Student Information

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Answer 1

a)

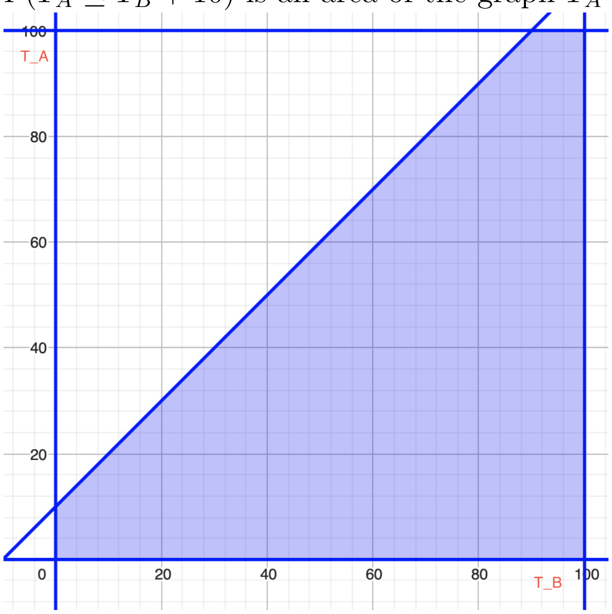
F(t_A, t_B)= P$(T_A \leq t_A)$P$(T_B \leq t_B)$= $\frac{t_A}{100} * \frac{t_B}{100} = \frac{t_A * t_B}{10000}$ for $0 \leq t_A, t_B \leq 100$. Otherwise it is 0.
f(t_A, t_B)= partial derivatives of F with respect to t_A, t_B. So, I take partial derivatives from F firstly with respect to $t_B = \frac{t_A}{10000}$ and then with respect to $t_A = \frac{1}{10000}$.
f(t_A, t_B)=$\frac{1}{10000}$ for $0 \leq t_A, t_B \leq 100$. Otherwise it is 0.

b)

P$(A \leq 30 \cup 40 \leq B \leq 60)$=$\int_{40}^{60} \int_{0}^{30} f(t_A, t_B)\ dt_A\ dt_B$= $\int_{40}^{60} \int_{0}^{30} \frac{1}{10000}\ dt_A\ dt_B$= $\int_{40}^{60} \frac{1}{10000} * (30 - 0)\ dt_B$= $\int_{40}^{60} \frac{3}{1000}\ dt_B = \frac{3}{1000} * (60 - 40)$
$= 0.06$

c)

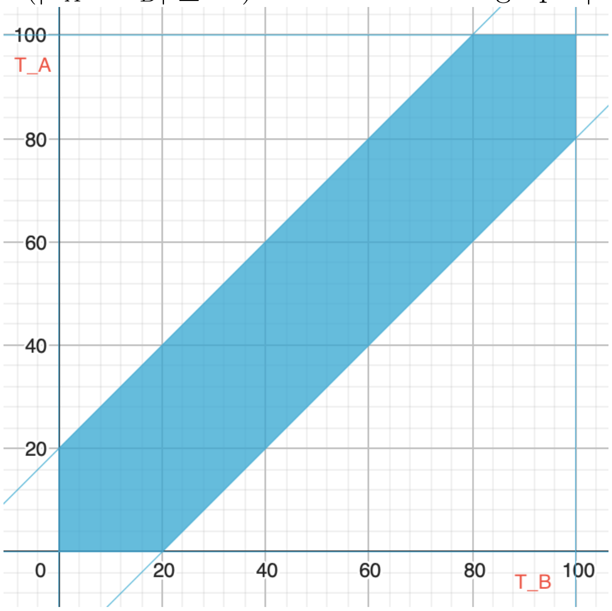
P$(T_A \leq T_B + 10)$ is an area of the graph $T_A <= T_B + 10$, T_A vs T_B.



area of shaded region=total area - unshaded region area
total area=100*100=10000
unshaded region area=$\frac{1}{2}(90 * 90)$=4050
area of shaded region=10000-4050=5950
P$(A \leq B + 10)$=$\frac{shadedarea}{totalarea} = \frac{5950}{10000} = 0.595$

d)

P($|T_A - T_B| \leq 20$) is an area of the graph $|T_A - T_B| \leq 20$, T_A vs T_B.



area of shaded region=total area - unshaded region area
total area=100*100=10000
unshaded region area=$2 * \frac{1}{2}(80 * 80)$=6400
area of shaded region=10000-6400=3600
P($|T_A - T_B| \leq 20$)=$\frac{shadedarea}{totalarea} = \frac{3600}{10000} = 0.36$

Answer 2

a)

X-number of frequent shoppers in sample.
As n=150 and p=0.6 we can assume that this follows Binomial Distribution. Also, we can use Central Limit Theorem to approximate X to Normal(standard normal) Distribution
X N(np,(1-p)np)=N(90,36)
P(X$\geq$ 0.65*150)=P(X$\geq$ 97.5)= P(Z$\geq \frac{97.5-90}{6}$)=P(Z$\geq$ 1.25)=1-P(Z$\leq$ 1.25)=1-0.89435= 0.10565

b)

X-number of rare shoppers in sample.
As n=150 and p=0.1 we can assume that this follows Binomial Distribution. Also, we can use Central Limit Theorem to approximate X to Normal(standard normal) Distribution
X N(np,(1-p)np)=N(15,13.5)
P(X$\leq$ 0.15*150)=P(X$\leq$ 22.5)= P(Z$\leq \frac{22.5-15}{\sqrt{13.5}}$)=P(Z$\leq$ 2.041)= 0.97938

Answer 3

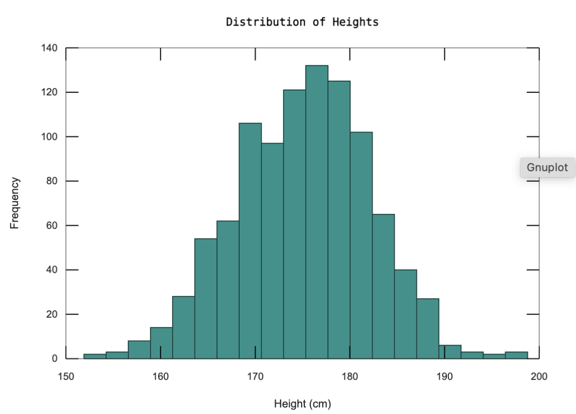
X-height of randomly chosen adult
P($170 < X < 180$)=P($\frac{170-175}{7} < Z < \frac{180-175}{7}$)=P($-0.7142 < Z < 0.7142$)=2 ϕ (0.7142)-1=0.52378

Answer 4

a)

```
mu = 175;
sigma = 7;
n = 1000;

x = normrnd(mu, sigma, n, 1);
xlabel('Height (cm)');
ylabel('Frequency');
title('Distribution of Heights');
hist(x, 20);
```



As it is seen in the graph, middle point i.e.mean is 175. According to this point the histogram is spread symmetrically. at y-axis there is Frequency and x-axis stands for Height

b)

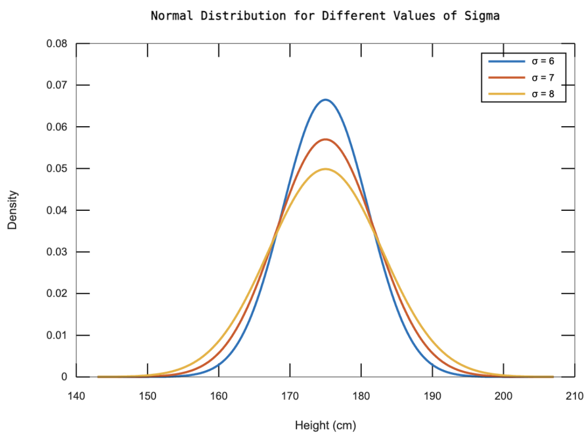
```
mu = 175;
sigma_values = [6, 7, 8];

xgrid = linspace(mu-4*max(sigma_values), mu+4*max(sigma_values), 1000);

for i = 1:length(sigma_values)
    sigma = sigma_values(i);
```

```
    ygrid = normpdf(xgrid, mu, sigma);
    plot(xgrid, ygrid, 'LineWidth', 2);
    hold on;
end

xlabel('Height (cm)');
ylabel('Density');
title('Normal Distribution for Different Values of Sigma');
legend('\sigma = 6', '\sigma = 7', '\sigma = 8');
hold off;
```



As it is represented in graph, according to the sigma value graph plot is changed.A larger value of sigma corresponds to a wider spread of the distribution, while a smaller value of sigma corresponds to a narrower spread.For example, sigma=8 is the widest but lower than others, sigma=7 is in the middle and sigma=6 is the narrowest but the highest

c)

```
mu = 175;
sigma = 7;
n = 150;
lower_bound = 170;
upper_bound = 180;
p = zeros(1, 1000);

for i = 1:1000
    heights = normrnd(mu, sigma, [1, n]);
    count = sum(heights >= lower_bound & heights <= upper_bound);
    proportion = count / n;
    p(i) = proportion;

p_45 = sum(p >= 0.45) / 1000;
```

```
p_50 = sum(p >= 0.5) / 1000;
p_55 = sum(p >= 0.55) / 1000;

fprintf('Probability of having at least 45% of adults with heights between %d
fprintf('Probability of having at least 50% of adults with heights between %d
fprintf('Probability of having at least 55% of adults with heights between %d
```

```
Probability of having at least 45% of adults with heights between 170 cm and 180 cm: 0.97
Probability of having at least 50% of adults with heights between 170 cm and 180 cm: 0.74
Probability of having at least 55% of adults with heights between 170 cm and 180 cm: 0.26
```

These results make sense because the normal distribution is a continuous probability distribution that is commonly used to model the heights of adult populations. The probability of having at least 45 percent of adults with heights between 170 cm and 180 cm is the highest because includes the probabilities of having at least 50 percent and 55 percent.