THE OHIO STATE UNIVERSITY

Representing Uncertainty in Power System Operations: Approaches, Challenges and Solutions

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What?

- Operational problems (day-ahead scheduling)
- Old good times!
- Main uncertainty sources
- Uncertainty representation
- Stochastic Programming
  - Challenges
  - Solutions
- Adaptive Robust Optimization
  - Challenges
  - Solutions
- Conclusions

Operational problems (day-ahead scheduling)

Day-ahead scheduling

Scheduling production units to securely supply the demand on an hourly basis one day in advance.

Satisfying production-unit constraints

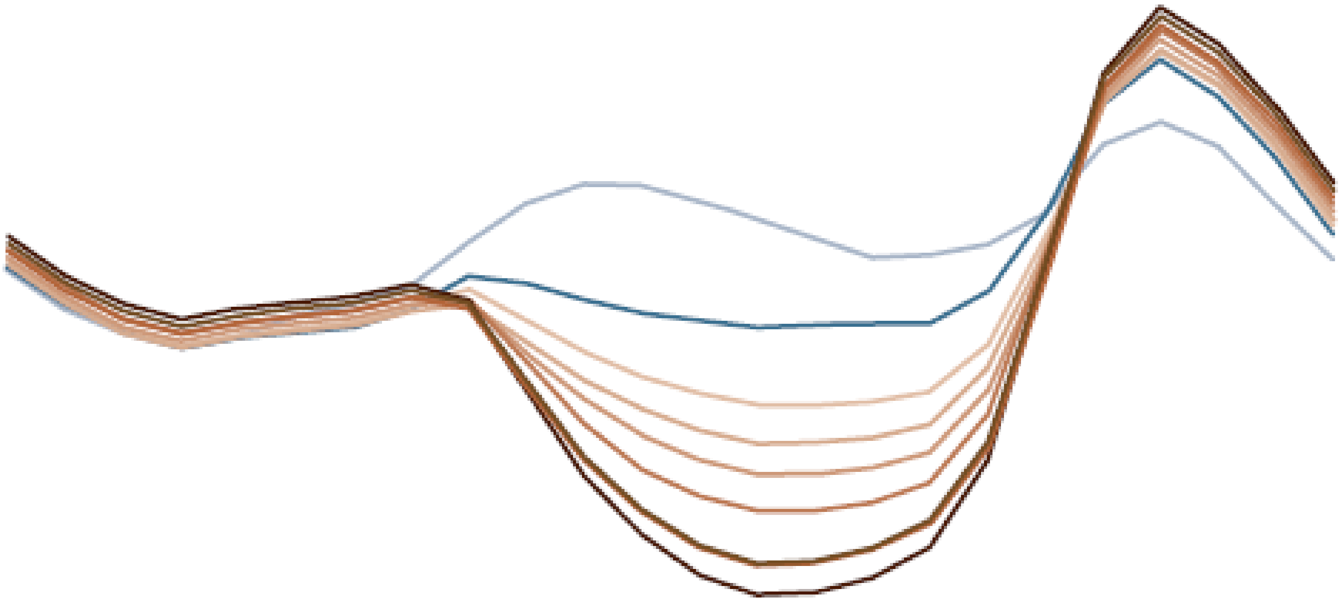
Satisfying transmission-network constraints

Old good times!

- Uncertainty pertaining to the demand: limited and easy to forecast
- Uncertainty pertaining to equipment failures: limited and controlled with deterministic security criteria (e.g., n-1)

Main uncertainty sources

- Demand altered by behind-the-meter devices
- Increasing weather-dependent production units



https://www.caiso.com/Documents/FlexibleResourcesHelpRenewables_FastFacts.pdf

Uncertainty representation

Many possible realizations $\Rightarrow$ Stochastic programming

Worst realization $\Rightarrow$ Adaptive robust optimization

Many possible realizations $\Rightarrow$ Stochastic programming

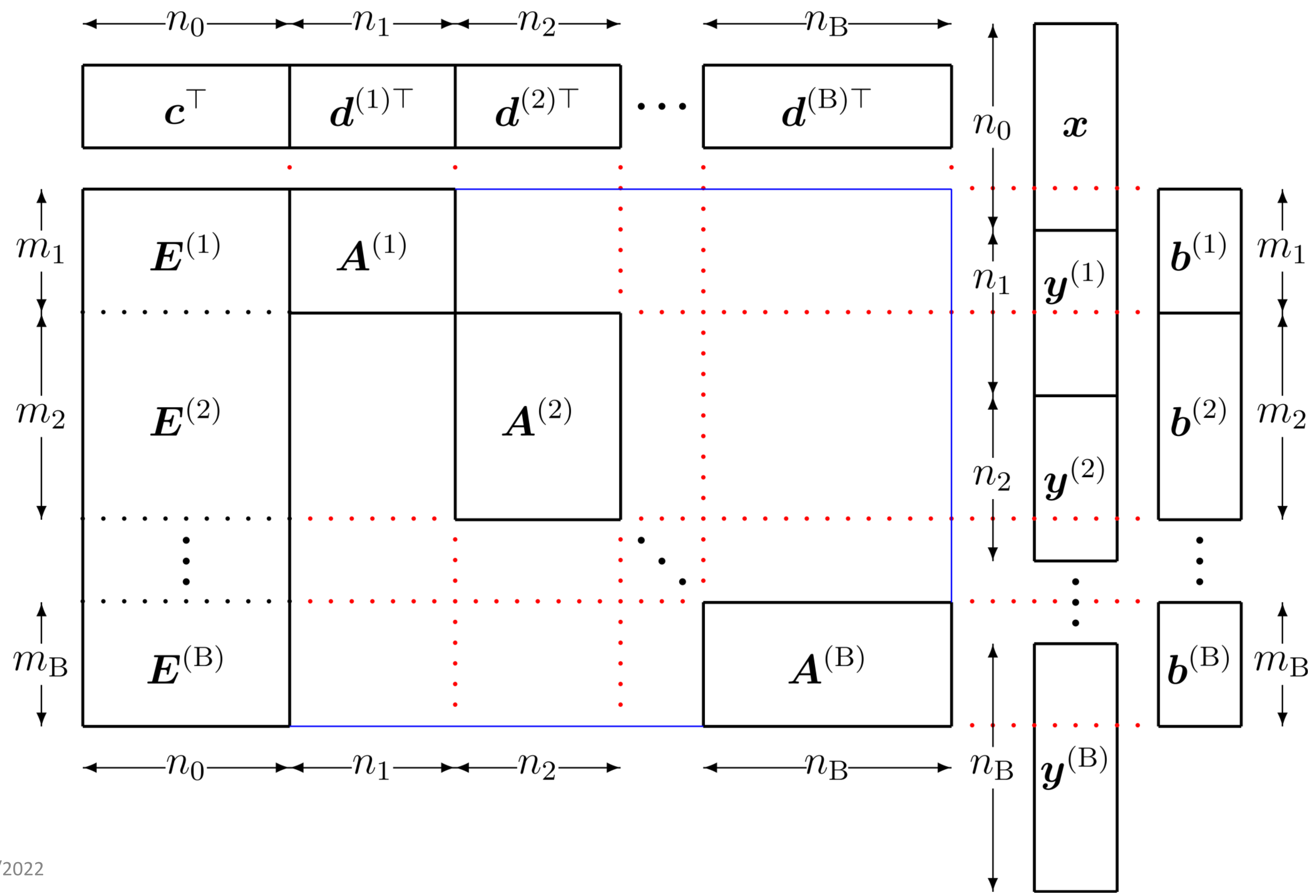
Many possible realizations $\Rightarrow$ Stochastic programming

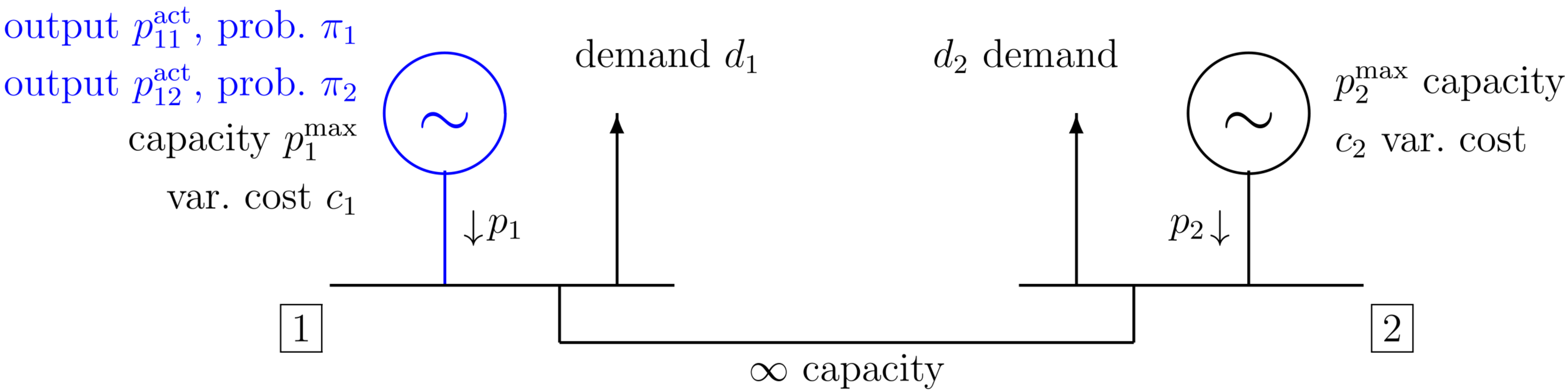
$$\min_{\mathbf{x}\in\mathcal{X}(\xi)} \quad \mathbb{E}_{\xi}\{f(\mathbf{x},\xi)\}$$

$$\min_{\mathbf{x}^1\in\mathcal{X}^1} \quad f^1(\mathbf{x}^1) + \mathbb{E}_{\xi^1}\left\{z^1_{\xi^1}\right\}, \text{where: } z^1_{\xi^1} = \left\{\min_{\mathbf{x}^2_{\xi^1}\in\mathcal{X}^2(\xi^1)} f^2(\mathbf{x}^1, \mathbf{x}^2_{\xi^1})\right\}$$

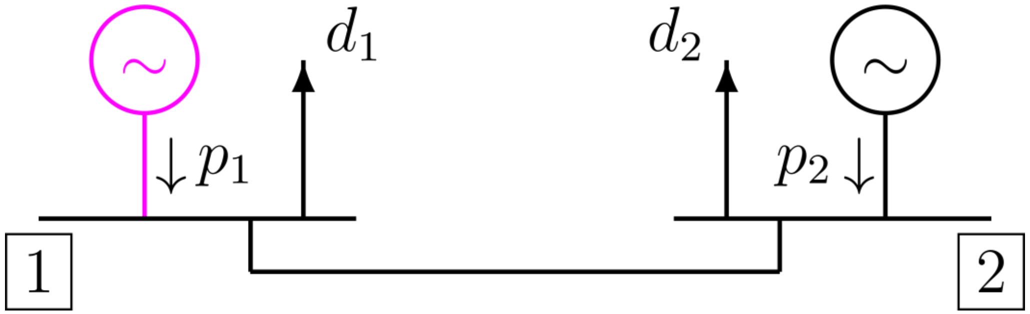
$$
\begin{aligned}
\min_{\boldsymbol{x};\boldsymbol{y}^{(k)},k=1,\ldots,B} \quad & z = \boldsymbol{c}^\top \boldsymbol{x} + \sum_{j=1}^{B} \boldsymbol{d}^{(k)\top} \boldsymbol{y}^{(k)} \\
\text{s. t.} \quad & \boldsymbol{E}^{(k)} \boldsymbol{x} + \boldsymbol{A}^{(k)} \boldsymbol{y}^{(k)} = \boldsymbol{b}^{(k)}, \quad k = 1, \ldots, B \\
& \boldsymbol{x} \in \mathbb{B}^{n_0}{}^{\dagger} \\
& \boldsymbol{y}^{(k)} \in \mathbb{R}^{n_k}, \quad k = 1, \ldots, B
\end{aligned}
$$

†or some of the components of $\boldsymbol{x} \in \mathbb{B}$


n_0
n_1
n_2
n_B
$c^\top$
$d^{(1)\top}$
$d^{(2)\top}$
$d^{(B)\top}$
x
$y^{(1)}$
$y^{(2)}$
$y^{(B)}$
$E^{(1)}$
$E^{(2)}$
$E^{(B)}$
$A^{(1)}$
$A^{(2)}$
$A^{(B)}$
$b^{(1)}$
$b^{(2)}$
$b^{(B)}$
m_1
m_2
m_B


output p_{11}^{act}, prob. π_1
output p_{12}^{act}, prob. π_2
capacity $p_1^{\max}$
var. cost c_1
$\downarrow p_1$
demand d_1
d_2 demand
$p_2^{\max}$ capacity
c_2 var. cost
$p_2 \downarrow$
1
2
∞ capacity

$$
\begin{aligned}
\min_{\substack{p_1,\, p_2 \ge 0 \\ s_{11},\, s_{12} \ge 0 \\ r_{21}^{U},\, r_{21}^{D},\, r_{22}^{U},\, r_{22}^{D} \ge 0}} \quad & c_1 p_1 + c_2 p_2 + \pi_1 (c_2^{U} r_{21}^{U} - c_2^{D} r_{21}^{D}) + \pi_2 (c_2^{U} r_{21}^{U} - c_2^{D} r_{21}^{D}) \\
\text{s. t.} \quad & p_1 + p_2 = d_1 + d_2 : \quad \lambda \\
& p_1 \le p_1^{\max} \\
& p_2 \le p_2^{\max} \\
& (r_{21}^{U} - r_{21}^{D}) + (p_{11}^{\text{act}} - p_1 - s_{11}) = 0 : \quad \gamma_1 \\
& 0 \le p_2 + (r_{21}^{U} - r_{21}^{D}) \le p_2^{\max} \\
& s_{11} \le p_{11}^{\text{act}} \\
& (r_{22}^{U} - r_{22}^{D}) + (p_{12}^{\text{act}} - p_1 - s_{12}) = 0 : \quad \gamma_2 \\
& 0 \le p_2 + (r_{22}^{U} - r_{22}^{D}) \le p_2^{\max} \\
& s_{12} \le p_{12}^{\text{act}}
\end{aligned}
$$



p_1 scheduled power output of producer 1

p_2 scheduled power output of producer 2

s_{11} power spillage of producer 1 in scenario 1

s_{12} power spillage of producer 1 in scenario 2

r_{21}^{U} deployed up-reserve of producer 2 in scenario 1

r_{21}^{D} deployed down-reserve of producer 2 in scenario 1

r_{22}^{U} deployed up-reserve of producer 2 in scenario 2

r_{22}^{D} deployed down-reserve of producer 2 in scenario 2

Challenges

- Stochastic programming problems might be intractable since its size depends on the number of considered scenarios.
- Stochastic programming allows for risk control, but risk-based formulations (e.g., those based on CVaR) might be computationally challenging.

Solution: Decomposition

$\boxed{\text{Master problem}}$

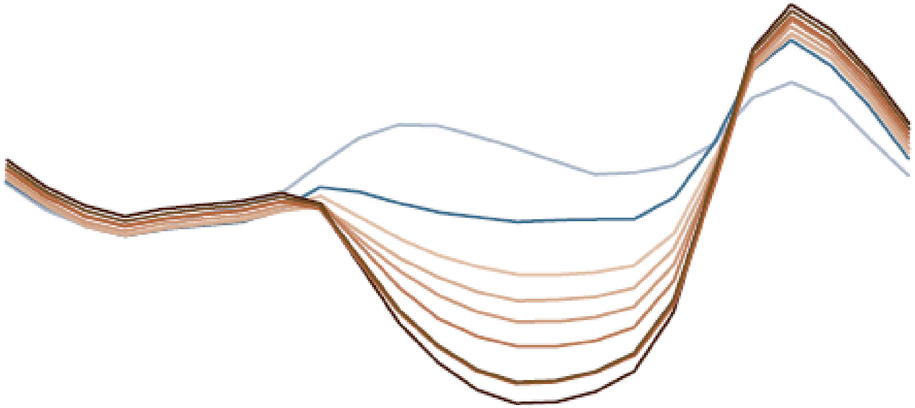
$$\Downarrow \qquad \Uparrow$$

$\boxed{\text{Subproblems}}$

Master problem

$\Rightarrow$ scheduling decisions

$\Rightarrow$ some operation decisions

$\Leftarrow$ 

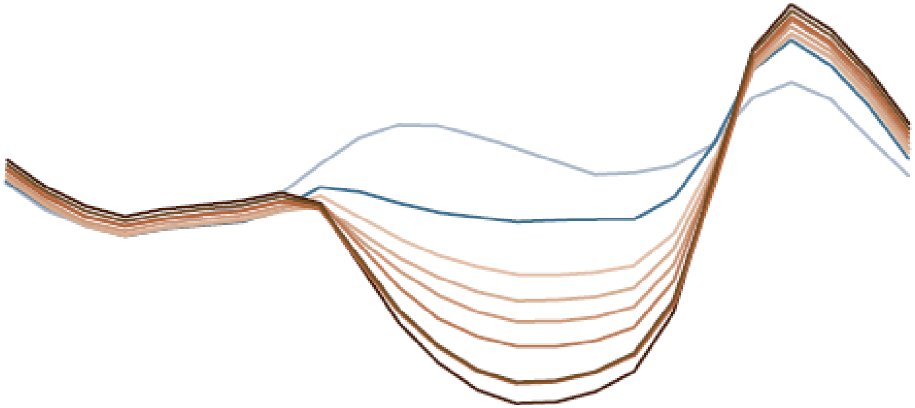
Commitment decisions $\Downarrow$

$\Uparrow$ 1) CCG:
Master operation information (extreme scenarios)

2) Benders' cuts:
Remaining operation information

Subproblems

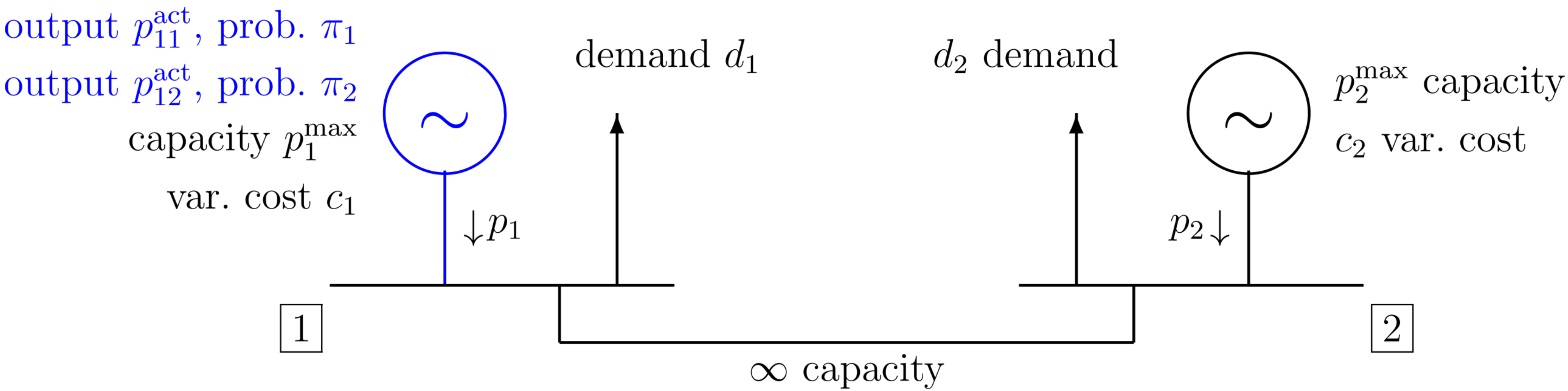
$\Rightarrow$ operation decisions

$\Leftarrow$ 

Worst realization $\Rightarrow$ Adaptive robust optimization

Worst realization $\Rightarrow$ Adaptive robust optimization

$$\min_{\mathbf{x}\in\mathcal{X}} \quad \max_{\mathbf{u}\in\mathcal{U}} \quad \min_{\mathbf{y}\in\mathcal{Y}(\mathbf{x},\mathbf{u})} \quad f(\mathbf{x},\mathbf{y},\mathbf{u})$$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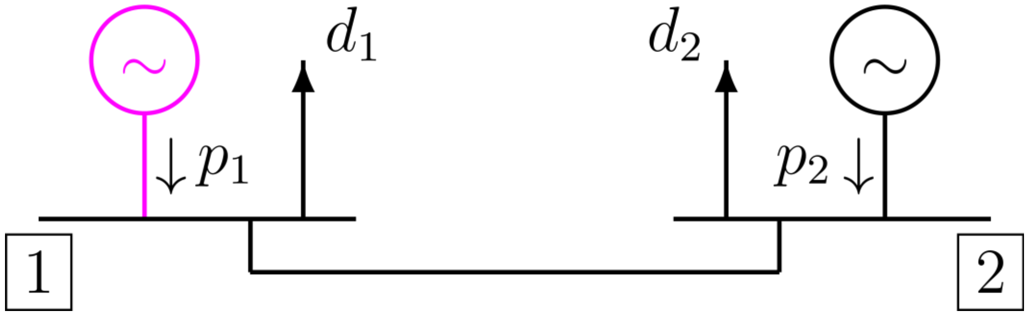
demand d_1
d_2 demand
$p_2^{\max}$ capacity
c_2 var. cost
$p_2 \downarrow$
1
2
∞ capacity

$$\min_{p_1,p_2\geq 0} \quad \max_{a_1\in\{p_{11}^{\text{act}},p_{12}^{\text{act}}\}} \quad \min_{s_1,r_2^{\text{U}},r_2^{\text{D}}\geq 0} \quad z = c_1p_1 + c_2p_2 + c_2^{\text{U}}r_2^{\text{U}} - c_2^{\text{D}}r_2^{\text{D}}$$

$$\text{s.t.} \quad (a_1 - s_1) + (p_2 + r_2^{\text{U}} - r_2^{\text{D}}) = d_1 + d_2$$

$$0 \leq p_2 + r_2^{\text{U}} - r_2^{\text{D}} \leq p_2^{\max}$$

$$s_1 \leq a_1$$



p_1 scheduled power output of producer 1

p_2 scheduled power output of producer 2

a_1 power output of producer 1 in the worst scenario

s_1 power spillage of producer 1 in the worst scenario

r_2^{U} deployed up-reserve of producer 2 in the worst scenario

r_2^{D} deployed down-reserve of producer 2 in the worst scenario

$$
\min_{p_1, p_2 \geq 0} \quad \max_{a_1 \in \{p_{11}^{\text{act}}, p_{12}^{\text{act}}\}} \quad \min_{s_1, r_2^{\text{U}}, r_2^{\text{D}} \geq 0} \quad z = c_1 p_1 + c_2 p_2 + c_2^{\text{U}} r_2^{\text{U}} - c_2^{\text{D}} r_2^{\text{D}}
$$

$$
\begin{aligned}
\text{s.t.} \quad & (a_1 - s_1) + (p_2 + r_2^{\text{U}} - r_2^{\text{D}}) = d_1 + d_2 \\
& 0 \leq p_2 + r_2^{\text{U}} - r_2^{\text{D}} \leq p_2^{\max} \\
& s_1 \leq a_1
\end{aligned}
$$

Challenges

- Robust optimization is particularly appropriate to identify optimal decisions under uncertainty if these decisions are *irreversible*.
- Robust optimization allows for robustness (risk) control.
- A robust optimization problem is generally tractable since its size does not generally depend on the accuracy of the uncertainty description, but they are mathematically complex and generally nonconvex.

Solution: dualization, merging and decomposition

$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & \max_{\boldsymbol{u}} \quad \min_{\boldsymbol{y}} \quad \boldsymbol{c}_{\mathrm{I}}^{\top}\boldsymbol{x} + [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{y} \\
\text{s.t.} \quad & \\
& \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\
& \boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0} \\
& \quad \text{s.t.} \\
& \quad \boldsymbol{u} \in \mathbb{U} \\
& \quad\quad \text{s.t.} \quad \boldsymbol{A}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} = \boldsymbol{b}(\boldsymbol{x},\boldsymbol{u}) : \quad \boldsymbol{\lambda} \\
& \quad\quad \phantom{\text{s.t.}} \quad \boldsymbol{D}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} \geq \boldsymbol{e}(\boldsymbol{x},\boldsymbol{u}) : \quad \boldsymbol{\mu}
\end{aligned}
$$

$$\min_{\boldsymbol{x}} \quad \text{s.t.} \quad \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0}$$

$$\boxed{\begin{array}{ll} \max\limits_{\boldsymbol{u}} & \boxed{\begin{array}{lll} \min\limits_{\boldsymbol{y}} & \boldsymbol{c}_{\mathrm{I}}^{\top}\boldsymbol{x} + [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{y} & \\ \text{s.t.} & \boldsymbol{A}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} = \boldsymbol{b}(\boldsymbol{x},\boldsymbol{u}) : & \boldsymbol{\lambda} \\ & \boldsymbol{D}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} \geq \boldsymbol{e}(\boldsymbol{x},\boldsymbol{u}) : & \boldsymbol{\mu} \end{array}} \\ \text{s.t.} & \\ \boldsymbol{u} \in \mathbb{U} & \end{array}}$$

Second-level problem and dual of the third-level problem merged

$$\boxed{\boxed{\begin{array}{ll} \max\limits_{\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\mu}} & [\boldsymbol{b}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{\lambda} + [\boldsymbol{e}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{\mu} \\ \text{s.t.} & \boldsymbol{u} \in \mathbb{U} \\ & [\boldsymbol{A}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{\lambda} + [\boldsymbol{D}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{\mu} = \boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u}) \\ & \boldsymbol{\lambda} : \quad \text{free} \\ & \boldsymbol{\mu} \geq \boldsymbol{0} \end{array}}}$$

Subproblem: $\boldsymbol{x} = \boldsymbol{x}^{(\nu-1)}$ fixed

$$
\begin{aligned}
\max_{\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & [\boldsymbol{b}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{e}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u})]^\top \boldsymbol{\mu} \\
\text{s.t.} \quad & \boldsymbol{u} \in \mathbb{U} \\
& [\boldsymbol{A}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{D}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u})]^\top \boldsymbol{\mu} = \boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u}) \\
& \boldsymbol{\lambda} : \quad \text{free} \\
& \boldsymbol{\mu} \geq \boldsymbol{0}
\end{aligned}
$$

$$\Downarrow$$

$$\boldsymbol{u}^{(\nu)}, \boldsymbol{\lambda}^{(\nu)}, \boldsymbol{\mu}^{(\nu)}$$

Master problem: $\boldsymbol{u} = \boldsymbol{u}^{(k)}, \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu},$ fixed

$$
\begin{array}{lll}
\displaystyle\min_{\boldsymbol{x},\boldsymbol{\eta},\boldsymbol{y}^{(k)},\boldsymbol{k}=1,\ldots,\boldsymbol{\nu}} & \boldsymbol{c}_{\mathbf{I}}^{\top}\boldsymbol{x} + \boldsymbol{\eta} & \\
\text{s.t.} & \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0} & \\
 & \boldsymbol{g}(\boldsymbol{x}) \leq \mathbf{0} & \\
 & \boldsymbol{\eta} \geq [\boldsymbol{c}_{\mathbf{O}}(\boldsymbol{x}, \boldsymbol{u}^{(k)})]^{\top}\boldsymbol{y}^{(k)} & \boldsymbol{k} = 1, \ldots, \boldsymbol{\nu} \\
 & \boldsymbol{A}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) \cdot \boldsymbol{y}^{(k)} = \boldsymbol{b}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) & \boldsymbol{k} = 1, \ldots, \boldsymbol{\nu} \\
 & \boldsymbol{D}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) \cdot \boldsymbol{y}^{(k)} \geq \boldsymbol{e}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) & \boldsymbol{k} = 1, \ldots, \boldsymbol{\nu}
\end{array}
$$

$$\Downarrow$$

$$\boldsymbol{x}^{(\nu)}, \boldsymbol{\eta}^{(\nu)}$$

$$(\& \quad \boldsymbol{y}^{(k)}, \boldsymbol{k} = 1, \ldots, \boldsymbol{\nu})$$

1. Solve the SP and get $\boldsymbol{u}^{(\boldsymbol{\nu})}$ (with previous or initial $\boldsymbol{x}^{(\boldsymbol{\nu}-\mathbf{1})}$)
2. Compute the upper bound $\mathrm{UB}_{\boldsymbol{\nu}}$
3. Solve the MP and get $\boldsymbol{x}^{(\boldsymbol{\nu})}$
4. Compute the lower bound $\mathrm{LB}_{\boldsymbol{\nu}}$
5. If $\mathrm{UB}_{\boldsymbol{\nu}} - \mathrm{LB}_{\boldsymbol{\nu}}$ is small enough, stop; otherwise continue in 1

Master problem

$\Rightarrow$ scheduling decisions

Scheduling decisions $\Downarrow$

$\Uparrow$ Worst uncertainty realization
Operation decisions

Subproblem

$\Rightarrow$ worst uncertainty realization

$\Rightarrow$ operation decisions

Conclusions

- Decision making under uncertainty renders large problems
- Decision making under uncertainty renders complex problems
- More often than not, decomposition is unavoidable
- Effective communication master-subproblem is a key requirement
- Interchanging both primal and dual information is generally effective

Thank you!