Student Information

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Answer 1

If 6^{2n-1} is divisible by both 5 and 7 then it is divisible by 5*7=35 (divisibility theorem).
Base step: for n=1, $6^{2*1} - 1 = 35$ and 35/35=1. $6^{2n} - 1 = 35\alpha$, $\alpha \in N^+$
Inductive hypothesis: assume n=k, $k \in N^+$, $35|6^{2k} - 1 = 35\beta$, $\beta \in N^+$
Induction: prove for n=k+1
$35|6^{2k+2} - 1 = 35\gamma$, $\gamma \in N^+$
$= 6^2k * 6^2 - 1$
$= (35\beta + 1) * 6^2 - 1$ By induction hypothesis
$= 35(36\beta - 1)$
$= 35\gamma$

By mathematical induction, it was proven that 6^{2n-1} is divisible by 35, hence divisible by 5 and 7 for any $n \in N^+$

Answer 2

Base step:n=0 $H_0 = 1 \leq 9^0$
$H_1 = 5 \leq 9^1$
$H_2 = 7 \leq 9^2$
Inductive hypothesis: assume $H_j \leq 9^j$ for $0 \leq j \leq k$ where $k \geq 2$
Induction: prove for $H_{k+1} \leq 9^{k+1}$
$H_{k+1} = 8H_k + 8H_{k-1} + 9H_{k-2}$ *
$8H_k + 8H_{k-1} + 9H_{k-2} \leq 9^{k+1}$
$8H_k + 8H_{k-1} + 9H_{k-2} \leq 8 * 9^k + 8 * 9^{k-1} + 9 * 9^{k-2}$
$8H_k + 8H_{k-1} + 9H_{k-2} \leq 9^{k-2} (8 * 9^2 + 8 * 9 + 9)$
729=9^3 $8H_k + 8H_{k-1} + 9H_{k-2} \leq 9^{k-2}$
By * $H_{k+1} \leq 9^{k+1}$

By strong induction, it was proven that $H_n \leq 9^n, n \in N$ for any integer n≥3.

Answer 3

Let a_n be the number of bit-strings of length n with 4 consecutive 0's. So, $a_n = 2a_{n-1} + 2^{n-5} - a_{n-5}$ because such a string can be taken by a string of length n-1(by putting 0 or 1 at the end) or from a string of length n-4 (by putting 10000 at the end).

Following that, $a_0 = a_1 = a_2 = a_3 = 0$ and $a_4 = 1$. Then by recursion, $a_5 = 3, a_6 = 8, a_7 = 20, a_8 = 48$.
The same logic is implemented to b_n, number of strings with 4 consecutive 1's.

In the above calculations, we added to the result strings such as 11110000,00001111.
So, the number of string with either 4 consecutive 0's or 4 consecutive 1's is $a_n + b_n - 2$= 48+48-2=94

Answer 4

1) Choose 1 star. There are $\binom{10}{1}$ ways.

2) Choose 2 habitable stars $\binom{20}{2}$ ways.

3) Choose 8 non-habitable stars $\binom{80}{8}$ ways.
There are 9 slots between non-habitable planets. For example, —X—X—X—X—X—X—X—X—
1st planet can only be in one of the first three slots.
If it is in 1st slot, there are 3 options for habitable 2.
If it is in 2nd slot, there are 2 options for habitable planet 2.
If it is in 3rd slot, there is only 1 option for habitable planet 2.
Hence,the quantity is $\binom{4}{2}$.

By product rule, $\binom{10}{1} * \binom{20}{2} * \binom{80}{8} * \binom{4}{2}$.

As planets are distinct, we should permute habitable and non-habitable.So, the final result is $\binom{10}{1} * \binom{20}{2} * \binom{80}{8} * \binom{4}{2} * 2! * 8!$

Answer 5

a) Consider a_n to be a recurrence relation
a_{n-1} 1 step
a_{n-2} 2 steps
a_{n-3} 3 steps
Then, $a_n = a_{n-1} + a_{n-2} + a_{n-3}$

b) a_1=(1). So there is 1 way
a_2=(1,1),(2). So there are 2 ways
a_3=(1,1,1),(1,2),(2,1),(3). So there are 4 ways

c) $a_9 = a_8 + a_7 + a_6$
$a_8 = a_7 + a_6 + a_5$
$a_7 = a_6 + a_5 + a_4$
$a_6 = a_5 + a_4 + a_3$
$a_5 = a_4 + a_3 + a_2$
$a_4 = a_3 + a_2 + a_1 = 1 + 2 + 4 = 7$
Traversing from down to up.
$a_5 = 7 + 4 + 2 = 13$
$a_6 = 13 + 7 + 4 = 24$
$a_7 = 24 + 13 + 7 = 44$
$a_8 = 44 + 24 + 13 = 81$
$a_9 = 81 + 44 + 24 = 149$
Answer is 149.