Homework 4

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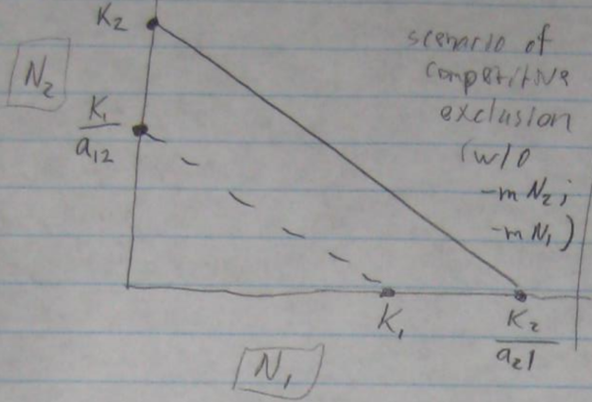
Hastings (7.1, 7.2, 7.4)



solve for N_2

$$\frac{dN_1}{dt} = r_1 N_1 (1 - a_{11} N_1 - a_{12} N_2) - m N_1$$

$$0 = r_1 N_1 (1 - N_1 - a_{12} N_2) - m N_1$$

$$m N_1 = r_1 N_1 (1 - N_1 - a_{12} N_2)$$

$$\frac{m N_1}{r_1 N_1} = 1 - N_1 - a_{12} N_2$$

$$\frac{m}{r_1} - 1 + N_1 = -a_{12} N_2$$

$$\frac{-1}{a_{12}} \left(\frac{m}{r_1} - 1 + N_1 \right) = N_2$$

$$\frac{-N_1}{a_{12}} + \frac{1}{a_{12}} - \frac{m}{r_1 a_{12}} = N_2$$

$$\frac{-N_1}{a_{12}} + \frac{1}{a_{12}} - \frac{m}{r_1 a_{12}} = 0$$

$$N_1 = \frac{m}{r_1} - 1 \quad \text{when } N_2 = 0$$

for $F(N_2)$

$$N_2 = 1 - \alpha_{21} N_1$$

when $N_2 = 0$

$$N_1 = \frac{1}{\alpha_{21}}$$

then, with $-mN_2$:

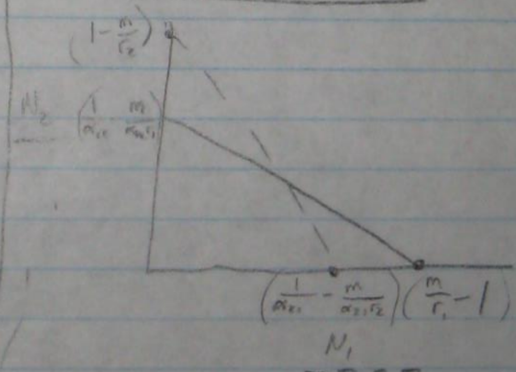
$$0 = r_2 N_2 (1 - N_2 - \alpha_{21} N_1) - m N_2$$

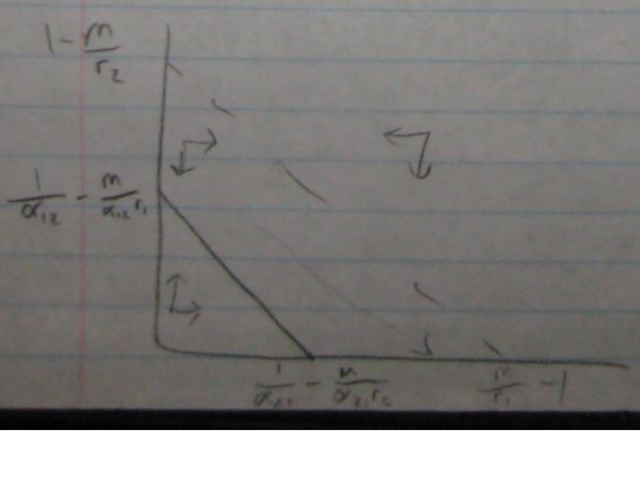
$$\frac{m N_2}{r_2 N_2} = 1 - N_2 - \alpha_{21} N_1$$

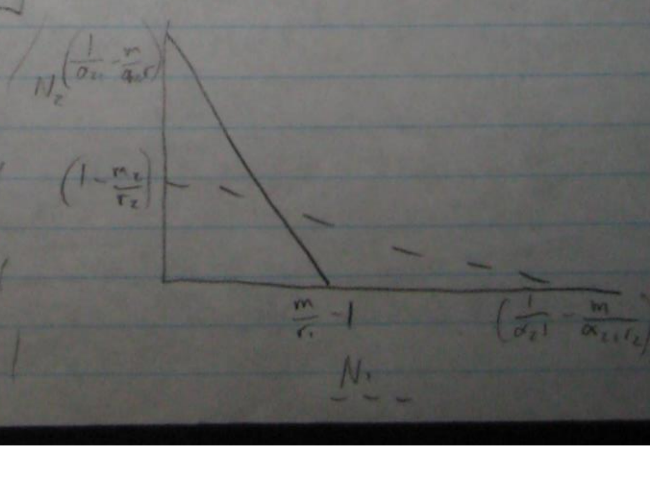
$$N_2 = -\alpha_{21} N_1 + 1 - \frac{m}{r_2}$$

when $N_2 = 0$ $\quad N_1 = \frac{1}{\alpha_{21}} - \frac{m}{\alpha_{21} r_2}$

Coexistence







7.2

c survival requires:

- $m_1 > e$ or $\frac{e}{m_1} < 1$ and $m_1 > m_2$

if e slowly increases then

$\frac{dp_2}{dt} = 0 \rightarrow P_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$ $\frac{dp_1}{dt} = 0 \Rightarrow P_1 = 1 - \frac{e}{m_1}$

- p_2 would be extinct first and
- p_1 would have a negative equilibrium @ highest rate of e

d When $\frac{dp_2}{dt} = 0$

$p_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$

so as $e\uparrow$ p_2 @ $\frac{dp_2}{dt} = 0$

p_2 = fraction of space occupied by spec. 2

If e slowly increases then the equilibrium of p_2 slowly increases as well.

- the space occupied by species 2 would have to be greater @ higher extinction rates to maintain equilibrium

$\boxed{7.2}$ eq 7.19 $\frac{dp_1}{dt} = m_1 p_1 (1-p_1) - ep_1$

$\boxed{a}$

$0 = m_1 p_1 (1-p_1) - ep_1$ solve for p_1

$0 = m_1 p_1 - m_1 p_1^2 - ep_1$

$0 = p_1 (m_1 - m_1 p_1 - e)$

$0 = m_1 - m_1 p_1 - e$

$\frac{m_1 p_1}{m_1} = \frac{m_1 - e}{m_1}$

$\boxed{p_1 = 1 - \frac{e}{m_1}}$

p_1 = fraction of patches occupied species 1
e = extinction rate
m_1 = colonization rate of species 1

extinction rate must be larger than colonization rate

$\boxed{b}$ eq 7.20 $\frac{dp_2}{dt} = m_2 p_2 (1-p_1-p_2) - m_1 p_1 p_2 - ep_2$

$= m_2 p_2 \left(1 - \left(1-\frac{e}{m_1}\right) - p_2\right) - \left(m_1 \left(1-\frac{e}{m_1}\right) p_2\right) - ep_2$

$= m_2 p_2 \left(1 - 1 + \frac{e}{m_1} - p_2\right) + \left((m_1 - e) p_2\right) - ep_2$

$= m_2 p_2 \left(\frac{e}{m_1} - p_2\right) - \left((p_2 m_1) ep_2\right) - ep_2$

$= m_2 p_2 \left(\frac{e}{m_1} - p_2\right) p_2 m_1 + \cancel{ep_2} - \cancel{ep_2}$

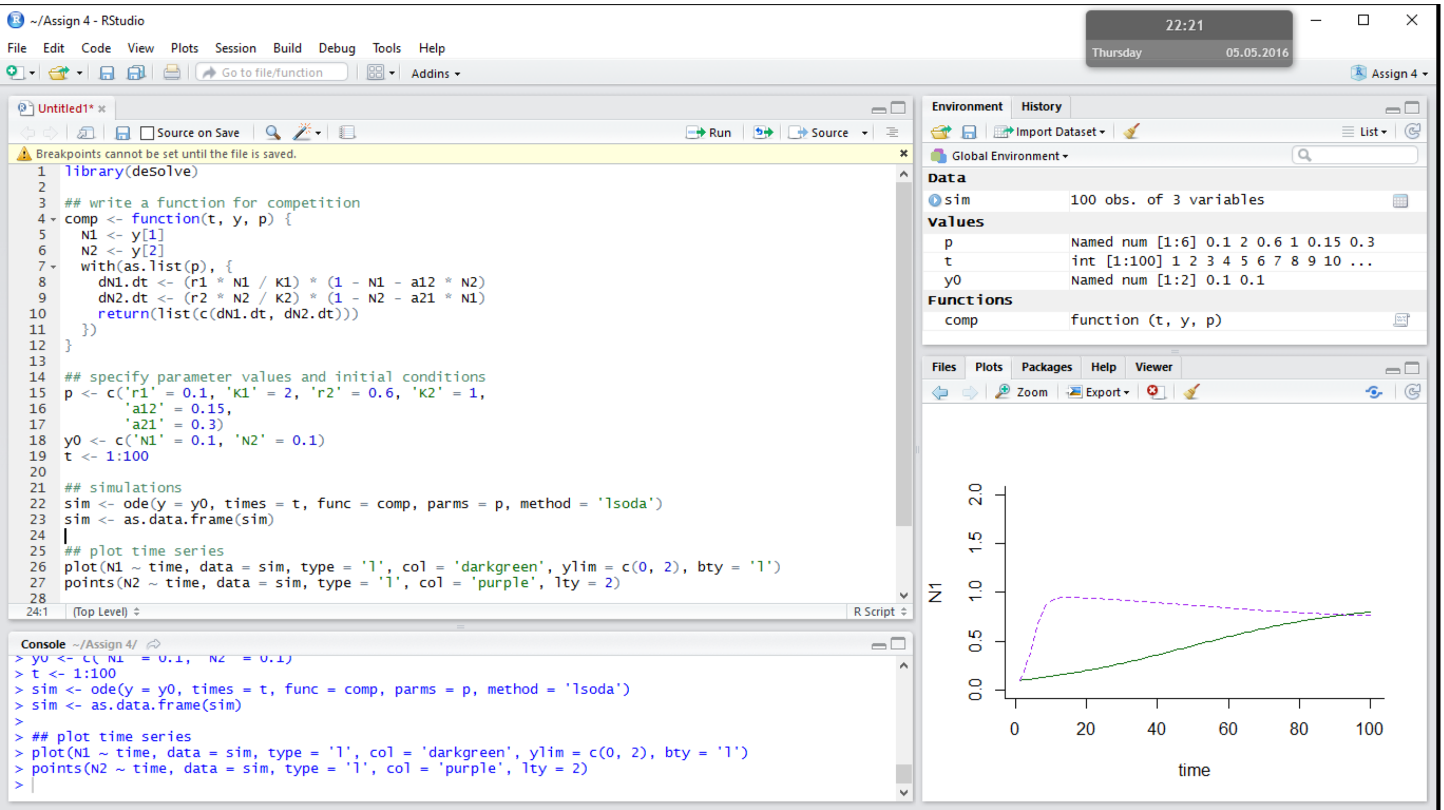
$= \frac{m_2 p_2 e}{m_1} - m_2 p_2^2 - p_2 m_1$

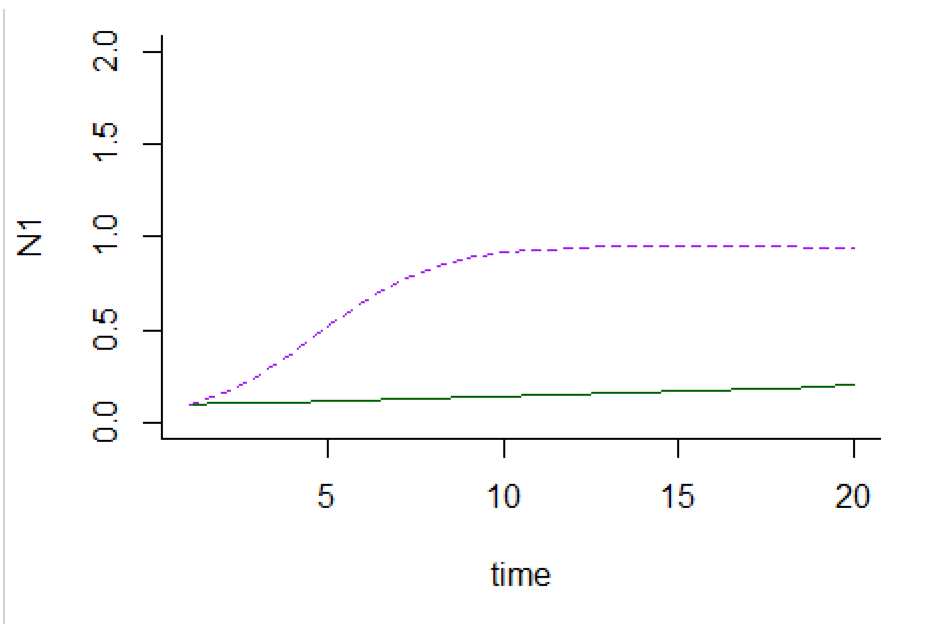
$0 = p_2 \left(\frac{m_2 e}{m_1} - m_2 p_2 - m_1\right)$

$m_2 p_2 = \frac{m_2 e}{m_1} - m_1$

$\boxed{p_2 = \frac{e}{m_1} - \frac{m_1}{m_2}}$

for both species to survive, $m_2 < m_1$


~/Assign 4 - RStudio
22:21
Thursday 05.05.2016
File Edit Code View Plots Session Build Debug Tools Help
Addins
Untitled1*
Source on Save
Run
Source
Breakpoints cannot be set until the file is saved.
library(deSolve)
write a function for competition
comp <- function(t, y, p) {
N1 <- y[1]
N2 <- y[2]
with(as.list(p), {
dN1.dt <- (r1 * N1 / K1) * (1 - N1 - a12 * N2)
dN2.dt <- (r2 * N2 / K2) * (1 - N2 - a21 * N1)
return(list(c(dN1.dt, dN2.dt)))
})
}
specify parameter values and initial conditions
p <- c('r1' = 0.1, 'K1' = 2, 'r2' = 0.6, 'K2' = 1,
'a12' = 0.15,
'a21' = 0.3)
y0 <- c('N1' = 0.1, 'N2' = 0.1)
t <- 1:100
simulations
sim <- ode(y = y0, times = t, func = comp, parms = p, method = 'lsoda')
sim <- as.data.frame(sim)
plot time series
plot(N1 ~ time, data = sim, type = 'l', col = 'darkgreen', ylim = c(0, 2), bty = 'l')
points(N2 ~ time, data = sim, type = 'l', col = 'purple', lty = 2)
24:1 (Top Level) R Script
Console ~/Assign 4/
> t <- 1:100
> sim <- ode(y = y0, times = t, func = comp, parms = p, method = 'lsoda')
> sim <- as.data.frame(sim)
>
> ## plot time series
> plot(N1 ~ time, data = sim, type = 'l', col = 'darkgreen', ylim = c(0, 2), bty = 'l')
> points(N2 ~ time, data = sim, type = 'l', col = 'purple', lty = 2)
Environment History
Import Dataset
List
Global Environment
Data
sim 100 obs. of 3 variables
Values
p Named num [1:6] 0.1 2 0.6 1 0.15 0.3
t int [1:100] 1 2 3 4 5 6 7 8 9 10 ...
y0 Named num [1:2] 0.1 0.1
Functions
comp function (t, y, p)
Files Plots Packages Help Viewer
Zoom
Export
N1
0.0
0.5
1.0
1.5
2.0
0
20
40
60
80
100
time


N1
0.0
0.5
1.0
1.5
2.0
5
10
15
20
time

7.4

(a) Lab experiments can be conducted where extrinsic factors are controlled and intrinsic growth rates and competition coefficients are determined for a competition system. Then these results can be considered in field research and any deviation from these values can be assumed to be a result of extrinsic (environmental factors) such as weather, food availability, etc.

(b) The species will coexist if their impact on each other is less than the impact on themselves.

(c) More overlap in resources results in a higher likelihood that one species will be driven to extinction when the species are sympatric.

(d) Controls are important as are replications of plots. Results should be considered and it should be asked whether or not they are applicable to ~~a~~ in situ settings.

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2. Conclusions: For a time period of 20 days, the populations co-exist in a stable state (it appears) although the abundance of species 2 is much higher than species 1.
- ~~If~~ If experiment goes for 100 days, the population of species 2 catches up to species 1 (okay, vice versa). This demonstrates that it takes time for populations to adjust to each other, so results from short term research projects shouldn't be applied to ~~applications~~ scenarios that are considering long-term populations' dynamics.

Research Topic – Growth Models for Eurasian Collared Dove populations in NA