Student Information

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Answer 1

a)

- not surjective, there is no x$\in$ R s.t. $x^2 = -1$
- not injective, $f(1) = 1 = f(-1), 1 \neq -1$

b)

- not surjective, there is no x$\in$ R^+ s.t. $x^2 < 0$
- injective, since every x is positive integer, its square will be unique. We cannot construct counter example.

c)

- surjective, $let\ \ f(x) = y, x^2 = y, y = \pm\sqrt{x} \in R$
- not injective, $f(1) = 1 = f(-1), 1 \neq -1$

d)

- surjective, $x = a \in R^+\ \ f(x) = x^2 = a * a = y\ \ y \in R^+$
- injective, $let\ \ x \in R^+\ \ y \in R^+\ \ f(x) = y, y = \sqrt{x}$ there is no possibility to y have $\pm\sqrt{x}$ since -$\sqrt{x}$ is out of range

Answer 2

a) Take x_0 and x are both $\in Z$, $\epsilon > 0\ \ \sigma = \frac{1}{2}\ \ |f(x) - f(x_0)| > \sigma = \frac{1}{2}$. So x=$x_0$ and f(x)=f(x_0) and $|f(x) - f(x_0)| = 0 < \epsilon$.
So all functions in given domain and range are continuous.

b) Since function is continuous it implies that for every x_0 we can find $\sigma > 0$. $|x - x_0| < \sigma\ , then |f(x) - f(x_0)| < \epsilon = \frac{1}{2}$. As range is integer set,it means f(x)=f(x_0) for all $|x - x_0| < 0$
So function is constant

Answer 3

a) Let's proof it for n=2
Let $A_1 = A$ and $A_2 = B$. A=$\{a_1, a_2, a_3...a_n\}$. B=$\{b_1, b_2, b_3...b_n\}$
AxB =

$$\begin{array}{lllll} a_1b_1 & a_1b_2 & a_1b_3 & ... & a_1b_n \\ a_2b_2 & a_2b_2 & a_2b_3 & ... & a_2b_n \\ a_3b_1 & a_3b_2 & a_3b_3 & ... & a_3b_n \\ . & & & & \\ . & & & & \\ . & & & & \\ a_nb_1 & a_nb_2 & a_nb_3 & & a_nb_n \end{array}$$

The matrix contains all elements of AxB, every element can be reached by traversing diagonals.For example, a_1b_1 is 1st element, a_2b_1 is 2nd, a_1b_2 is 3rd and so on.So every element of the set can be uniquely enumerated and AxB is one-to-one correspondence, implying that AxB is countable.

b) Assume $\{0, 1\}$x$\{0, 1\}$ is countable
Then
$N_1 = x_{11}\ \ x_{12}\ \ x_{13}...$
$N_2 = x_{21}\ \ x_{22}\ \ x_{23}...$
Take sequence y=$\{y_1, y_2, y_3...\}$ where y_i=1-x_{ii}. Then there will not be any N_k that is equal to y by diagonal traversing method.By contradiction,set is uncountable.

$\{0, 1\}$ is a subset of any sequence of countable sets, assuming every set has cardinality ≥ 2, infinite product of $\{0, 1\}$ with itself is uncountable.

Answer 4

Order: $(n!)^2 \qquad 5^n \qquad 2^n \qquad n^{51} + n^{49} \qquad n^{50} \qquad \sqrt{n}logn \qquad (logn)^2$
$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0 \to \quad f(x) \in O(g(x))$

a) $\lim_{n\to\infty} \frac{5^n}{(n!)^2} = \lim_{n\to\infty} \left(\frac{5^n}{(n!)} * \frac{1}{n!}\right) = 0$, because $\frac{1}{n!} = 0$

b) $\lim_{n\to\infty} \frac{2^n}{5^n} = \lim_{n\to\infty} \left(\frac{2}{5}\right)^n = 0$ because $\frac{2}{5} < 1$

c) $\lim_{n\to\infty} \frac{n^{51}+n^{49}}{2^n} = \lim_{n\to\infty}\left(\frac{n^{51}}{2^n}+\frac{n^{49}}{2^n}\right) = 0$, applying L'hospitals Rule $\frac{51!}{ln^{51}(2)} * \lim_{n\to\infty} \frac{1}{2^n} = 0$ and $\frac{49!}{ln^{49}(2)} * \lim_{n\to\infty} \frac{1}{2^n} = 0$

d) $\lim_{n\to\infty} \frac{n^{50}}{n^{51}+n^{49}} = 0$ because $\frac{n^{50}}{n^{51}+n^{49}} < 1$

e) $\lim_{n\to\infty} \frac{\sqrt{n}logn}{n^{50}} = 0$ because $\frac{\sqrt{n}logn}{n^{50}} < 1$

f) $\lim_{n\to\infty} \frac{(logn)^2}{\sqrt{n}logn} = \lim_{n\to\infty}\left(\frac{logn}{\sqrt{n}} * \frac{logn}{logn}\right) = \lim_{n\to\infty} \frac{logn}{\sqrt{n}} = 0$,apply L'hospitals $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = 0$

Answer 5

a) By Euclidean algorithm if $a < b$ gcd(a,b)=gcd(b,a)=gcd(a,b mod a)
gcd(134,94)=gcd(94,40)=gcd(40,14)=gcd(14,12)=gcd(12,2)=gcd(2,0)=2
Details:

$\frac{94}{40} = 2\frac{14}{40}$

$\frac{40}{14} = 2\frac{12}{14}$

$\frac{14}{12} = 1\frac{2}{12}$

$\frac{12}{2} = 6$

b) Assume that even numbers greater than 2 are sum of two primes, x and y. Take number $n > 5$.
-if n is EVEN, then even number $n - 2 > 2$ and n-2=x+y $\rightarrow n = x + y + 2$. n is a sum of THREE PRIMES.
-if n is ODD, then even number $n - 3 > 2$ and n-3=x+y $\rightarrow n = x + y + 3$. again n is sum of THREE PRIMES.

Now, assume even integers greater than 5 be a sum of three primes(z,t,s) and take $n > 2$ is even.
-even number $n + 2 > 5$ and n+2=z+t+s.
- if z,t,s are all odd, then n+2 must be odd which contradicts our assumption.
So, one of the z,t,s must be even prime number which is only 2.
n+2=z+t+2 n=z+t
Therefore, n is sum of 2 primes.
Also, any prime number greater than 2 is odd (2n+1). Sum of two odd numbers is even. So, if we take x and y as odd prime numbers and c is their sum. c=x+y=(2n+1)+(2n+1)=4n+2> 5 if n starts from 1. Next even number after c is d=c+2=(4n+2)+2 which can be rewritten as d=(2n+1)+(2n+1)+2=x+y+2. So d is sum of three prime numbers.