CENG 371
HW 3

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1. 1.3 For the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

largest eigenpair and smallest eigenpair are

$$\lambda_{\max} = 3.7321, \qquad v_{\max} = \begin{bmatrix} 0.2887 \\ -0.5000 \\ 0.5774 \\ -0.5000 \\ 0.2887 \end{bmatrix}$$

$$\lambda_{\min} = 0.2679, \qquad v_{\min} = \begin{bmatrix} -0.2887 \\ -0.5000 \\ -0.5774 \\ -0.5000 \\ -0.2887 \end{bmatrix}$$

1.4) For

$$A = \begin{bmatrix} 0.2 & 0.3 & -0.5 \\ 0.6 & -0.8 & 0.2 \\ -1.0 & 0.1 & 0.9 \end{bmatrix}, v_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Using power method computation by hand,

$$v_1 = Av_0 = \begin{bmatrix} 0.2 & 0.3 & -0.5 \\ 0.6 & -0.8 & 0.2 \\ -1.0 & 0.1 & 0.9 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

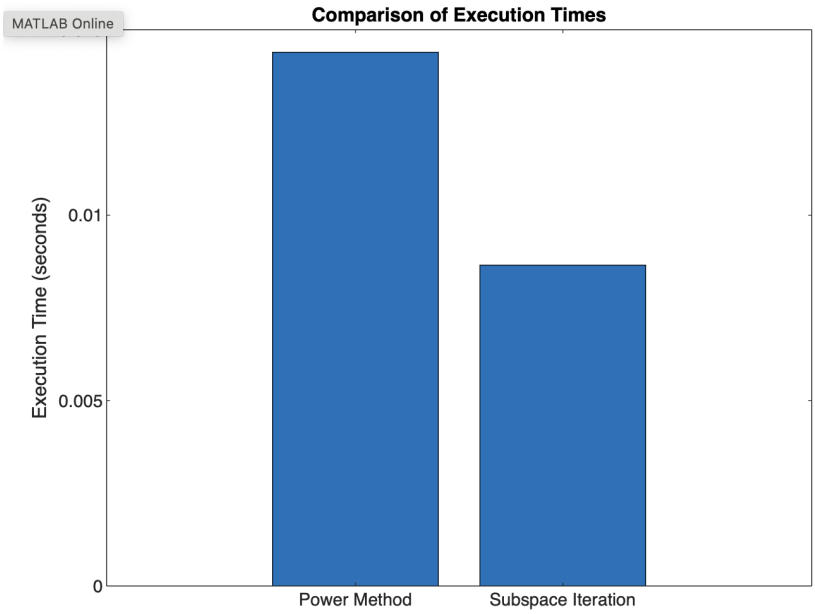
$$v_2 = Av_1 = \begin{bmatrix} 0.2 & 0.3 & -0.5 \\ 0.6 & -0.8 & 0.2 \\ -1.0 & 0.1 & 0.9 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As $v_1 = v_2$, v converges to 0, meaning power method by hand fails to find eigenpairs. However in matlab, the eigenpair are computed as

$$(\lambda, v) = \left(1.3427, \begin{bmatrix} -0.4070 \\ -0.0288 \\ 0.9130 \end{bmatrix} \right).$$

It is obvious that the result gained from matlab is not equal to the result obtained by hand computation. This can be caused by cancellation errors, as the v converges without getting fully canceled out.

2. 2.1 The power method is a simple way to find the largest eigenvalue and its eigenvector of a matrix A. You start with a random vector v0 and repeatedly multiply it by the matrix A. Each time, you normalize the resulting vector so it doesn't get too large. After several iterations, the vector will point in the same direction as the dominant eigenvector, and you can use the Rayleigh quotient, $\lambda_1 = \frac{v_i A v_i^T}{v_i^T v_i}$ to estimate the largest eigenvalue. To find the next largest eigenvalue, you subtract the contribution of the first eigenvector from the matrix by using $A' = A - \lambda_1 \frac{v_1 v_1^T}{v_1^T v_1}$ and repeat the process to find the next eigenvalue and eigenvector.



2.4

The Power Method takes more time compared to the Subspace Iteration Method. This is expected because it computes each eigenvalue sequentially, requiring k iterations of the matrix-vector multiplication process and a deflation step after every eigenvalue calculation. The execution time grows linearly with k, as the process must be repeated k times for k eigenvalues. The Subspace Iteration Method is faster as it computes all k eigenvalues and eigenvectors simultaneously. By iterating on a subspace (spanned by k vectors), it avoids the sequential deflation process of the Power Method. It takes advantage of QR factorization to re-orthogonalize the vectors, which is computationally efficient for small to moderate k.