```
In [2]: import numpy as np
    import pandas as pd
    import matplotlib.pyplot as plt
    import seaborn as sns
    from statsmodels.nonparametric.smoothers_lowess import lowess
    from scipy.interpolate import CubicSpline, interp1d
```

```
In [3]: # --- Adjust parameters and Load Data---
lower_limit_G, upper_limit_G, color_index_upper_lim = 20, 10, 1.5 #hard limi
bprp, gmag = np.loadtxt("NGC6397.txt", unpack=True, skiprows=1) #input data,
```

Data Preparation

In this section we search for the bluest point in the color-magnitude diagram (CMD), This is useful for example in isochrone fitting, where we need to know the position of the main sequence turn-off (MSTO) point. The LOWESS (Locally Weighted Scatterplot Smoothing) method is applied to determine the primary ridgeline of the CMD. Then the numerical derivative of the ridge line is calculated, which is used to determine the slope of the ridgeline. This slope is used to rotate the points in the CMD, so that the ridgeline is horizontal in the transformed coordinate system.

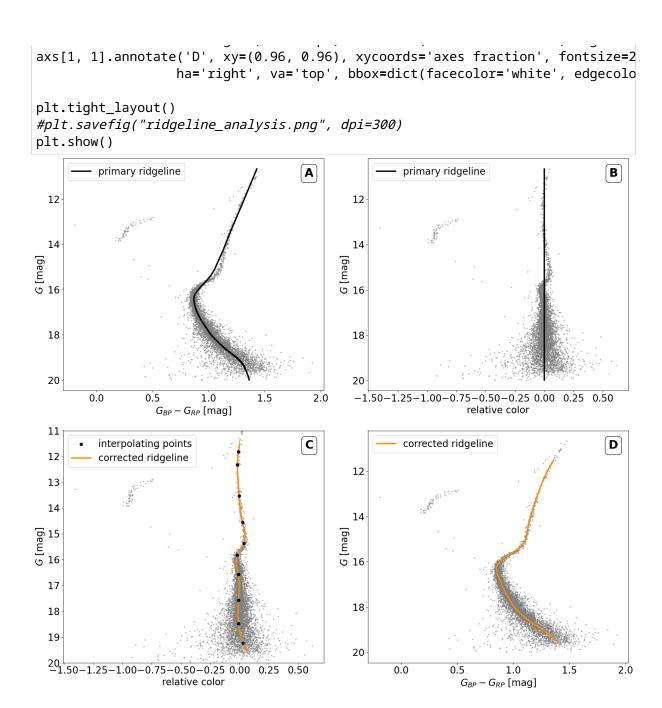
```
In [4]: arr = np.column_stack((bprp, gmag))
        arr = arr[(arr[:, 1] < lower_limit_G) & ~((arr[:, 0] < color_index_upper_lim)</pre>
        df = pd.DataFrame(arr, columns=['bprp', 'gmag']).dropna().sort_values('gmag')
        df[['ridge_bprp', 'ridge_gmag']] = lowess(df['bprp'], df['gmag'], frac=0.2)[:
        blue_x, blue_y = df.loc[df['ridge_bprp'].idxmin(), ['ridge_bprp', 'ridge_gmag
        #cut2_points_x, cut2_points_y = blue_x + 0.3, blue_y - 2
        def numerical_derivative(x, y):
            x, y = np.asarray(x), np.asarray(y)
            d = np.zeros_like(y)
            d[0] = (y[1] - y[0]) / (x[1] - x[0])
            d[1:-1] = (y[2:] - y[:-2]) / (x[2:] - x[:-2])
            d[-1] = (y[-1] - y[-2]) / (x[-1] - x[-2])
            return d
        def rotate_points(x, y, rx, ry, inverse=False):
            a = np.arctan(numerical_derivative(ry, rx))
            if inverse:
                a = -a
            c, s = np.cos(a), np.sin(a)
            x_rot = x * c - y * s
            y_rot = x * s + y * c
            if not inverse:
                y_rot += ry
            return x rot, y rot
        # --- Rotation and Binning ---
        df[['x_new', 'y_new']] = np.column_stack(
            rotate_points(df['bprp']-df['ridge_bprp'], df['qmag']-df['ridge_qmag'], d
        df["x_new0"] = 0
        df = df.sort_values(by='y_new')
```

We separate the datapoints into N number of bins along the Y axis. Then we calculate KDE (Kernel Density Estimate) values along the X axis of each bin and calculate the median values along the Y axis. The maximum values of the KDEs became the X coordinate and the median values became the Y coordinates of the N well-defined bin points. We connect these points with a cubic spline. Then we transform everything back into the original coordinate system where the interpolated cubic spline becomes the new ridgeline. This way, the new ridgeline now closely follows the turnoff point and the RGB.

```
In [5]: bins = np.array([11.5,12,13,14,15,15.6,16,17,18,19,19.5]) #Adjust the bins as
        bin_labels = [str(b) for b in bins[:-1]]
        df['bin'] = pd.cut(df['y new'], bins=bins, labels=bin labels, right=False)
        # --- KDE Calculation ---
        def calculate_kde(df, column, y_column, bandwidth='scott'):
            for bin_label, group in df.groupby('bin', observed=False):
                if group[column].empty:
                    continue
                fig, ax = plt.subplots()
                kde_plot = sns.kdeplot(group[column], bw_method=bandwidth, ax=ax)
                xxx, yyy = kde_plot.lines[0].get_data()
                plt.close(fig)
                idx_max = pd.Series(yyy).idxmax()
                results.append({
                    'bin': bin_label,
                    'x_at_max_density': xxx[idx_max],
                    'median_y': group[y_column].median()
                })
            return results
        kde_results = calculate_kde(df, 'x_new', 'y_new')
        results_df = pd.DataFrame(kde_results).sort_values(by='median_y')
        y_sorted = results_df['median_y'].values
        x sorted = results df['x at max density'].values
        # --- Interpolation and Inverse Rotation ---
        cs = CubicSpline(y_sorted, x_sorted)
        mask = (df['y_new'] >= bins[0]) & (df['y_new'] < bins[-1])
        df.loc[mask, 'x_interpolate'] = cs(df.loc[mask, 'y_new'])
        df.loc[~mask, 'x_interpolate'] = np.nan
        df['x_back2'], df['y_back2'] = rotate_points(
            df['x_interpolate'], df['y_new'] - df['ridge_gmag'], df['ridge_bprp'], df
        df['x_back2'] += df['ridge_bprp']
        df['y back2'] += df['ridge gmag']
        # --- Final Sorting and Linear Interpolation ---
        df_sorted = df[['x_back2', 'y_back2']].sort_values(by='y_back2').dropna().res
        x_sorted = df_sorted['x_back2'].values
        y_sorted = df_sorted['y_back2'].values
        linear interp = interp1d(y sorted, x sorted, kind='linear', fill value="extra
        y_fine2 = np.linspace(y_sorted.min(), y_sorted.max(), 10000)
        x_fine2 = linear_interp(y_fine2)
        /tmp/ipykernel 3341/3890085491.py:13: RuntimeWarning: invalid value encounte
        red in divide
          d[1:-1] = (y[2:] - y[:-2]) / (x[2:] - x[:-2])
```

Visualization

```
In [ ]: # Set the font size and family for the plot
        plt.rcParams.update({'font.size': 20})
        #plt.rcParams["font.family"] = "Liberation Serif" ## specital font family, ca
        # Set the point size and transparency
        s = 3
        alpha = 0.5
        fig, axs = plt.subplots(2, 2, figsize=(18, 16))
        # Top left: CMD with primary ridgeline
        axs[0, 0].invert_yaxis()
        #axs[0, 0].set_xlim(0.75, 1.5)
        axs[0, 0].scatter(df['bprp'], df['gmag'], s=s, c='grey', alpha=alpha)
        axs[0, 0].plot(df['ridge_bprp'], df["ridge_gmag"], c="black", lw=3, label="pr
        axs[0, 0].set_xlabel('$G_{BP}-G_{RP}$ [mag]')
        axs[0, 0].set_ylabel('$G$ [maq]')
        axs[0, 0].legend(loc='upper left')
        # Top right: Rotated CMD with primary ridgeline
        axs[0, 1].invert_yaxis()
        #axs[0, 1].set_xlim(-0.6, 0.4)
        axs[0, 1].scatter(df['x_new'], df['y_new'], s=s, color='grey', alpha=alpha)
        axs[0, 1].plot(df['x_new0'], df['y_new'], c="black", lw=3, label="primary rid
        axs[0, 1].set_xlabel('relative color')
        axs[0, 1].set_ylabel('$G$ [mag]')
        axs[0, 1].legend(loc='upper left')
        # Bottom left: Rotated CMD with KDE maxima and interpolated curve
        axs[1, 0].invert_yaxis()
        #axs[1, 0].set_xlim(-0.6, 0.4)
        axs[1, 0].set_ylim(20, 11)
        axs[1, 0].scatter(df['x_new'], df['y_new'], s=s, color='grey', alpha=alpha)
        axs[1, 0].scatter(results_df['x_at_max_density'].values, results_df['median_y
        axs[1, 0].plot(df['x_interpolate'], df['y_new'], c="darkorange", zorder=1, lw
        axs[1, 0].set_xlabel('relative color')
        axs[1, 0].set_ylabel('$G$ [mag]')
        axs[1, 0].legend(loc='upper left')
        # Bottom right: CMD with corrected ridgeline
        axs[1, 1].invert_yaxis()
        #axs[1, 1].set_xlim(0.75, 1.5)
        axs[1, 1].scatter(df['bprp'], df['gmag'], s=s, color='grey', alpha=alpha)
        axs[1, 1].plot(df['x_back2'], df['y_back2'], color='darkorange', lw=3, label=
        axs[1, 1].set_xlabel('$G_{BP}-G_{RP}$ [mag]')
        axs[1, 1].set_ylabel('$G$ [mag]')
        axs[1, 1].legend()
        ## Add annotation for the panels a, b, c, d
        axs[0, 0].annotate('A', xy=(0.96, 0.96), xycoords='axes fraction', fontsize=2
                          ha='right', va='top', bbox=dict(facecolor='white', edgecolo
        axs[0, 1].annotate('B', xy=(0.96, 0.96), xy=(0.96, 0.96), xy=(0.96, 0.96)
                          ha='right', va='top', bbox=dict(facecolor='white', edgecolo
        axs[1, 0].annotate('C', xy=(0.96, 0.96), xycoords='axes fraction', fontsize=2
                          ha='right', va='top', bbox=dict(facecolor='white', edgecolo
```



Supplementary features

Only the final ridgeline plot

```
In [20]: fig = plt.figure(figsize=(8,8))
    plt.gca().invert_yaxis()

## Adjust limits according to your data
    plt.xlim(0.7,1.5)

plt.scatter(df['bprp'],df['gmag'], s=s, alpha=alpha, color='grey')
    #plt.plot(df['ridge_bprp'], df["ridge_gmag"], c="darkorange", lw=2, label="pr
    plt.plot(df['x_back2'], df['y_back2'],color='navy',lw=2, label="corrected rid

plt.xlabel('$G_{BP}-G_{RP}$ [mag]', fontsize=20)
    plt.ylabel('$G$ [mag]', fontsize=20)
    plt.legend()

#plt.savefig("ridge_NGC6379.png", dpi=300)
    plt.show()
```

