Grapheme: An Online Graphing Calculator

Timothy Herchen

October 2020

Contents

1 Floating-point Operations **2**
1.1 Directed Rounding 2

2 Interval Arithmetic **2**

1 Floating-point Operations

Grapheme uses double-precision floating-point arithmetic for most calculations, since this functionality is provided by JS directly and is highly optimized. When the calculator is directed to evaluate $3 \cdot 4$, it uses the JS $*$ operator which maps directly to a machine instruction. There is no point in using single-precision arithmetic, as these are the same speed on modern processors and JS has no facilities besides asm.js to use this format.

There are some important limitations in double-precision FP. Some of the most obvious are the inability to express integers greater than $2^{53} \approx 9.007 \cdot 10^{15}$, numbers greater than about $2^{1023} \approx 1.798 \cdot 10^{308}$, and positive numbers smaller than $2^{-1074} \approx 4.941 \cdot 10^{-324}$. While arbitrary-precision arithmetic may be eventually implemented, this is difficult and thus we will try to do our best using the existing system.

Some conventions:

1. $\pm\infty$ and NaN are known as *special numbers*.
2. Floating-point numbers that are not special numbers are *finite numbers*.
3. Denormal numbers and normal numbers are named as usual.
4. NaN $\neq$ NaN, contrary to the mathematical definition of equality. However, NaN $\simeq$ NaN. For all other purposes, $=$ and $\simeq$ are equivalent.
5. There is only one NaN value, because the standard does not specify the existence of qNaNs, sNaNs and the like.
6. The set of all double-precision floating-point numbers, including the special numbers, is denoted $\mathbb{F}_{\text{all}}$.
7. $\mathbb{F}_{\text{all}}$ without the special numbers is denoted $\mathbb{F}$. Without only NaN, it is denoted $\mathbb{F}_\infty$. Thus, $\mathbb{F} \subset \mathbb{F}_\infty \subset \mathbb{F}_{\text{all}}$.

1.1 Directed Rounding

Per the ECMAScript standard, JS operations all use round-to-nearest, ties-to-even. That means that if the mathematical result of an operation is, say, 3.261, and the nearest permitted floats are 3.26 and 3.27, the operation will return 3.26. Unfortunately, JS does not provide facilities to set the rounding mode, which is understandable given the niche use of these modes.

2 Interval Arithmetic