Q1

a)

p	q	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
F	F	F	T	F
F	T	F	T	F
T	F	F	T	F
T	T	T	F	F

Statement is contradiction.

b)

1. $p \rightarrow (T \rightarrow (p \wedge q))$ Table 6, negation law, $q \vee \neg q \equiv T$

2. $p \rightarrow (\neg T \vee (p \wedge q))$ Table 7, $T \rightarrow (p \wedge q) \equiv \neg T \vee (p \wedge q)$

3. $p \rightarrow (F \vee (p \wedge q))$ By given $\neg T \equiv F$

4. $p \rightarrow (p \wedge q)$ Table 6, identity law, $F \vee (p \wedge q) \equiv (p \wedge q)$

5. $\neg p \vee (p \wedge q)$ Table 7, $p \rightarrow (p \wedge q) \equiv \neg p \vee (p \wedge q)$

6. $(\neg p \vee p) \wedge (\neg p \vee q)$ Table 6, distributive law, $\neg p \vee (p \wedge q) \equiv (\neg p \vee p) \wedge (\neg p \vee q)$

7. $T \wedge (\neg p \vee q)$ Table 6, negation law, $\neg p \vee p \equiv T$

8. $\neg p \vee q$ Table 6, identity law, $T \wedge (\neg p \vee q) \equiv \neg p \vee q$

Q2

1. $\forall x \exists y W(x, y)$

2. $\neg \forall y \exists x F(x, y)$

3. $\forall x (W(x, P) \rightarrow A(Ali, x))$

4. $\exists y (W(Bursa, y) \wedge F(Tubitak, y))$

5. $\exists y \exists x \exists z (T(x, y) \wedge T(z, y) \wedge x \neq z)$

6. $\neg \exists x \exists y \exists z \left(W\left(x,z \right) \wedge W\left(y,z \right) \wedge x \neq y \right)$

7. $\exists x \exists y \exists z \left(W\left(x,z \right) \wedge W\left(y,z \right) \wedge x \neq y \wedge \forall t \left(W\left(t,z \right) \rightarrow t \neq x \wedge t \neq y \right) \right)$

Q3

1	$p \rightarrow q$	premise
2	$(q \wedge \neg r) \rightarrow s$	premise
3	$\neg s$	premise
4	p	assume
5	q	$\rightarrow e, 1$
6	$\neg r$	assume
7	$q \wedge \neg r$	$\wedge i, 5, 6$
8	s	$\rightarrow e, 2$
9	$\neg s$	copy 3
10	$\bot$	$\neg e, 8, 9$
11	r	$\neg e, 6 - 10$
12	$p \rightarrow r$	$\rightarrow i, 4 - 11$

Q4

Consider $\neg (s \rightarrow \neg q) \equiv s \wedge q$ by Table 7

1	p	premise
2	$p \rightarrow (q \wedge r)$	premise
3	$r \rightarrow s$	premise
4	$q \wedge r$	$\rightarrow e, 2, 1$
5	q	$\wedge e, 4$
6	r	$\wedge e, 4$
7	s	$\rightarrow e, 3$
8	$s \wedge q$	$\wedge i, 5, 7$

Q5

Consider "a" as $\forall i$ constant

1	$\forall x\,(Px \to (Qx \to Rx))$	premise
2	$\exists x Px$	premise
3	$\forall x \neg Rx$	premise
4	$Pa \to (Qa \to Ra)$	$\forall e, 1$
5	$\neg Ra$	$\forall e, 3$
6	Pa	assume
7	$Qa \to Ra$	$\to e, 4, 6$
8	Qa	assume
9	Ra	$\to e, 7, 8$
10	$\bot$	$\neg e, 5, 9$
11	$\neg Qa$	$\neg i, 8 - 10$
12	$\exists x \neg Qx$	$\exists i, 11$
13	$Pa \to \exists x \neg Qx$	$\to i, 6 - 12$
14	$\exists x \neg Qx$	$\exists e, 2, 13$