Hmwk Week 2

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Pop. Eco

1. $\boxed{4.1}$

(a) $\frac{dN}{dt} = rN[1-(\frac{N}{k})^{\theta}]$ * find equilibria and determine their stability.

$0 = rN[1-(\frac{N}{k})^{\theta}]$ equilibria means equal to 0

$$\frac{dn}{dt} \approx n \left.\frac{dF}{dN}\right|_{N=\hat{N}}$$

$$F(N) = rN(1-N/k)^{\theta} = rN - r(\frac{N^2}{k})^{\theta}$$

$$\frac{dF}{dN} = r - 2\theta rN/k$$

near $\hat{N}=0$

$$\frac{dn}{dt} = n(r-2\theta rN/k)\Big|_{N=0}$$

$$\boxed{\frac{dn}{dt} = nr \quad \text{near} \quad \hat{N}=0}$$ $\lambda = r$

for $\hat{N}=k$

$$\frac{dn}{dt} \approx ((r-2r\theta N/k)\big|_{N=k})n$$

$$(r-2r\theta)n \qquad \lambda = r-2r\theta$$

if θ is positive then λ is negative

$\boxed{\text{The equilibrium is unstable near } \hat{N}\approx 0 \text{ and stable where } \hat{N}\approx k}$

b. $r=1$, $k=1$, $N(0)=0.01$

different θ-values

$$\frac{dN/dt}{N} = r\left[1-(N/K)^{\theta}\right] = 1\left[1-(N/1)^{.5,1,2}\right]$$

~~- Bigger θ-values result in bigger rates of negative change. The graphs all appear to be linear but their slopes increase (negatively) as θ increases.~~

- As N increases, the $\frac{dN}{dt}$ decreases:
  - at an increasing rate for $\theta = .5$ $(\theta < 1)$
  - at a constant rate for $\theta = 1$
  - at a decreasing rate for $\theta > 1$ (e.g. $\theta = 2$)

C. The advantage of using θ-logistic growth compared to pure logistic growth is that it allows you to change the sensitivity to density-dependence. Higher θ values have faster initial growth and then a sudden plateau at k; Lower θ values have slower initial growth and take a long time to reach K. Fast growing species such as bacteria or weeds might have higher θ values; while more slow "growing" taxa might have lower θ. These latter taxa may have lower reproductive rates or dispersal rates.

4.3 (a) original logistic: $\frac{dN}{dt} = rN(1-N/K)$

(b) logistic w/ Allee effect: $\frac{dN}{dt} = rN(N-a)[1-(N/K)]$

$\hat{N}$ near a

$\frac{dN}{dt} = rN[N-a][1-(N/K)] = 0$

1st equilibrium

$\frac{dn}{dt} = nr$ near $\hat{N} > a$

↳ slightly greater

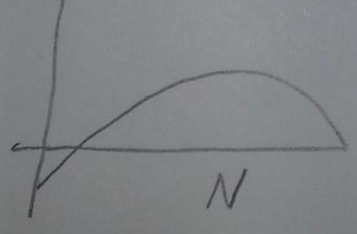
*It's unstable

$\hat{N}$ near K:

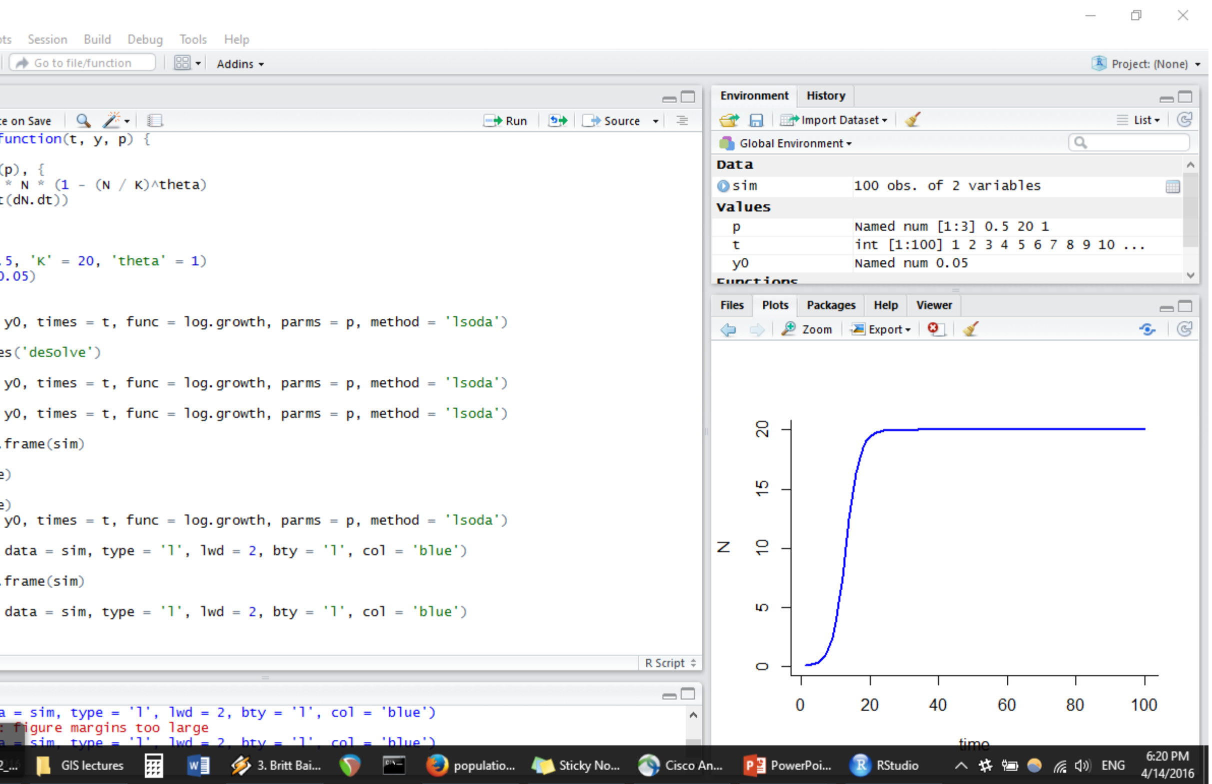
$\frac{dn}{dt} = rN\underbrace{[N-a]}_{\text{approaches } N}[1-(N/K)] = 0$

- 2nd Equilibrium is near $N \approx K$; it's stable.

(c) $\frac{dN/dt}{N} = rN(N-a)[1-(N/K)]$



(d) There is negative population growth at low populations w/ an allee effect, while w/ standard logistic growth, the rate of growth will always be highest at this point.


Session Build Debug Tools Help
Go to file/function
Addins
Project: (None)
Run
Source
function(t, y, p) {
(p), {
* N * (1 - (N / K)^theta)
(dN.dt))
5, 'K' = 20, 'theta' = 1)
0.05)
y0, times = t, func = log.growth, parms = p, method = 'lsoda')
es('deSolve')
y0, times = t, func = log.growth, parms = p, method = 'lsoda')
y0, times = t, func = log.growth, parms = p, method = 'lsoda')
frame(sim)
y0, times = t, func = log.growth, parms = p, method = 'lsoda')
data = sim, type = 'l', lwd = 2, bty = 'l', col = 'blue')
frame(sim)
data = sim, type = 'l', lwd = 2, bty = 'l', col = 'blue')
R Script
a = sim, type = 'l', lwd = 2, bty = 'l', col = 'blue')
figure margins too large
Environment History
Import Dataset
List
Global Environment
Data
sim 100 obs. of 2 variables
Values
p Named num [1:3] 0.5 20 1
t int [1:100] 1 2 3 4 5 6 7 8 9 10 ...
y0 Named num 0.05
Files Plots Packages Help Viewer
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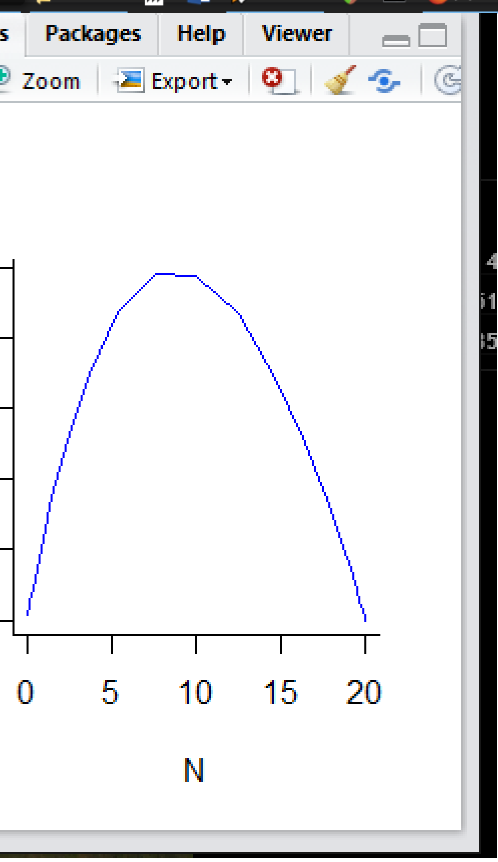
6:20 PM
4/14/2016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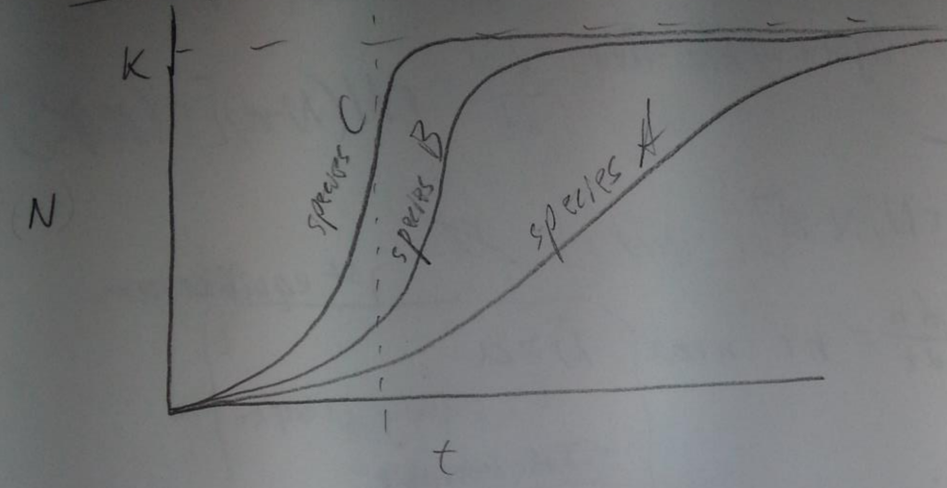
3. fishery



All species will eventually reach the same abundance after enough time if each has the same carrying capacity and assuming they reach the carrying capacity.
If they don't reach carrying capacity (K), then at any given point (t): $N_C > N_B > N_A$.

x-credit

Density depend. Pop growth

$$\frac{dN}{dt} = r\left(1-\frac{N}{K}\right)N$$

Mission: Solve for $N(t)$

1. Separate variables (N, t)

$$\frac{dN}{N(1-\frac{N}{K})} = r\,dt$$

2. Integrate both sides

$$\int \frac{dN}{N(1-\frac{N}{K})} = \int r\,dt$$

→ partial fractions trick.

$$\frac{1}{N(1-\frac{N}{K})} = \frac{1}{N} - \frac{1}{N-K} \quad (a)$$

ample:

$$\boxed{\int \frac{dN}{N-N_0} = \ln(N-N_0) \quad (b)}$$

continue on your own for bonus homework

$$\int_{N(0)}^{N(T)} \frac{1}{N} + \frac{1/K}{1-N/K}\,dN = \int_{N(0)}^{N(T)} r\,dt\ dN \ldots = rT$$

$$= \left[\ln(N) - \ln(1-N/K)\right]_{N(0)}^{N(T)}$$

$$= \ln(N(T)) - (\ln(1-N(T)/K) - \ln(N(0)) + \ln(1-N(0)/K))$$

3. Evaluate at ($t=0$ to $t=T$

→ $= rT$ → exponential of both sides.

$$= \frac{N(T)(1-N(0)/K)}{(1-N(T)/K)N(0)} = e^{rt}$$

solve for $N(T)$

$$N(T) = \frac{N(0)e^{rt}}{1+N(0)(e^{rT}-1)/K}$$

get to here

$$\boxed{\text{Solution: } N(T) = \frac{N(0)\,e^{rT}}{1+\frac{N(0)(e^{rt}-1)}{K}}}$$