Student Information

Full Name : Aneliya Abdimalik Id Number : 2547651

Answer 1

A(x)=$\Sigma_{k=0}^{\infty} a_k x^k$
$\Sigma_{k=2}^{\infty} a_k x^k = 3x\Sigma_{k=2}^{\infty} a_{k-1} x^{k-1} + 4x^2\Sigma_{k=2}^{\infty} a_{k-2} x^{k-2}$
A(x)-($a_0 + a_1 x$)=3x(A(x)-a_0)+4x^2A(x)
A(x)-1-x=3xA(x)-3x+4x^2A(x)
A(x)-3xA(x)-4x^2A(x)=1-2x

A(x)=$\frac{1-2x}{1-3x-4x^2} = \frac{x-\frac{1}{2}}{(x-1)\left(x+\frac{1}{4}\right)} = \frac{A}{x-1} + \frac{B}{x+\frac{1}{4}}$

A($x + \frac{1}{4}$)+B(x-1)=(x-$\frac{1}{2}$)

A+B=1
$\frac{A}{4} - B = \frac{-1}{2}$

B=$\frac{3}{5}$
A=$\frac{2}{5}$

A(x)=$\frac{\frac{2}{5}}{x-1} + \frac{\frac{3}{5}}{x+\frac{1}{4}} = \Sigma_{k=0}^{\infty} - \frac{2}{5}x^k + \Sigma_{k=0}^{\infty} \frac{3}{5} * 4 * (-4)^k x^k = \Sigma_{k=0}^{\infty} \left(-\frac{2}{5} + \frac{12}{5}(-4)^k\right) x^k$

$a_k = -\frac{2}{5} + \frac{12}{5}(-4)^k$

Answer 2

a)

$2 + 5x + 11x^2 + 29x^3 + 83x^4 + 245x^5 + ...$
=2+(2x+3x)+($2x^2 + 9x^2$)+($2x^3 + 27x^3$)+($2x^4 + 81x^4$)+($2x^5 + 243x^5$)+...
=($2 + 2x + 2x^2 + 2x^3 + 2x^4 + 2x^5 + ...$)+($3x + 9x^2 + 27x^3 + 81x^4 + 243x^5 + ...$)
=2($1 + x + x^2 + x^3 + x^4 + x^5 + ...$)+3x($1 + 3x + 9x^2 + 27x^3 + 81x^4 + ...$)

Using $\Sigma_{k=0}^{\infty} a^k x^k = \frac{1}{1-ax}$

$\frac{2}{1-x} = 2\Sigma_{k=0}^{\infty} x^k$

$\frac{3x}{1-3x} = 3x\Sigma_{k=0}^{\infty} 3^k x^k$

$\frac{2}{1-x} + \frac{3x}{1-3x} = \frac{2(1-3x)+3x(1-x)}{(1-x)(1-3x)} = \frac{-3x^2-3x+2}{3x^2-4x+1}$

b)

$\frac{7-9x}{1-3x+2x^2} = \frac{7-9x}{2x^2-x-2x+1} = \frac{7-9x}{x(2x-1)-(2x-1)} = \frac{7-9x}{(2x-1)(x-1)} = \frac{A}{2x-1} + \frac{B}{x-1}$

A(x-1)+B(2x-1)=7-9x
A+2B=-9
-A-B=7
A=-5 B=-2

$-\frac{5}{2x-1} - \frac{2}{x-1} = \frac{5}{1-2x} + \frac{2}{1-x}$

Using $\Sigma_{k=0}^{\infty} a^k x^k = \frac{1}{1-ax}$

$\Sigma_{k=0}^{\infty} 5 * (2x)^k + \Sigma_{k=0}^{\infty} 2 * x^k = \Sigma_{k=0}^{\infty} \left(5 * (2x)^k + 2 * x^k\right) = \Sigma_{k=0}^{\infty} x^k (5 * 2^k + 2)$

Sequence:7+12x+22$x^2 + 42x^3 + 82x^4$+...

Answer 3

a)

reflexive: Let a∈Z, aRa. n=$\sqrt{a^2 + a^2} = a\sqrt{2}$ by Pythagorean theorem. $a\sqrt{2} \notin Z$
As reflexive case falls, the relation is not equivalence.

b)

reflexive: $(x_1, y_1) \in R \times R$, (x_1, y_1)R(x_2, y_2), $2x_1 + y_1 = 2x_1 + y_1 \in R$

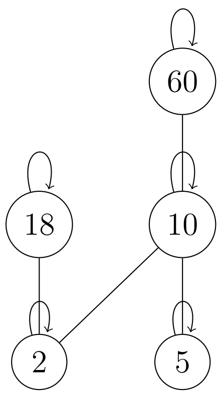
symmetric: $(x_1, y_1), (x_2, y_2) \in R \times R$
$(x_1, y_1)\, R\, (x_2, y_2)$, $2x_1 + y_1 = 2x_2 + y_2 \in R$
$(x_2, y_2)\, R\, (x_1, y_1)$, $2x_2 + y_2 = 2x_1 + y_1 \in R$

transitive: $(x_1, y_1), (x_2, y_2), (x_2, y_2) \in R \times R$
$(x_1, y_1)\, R\, (x_2, y_2)$,$2x_1 + y_1 = 2x_2 + y_2$
$(x_2, y_2)\, R\, (x_3, y_3)$,$2x_2 + y_2 = 2x_3 + y_3$
$2x_1 + y_1 = 2x_2 + y_2 = 2x_3 + y_3 \rightarrow 2x_1 + y_1 = 2x_3 + y_3$

In Cartesian coordinates, it represents all points lying on the line which passes through P(x,y) and has slope=-2
(1,-2)=$(x, y) : y = -2x$

Answer 4

a)



b)

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

$R^{-1} = (b, a) : (a, b) \in R =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Symmetric closure $= R \cup R^{-1}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

d)

If we remove only one element, then it is impossible to create a total ordering. For example, remove 18, but 2 is not divisible by 5, and 5 is not divisible by 2. Or remove 5, but 18 is not divisible by

10 and 60, and vice versa is also true.
If we remove 2 elements, namely 5 and 18, then it is possible to create a total ordering by adding 30.