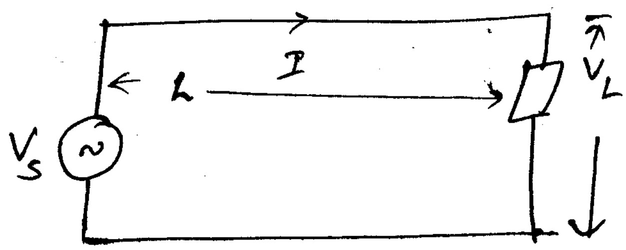
Transformers

Transformers

→ Electrical energy is generated at places where it is easier to get water head, coal, wind etc for hydro electric diesel or thermal power stations ~~...~~ and wind power stations respectively.

→ This energy is to be transmitted at considerable distances for use in villages, town and cities located at distant places.

→



Let current density of the conductor $= \delta \; A/m^2 = \dfrac{I}{a}$

$R = \dfrac{\rho l}{a}$

ρ = resistivity of the conductor material
l = length of conductor
a = area of cross section of conductor

Total cu losses(W) $= 2I^2R = 2(\delta a)^2\left(\dfrac{\rho l}{a}\right)$ ∵ 2 conductors of length L

$V_L = V_s - 2IR = V_s - 2\delta \cancel{a} \cdot \dfrac{\rho l}{\cancel{a}} = V_s - 2\delta\rho l$

$P = VI = V\delta a.$

If $V_2 = KV_1 \Rightarrow$ For the same power $P = V_2 I_2 = KV_1\delta a_2$
$V_1 I_1 = V_1 \delta a_1$

$$\Rightarrow a_2 = \frac{a_1}{K}$$

Total cu loses $= 2 I_2^2 R_2 = 2(\delta a_2)^2\left(\dfrac{\rho l}{a_2}\right)$

$= 2\left(\delta \dfrac{a_1}{K}\right)^2\left(\dfrac{\rho l}{a_1/K}\right)$

$= \dfrac{1}{K}\, 2(\delta a_1)^2 \dfrac{\rho l}{a_1} = \dfrac{1}{K}(W_1)$

$$V_L = V_S - 2\delta Pl.$$

As $V \uparrow \Rightarrow$ cu losses $\downarrow$, size of conductor $\downarrow$.

$\downarrow$

cost of conductor $\downarrow$.

Insulation $\uparrow$.

But we need less voltage at the load due to safety concern.

So a device is required to increase the voltage at the generating station and decrease the voltage at the consumer end.

This is done using a static electrical machine called transformer.

→ Electrical power is generated at 6.6, 11 or 33 KV, stepped upto 132, 220, 400 (or) 765 KV with the help of step up transformer for transmission and then stepped down to 66 KV or 33 KV at grid substations for feeding various substations, which further step down the voltage to 11 KV for feeding distributing transformers stepping down the voltage further to 400 / 230 V for consumer uses.

Applications of Transformer:-

1) Impedance matching T/F → In electronic circuits in last stage.

2) Isolation Transformer [1:1] → Blocking d.c from transferring from one ckt to other.

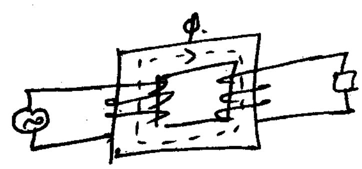
3) Power Transformer → change voltage levels in transmission lines.

→ Transformer is an ac machines that

(i) Transfers electrical energy from one electrical circuit to another which out electrical connection between the 2 circuits

(ii) has electrical circuits that are linked by common magnetic circuit

(iii) works on the principle of electromagnetic induction.



(iv) singly excited device.
ie power is given to the primary and load is connected to the secondary.

(v) changes voltage level but power and frequency remains same.

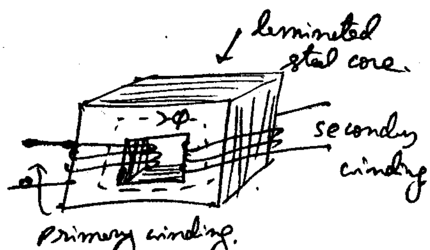
(vi) there is electromagnetic energy conversion internally but not energy conversion device externally.

(vii) Has no moving parts and hence more efficient (>96%).

Basic construction and working principle of Transformer:-

→ The elementary transformer consists of a soft iron or silicon steel core and 2 windings placed on it

→ The windings are insulated from core and each other



→ The core is made up of thin silicon steel laminations to provide a path for low reluctance to the magnetic flux.

→ The winding connected to the supply main is called primary and the winding connected to the load circuit is called the secondary.

→ Transformer works on the principle of mutual induction
↓
statically induced emf

Basic requirement :-

(1) magnetic field.

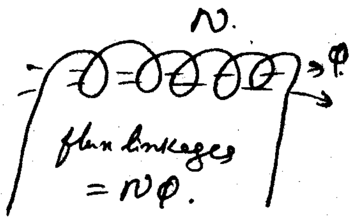
(2) set of conductors

(3) Relative space variation (or) time variation between set of conductors & magnetic field.

Statically induced emf :-

This is the emf induced in a set of stationary conductors which are placed in time varying magnetic field.

$\mathcal{E}_S \propto$ Rate of change of flux linkages

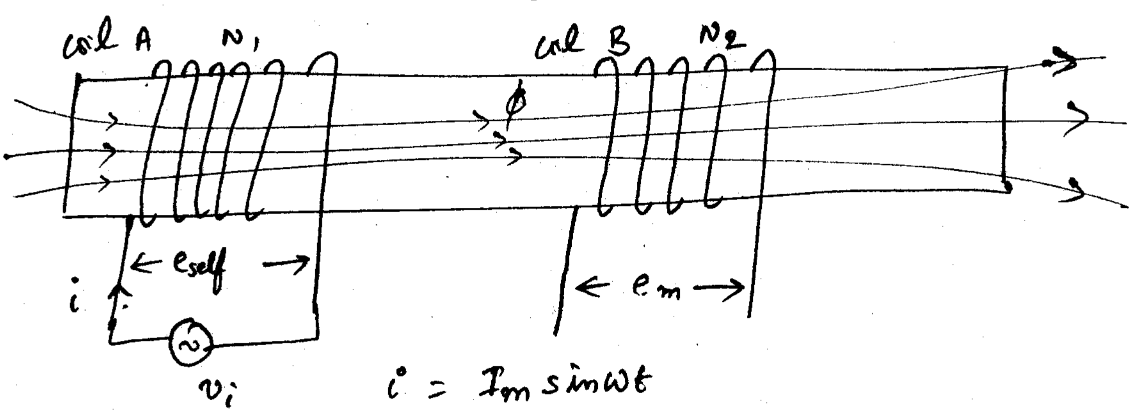


$\propto \frac{d}{dt}(N\phi)$

$\mathcal{E}_S = -N\frac{d\phi}{dt}$

→ magnitude by faradays second law.

sign by lenz's law.



$i = I_m \sin\omega t$

$MMF = N_1 i = N_1 I_m \sin\omega t$ ← time varying MMF

↑
magnetomotive force

$\text{flux} = \frac{MMF}{\text{Reluctance}} = \frac{N_1 I_m \sin\omega t}{R} = \phi_m \sin\omega t.$

→ self induced emf is the emf induced in a coil due to time varying nature of current flowing through its own coil

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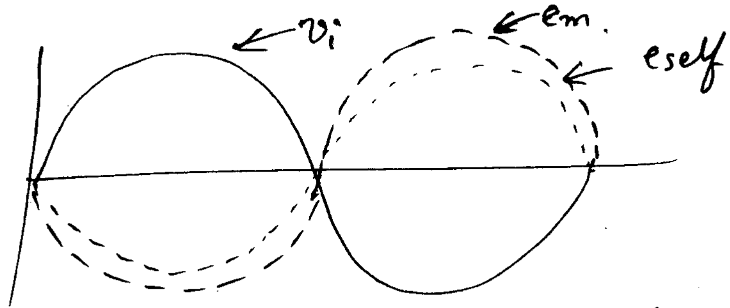
$$e_{self} = -N_1 \frac{d\phi}{dt} = -\left(N_1 \frac{d\phi}{di_1}\right) \cdot \frac{di_1}{dt}$$

$$= -L_{self} \frac{di_1}{dt}$$

→ self inductance (L_{self}) of a coil may be defined as rate of change of flux linkages at a coil with respect to time varying nature of current flowing through its own coil.

$$\text{self induced emf } (e_{self}) \propto \frac{di_1}{dt}$$



$$v_1 = -e_{self}$$

To satisfy the lenz's law, self induced emf always opposes the change in applied voltage so that they are 180° out of phase with each other.

→ mutually induced emf is the emf induced in a coil due to time varying nature of current flowing through another coil which is magnetically coupled to the first coil.

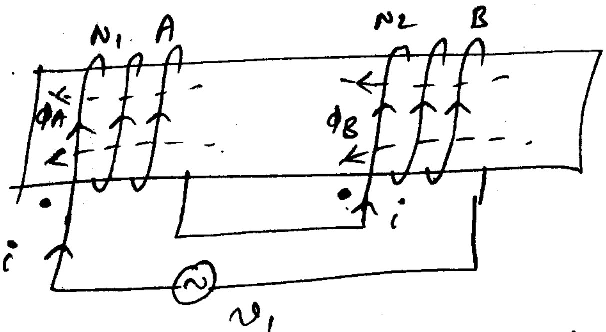
$$e_m = -N_2 \frac{d\phi}{dt} = -\left(N_2 \frac{d\phi}{di_1}\right)\frac{di_1}{dt} = -M \frac{di_1}{dt}$$

→ mutual inductance (m) between two coils may be defined as rate of change of flux linkages at a coil w.r.t the time varying nature of current flowing through another coil which is magnetically coupled to the 1st coil.

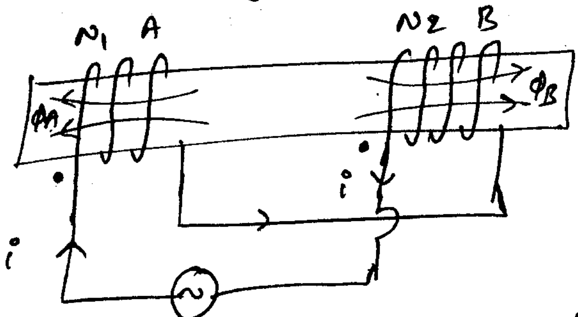
→ mutually induced emf also opposes the change in applied voltage to satisfy lenz's law.

Dot notation:-

↳ To predict the orientation of flux internally



Two coils are said to be +vely coupled if the flux produced by them are aiding one another in the common magnetic ckt



Two coils are said to be -vely coupled if the flux produced by them in common magnetic circuit is opposing one another.

Ideal Transformer:-

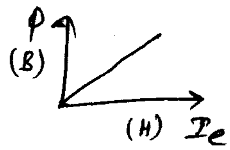
1, permeability of Transformer core is infinite
[No MMF is required to produce flux) ($I_e = 0$)

2, Iron losses in the transformer core is zero.

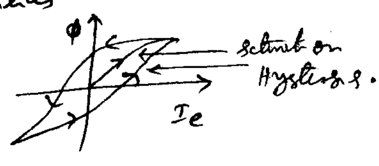
3, Resistances of transformer winding is zero.

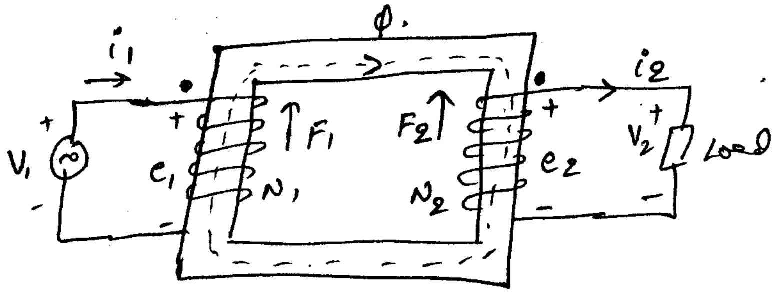
4, No magnetic leakage flux in the transformer.

5, magnetisation of transformer core is linear



Practically there are 2 non linearities
(i) saturation non linearity
(2) Hysteresis non linearity





when the voltage is applied to the primary winding flux is produced in the core. As Reluctance $= 0 \Rightarrow$ mmf $= 0 \Rightarrow$ no magnetising current is required from the supply.

Let $\quad \phi = \phi_m \sin \omega t \qquad \omega = 2\pi f$ rad/sec

The emf induced in primary winding balances the applied voltage as per KVL

$$v_1 = -e_1 = N_1 \frac{d\phi}{dt} = \omega N_1 \phi_m \cos \omega t = -\omega N_1 \phi_m \sin(\omega t - 90)$$

The secondary induced voltage is equal to the load voltage

$$v_2 = e_2 = \omega N_2 \phi_m \cos \omega t$$

The r.m.s values of voltages are $\qquad \because V = \frac{V_m}{\sqrt{2}}$

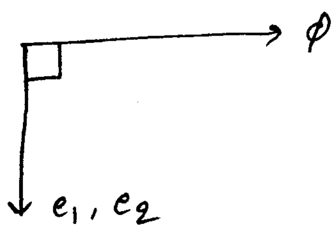
$$V_1 = E_1 = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = \sqrt{2}\, \pi f N_1 \phi_m$$
$$= 4.44 f N_1 \phi_m$$

$$V_2 = E_2 = 4.44 f N_2 \phi_m.$$

$$e_1 = -\omega N_1 \phi_m \cos \omega t = +\omega N_1 \phi_m \sin(\omega t - 90)$$
$$= 2\pi f N_1 \phi_m \sin(\omega t - 90)$$
$$= e_m \sin(\omega t - 90).$$

as $\quad \phi = \phi_m \sin \omega t$

this implies that the induced voltage lags the flux by $90°$



From above equations voltage transformation ratio of rms value is

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \text{ (turns ratio)}$$

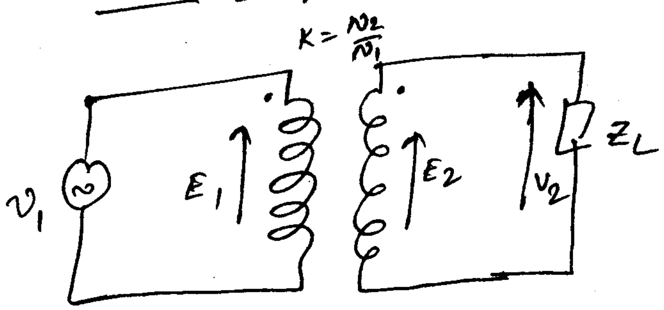
$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K \text{ (transformation ratio)}$$

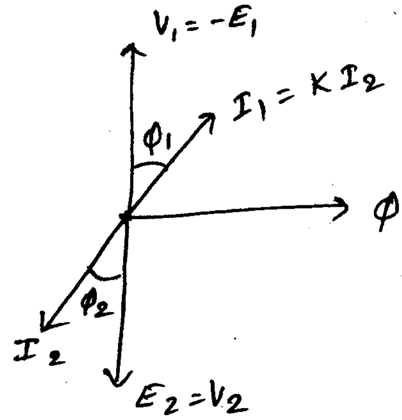
and $$\phi_m = \frac{E_1}{\sqrt{2}\pi f N_1} = \frac{E_2}{\sqrt{2}\pi f N_2}$$

$$= \frac{V_1}{\sqrt{2}\pi f N_1} = \frac{V_2}{\sqrt{2}\pi f N_2} \qquad \because E_1 = V_1$$

It can be seen that ϕ_{max} is determined by applied voltage and frequency and is independent of current.

Ideal transformer connected to load:-





When load is connected to the secondary of the transformer, current will pass through the secondary $I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$

The current may lag or lead V_2 depending on the type of the load.

The secondary current I_2 sets up an mmf $I_2 N_2$ which produces a flux in the opposite direction to main flux as per Lenz's law.

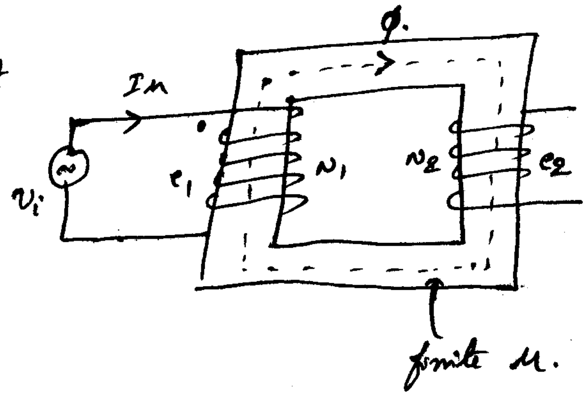
Due to this net flux in the core decreases $\Rightarrow \phi\downarrow \Rightarrow E_1\downarrow$. to satisfy KVL ie $V_1 = -E_1$ extra current will be drawn from the supply to counterbalance the secondary mmf.

$$\therefore \quad N_1 I_1 = N_2 I_2$$

$$I_1 = \frac{N_2}{N_1} I_2 = K I_2$$

Irrespective of the load, the flux in the transformer core is always constant = no load flux.

Transformer with finite permeability core :-



I_μ = magnetising component of current

$i_\mu = I_m \sin\omega t$ ← reference

primary mmf = $N_1 i_\mu = N_1 I_m \sin\omega t$

$$\text{primary flux (main field flux)} = \frac{mmf}{R} = \frac{N_1 I_m \sin\omega t}{R} = \phi_m \sin\omega t.$$

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t) = -N_1 \phi_m \omega \cos\omega t$$

$$= N_1 \omega \phi_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

the primary induced emf lags the flux by 90°

$$\text{at } \omega t = \pi \Rightarrow \quad e_1 = N_1 \phi_m \omega \sin\left(\pi - \frac{\pi}{2}\right) = \underbrace{N_1 \phi_m \omega}_{E_{1max}} = 2\pi f N_1 \phi_m$$

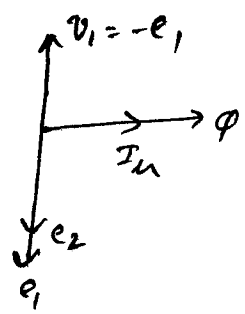
$$= 4.44\, N_1 \underbrace{\phi_m}_{B_m A_n} f$$

A_n ← net cross section area of core.

$$e_2 = -N_2 \frac{d\phi}{dt} = -N_2 \frac{d}{dt}(\phi_m \sin\omega t)$$

$$= N_2 \phi_m \omega \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$E_2 = 4.44\, N_2\, B_m A_n f$$



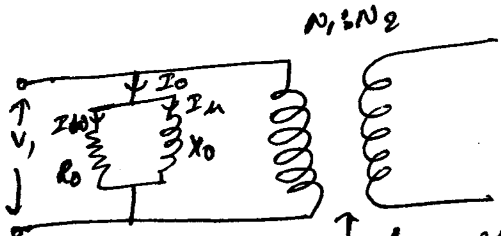
Transformer with iron losses:-

The losses that takes place in the transformer core is called as core losses or iron losses — hysteresis loss, eddy current loss.

→ since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it.

→ The loss is dissipated in the form of heat and can be represented by a resistance.

→ The iron losses also depends on voltage and independent of load current.



lines should not be placed as they represent transformer core.

I_w = iron loss component of current

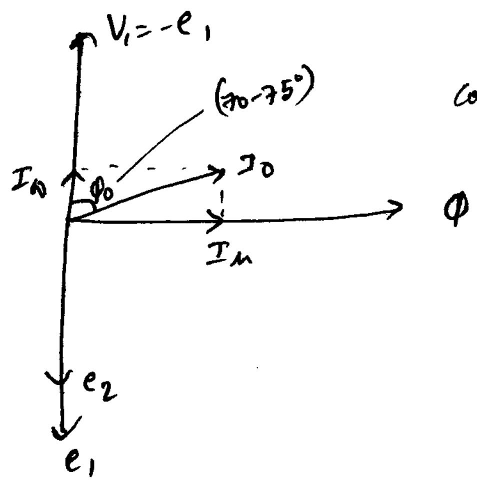
I_μ = magnetising component of current

$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w$$

I_0 ↓ 5 to 8% of I_{fl}; I_μ ↓ 4 to 6% of I_{fl}; I_w ↓ 1 to 2% of I_{fl}

I_μ → quadrature with the applied voltage ← wattless or reactive component

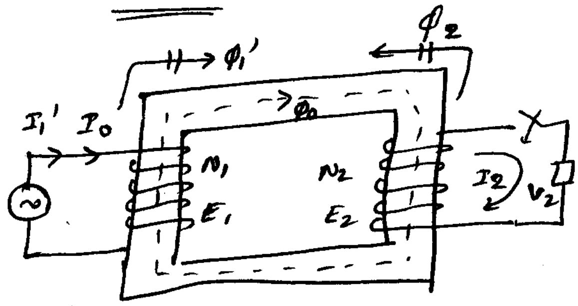
I_w → in phase with the applied voltage ← wattful or active component



$\cos\phi_0$ = no load p.f of T/F $\simeq 0.2$ lag.

Semi ideal transformer under loaded condition:

$N_1 I_\mu$ = primary MMF $\Rightarrow \phi_0$ = primary working flux



I_2 always opposes the main field flux.

The direction of I_2 can be found by lenz's law.

$\phi_1' = \phi_2$

$\phi_R = \phi_0$

$N_2 I_2$ = secondary MMF $\Rightarrow \phi_2$ = secondary flux.

$N_1 I_1'$ = load component of primary MMF = compensating MMF

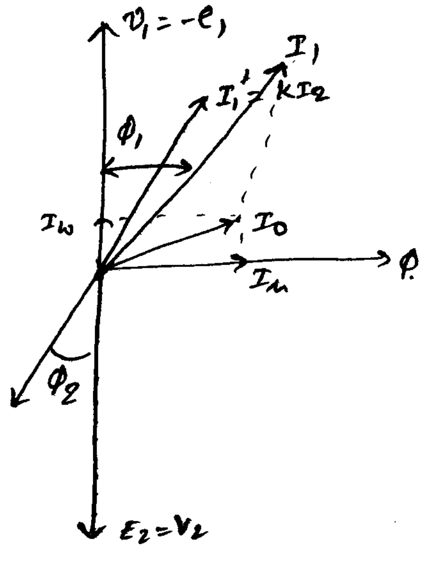
$\Rightarrow \phi_1'$ = compensating flux (or) load component of primary flux.

$\phi_R = \phi_0 \leftarrow$ always as $\phi_1' = \phi_2 \Rightarrow N_1 I_1' = N_2 I_2$

T/F is a const flux device

$\bar{I}_1 = \bar{I}_1' + \bar{I}_0$

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1'}{I_2} = K.}$$



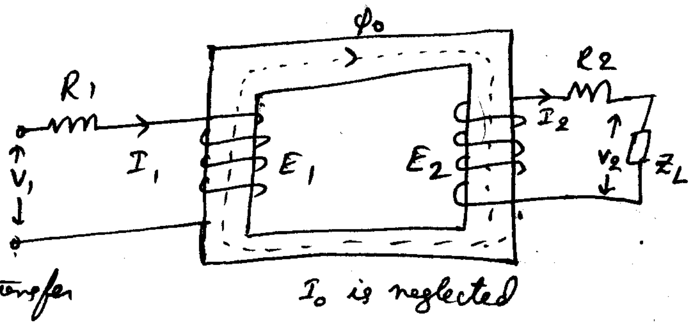
$\cos\phi_2$ = secondary (or) load P.f

$\cos\phi_1$ = primary P.f

Transformer with winding resistance:-

Since the winding consists of copper conductors, both primary and secondary has winding resistance.

The resistance acts in series with respective windings as the loss due to resistance depends on current through that winding.



I_o is neglected

$$\bar{V}_1 = -\bar{E}_1 + \bar{I}_1 R_1$$
$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 R_2$$

Total cu losses $= I_1^2 R_1 + I_2^2 R_2$

condition to refer resistance from one side to other is

cu losses before transfer = cu losses after transfer

$$I_2^2 R_2 = I_1^2 R_2'$$

$$\Rightarrow \boxed{R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \frac{R_2}{K^2}}$$ ← equivalent of secondary side resistance referred to primary.

$$I_1^2 R_1 = I_2^2 R_1' \Rightarrow \boxed{R_1' = \left(\frac{I_1}{I_2}\right)^2 R_1 = K^2 R_1}$$

Total resistance referred to primary $= R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$

" " secondary $= R_{02} = R_1' + R_2 = K^2 R_1 + R_2$

Total cu losses $= I_1^2 R_{01} = I_2^2 R_{02}$

$$I_1^2 R_{01} = I_1^2\left(R_1 + \frac{R_2}{K^2}\right) = K^2 I_2^2\left(R_1 + \frac{R_2}{K^2}\right) = I_2^2\left(K^2 R_1 + R_2\right) = I_2^2 R_{02}$$

$$\because \frac{I_1}{I_2} = K.$$

p.u primary resistance drop $= \dfrac{\text{primary voltage drop}}{\text{primary induced voltage}} = \dfrac{I_1 R_1}{E_1}$

p.u secondary resistance drop $= \dfrac{I_2 R_2}{E_2}$

p.u resistance drop referred to primary $= \dfrac{I_1 R_{01}}{E_1} \Rightarrow \dfrac{I_1}{E_1} \cdot \dfrac{R_{02}}{K^2}$

$$\Rightarrow \frac{I_1}{E_1} \cdot \frac{R_{02}}{\left(\frac{E_2}{E_1}\right)\left(\frac{I_1}{I_2}\right)} \Rightarrow \frac{I_2 R_{02}}{E_2}$$

p.u resistance drop referred to primary = p.u resistance drop referred to secondary

% Resistance drop = % Resistance.

Transformer with Magnetic leakage flux:-

→ Both primary and secondary currents produce flux. The flux ϕ which links both the windings is the useful flux and is called useful flux.

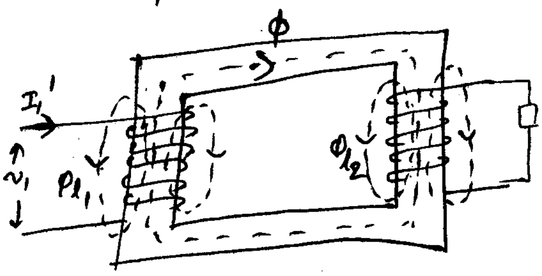
→ However primary current would produce some flux ϕ_1 which would not link the secondary winding.

Similarly, secondary winding would produce some flux ϕ_2 that would not link the primary winding.

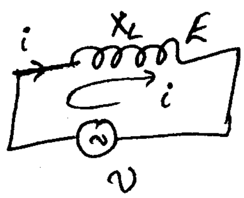
→ This flux ϕ_1 & ϕ_2 links only one winding and hence not useful from power transfer from one winding to another.

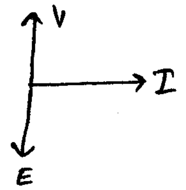
→ This leakage flux is represented in the form of reactance in series with each winding as leakage flux depends on current and it does not consume active power.

$$\begin{array}{cccc} \phi & \phi_{l1} & \phi & \phi_{l2} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ E_1 & E_{x1} & E_1 & E_{x2} \end{array}$$



The leakage flux at the windings always induces some additional emf's in the respective windings and are lagging behind the respective load currents by exactly 90°.



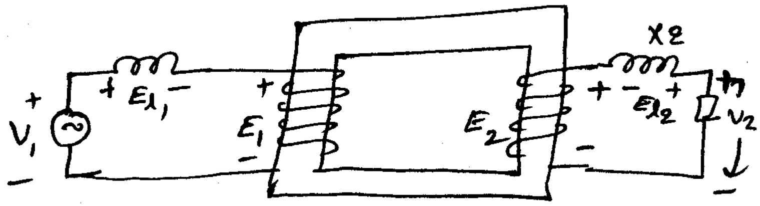


$$\bar{V} + \bar{E} = 0$$

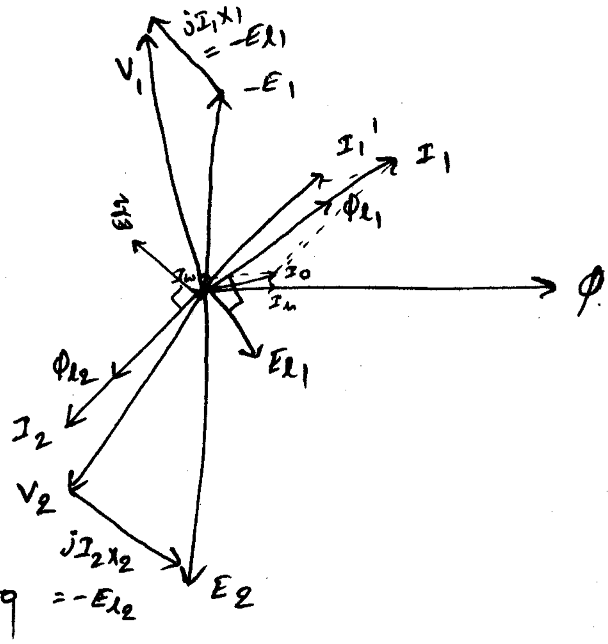
$$\bar{V} = -\bar{E}$$

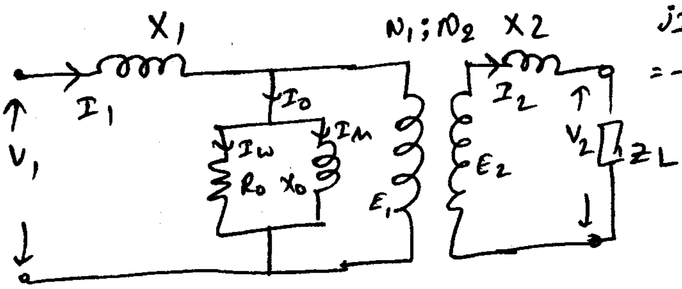
$$= j\omega L \bar{I}$$

$$\bar{V}_1 = -\bar{E}_1 - \bar{E}_{l_1}$$
$$= -\bar{E}_1 + jI_1X_1$$



$$\bar{V}_2 = \bar{E}_2 + \bar{E}_{l_2}$$
$$= \bar{E}_2 - jI_2X_2$$
$$E_2 = \bar{V}_2 + jI_2X_2$$





Equivalent circuit with leakage reactance & no load elements.

p.u reactance drop before transfer = p.u reactance drop after transfer

$$\frac{I_1X_1}{E_1} = \frac{I_2X_1'}{E_2} \Rightarrow X_1' = \left(\frac{I_1}{I_2}\right)\left(\frac{E_2}{E_1}\right)X_1 = K^2X_1$$

$$\frac{I_2X_2}{E_2} = \frac{I_1X_2'}{E_1} \Rightarrow X_2' = \frac{X_2}{K^2}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} \quad ; \quad X_{02} = X_1' + X_2 = K^2X_1 + X_2$$

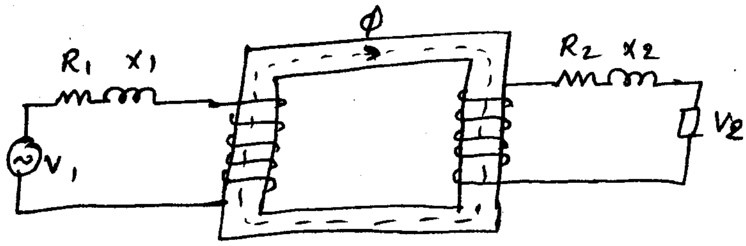
p.u reactance drop referred to primary $= \dfrac{I_1X_{01}}{E_1}$

" secondary $= \dfrac{I_2X_{02}}{E_2}$

$$\frac{I_1X_{01}}{E_1} = \frac{I_1}{E_1}\cdot\frac{X_{02}}{K^2} = \frac{I_1}{E_1}\cdot\frac{X_{02}}{\left(\frac{E_2}{E_1}\right)\left(\frac{I_1}{I_2}\right)} = \frac{I_2X_{02}}{E_2}$$

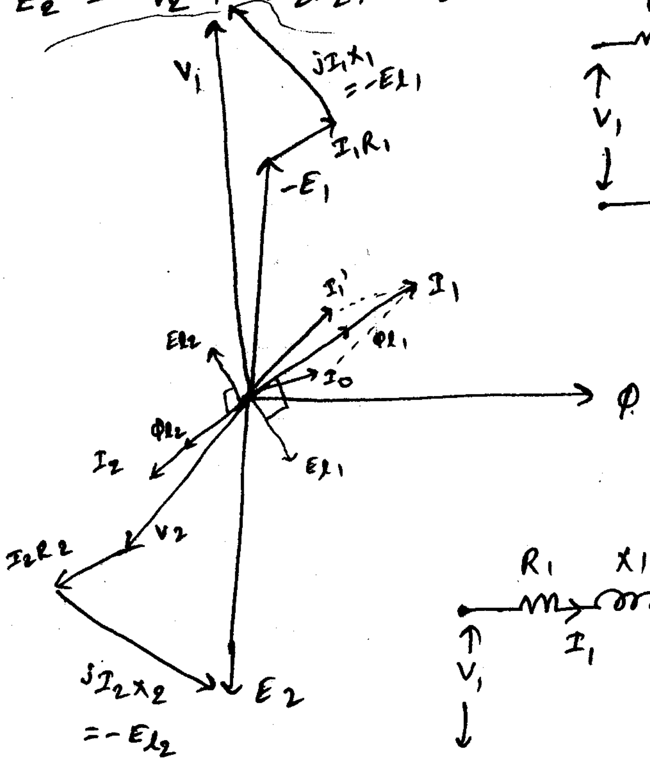
p.u reactance drop referred to primary = p.u reactance drop referred to secondary

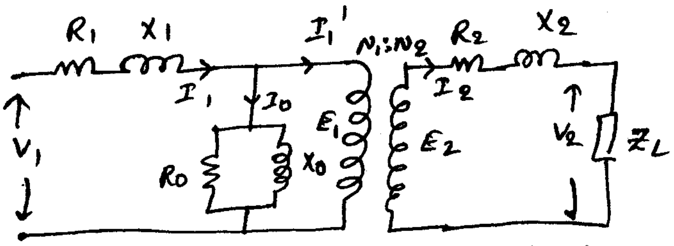
Including winding resistance :-



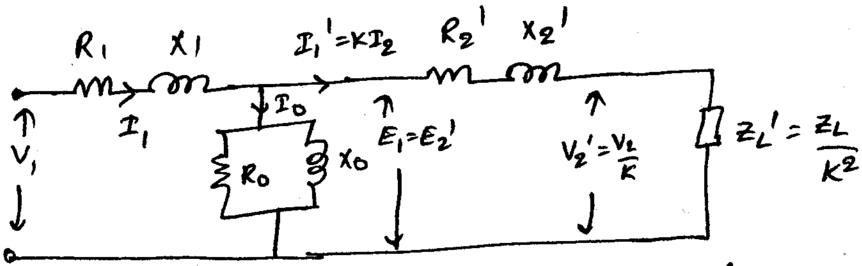
$$\bar{V}_1 = -\bar{E}_1 + \bar{I}_1 R_1 + j\bar{I}_1 X_1$$

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 R_2 + j\bar{I}_2 X_2$$

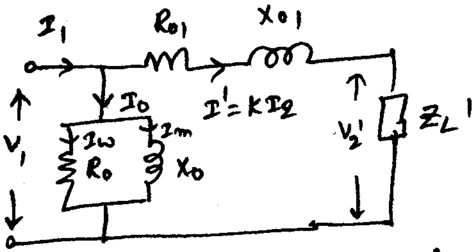




Exact equivalent circuit of transformer.



Equivalent ckt refered to Primary side.



approximate equivalent circuit

← the no load primary impedance drop is neglected

no load primary cu loss is neglected

As $V_1 > E_1 \Rightarrow I_0$ & I_m, I_w are more than actual values.

As $I_w \uparrow \Rightarrow$ Iron losses > actual values.

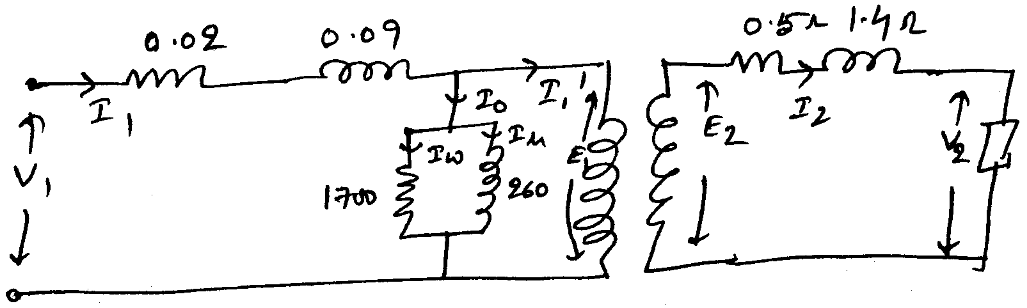
$$I_w = I_0 \cos\phi_0 \; ; \; I_m = I_0 \sin\phi_0 \; ; \; I_0 = \sqrt{I_m^2 + I_w^2}$$

$$R_0 = \frac{V_1}{I_w} \qquad X_0 = \frac{V_1}{I_m}$$

ex: A 200/1000 V, 100 KVA, 50 Hz transformer has the following parameters $R_1 = 0.02\Omega$, $R_2 = 0.5\Omega$, $R_0 = 1700\Omega$, $X_1 = 0.09\Omega$, $X_2 = 1.4\Omega$, $X_0 = 260\Omega$

The transformer is supplying ~~the~~ full load at a power factor of 0.8 lag. Using (i) Exact (ii) Approximate equivalent circuits, find the input current.

Sol: (a) Exact Equivalent circuit:-



$K = \frac{1000}{200} = 5$

taking load voltage as reference $V_2 = 1000\angle 0^\circ$

$$I_2 = \frac{100 \times 10^3}{1000} = 100\,A.$$

$\therefore$ load p.f $= 0.8$ lag $\Rightarrow \phi = -36.9^\circ$

$$I_2 = 100\angle -36.9^\circ$$

$$Z_2 = (0.5 + j1.4) = 1.4866\angle 70.35^\circ$$

$$\bar{E_2} = \bar{V_2} + \bar{I_2}\bar{Z_2} = 1000\angle 0^\circ + 100\angle -36.9 * 1.486\angle 70.35$$

$$= (1000\angle 0) + 148.6\angle 33.45^\circ$$

$$= (1000 + j0) + (123.99 + j\,81.91)$$

$$= 1123.99 + j\,81.91$$

$$= 1126.97\angle 4.168^\circ.$$

$$\bar{E_1} = \frac{\bar{E_2}}{K} = \frac{1126.97\angle 4.168}{5} = 225.394\angle 4.168$$

$$I_1' = K I_2 = 5 * 100\angle -36.9 = 500\angle -36.9^\circ.$$

$$= 399.84 - j\,300.21$$

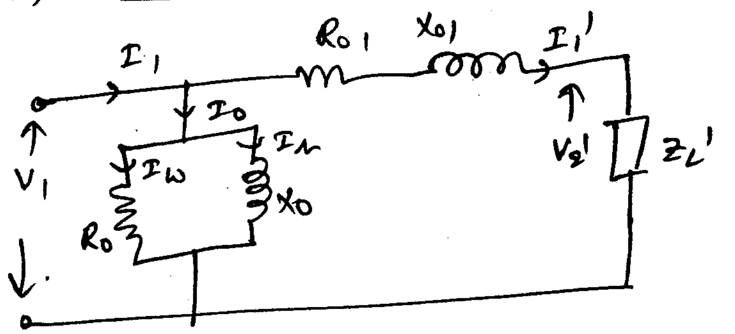
$$I_w = \frac{E_1}{R_0} = \frac{225.394 \angle 4.168}{1700 \angle 0^\circ} = 0.133 \angle 4.168^\circ$$
$$= 0.133 + j0.01$$

$$I_\mu = \frac{E_1}{X_0 \; (j260\Omega)} = \frac{225.394 \angle 4.168}{260 \angle 90^\circ} = 0.8669 \angle -85.832$$
$$= 0.063 - j\,0.865$$

$$\bar{I}_0 = \bar{I}_w + \bar{I}_\mu = 0.196 - j\,0.855$$

$$\bar{I}_1 = I_1' + I_0 = (399 - j300.21) + (0.196 - j0.855)$$
$$= 399.196 - j\,301.065$$
$$= 499.998 \angle -37.022$$

(ii) approximate equivalent circuit:



$$K = \frac{1000}{200} = 5$$

$$V_2 = 1000 \angle 0^\circ$$

$$V_2' = \frac{V_2}{K} = 200 \angle 0^\circ$$

$$I_2 = \frac{100 \times 10^3}{1000} = 100 \angle -36.9^\circ$$

$$I_1' = K\, I_2 = 500 \angle -36.9^\circ.$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.02 + \frac{0.5}{25} = 0.04\,\Omega$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 0.09 + \frac{1.4}{25} = 0.146\,\Omega$$

$$Z_{01} = 0.04 + j0.146 = 0.151 \angle 74.68$$

$$V_1 = V_2' + I_1' Z_{01} = 200 \angle 0^\circ + (500 \angle -36.9)(0.151 \angle 74.68)$$
$$= (200 + j0) + 75.5 \angle 37.78$$
$$= (200 + j0) + (59.67 + j\,46.25)$$
$$= 259.67 + j\,46.25$$
$$= 263.76 \angle 10.1^\circ$$

$$I_w = \frac{V_1}{R_0} = \frac{263.76 \angle 10.1}{1700 \angle 0^\circ} = 0.155 \angle 10.1^\circ$$

$$I_\mu = \frac{V_1}{X_0} = \frac{263.76 \angle 10.1}{260 \angle 90^\circ} = 1.014 \angle -79.9^\circ$$

$$I_0 = I_w + I_\mu = 0.155 \angle 10.1^\circ + 1.014 \angle -79.9^\circ$$

$$= (0.153 + j\,0.027) + (0.178 - j\,0.998)$$

$$= 0.331 - j\,0.971$$

$$I_1 = I_1' + I_0$$

$$= 500 \angle -36.9 + I_0$$

$$= (399.84 - j\,300.21) + (0.331 - j\,0.971)$$

$$= 400.171 - j\,301.18$$

$$= 500.85 \angle -36.97^\circ$$

Voltage Regulation of Transformer:-

It is the change in secondary terminal voltage from no load to full load at some specific power factor and is expressed as percentage (or) fraction of either no load secondary terminal voltage (or) full load secondary terminal voltage.

Let E_2 = No load secondary terminal voltage $= V_1' = KV_1$.

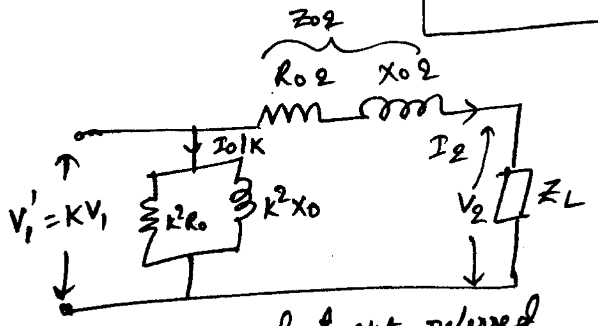
V_2 = Full load secondary terminal voltage

$$\text{Regulation} = \left.\frac{E_2 - V_2}{E_2}\right|_{\cos\phi_2 = \text{const}} \leftarrow \text{Regulation down}$$

$$= \left.\frac{E_2 - V_2}{V_2}\right|_{\cos\phi_2 = \text{const}} \leftarrow \text{Regulation up}$$

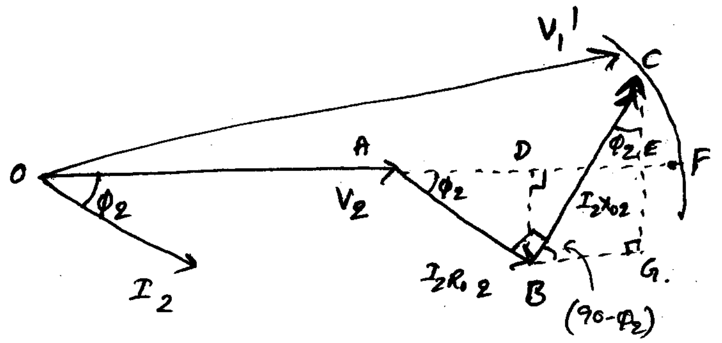
unless specifically mentioned simply the regulation in the sence regulation down only

$$\therefore \boxed{\text{Regulation} = \frac{E_2 - V_2}{E_2}}$$



Approx equivalent ckt referred to secondary side.

$$\bar{V}_1' = \bar{V}_2 + \bar{I}_2 (R_{02} + jX_{02})$$



$$OC = OF = \sqrt{OE^2 + EC^2}$$

$$= \sqrt{(OA + AD + DE)^2 + (GC - GE)^2}$$

$OA = V_2$; $AD = I_2R_{02}\cos\phi_2$

$DE = BG = I_2X_{02}\sin\phi_2$

$GC = I_2X_{02}\cos\phi_2$

$GE = BD = I_2R_{02}\sin\phi_2$

$$V_1' = \sqrt{(V_2 + I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2)^2 + (I_2X_{02}\cos\phi_2 - I_2R_{02}\sin\phi_2)^2}$$

Generally EF is very less and can be neglected.

$$\therefore V_1' = OA + AD + DE = V_2 + I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$$

$$(V_1' - V_2) = I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$$

$$\therefore \text{approximate voltage drop} = [I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2]$$

$+ \rightarrow$ lag p.f

$- \rightarrow$ lead p.f

$$\text{Approximate voltage regulation} = \frac{I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2}{V_1'}$$

$$= \left(\frac{I_2R_{02}}{V_1'}\right)\cos\phi_2 \pm \left(\frac{I_2X_{02}}{V_1'}\right)\sin\phi_2$$

$$\boxed{\text{Voltage Reg} = \varepsilon_r \cos\phi_2 \pm \varepsilon_x \sin\phi_2}$$

ε_r = p.u resistance drop = p.u resistance.

ε_x = p.u reactance drop = p.u reactance.

% reg = (% Resistance) $\cos\phi_2$ $\pm$ (% reactance) $\sin\phi_2$

Exact voltage regulation = $(\varepsilon_r \cos\phi_2 \pm \varepsilon_x \sin\phi_2) + \frac{1}{2}(\varepsilon_x \cos\phi_2 \mp \varepsilon_r \sin\phi_2)^2$

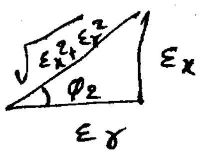
Condition for maximum voltage regulation:-

$$Reg = \varepsilon_r \cos\phi_2 \pm \varepsilon_x \sin\phi_2$$

$\frac{\partial reg}{\partial \phi_2} = 0$ Assume that ~~reg~~ max voltage regulation occurs at lag p.f

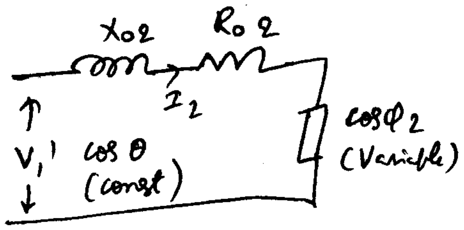
[If the sign changes in answer then assumption is wrong]

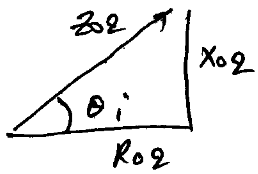
$$-\varepsilon_r \sin\phi_2 + \varepsilon_x \cos\phi_2 = 0 \Rightarrow \tan\phi_2 = \frac{\varepsilon_x}{\varepsilon_r}$$



p.f corresponding to max voltage reg $\Rightarrow \cos\phi_2 = \frac{\varepsilon_r}{\sqrt{\varepsilon_x^2+\varepsilon_r^2}}$

$$\Rightarrow \boxed{\cos\phi_2 = \frac{R_{02}}{Z_{02}}}$$

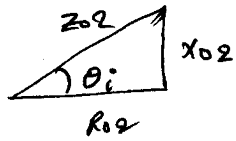




$\therefore$ Regulation is max when load p.f = internal impedance angle of transformer

$$\boxed{\phi_2 = \theta_i}$$

Value of maximum voltage Regulation:-



$$Reg = \frac{I_2 R_{02}}{V_1'}\cos\phi_2 + \frac{I_2 X_{02}}{V_1'}\sin\phi_2$$

$\phi_2 = \theta_i$

$\cos\phi_2 = \frac{R_{02}}{Z_{02}}$

$\sin\phi_2 = \frac{X_{02}}{Z_{02}}$

$$= \frac{I_2 R_{02}}{V_1'}\,\frac{R_{02}}{Z_{02}} + \frac{I_2 X_{02}}{V_1'}\,\frac{X_{02}}{Z_{02}}$$

$$= \frac{I_2}{V_1' Z_{02}}\left[R_{02}^2 + X_{02}^2\right] = \frac{I_2 Z_{02}}{V_1'} = \text{p.u impedance of T/F}$$

Regulation at UPf:-

$\cos\phi_2 = 1, \quad \sin\phi_2 = 0$

$$Reg = \frac{I_2 R_{02}}{V_1'} \times 1 + 0 = \varepsilon_r = \text{p.u resistance} = \text{p.u F.L cu loss.}$$

$$\boxed{Reg_{UPf} = \text{p.u resistance} = \text{p.u full load cu loss}}$$

Regulation at any fraction x of F.L:-

$$Reg_{F.L} = \underbrace{\frac{I_2 R_{02}}{V_1'}}_{\varepsilon_r}\cos\phi_2 \pm \underbrace{\frac{I_2 X_{02}}{V_1'}}_{\varepsilon_x}\sin\phi_2$$

$$\boxed{Reg_{x.F.L} = x\left(\varepsilon_r\cos\phi_2 \pm \varepsilon_x\sin\phi_2\right) = x * \text{F.L regulation}}$$

condition for zero voltage regulation

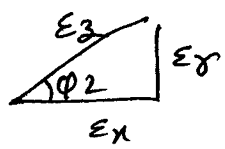
assume Reg = 0 at lag p.f

$$\varepsilon_r\cos\phi_2 + \varepsilon_x\sin\phi_2 = 0$$

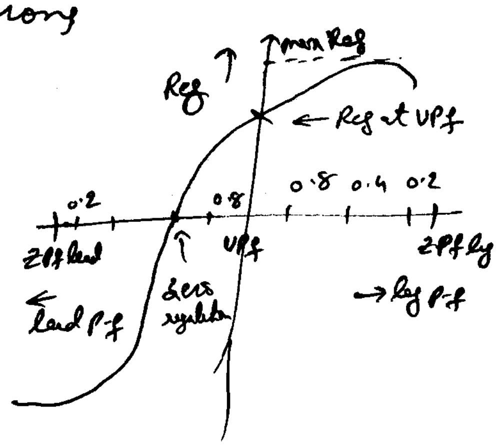
$$\tan\phi_2 = -\frac{\varepsilon_r}{\varepsilon_x}.$$

-ve sign indicates our assumption is wrong

$$\phi_{2\,lead} = \tan^{-1}\left(\frac{\varepsilon_r}{\varepsilon_x}\right)$$



$$\boxed{\cos\phi_2 = \frac{X_{02}}{Z_{02}}}\ \text{lead.}$$



Testing of Transformers < open circuit test / short circuit test

1, open circuit test :-

objectives :- (i) To find R_0 & X_0

(ii) To measure constant losses in transformer

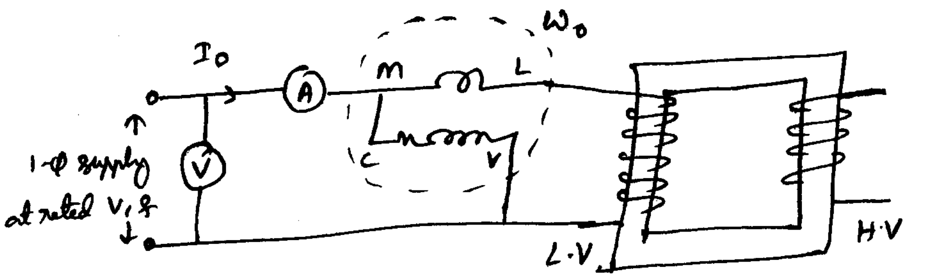
→ one winding terminals are kept open and other winding is supplied at rated voltage & frequency

→ It is convenient to conduct this test on LV side because

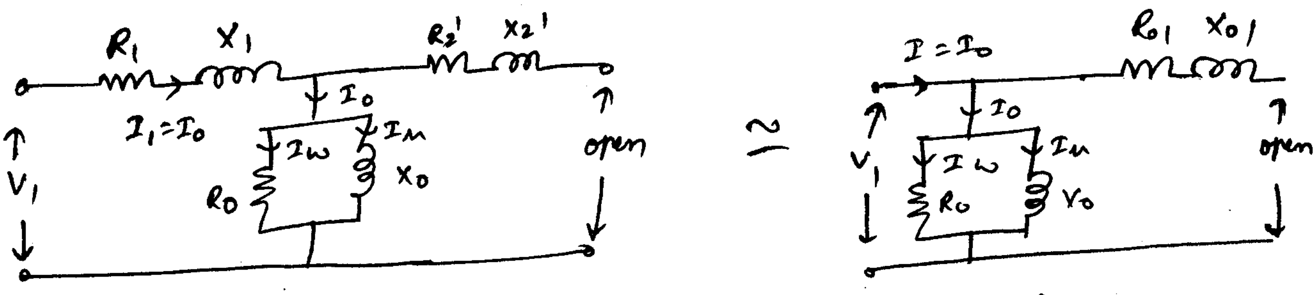
(i) No load current on L.V side is greater than HV side which can be accurately measured.

$$E_1 I_1 = E_2 I_2 \Rightarrow E_{HV} > E_{LV} \Rightarrow I_{LV} > I_{HV}.$$

(ii) The rated voltage on L.V side is less than HV side which can be conviniently applied & measured.



obtaind :- V_1, I_0 & W_0



W_0 = Iron loss + dielectric loss + no load primary cu loss ($I_0^2 R_1$).

If dielectric loss and small amount of cu loss is neglected then W_0 = iron loss.

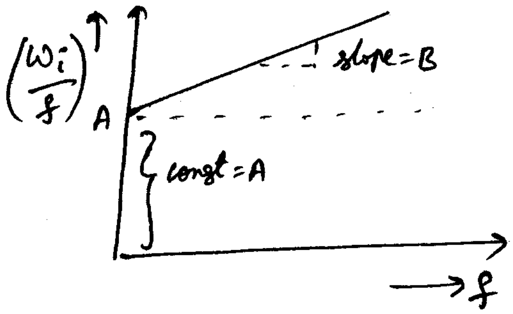
$$R_0 = \frac{V_1}{I_W} \qquad X_0 = \frac{V_1}{I_\mu} \qquad W_0 = V_1 I_0 \cos\phi_0$$

$$I_W = I_0 \cos\phi_0 , \quad I_\mu = I_0 \sin\phi_0 \qquad \cos\phi_0 = \frac{W_0}{V_1 I_0}$$

Seperation of iron losses:-

$$\frac{V_1}{f} = \text{const} \Rightarrow B_{max} = \text{const} \Rightarrow \begin{matrix} W_h = Af \leftarrow \boxed{W_h = \eta B_m^{1.6} f V} \\ W_e = Bf^2 \leftarrow \boxed{W_e = K B_m^2 f^2 t^2} \end{matrix}$$

$$W_i = Af + Bf^2 \Rightarrow \frac{W_i}{f} = A + Bf$$



W_h at rated $f = A * f_{rated}$

W_e at rated $f = B * f^2_{rated}$

2, Short circuit test:-

objectives :- (i) To measure variable losses in transformer

(ii) To find out total resistance & reactance of T/F referred to the side in which instrument is placed.

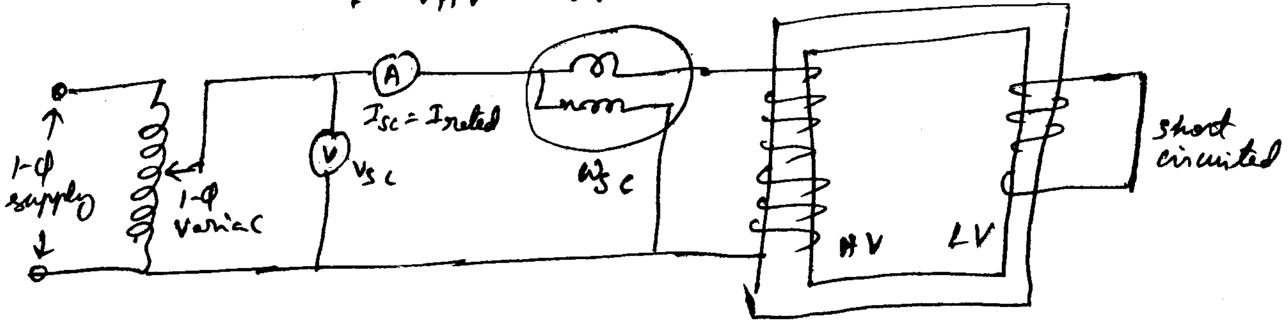
→ As this test is conducted at rated current, it is convenient to conduct this test on HV side by short circuiting the LV side terminals.

$\because I_{rated\ HV} < I_{rated\ LV} \Rightarrow$ easy to supply

→ As LV side wdg is short circuited, about 5 to 8% of rated voltage is enough to produce the rated current

∴ It is convenient to measure in HV side

$\because V_{HV} > V_{LV}$



W_{SC} = losses in T/F under short circuit condition

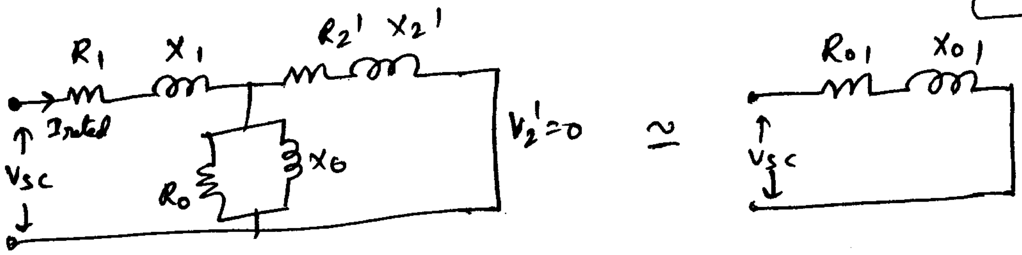
= F.L cu loss + stray load loss + Iron loss corresponding to V_{SC}

In o.c test $\rightarrow V \rightarrow W_0$

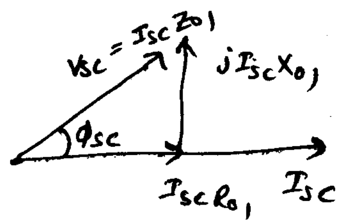
for $V_{SC} \Rightarrow \quad W_0 \times \left(\frac{V_{SC}}{V_1}\right)^2 << W_0 \quad (\because W_i \propto V^2) \leftarrow$ iron losses.

As iron losses & stray load losses are negligible.

$\therefore \quad W_{SC} \simeq$ F.L Cu losses $\simeq I_{SC}^2 R_{01} \Rightarrow \boxed{R_{01} = \frac{W_{SC}}{I_{SC}^2}}$



$$V_{SC} = I_{SC} R_{01} + j I_{SC} X_{01} = I_{SC} Z_{01}$$



$$\boxed{Z_{01} = \frac{V_{SC}}{I_{SC}}}$$

$$\boxed{X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}}$$

$$\phi_{SC} = \tan^{-1}\left(\frac{X_{01}}{R_{01}}\right)$$

$$\cos\phi_{SC} = \frac{R_{01}}{Z_{01}}$$

ex: The test results on a 50 KVA, 1000/100 V transformer

o.c test (LV side) : 200 W, 5 A, 100 V.

s.c test (HV side) : 800 W, 20 A, 60V

calculate the parameters of the equivalent circuit.

Sol:

$$K = \frac{V_2}{V_1} = \frac{100}{1000} = \frac{1}{10}$$

$$I_0 = 5A, \quad P_0 = 200\,W, \quad V_2 = 100\,V.$$

$$\cos\phi_0 = \frac{P_0}{V_2 I_0} = \frac{200}{100 \times 5} = 0.4 \text{ lag.}$$

$$I_W = I_0 \cos\phi_0 = 5 \times 0.4 = 2\ A$$

$$I_\mu = \sqrt{I_0^2 - I_W^2} = \sqrt{5^2 - 2^2} = 4.58\ A$$

$$R_0 = \frac{V_2}{I_w} = \frac{100}{2} = 50\,\Omega$$

$$X_0 = \frac{V_2}{I_m} = \frac{100}{4.58} = 21.83\,\Omega$$

Referred to HV side is

$$R_0' = \frac{R_0}{K^2} = \frac{50}{(1/10)^2} = \underline{5\,k\Omega}$$

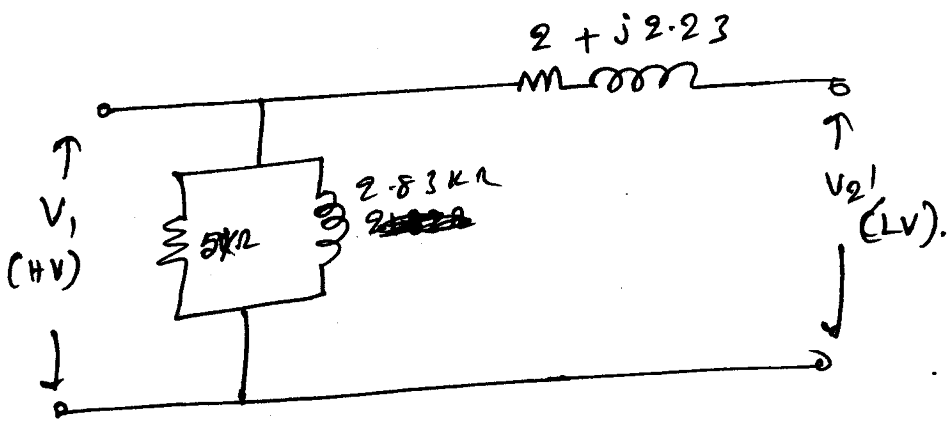
$$X_0' = \frac{X_0}{K^2} = \frac{21.83}{(1/10)^2} = \underline{2.18\,k\Omega}.$$

$V_{sc} = 60V$, $I_{sc} = 20A$, $P_{sc} = 800W$.

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{60}{20} = \underline{3.0}\,\Omega.$$

$$R_{01} = \frac{P_{sc}}{I_{sc}^2} = \frac{800}{(20)^2} = \underline{2\,\Omega}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \underline{2.23\,\Omega}$$



Efficiency of Transformer:-

→ The rating of any machine is limited by rate of rise of heat.

∴ In T/F the losses are → Iron losses → V_1
→ cu losses → I_1

∴ Total losses → $V_1 I_1$ and not on $\cos\phi$.

∴ T/F rating is only in KVA.

$$\text{Efficiency}(\eta) = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

at full load

$$\eta_{F.L} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + \underbrace{\text{Variable losses}}_{\text{cu loss} \propto I_2^2 = C I_2^2} + \underbrace{\text{const losses}}_{W_C}}$$

C ← const

W_C ← iron losses

$$\eta_{F.L} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + C I_2^2 + W_C}$$

$$\eta_{x \text{ of } F.L} = \frac{x V_2 I_2 \cos\phi}{x V_2 I_2 \cos\phi + x^2 (C I_2^2) + W_C}$$

Let $V_2 I_2 \cos\phi = P$

$C I_2^2 = P_c$

$W_C = P_i$

$$= \frac{xP}{xP + x^2 P_c + P_i}$$

condition for max efficiency:

$$\left.\frac{d\eta}{dx}\right|_{\phi = \text{const}} = 0$$

$$\left.\frac{d\eta}{d\phi}\right|_{xI_2 = \text{const}} = 0.$$

(i) $$\frac{d\eta}{dx} = \frac{(xP + x^2 P_c + P_i)P - xP(P + 2xP_c)}{(xP + x^2 P_c + P_i)^2} = 0$$

$$\cancel{xP^2} + x^2 P_c P + P_i P - \cancel{xP^2} - 2x^2 P P_c = 0$$

$$P(P_i - x^2 P_c) = 0$$

$$P \neq 0; \quad P_i = x^2 P_c \Rightarrow x = \sqrt{\frac{P_i}{P_c}}$$

→ By suitably adjusting the value of P_i & P_c in design, we can set maximum efficiency at any 'x' of F-L.

Put $xI_2 = I_m$ & $x^2 P_c = P_i$

$$x^2 I_2^2 R_{02} = P_i$$

$$I_m^2 R_{02} = P_i \Rightarrow I_m = \sqrt{\frac{P_i}{R_{02}}}$$

$$\frac{E_2 I_m}{1000} = \frac{E_2}{1000}\sqrt{\frac{P_i}{R_{02}}}$$

$$(KVA)_{max\,\eta} = \frac{E_2 I_2}{1000}\sqrt{\frac{W_i}{I_2^2 R_{02}}}$$

$$\boxed{(KVA)_{max\,\eta} = KVA_{F.L}\sqrt{\frac{W_i}{F.L\ cu\ loss}}}$$

(ii) $$\left.\frac{d\eta}{d\phi_2}\right|_{xI_2 = const} = 0$$

$$(x V_2 I_2 \cos\phi_2 + x^2 P_c + P_i)(-x V_2 I_2 \sin\phi_2)$$

$$= x V_2 I_2 \cos\phi_2 \,(-x V_2 I_2 \sin\phi_2).$$

$$\Rightarrow x V_2 I_2 \sin\phi_2 = 0. \Rightarrow \sin\phi_2 = 0 \Rightarrow \phi_2 = 0$$

$$\Rightarrow \cos\phi_2 = 1$$

→ By keeping load current constant, if load p.f is varied, then we get maximum efficiency at unity p.f ie $\cos\phi = 1$

<u>If losses are given in p.u values:-</u>

Chose KVA rating of T/F as base KVA

p.u F.L cu loss = W_{cu} & p.u iron loss = W_i

$$\eta_{F.L} = \frac{1\,p.u \cos\phi_2}{1\,p.u \cos\phi_2 + W_{cu} + W_i} \qquad \eta_{xF.L} = \frac{x\cos\phi_2}{x\cos\phi_2 + x^2 W_{cu} + W_i}$$

All day efficiency:-

→ used for distribution transformers {Rating ≤ 1 MVA, low voltage}

→ As load on consumers varies through out the day, commercial efficiency cannot be used.

$$\text{All day efficiency} = \left.\frac{\text{output in KWh}}{\text{input in KWh}}\right|_{24\text{ hrs}}.$$

→ As T/F is always connected to supply irrespective of the load, iron losses occurs through out the day.

∴ iron losses should be minimized.

→ average load = 70 to 75% of F.L

$$\eta_{max} = 0.7\ F.L$$

⇒ F.L cu losses ≃ 2 F.L iron losses.