Boolean Alzebra

(i) AND laws

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A = A$

$A \cdot \overline{A} = 0$

(ii) OR law.

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + \overline{A} = 1$

→ Identity element of AND operation is '1' $\because A \cdot 1 = A$.

Identity element of OR operation is '0' $\because A + 0 = A$

(1) commutative laws:-

$A + B = B + A$

$A \cdot B = B \cdot A$

(2) Associative laws:-

$(A + B) + C = A + (B + C)$

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

1) Determine the commutative & associative properties of the following (a) NAND operation $\uparrow$ (b) Inhibition operation

[$x/y = x\overline{y}$ ie "x but not y"]

Sol:

(a) (i) LHS $A \uparrow B = \overline{AB}$

RHS $B \uparrow A = \overline{BA}$

$\therefore$ LHS = RHS.

(ii) LHS:- $(A \uparrow B) \uparrow C = \overline{AB} \uparrow C = (\overline{\overline{AB} \cdot C})$

RHS:- $A \uparrow (B \uparrow C) = A \uparrow (\overline{BC}) = (\overline{A \cdot \overline{BC}})$

LHS $\neq$ RHS.

→ The NAND operation is commutative but not associative.

(b) (i) $LHS = A/B = A\bar{B}$

$RHS = B/A = B\bar{A}$

$LHS \neq RHS.$

(ii) $LHS = (A/B)/C = (A\bar{B})/C = A\bar{B}\bar{C}$

$RHS = A/(B/C) = A/(B\bar{C}) = A\cdot(\overline{B\bar{C}}) = A(\bar{B}+C)$

$LHS \neq RHS.$

The Inhibition operation is neither commutative nor Associative.

(3) Distribution laws:-

$$A(B+C) = AB+AC$$

Dual → $A + B\cdot C = (A+B)\cdot(A+C)$

ex:-

$$A+\bar{A}B = (A+\bar{A})(A+B) = A+B$$

$$\bar{A}+AB = (\bar{A}+A)(\bar{A}+B) = \bar{A}+B.$$

(4) Absorption laws:-

$$A+AB = A$$

$$A\cdot(A+B) = A$$

(5) consensus laws:-

$$AB+\bar{A}C+BC = AB+\bar{A}C$$

$$(A+B)\cdot(\bar{A}+C)\cdot(B+C) = (A+B)\cdot(\bar{A}+C)$$

L.H.S:-

$$AB+\bar{A}C+BC(A+\bar{A})$$

$$= AB+\bar{A}C+BCA+BC\bar{A}$$

$$= AB(1+C)+\bar{A}C(1+B)$$

$$= AB+\bar{A}\cdot C = R.H.S.$$

ex:- $xy + \bar{y}z + wxyz$

$= xy + \bar{y}z.$

(6) Transposition laws:-

$$AB + \bar{A}C = (A+C)(\bar{A}+B)$$

$$(A+B)\cdot(\bar{A}+C) = AC + \bar{A}B.$$

proof:-

R.H.S:-

$A\bar{A} + AB + C\bar{A} + CB.$

$\hookrightarrow 0$; $\hookrightarrow \times$ as per concensus laws.

$AB + \bar{A}C$

(7) De morgans laws:-

$$\overline{A+B+C+\cdots} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdots$$

$$\overline{A\cdot B\cdot C\cdots} = \bar{A} + \bar{B} + \bar{C} + \cdots$$

(8) Duality property:-

All the alzebraic expressions are valid if their operators and identity elements are interchanged.

ex:- $A\cdot 1 = A$ then $A + 0 = A$.

*Note:- To find the complement of the funtion F

(i) Find the duel of function ie F_D

(ii) complement all variables $\rightarrow \bar{F}$.

ex:- $F = xy\bar{z} + \bar{x}\bar{y} + x\bar{y}z$ then $\bar{F} = ?$

sol:- $F_D = (x+y+\bar{z})\cdot(\bar{x}+\bar{y})\cdot(x+\bar{y}+z)$

$\bar{F} = (\bar{x}+\bar{y}+z)(x+y)(\bar{x}+y+\bar{z})$

$$= (0 + \bar{x}y + \bar{y}x + 0 + zx + zy)(\bar{x}+y+\bar{z})$$

$$= (x\bar{y} + yz)\quad(\bar{x} + y + \bar{z})$$

$$\bar{x}y + xy\bar{z} + x\bar{y}\bar{z} + yz + \cancel{yz\bar{x}}$$

$$= \cancel{x\bar{y}\bar{z}} \quad \cancel{+\bar{x}yz + yz}$$

$$F = x\bar{y}\bar{z} + yz + \bar{x}y$$

(2) simplify following expression to 3 literals.

$$F = AB + \bar{A}CD + \bar{C}D + \bar{B}CD.$$

Sol: literal $\rightarrow$ variable (or) complement of a variable.

$$= AB + CD(\bar{A}+\bar{B}) + \bar{C}D.$$

$$= \underbrace{AB}_{x} + \underbrace{\overline{AB}}_{\bar{x}} \cdot \underbrace{CD}_{y} + \bar{C}D.$$

$\because x + \bar{x}y = x + y$

$$= AB + CD + \bar{C}D.$$

$$= AB + D(C+\bar{C})$$

$$= AB + D$$

(3) $F = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

Sol:- $= AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC$

$\because x + x + x = x.$

$$= AB(C+\bar{C}) + AC(\bar{B}+B) + BC(\bar{A}+A)$$

$$= AB + AC + BC$$

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13) $F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$

Sol:

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}BC + \bar{A}BC + ABC$$

$$= \bar{A}C + \bar{A}B + BC .$$

14) Implement EX-OR gate using minimum no of (i) NAND gates (ii) NOR gates.

Sol:

(i) $Y = \bar{A}B + A\bar{B}$, NAND $\Rightarrow Y = \overline{AB}$

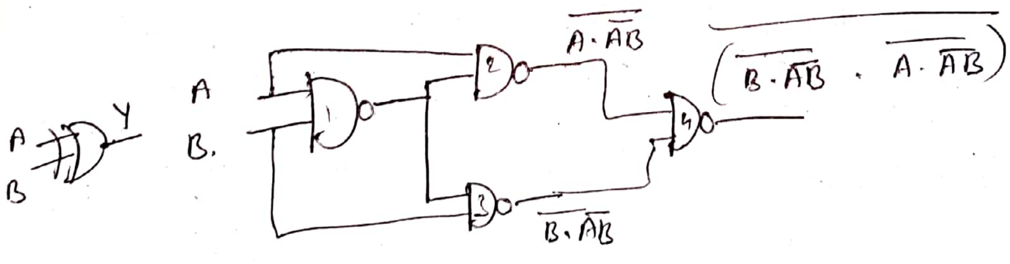
$$Y = \bar{A}B + A\bar{B} + A\bar{A} + B\bar{B}$$

$$= B(\bar{A}+\bar{B}) + A(\bar{A}+\bar{B})$$

$$= B(\overline{AB}) + A(\overline{AB})$$

$$Y = \bar{\bar{Y}} = \overline{\overline{B \cdot \overline{AB} + A \cdot \overline{AB}}}$$

$$= \overline{\overline{B \cdot \overline{AB}} \cdot \overline{A \cdot \overline{AB}}}$$

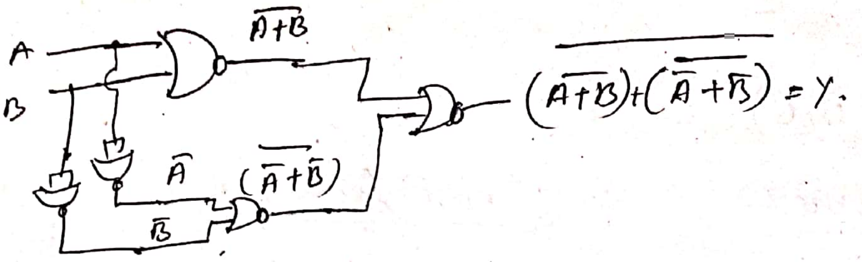


(ii) $Y = \bar{A}B + A\bar{B} + A\bar{A} + B\bar{B}$ NOR $Y = \overline{A+B}$

$$= B(\bar{A}+\bar{B}) + A(\bar{A}+\bar{B})$$

$$= (\bar{A}+\bar{B}) \cdot (B+A)$$

$$Y = \bar{\bar{Y}} = \overline{\overline{(\bar{A}+\bar{B})} + \overline{(B+A)}}$$



(5) Implement the Equivalence gate (EX-NOR) using min no of NOR gates.

Sol:

$$Y = AB + \bar{A}\bar{B}$$

(or)

$$\overline{\bar{A}B + A\bar{B}}$$

~~AB ⊙ AB~~
$A \odot B$

NOR
$Y = \overline{A+B}$