \\ f_{1_S}^* \\ f_{2_S}^* \\ f_{3_S}^* \\ f_{4_S}^* \\ f_{5_S}^* \\ f_{6_S}^* \end{bmatrix}.$$

Nakon unosa ranije navedenih funkcija ostatak proračuna vrši se pomoću modelSolver-a. Kod koji je korišten za proračun naveden je u prilogu.

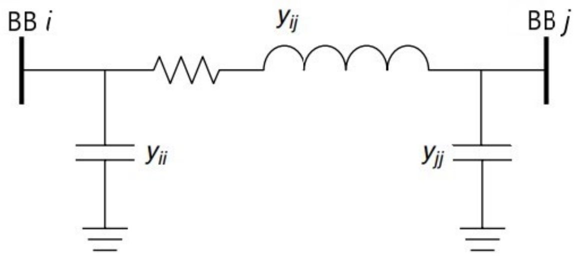
1.2.2 Post-procesiranje

Nakon dobivenih vrijednosti kompleksnih napona čvorova računaju se struje i snage kroz grane koristeći ekvivalente prenosnih elemenata grana (pozitivna sekvenca). Nakon dobivenih vrijednosti kompleksnih napona čvorova računaju se struje i snage kroz otočne shunt-ove koristeći vrijednosti admitansi otočnih grana (pozitivna sekvenca). Formule za proračun snaga sabirnica su:

$$f_i = S_i - v_i(\sum_{j=1}^{n} y_{ij} v_i)^* = 0 \tag{1.7}$$

$$f_i^* = S_i^* - v_i^* \sum_{j=1}^{n} y_{ij} v_i = 0 \tag{1.8}$$

Snage čvorova su prethodno navedene i njihovo izvođenje je intuitivno (objašnjeno u dijelu (1.1)). Za snage grana, odnosno linija potrebno je analizirati model linije i na osnovu njega izvesti jednačine snaga linije. Posmatrajući struju linije ij mjerenu na čvoru i, sa slike 1.2



Slika 1.2: Π ekvivalent (model) linije

vrijedi:

$$i_{ij} = v_i y_{ii} + (v_i - v_j) y_{ij}$$

pa je snaga linije posmatrana sa čvora i:

$$s_{ij} = v_i i_{ij}^* = v_i (v_i y_{ii} + (v_i - v_j) y_{ij})^*$$

Analogno, mjerenjem na čvoru j linije ji dobija se:

$$i_{ji} = v_j y_{jj} + (v_j - v_i) y_{ij}$$

$$s_{ji} = v_j i_{ji}^* = v_j (v_j y_{jj} + (v_j - v_i) y_{ij})^*$$

pa se snage linija izračunavaju pomoću formula:

$$f_{ij} = S_{ij} - v_i (\frac{y_{sh_{ij}}}{2} v_i + y_{ij}(v_i - v_j))^* = 0 \tag{1.9}$$

$$f_{ij}^* = S_{ij}^* - v_i^* \frac{y_{sh_{ij}}}{2} v_i + y_{ij}(v_i - v_j) = 0 \tag{1.10}$$

$$f_{ji} = S_{ji} - v_j (\frac{y_{sh_{ij}}}{2} v_j + y_{ij}(v_j - v_i))^* = 0 \tag{1.11}$$

$$f_{ji}^* = S_{ji}^* - v_j^* \frac{y_{sh_{ij}}}{2} v_j + y_{ij}(v_j - v_i) = 0 \tag{1.12}$$

Gubitke na linijama moguće je izračunati pomoću formula:

$$f_{ij} = S_{ij_{loss}} - (S_{ij} + S_{ji}) = 0 \tag{1.13}$$

$$f_{ij}^* = S_{ij_{loss}}^* - (S_{ij} + S_{ji})^* = 0 \tag{1.14}$$

Rješavani elektroenergetski sistem posjeduje 6 sabirnica, i za njih se pišu jednačine prema formulama koje su ranije navedene.
Za sabirnicu BB1 vrijedi:

$$f_1 = S_1 - v_1 (\sum_{j=1}^{6} y_{1j} v_1)^* = 0$$

$$f_1^* = S_1^* - v_1^* \sum_{j=1}^{6} y_{1j} v_1 = 0$$

Za sabirnicu BB2 vrijedi:

$$f_2 = S_2 - v_2 (\sum_{j=1}^{6} y_{2j} v_2)^* = 0$$

$$f_2^* = S_2^* - v_2^* \sum_{j=1}^{6} y_{2j} v_2 = 0$$

Za sabirnicu BB3 vrijedi:

$$f_3 = S_3 - v_3 (\sum_{j=1}^{6} y_{3j} v_3)^* = 0$$

$$f_3^* = S_3^* - v_3^* \sum_{j=1}^{6} y_{3j} v_3 = 0$$

Za sabirnicu BB4 vrijedi:

$$f_4 = S_4 - v_4 (\sum_{j=1}^{6} y_{4j} v_4)^* = 0$$

$$f_4^* = S_4^* - v_4^* \sum_{j=1}^{6} y_{4j} v_4 = 0$$

Za sabirnicu BB5 vrijedi:

$$f_5 = S_5 - v_5 (\sum_{j=1}^{6} y_{5j} v_5)^* = 0$$

$$f_5^* = S_5^* - v_5^* \sum_{j=1}^{6} y_{5j} v_5 = 0$$

Za sabirnicu BB6 vrijedi:

$$f_6 = S_6 - v_6 (\sum_{j=1}^{6} y_{6j} v_6)^* = 0$$

$$f_6^* = S_6^* - v_6^* \sum_{j=1}^{6} y_{6j} v_6 = 0$$

Nakon što se napišu sve navedene formule, potrebno je još napisati formule vezane za grane i gubitke na istim.
Za liniju 1-2 vrijedi:

$$f_{12} = S_{12} - v_1 (\frac{y_{sh_{12}}}{2} v_1 + y_{12}(v_1 - v_2))^* = 0$$

$$f_{12}^* = S_{12}^* - v_1^* \frac{y_{sh_{12}}}{2} v_1 + y_{12}(v_1 - v_2) = 0$$

Za liniju 2-1 vrijedi:

$$f_{21} = S_{21} - v_2 (\frac{y_{sh_{21}}}{2} v_2 + y_{21}(v_2 - v_1))^* = 0$$

$$f_{21}^* = S_{21}^* - v_2^* \frac{y_{sh_{21}}}{2} v_2 + y_{21}(v_2 - v_1) = 0$$

$$f_{12} = S_{12_{loss}} - (S_{12} + S_{21}) = 0$$

$$f_{12}^* = S_{12_{loss}}^* - (S_{12} + S_{21})^* = 0$$

Za liniju 1-4 vrijedi:

$$f_{14} = S_{14} - v_1 (\frac{y_{sh_{14}}}{2} v_1 + y_{14}(v_1 - v_4))^* = 0$$

$$f_{14}^* = S_{14}^* - v_1^* \frac{y_{sh_{14}}}{2} v_1 + y_{14}(v_1 - v_4) = 0$$

Za liniju 4-1 vrijedi:

$$f_{41} = S_{41} - v_4(\frac{y_{sh_{41}}}{2}v_4 + y_{41}(v_4 - v_1))^* = 0$$

$$f_{41}^* = S_{41}^* - v_4^*\frac{y_{sh_{41}}}{2}v_4 + y_{41}(v_4 - v_1) = 0$$

$$f_{14} = S_{14_{loss}} - (S_{14} + S_{41}) = 0$$

$$f_{14}^* = S_{14_{loss}}^* - (S_{14} + S_{41})^* = 0$$

Za liniju 1-5 vrijedi:

$$f_{15} = S_{15} - v_1(\frac{y_{sh_{15}}}{2}v_1 + y_{15}(v_1 - v_5))^* = 0$$

$$f_{15}^* = S_{15}^* - v_1^*\frac{y_{sh_{15}}}{2}v_1 + y_{15}(v_1 - v_5) = 0$$

Za liniju 5-1 vrijedi:

$$f_{51} = S_{51} - v_5(\frac{y_{sh_{51}}}{2}v_5 + y_{51}(v_5 - v_1))^* = 0$$

$$f_{51}^* = S_{51}^* - v_5^*\frac{y_{sh_{51}}}{2}v_5 + y_{51}(v_5 - v_1) = 0$$

$$f_{15} = S_{15_{loss}} - (S_{15} + S_{51}) = 0$$

$$f_{15}^* = S_{15_{loss}}^* - (S_{15} + S_{51})^* = 0$$

Za liniju 2-3 vrijedi:

$$f_{23} = S_{23} - v_2(\frac{y_{sh_{23}}}{2}v_2 + y_{23}(v_2 - v_3))^* = 0$$

$$f_{23}^* = S_{23}^* - v_2^*\frac{y_{sh_{23}}}{2}v_2 + y_{23}(v_2 - v_3) = 0$$

Za liniju 3-2 vrijedi:

$$f_{32} = S_{32} - v_3(\frac{y_{sh_{32}}}{2}v_3 + y_{32}(v_3 - v_2))^* = 0$$

$$f_{32}^* = S_{32}^* - v_3^*\frac{y_{sh_{32}}}{2}v_3 + y_{32}(v_3 - v_2) = 0$$

$$f_{23} = S_{23_{loss}} - (S_{23} + S_{32}) = 0$$

$$f_{23}^* = S_{23_{loss}}^* - (S_{23} + S_{32})^* = 0$$

Za liniju 2-4 vrijedi:

$$f_{24} = S_{24} - v_2(\frac{y_{sh_{24}}}{2}v_2 + y_{24}(v_2 - v_4))^* = 0$$

$$f_{24}^* = S_{24}^* - v_2^*\frac{y_{sh_{24}}}{2}v_2 + y_{24}(v_2 - v_4) = 0$$

Za liniju 4-2 vrijedi:

$$f_{42} = S_{42} - v_4(\frac{y_{sh_{42}}}{2}v_4 + y_{42}(v_4 - v_2))^* = 0$$

$$f_{42}^* = S_{42}^* - v_4^*\frac{y_{sh_{42}}}{2}v_4 + y_{42}(v_4 - v_2) = 0$$

$$f_{24} = S_{24_{\text{loss}}} - (S_{24} + S_{42}) = 0$$

$$f_{24}^* = S_{24_{\text{loss}}}^* - (S_{24} + S_{42})^* = 0$$

Za liniju 2-5 vrijedi:

$$f_{25} = S_{25} - v_2\left(\frac{y_{sh_{25}}}{2}v_2 + y_{25}(v_2 - v_5)\right)^* = 0$$

$$f_{25}^{*} = S_{25}^{*} - v_2^{*}\frac{y_{sh_{25}}}{2}v_2 + y_{25}(v_2 - v_5) = 0$$

Za liniju 5-2 vrijedi:

$$f_{52} = S_{52} - v_5\left(\frac{y_{sh_{52}}}{2}v_5 + y_{52}(v_5 - v_2)\right)^{*} = 0$$

$$f_{52}^{*} = S_{52}^{*} - v_5^{*}\frac{y_{sh_{52}}}{2}v_5 + y_{52}(v_5 - v_2) = 0$$

$$f_{25} = S_{25_{\text{loss}}} - (S_{25} + S_{52}) = 0$$

$$f_{25}^{*} = S_{25_{\text{loss}}}^{*} - (S_{25} + S_{52})^{*} = 0$$

Za liniju 2-6 vrijedi:

$$f_{26} = S_{26} - v_2\left(\frac{y_{sh_{26}}}{2}v_2 + y_{26}(v_2 - v_6)\right)^{*} = 0$$

$$f_{26}^{*} = S_{26}^{*} - v_2^{*}\frac{y_{sh_{26}}}{2}v_2 + y_{26}(v_2 - v_6) = 0$$

Za liniju 6-2 vrijedi:

$$f_{62} = S_{62} - v_6\left(\frac{y_{sh_{62}}}{2}v_6 + y_{62}(v_6 - v_2)\right)^{*} = 0$$

$$f_{62}^{*} = S_{62}^{*} - v_6^{*}\frac{y_{sh_{62}}}{2}v_6 + y_{62}(v_6 - v_2) = 0$$

$$f_{26} = S_{26_{\text{loss}}} - (S_{26} + S_{62}) = 0$$

$$f_{26}^{*} = S_{26_{\text{loss}}}^{*} - (S_{26} + S_{62})^{*} = 0$$

Za liniju 3-5 vrijedi:

$$f_{35} = S_{35} - v_3\left(\frac{y_{sh_{35}}}{2}v_3 + y_{35}(v_3 - v_5)\right)^{*} = 0$$

$$f_{35}^{*} = S_{35}^{*} - v_3^{*}\frac{y_{sh_{35}}}{2}v_3 + y_{35}(v_3 - v_5) = 0$$

Za liniju 5-3 vrijedi:

$$f_{53} = S_{53} - v_5\left(\frac{y_{sh_{53}}}{2}v_5 + y_{53}(v_5 - v_3)\right)^{*} = 0$$

$$f_{53}^{*} = S_{53}^{*} - v_5^{*}\frac{y_{sh_{53}}}{2}v_5 + y_{53}(v_5 - v_3) = 0$$

$$f_{35} = S_{35_{\text{loss}}} - (S_{35} + S_{53}) = 0$$

$$f_{35}^{*} = S_{35_{\text{loss}}}^{*} - (S_{35} + S_{53})^{*} = 0$$

Za liniju 3-6 vrijedi:

$$f_{36} = S_{36} - v_3\left(\frac{y_{sh_{36}}}{2}v_3 + y_{36}(v_3 - v_6)\right)^{*} = 0$$

$$f_{36}^{*} = S_{36}^{*} - v_3^{*}\frac{y_{sh_{36}}}{2}v_3 + y_{36}(v_3 - v_6) = 0$$

Za liniju 6-3 vrijedi:

$$f_{63} = S_{63} - v_6\left(\frac{y_{sh_{63}}}{2}v_6 + y_{63}(v_6 - v_3)\right)^{*} = 0$$

$$f_{63}^* = S_{63}^* - v_6^* \frac{y_{sh_{63}}}{2} v_6 + y_{63}(v_6 - v_3) = 0$$

$$f_{36} = S_{36_{\text{loss}}} - (S_{36} + S_{63}) = 0$$

$$f_{36}^* = S_{36_{\text{loss}}}^* - (S_{36} + S_{63})^* = 0$$

Za liniju 4-5 vrijedi:

$$f_{45} = S_{45} - v_4 \left(\frac{y_{sh_{45}}}{2} v_4 + y_{45}(v_4 - v_5) \right)^* = 0$$

$$f_{45}^* = S_{45}^* - v_4^* \frac{y_{sh_{45}}}{2} v_4 + y_{45}(v_4 - v_5) = 0$$

Za liniju 5-4 vrijedi:

$$f_{54} = S_{54} - v_5 \left(\frac{y_{sh_{54}}}{2} v_5 + y_{54}(v_5 - v_4) \right)^* = 0$$

$$f_{54}^* = S_{54}^* - v_5^* \frac{y_{sh_{54}}}{2} v_5 + y_{54}(v_5 - v_4) = 0$$

$$f_{45} = S_{45_{\text{loss}}} - (S_{45} + S_{54}) = 0$$

$$f_{45}^* = S_{45_{\text{loss}}}^* - (S_{45} + S_{54})^* = 0$$

Za liniju 5-6 vrijedi:

$$f_{56} = S_{56} - v_5 \left(\frac{y_{sh_{56}}}{2} v_5 + y_{56}(v_5 - v_6) \right)^* = 0$$

$$f_{56}^* = S_{56}^* - v_5^* \frac{y_{sh_{56}}}{2} v_5 + y_{56}(v_5 - v_6) = 0$$

Za liniju 6-5 vrijedi:

$$f_{65} = S_{65} - v_6 \left(\frac{y_{sh_{65}}}{2} v_6 + y_{65}(v_6 - v_5) \right)^* = 0$$

$$f_{65}^* = S_{65}^* - v_6^* \frac{y_{sh_{65}}}{2} v_6 + y_{65}(v_6 - v_5) = 0$$

$$f_{56} = S_{56_{\text{loss}}} - (S_{56} + S_{65}) = 0$$

$$f_{56}^* = S_{56_{\text{loss}}}^* - (S_{56} + S_{65})^* = 0$$

Ovim su napisane sve potrebne formule za post-procesiranje koje se dalje rješavaju pomoću modelSolver-a.

1.2.3 Rezultati problema

Za pokretanje modelSolver-a koji će rješiti zadanu problematiku, potrebno je napisati skriptu sa svim formulama koje su navedene ranije. Nakon kreiranja skripte i pokretanja solvera, rješenja su prikazana u tabeli 1.2.
Problem je riješen u 3 iteracije sa preciznošću $\varepsilon = 1.50846 \times 10^{-7}$.

Parametar	Vrijednost ([rad])	Vrijednost ([°])
v1	$(1.05,0)$	$(1.05,0)$
v2	$(1.04785,-0.0672314)$	$(1.05,-3.67116)$
v3	$(1.06703,-0.0797294)$	$(1.07,-4.27327)$
v4	$(0.986721,-0.0723879)$	$(0.989373,-4.19582)$
v5	$(0.981269,-0.0906217)$	$(0.985445,-5.27639)$
v6	$(0.999019,-0.104075)$	$(1.00443,-5.94745)$
S1	$(1.07875,0.159562)$	$(1.09049,8.41379)$
S2	$(0.5,0.743565)$	$(0.89604,56.0817)$
S3	$(0.6,0.896268)$	$(1.07856,56.2)$
S4	$(0.7,0.7)$	$(0.989949,45)$
S5	$(0.7,0.7)$	$(0.989949,45)$
S6	$(0.7,0.7)$	$(0.989949,45)$
S12	$(0.286897,-0.154187)$	$(0.325704,-28.2549)$
S21	$(-0.277847,0.128185)$	$(0.305991,155.234)$
S12_loss	$(0.00904939,-0.0260012)$	$(0.027531,-70.8102)$
S14	$(0.435849,0.201201)$	$(0.480049,24.7795)$
S41	$(-0.424974,-0.199326)$	$(0.469397,-154.872)$
S14_loss	$(0.0108755,0.00187502)$	$(0.011036,9.78204)$
S15	$(0.356009,0.112547)$	$(0.373375,17.5436)$
S51	$(-0.345273,-0.134497)$	$(0.370544,-158.717)$
S15_loss	$(0.0107354,-0.0219501)$	$(0.0244348,-63.9375)$
S23	$(0.0293032,-0.122687)$	$(0.126138,-76.5669)$
S32	$(-0.0289,0.0572812)$	$(0.0641587,116.772)$
S23_loss	$(0.000403132,-0.0654063)$	$(0.0654076,-89.6469)$
S24	$(0.330909,0.460541)$	$(0.567097,54.3019)$
S42	$(-0.315858,-0.451252)$	$(0.550813,-124.99)$
S24_loss	$(0.015051,0.00928844)$	$(0.0176864,31.68)$
S25	$(0.155145,0.153532)$	$(0.21827,44.7005)$
S52	$(-0.150166,-0.180065)$	$(0.234464,-129.827)$
S25_loss	$(0.00497949,-0.0265336)$	$(0.0269968,-79.3711)$
S26	$(0.262489,0.123995)$	$(0.290302,25.2851)$
S62	$(-0.256656,-0.160113)$	$(0.302504,-148.042)$
S26_loss	$(0.00583303,-0.0361185)$	$(0.0365864,-80.8261)$
S35	$(0.191168,0.231745)$	$(0.300418,50.4805)$
S53	$(-0.180232,-0.26095)$	$(0.317142,-124.632)$
S35_loss	$(0.0109357,-0.0292059)$	$(0.0311862,-69.4723)$
S36	$(0.437732,0.607242)$	$(0.748567,54.214)$
S63	$(-0.427698,-0.57861)$	$(0.719525,-126.471)$
S36_loss	$(0.0100338,0.0286315)$	$(0.0303388,70.6871)$
S45	$(0.0408324,-0.0494214)$	$(0.0641074,-50.4362)$
S54	$(-0.0404702,-0.0278527)$	$(0.0491285,-145.463)$
S45_loss	$(0.000362195,-0.077274)$	$(0.0772749,-89.7314)$
S56	$(0.0161416,-0.0966345)$	$(0.0979734,-80.517)$
S65	$(-0.0156456,0.0387235)$	$(0.0417647,112)$
S56_loss	$(0.000496034,-0.057911)$	$(0.0579132,-89.5092)$

Tabela 1.2: Rješenje razmatranog problema

Prilozi

Prilog A

XML opis (kod) za modelSolver

```xml
<?xml version="1.0" encoding="utf-8"?>
<Model type="NR" domain="cmplx" name="SEM_6Nodes">
   <Vars conj="true">
      <Var name="v1" val="V1_sp"/>
      <Var name="v2" val="V2_sp"/>
      <Var name="v3" val="V3_sp"/>
      <Var name="v4" val="V1_sp"/>
      <Var name="v5" val="V1_sp"/>
      <Var name="v6" val="V1_sp"/>
      <Var name="S1"/>
      <Var name="S2" val="P2_inj"/>
      <Var name="S3" val="P3_inj"/>
      <Var name="S4" val="S4_inj"/>
      <Var name="S5" val="S5_inj"/>
      <Var name="S6" val="S6_inj"/>
      <Var name="S12"/>
      <Var name="S21"/>
      <Var name="S12_loss"/>
      <Var name="S14"/>
      <Var name="S41"/>
      <Var name="S14_loss"/>
      <Var name="S15"/>
      <Var name="S51"/>
      <Var name="S15_loss"/>
      <Var name="S23"/>
      <Var name="S32"/>
      <Var name="S23_loss"/>
      <Var name="S24"/>
      <Var name="S42"/>
      <Var name="S24_loss"/>
      <Var name="S25"/>
      <Var name="S52"/>
      <Var name="S25_loss"/>
      <Var name="S26"/>
      <Var name="S62"/>
```

```
      <Var name="S26_loss"/>
      <Var name="S35"/>
      <Var name="S53"/>
      <Var name="S35_loss"/>
      <Var name="S36"/>
      <Var name="S63"/>
      <Var name="S36_loss"/>
      <Var name="S45"/>
      <Var name="S54"/>
      <Var name="S45_loss"/>
      <Var name="S56"/>
      <Var name="S65"/>
      <Var name="S56_loss"/>
   </Vars>
   <Params>
      <Param name="V1_sp" val="1.05"/>
      <Param name="V2_sp" val="1.05"/>
      <Param name="V3_sp" val="1.07"/>
      <Param name="P2_inj" val="0.5"/>
      <Param name="P3_inj" val="0.6"/>

      <Param name="z12" val="0.1+0.2i"/>
      <Param name="z14" val="0.05+0.2i"/>
      <Param name="z15" val="0.08+0.3i"/>
      <Param name="z23" val="0.05+0.25i"/>
      <Param name="z24" val="0.05+0.1i"/>
      <Param name="z25" val="0.1+0.3i"/>
      <Param name="z26" val="0.07+0.2i"/>
      <Param name="z35" val="0.12+0.26i"/>
      <Param name="z36" val="0.02+0.1i"/>
      <Param name="z45" val="0.2+0.4i"/>
      <Param name="z56" val="0.1+0.3i"/>

      <Param name="ysh12" val="0+0.04i"/>
      <Param name="ysh14" val="0+0.04i"/>
      <Param name="ysh15" val="0+0.06i"/>
      <Param name="ysh23" val="0+0.06i"/>
      <Param name="ysh24" val="0+0.02i"/>
      <Param name="ysh25" val="0+0.04i"/>
      <Param name="ysh26" val="0+0.05i"/>
      <Param name="ysh35" val="0+0.05i"/>
      <Param name="ysh36" val="0+0.02i"/>
      <Param name="ysh45" val="0+0.08i"/>
      <Param name="ysh56" val="0+0.06i"/>

      <!-- NODE 1 -->
      <Param name="y12" val="1/z12"/>
      <Param name="y14" val="1/z14"/>
```

```
    <Param name="y15" val="1/z15"/>

    <!-- NODE 2 -->
    <Param name="y21" val="1/z12"/>
    <Param name="y23" val="1/z23"/>
    <Param name="y24" val="1/z24"/>
    <Param name="y25" val="1/z25"/>
    <Param name="y26" val="1/z26"/>

    <!-- NODE 3 -->
    <Param name="y32" val="1/z23"/>
    <Param name="y35" val="1/z35"/>
    <Param name="y36" val="1/z36"/>

    <!-- NODE 4 -->
    <Param name="y41" val="1/z14"/>
    <Param name="y42" val="1/z24"/>
    <Param name="y45" val="1/z45"/>

    <!-- NODE 5 -->
    <Param name="y51" val="1/z15"/>
    <Param name="y52" val="1/z25"/>
    <Param name="y53" val="1/z35"/>
    <Param name="y54" val="1/z45"/>
    <Param name="y56" val="1/z56"/>

    <!-- NODE 6 -->
    <Param name="y62" val="1/z26"/>
    <Param name="y63" val="1/z36"/>
    <Param name="y65" val="1/z56"/>

    <Param name="y11" val="y21+y41+y51+1/2*ysh12+1/2*ysh14
       +1/2*ysh15"/>
    <Param name="y22" val="y21+y23+y24+y25+y26+1/2*ysh12+1/2*
       ysh23+1/2*ysh24+1/2*ysh25+1/2*ysh26"/>
    <Param name="y33" val="y23+y35+y36+1/2*ysh23+1/2*ysh35
       +1/2*ysh36"/>
    <Param name="y44" val="y41+y24+y45+1/2*ysh14+1/2*ysh24
       +1/2*ysh45"/>
    <Param name="y55" val="y51+y25+y35+y45+y56+1/2*ysh15+1/2*
       ysh25+1/2*ysh35+1/2*ysh45+1/2*ysh56"/>
    <Param name="y66" val="y26+y36+y56+1/2*ysh26+1/2*ysh36
       +1/2*ysh56"/>

    <Param name="S4_inj" val="0.7+0.7i"/>
    <Param name="S5_inj" val="0.7+0.7i"/>
    <Param name="S6_inj" val="0.7+0.7i"/>
</Params>
```

```
<NLEqs>
   <!-- PQ nodes-->
   <!-- node 4 -->
   <Eq fx="v4*conj(-y41*v1-y24*v2+y44*v4-y45*v5) + S4_inj=0"
      />
   <Eq fx="conj(v4)*(-y41*v1-y24*v2+y44*v4-y45*v5) + conj(
      S4_inj)=0"/>
   <!-- node 5 (ZI) -->
   <Eq fx="v5*conj(-y51*v1-y25*v2-y35*v3-y45*v4+y55*v5-y56*
      v6) + S5_inj=0"/>
   <Eq fx="conj(v5)*(-y51*v1-y25*v2-y35*v3-y45*v4+y55*v5-y56
      *v6) + conj(S5_inj)=0"/>
   <!-- node 6 -->
   <Eq fx="v6*conj(-y26*v2-y36*v3-y56*v5+y66*v6) + S6_inj=0"
      />
   <Eq fx="conj(v6)*(-y26*v2-y36*v3-y56*v5+y66*v6) + conj(
      S6_inj)=0"/>

   <!-- PV nodes-->
   <!-- node 2 -->
   <Eq fx="1/2*(v2*conj(-y21*v1+y22*v2-y23*v3-y24*v4-y25*v5-
      y26*v6)+conj(v2)*(-y21*v1+y22*v2-y23*v3-y24*v4-y25*v5-
      y26*v6))-P2_inj=0"/>
   <Eq fx="v2*conj(v2)-V2_sp^2=0"/>
   <!-- node 3 -->
   <Eq fx="1/2*(v3*conj(-y23*v2+y33*v3-y35*v5-y36*v6)+conj(
      v3)*(-y23*v2+y33*v3-y35*v5-y36*v6))-P3_inj=0"/>
   <Eq fx="v3*conj(v3)-V3_sp^2=0"/>

   <!-- SLACK NODE -->
   <Eq fx="1/2*(v1+conj(v1))-V1_sp=0"/>
   <Eq fx="1i*1/2*(conj(v1)-v1)=0"/>

   <!-- Postprocessing power calculation -->

   <!-- BUS power -->
   <!-- node 1 -->
   <Eq fx="S1 - v1*conj(y11*v1-y21*v2-y41*v4-y51*v5)=0"/>
   <Eq fx="conj(S1) - conj(v1)*(y11*v1-y21*v2-y41*v4-y51*v5)
      =0"/>
   <!-- node 2 -->
   <Eq fx="S2 - v2*conj(-y21*v1+y22*v2-y23*v3-y24*v4-y25*v5-
      y26*v6)=0"/>
   <Eq fx="conj(S2) - conj(v2)*(-y21*v1+y22*v2-y23*v3-y24*v4
      -y25*v5-y26*v6)=0"/>
   <!-- node 3 -->
   <Eq fx="S3 - v3*conj(-y23*v2+y33*v3-y35*v5-y36*v6)=0"/>
```

```
<Eq fx="conj(S3) - conj(v3)*(-y23*v2+y33*v3-y35*v5-y36*v6)=0"/>
<!-- node 4 -->
<Eq fx="S4 + v4*conj(-y41*v1-y24*v2+y44*v4-y45*v5)=0"/>
<Eq fx="conj(S4) + conj(v4)*(-y41*v1-y24*v2+y44*v4-y45*v5)=0"/>
<!-- node 5 -->
<Eq fx="S5 + v5*conj(-y51*v1-y25*v2-y35*v3-y45*v4+y55*v5-y56*v6)=0"/>
<Eq fx="conj(S5) + conj(v5)*(-y51*v1-y25*v2-y35*v3-y45*v4+y55*v5-y56*v6)=0"/>
<!-- node 6 -->
<Eq fx="S6 + v6*conj(-y26*v2-y36*v3-y56*v5+y66*v6)=0"/>
<Eq fx="conj(S6) + conj(v6)*(-y26*v2-y36*v3-y56*v5+y66*v6)=0"/>

<!-- BRANCH power -->
<!-- line 1-2 -->
<Eq fx="S12 - v1*conj(1/2*ysh12*v1+y21*(v1-v2))=0"/>
<Eq fx="conj(S12) - conj(v1)*(1/2*ysh12*v1+y21*(v1-v2))=0"/>
<Eq fx="S21 - v2*conj(1/2*ysh12*v2+y21*(v2-v1))=0"/>
<Eq fx="conj(S21) - conj(v2)*(1/2*ysh12*v2+y21*(v2-v1))=0"/>
<!-- line 1-4 -->
<Eq fx="S14 - v1*conj(1/2*ysh14*v1+y41*(v1-v4))=0"/>
<Eq fx="conj(S14) - conj(v1)*(1/2*ysh14*v1+y41*(v1-v4))=0"/>
<Eq fx="S41 - v4*conj(1/2*ysh14*v4+y41*(v4-v1))=0"/>
<Eq fx="conj(S41) - conj(v4)*(1/2*ysh14*v4+y41*(v4-v1))=0"/>
<!-- line 1-5 -->
<Eq fx="S15 - v1*conj(1/2*ysh15*v1+y51*(v1-v5))=0"/>
<Eq fx="conj(S15) - conj(v1)*(1/2*ysh15*v1+y51*(v1-v5))=0"/>
<Eq fx="S51 - v5*conj(1/2*ysh15*v5+y51*(v5-v1))=0"/>
<Eq fx="conj(S51) - conj(v5)*(1/2*ysh15*v5+y51*(v5-v1))=0"/>
<!-- line 2-3 -->
<Eq fx="S23 - v2*conj(1/2*ysh23*v2+y23*(v2-v3))=0"/>
<Eq fx="conj(S23) - conj(v2)*(1/2*ysh23*v2+y23*(v2-v3))=0"/>
<Eq fx="S32 - v3*conj(1/2*ysh23*v3+y23*(v3-v2))=0"/>
<Eq fx="conj(S32) - conj(v3)*(1/2*ysh23*v3+y23*(v3-v2))=0"/>
<!-- line 2-4 -->
<Eq fx="S24 - v2*conj(1/2*ysh24*v2+y24*(v2-v4))=0"/>
```

```
<Eq fx="conj(S24) - conj(v2)*(1/2*ysh24*v2+y24*(v2-v4))=0
   "/>
<Eq fx="S42 - v4*conj(1/2*ysh24*v4+y24*(v4-v2))=0"/>
<Eq fx="conj(S42) - conj(v4)*(1/2*ysh24*v4+y24*(v4-v2))=0
   "/>
<!-- line 2-5 -->
<Eq fx="S25 - v2*conj(1/2*ysh25*v2+y25*(v2-v5))=0"/>
<Eq fx="conj(S25) - conj(v2)*(1/2*ysh25*v2+y25*(v2-v5))=0
   "/>
<Eq fx="S52 - v5*conj(1/2*ysh25*v5+y25*(v5-v2))=0"/>
<Eq fx="conj(S52) - conj(v5)*(1/2*ysh25*v5+y25*(v5-v2))=0
   "/>
<!-- line 2-6 -->
<Eq fx="S26 - v2*conj(1/2*ysh26*v2+y26*(v2-v6))=0"/>
<Eq fx="conj(S26) - conj(v2)*(1/2*ysh26*v2+y26*(v2-v6))=0
   "/>
<Eq fx="S62 - v6*conj(1/2*ysh26*v6+y26*(v6-v2))=0"/>
<Eq fx="conj(S62) - conj(v6)*(1/2*ysh26*v6+y26*(v6-v2))=0
   "/>
<!-- line 3-5 -->
<Eq fx="S35 - v3*conj(1/2*ysh35*v3+y35*(v3-v5))=0"/>
<Eq fx="conj(S35) - conj(v3)*(1/2*ysh35*v3+y35*(v3-v5))=0
   "/>
<Eq fx="S53 - v5*conj(1/2*ysh35*v5+y35*(v5-v3))=0"/>
<Eq fx="conj(S53) - conj(v5)*(1/2*ysh35*v5+y35*(v5-v3))=0
   "/>
<!-- line 3-6 -->
<Eq fx="S36 - v3*conj(1/2*ysh36*v3+y36*(v3-v6))=0"/>
<Eq fx="conj(S36) - conj(v3)*(1/2*ysh36*v3+y36*(v3-v6))=0
   "/>
<Eq fx="S63 - v6*conj(1/2*ysh36*v6+y36*(v6-v3))=0"/>
<Eq fx="conj(S63) - conj(v6)*(1/2*ysh36*v6+y36*(v6-v3))=0
   "/>
<!-- line 4-5 -->
<Eq fx="S45 - v4*conj(1/2*ysh45*v4+y45*(v4-v5))=0"/>
<Eq fx="conj(S45) - conj(v4)*(1/2*ysh45*v4+y45*(v4-v5))=0
   "/>
<Eq fx="S54 - v5*conj(1/2*ysh45*v5+y45*(v5-v4))=0"/>
<Eq fx="conj(S54) - conj(v5)*(1/2*ysh45*v5+y45*(v5-v4))=0
   "/>
<!-- line 5-6 -->
<Eq fx="S56 - v5*conj(1/2*ysh56*v5+y56*(v5-v6))=0"/>
<Eq fx="conj(S56) - conj(v5)*(1/2*ysh56*v5+y56*(v5-v6))=0
   "/>
<Eq fx="S65 - v6*conj(1/2*ysh56*v6+y56*(v6-v5))=0"/>
<Eq fx="conj(S65) - conj(v6)*(1/2*ysh56*v6+y56*(v6-v5))=0
   "/>
```

```xml
      <!-- BRANCH power loss -->
      <!-- line 1-2 -->
      <Eq fx="S12_loss - (S12+S21)=0"/>
      <Eq fx="conj(S12_loss) - conj(S12+S21)=0"/>
      <!-- line 1-4 -->
      <Eq fx="S14_loss - (S14+S41)=0"/>
      <Eq fx="conj(S14_loss) - conj(S14+S41)=0"/>
      <!-- line 1-5 -->
      <Eq fx="S15_loss - (S15+S51)=0"/>
      <Eq fx="conj(S15_loss) - conj(S15+S51)=0"/>
      <!-- line 2-3 -->
      <Eq fx="S23_loss - (S23+S32)=0"/>
      <Eq fx="conj(S23_loss) - conj(S23+S32)=0"/>
      <!-- line 2-4 -->
      <Eq fx="S24_loss - (S24+S42)=0"/>
      <Eq fx="conj(S24_loss) - conj(S24+S42)=0"/>
      <!-- line 2-5 -->
      <Eq fx="S25_loss - (S25+S52)=0"/>
      <Eq fx="conj(S25_loss) - conj(S25+S52)=0"/>
      <!-- line 2-6 -->
      <Eq fx="S26_loss - (S26+S62)=0"/>
      <Eq fx="conj(S26_loss) - conj(S26+S62)=0"/>
      <!-- line 3-5 -->
      <Eq fx="S35_loss - (S35+S53)=0"/>
      <Eq fx="conj(S35_loss) - conj(S35+S53)=0"/>
      <!-- line 3-6 -->
      <Eq fx="S36_loss - (S36+S63)=0"/>
      <Eq fx="conj(S36_loss) - conj(S36+S63)=0"/>
      <!-- line 4-5 -->
      <Eq fx="S45_loss - (S45+S54)=0"/>
      <Eq fx="conj(S45_loss) - conj(S45+S54)=0"/>
      <!-- line 5-6 -->
      <Eq fx="S56_loss - (S56+S65)=0"/>
      <Eq fx="conj(S56_loss) - conj(S56+S65)=0"/>
   </NLEqs>
</Model>
```