International Investing
Diversification and Beyond

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Motivation

- Is international investing useful? In what sense?
- Conventional wisdom suggests "good for diversification"
- Recent studies question this (Bae, Elkamhi, and Simutin (2019)), due to **increased integration**
- Often studies (e.g. Griffin (2002); Fama and French (2017)) *assume* a version of Fama and French (1993) for each country
- Aim for a more agnostic approach to factor structure
- **Develop latent factor models in a cross-country setting**

Literature

- International Diversification
  - Grubel (1968); Solnik (1974); Quinn and Voth (2008); Bae et al. (2019) and many more
- Integration (broadly defined)
  - Cho et al. (1986); Harvey (1991); De Santis and Gerard (1997); Bekaert and Harvey (1995); Karolyi and Stulz (2003); Carrieri et al. (2007); Pukthuanthong and Roll (2009); Bekaert et al. (2011); Chaieb et al. (2021) and many more
- Factor models
  - Chamberlain and Rothschild (1983); Connor and Korajczyk (1986, 1988); Connor et al. (2012); Fan et al. (2016); Kelly et al. (2019); Andreou et al. (2019); Kim et al. (2021), Zaffaroni (2025), Ferson et al. (2025) and many more

Simple Example I

- Excess returns follow a factor model
- Investor is allocated to country 1 – "home country" (USA)

$$R_{1it} = \underbrace{\beta_{1i}(\lambda_1^c + f_t)}_{\text{common to the pair}} + \underbrace{\delta_{1i}(\lambda_1^s + g_{1t})}_{\text{country specific}} + e_{1it}$$

- Similarly for country 2

$$R_{2it} = \underbrace{\beta_{2i}(\lambda_2^c + f_t)}_{\text{common to the pair}} + \underbrace{\delta_{2i}(\lambda_2^s + g_{2t})}_{\text{country specific}} + e_{2it}$$

Simple Example II

- **How should an investor allocate toward country 2 (if possible)?**
- Exposure to foreign country risk g_{2t}?
  - Expand investment opportunity set
- Possibly exploit different prices of risk for f_t – λ_1^c v.s. λ_2^c?
  - Segmentation premia
- Throughout assume large economies, i.e. $N_1, N_2 \to \infty$
- **How can we achieve this?**
- Single common factor, single country specific factor
- Factors loadings are known
- Factor loadings are cross-sectionally orthogonal ($\beta'\delta = 0$)

Simple Example III

- Consider $w_i^s = \frac{\delta_{2i}}{N}$, then (up to normalization)

$$\sum_{i=1}^{N} w_i^s R_{2it} = \sum_{i=1}^{N} \frac{\delta_{2i}\beta_{2i}}{N} \left(\lambda_2^c + f_t\right) + \sum_{i=1}^{N} \frac{\delta_{2i}^2}{N} \left(\lambda_2^s + g_{2t}\right) + \sum_{i=1}^{N} \frac{\delta_{2i} e_{2it}}{N} \to^P \left(\lambda_2^s + g_{2t}\right).$$

- Interesting if $\lambda_2^s \neq 0$ and g_t not spanned by f_t
- Consider $w_i^c = \frac{\beta_{2i}}{N}$, then (up to normalization)

$$\sum_{i=1}^{N} w_i^c R_{2it} = \sum_{i=1}^{N} \frac{\beta_{2i}^2}{N} \left(\lambda_2^c + f_t\right) + \sum_{i=1}^{N} \frac{\beta_{2i}\delta_{2i}}{N} \left(\lambda_2^s + g_{2t}\right) + \sum_{i=1}^{N} \frac{\beta_{2i} e_{2it}}{N} \to^P \left(\lambda_2^c + f_t\right)$$

- Segmentation premia if $\lambda_1 \neq \lambda_2$ (segmentation portfolio)

Challenges

- Factors not observed – f_t and g_t not given
- Factor loadings not observed – β and δ not given
- Generally, $\beta'\delta \neq 0$ even if we assume f and g known
  - Local Fama and French (1993) factors and their global counterparts strongly correlated
- **Can we still implement these portfolios?**
- **What are their empirical properties?**

This Paper

- Cross-country *latent factor model*
- Develop feasible estimation for
  - Common factors f_t
  - Country specific factors g_t
  - Common factor loadings β_1 and β_2
  - Country specific loadings δ_2
- Large N, but fixed T
- Implement portfolios empirically

Preview of Results

- **Country-specific portfolios** (exposure to foreign-specific risk)
  - Sharpe ratios from 0.49 (Japan) to 1.16 (Germany) among G7
  - Nearly **quadruple** the US market Sharpe ratio when combined
  - Virtually uncorrelated with US market
- **Segmentation portfolios** (exploiting risk premia differences)
  - Sharpe ratios 0.67–1.32 even among G7 countries
  - Challenges view of integrated developed markets
- **Naive diversification** (country indices) yields SR of only 0.46–0.54
- **Combined approach**: SR up to 2.23

Setup: Returns within a Country I

- Consider a pair of two countries, indexed by $g = 1, 2$
- Returns follow a $K_g \left(= K^c + K_g^s\right)$-factor model

$$\mathbf{R}_g = \underbrace{\mathbf{B}_g \left(\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}'\right)}_{\text{Common factors}} + \underbrace{\mathbf{D}_g \left(\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g'\right)}_{\text{Country-Specific factors}} + \mathbf{E}_g$$

- Can we separately identify common and country specific factors?
- Allow for segmentation and country specific risks
- Show how to isolate
  - Possible differences in $\boldsymbol{\lambda}_g^c$
  - Distinct premium for each country $\boldsymbol{\lambda}_g^s$

Setup: Returns within a Country II

- Consider a pair of two countries, indexed by $g = 1, 2$
- Returns follow a $K_g \left(= K^c + K_g^s\right)$-factor model

$$\mathbf{R}_g = \underbrace{\mathbf{B}_g \left(\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}'\right)}_{\text{Common factors}} + \underbrace{\mathbf{D}_g \left(\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g'\right)}_{\text{Country-Specific factors}} + \mathbf{E}_g$$

- Premia, factors, and factor loadings **not observed**
- If we knew loadings or could estimate them all problems solved
- Intuition as in simple example (with more complicated notation)

Characteristic Based Factor Model

Characteristics map to factor loadings $[\mathbf{B}_g \ \mathbf{D}_g]$:

$$\underbrace{\mathbf{B}_g = \mathbf{X}_g \boldsymbol{\Theta}_g^c + \Gamma_g^c}_{\text{common factor loadings}} \qquad \underbrace{\mathbf{D}_g = \mathbf{X}_g \boldsymbol{\Theta}_g^s + \Gamma_g^s}_{\text{country specific loadings}}$$

- Use $\mathbf{X}_g \boldsymbol{\Theta}_g^{c,s}$ instead of $\mathbf{B}_g$ and $\mathbf{D}_g$
- Learn loadings from cross-section rather than time series
- Γ_g^c and Γ_g^s sources of beta not attributable to characteristics
- Mapping can be different across countries and over time
- Common factors are orthogonal to country-specific factors (identification restriction)

Estimation Roadmap

Recall Objectives

- **Isolate Country Specific Risk**
  - Learn $\mathbf{\Theta}_g^s$ for each country to obtain $\mathbf{D}_g^s$
- **Segmentation Portfolio**
  - Learn $\mathbf{\Theta}_g^c$ for each country to obtain $\mathbf{B}_g$
- Rather than pursue separate estimation, joint estimation easier

Estimation Steps

1. Estimate common and country specific factors ($\mathbf{F}$ and $\mathbf{G}$)
2. Estimate common and country specific factor loadings ($\mathbf{\Theta}_g^c$ and $\mathbf{\Theta}_g^s$)
3. Form portfolios
   - Segmentation portfolio ($\mathbf{r}^{\Delta\lambda} = \mathbf{w}_1'\mathbf{R}_1 + \mathbf{w}_2'\mathbf{R}_2$)
   - Country specific portfolio ($\mathbf{r}^{cs} = \mathbf{w}_2'\mathbf{R}_2$)

Step 1 – Estimate All Factors

- De-mean returns ($\mathbf{J}_T = \mathbf{I}_T - \frac{1}{T}\mathbf{1}_T\mathbf{1}_T'$)
- Information in characteristics (Fan et al. (2016); Kim et al. (2021)) ($\mathbf{P}_g = \mathbf{X}_g\left(\mathbf{X}_g'\mathbf{X}_g\right)^{-1}\mathbf{X}_g'$)

$$
\begin{aligned}
\widehat{\mathbf{R}}_g \equiv & \mathbf{P}_g\mathbf{R}_g\mathbf{J}_T \\
= & \left(\mathbf{P}_g\mathbf{X}_g\Theta_g^c + \mathbf{P}_g\Gamma_g^c\right)\mathbf{F}'\mathbf{J}_T + \left(\mathbf{P}_g\mathbf{X}_g\Theta_g^s + \mathbf{P}_g\Gamma_g^s\right)\mathbf{G}_g'\mathbf{J}_T + \mathbf{P}_g\mathbf{E}_g\mathbf{J}_T \\
\approx & \mathbf{X}_g\Theta_g^c\mathbf{F}'\mathbf{J}_T + \mathbf{X}_g\Theta_g^s\mathbf{G}_g'\mathbf{J}_T
\end{aligned}
$$

Consistency for $\left[\mathbf{F}'\mathbf{J}_T\ \mathbf{G}_g'\mathbf{J}_T\right]$

- Apply PCA to $\frac{\widehat{\mathbf{R}}_g'\widehat{\mathbf{R}}_g}{N_g}$
- eigenvectors $\rightarrow^P \mathcal{O}_g\left[\mathbf{F}'\mathbf{J}_T\ \mathbf{G}_g'\mathbf{J}_T\right]$ for a $(K_g \times K_g)$ matrix $\mathcal{O}_g$
- **F** and **G** are **not separately identified here**

Step 2: Separate $\mathbf{F}'\mathbf{J}_T$ and $\mathbf{G}_g'\mathbf{J}_T$ I

- Consistent estimators *for each* country

$$\text{eigenvectors of } \frac{\widehat{\mathbf{R}}_1'\widehat{\mathbf{R}}_1}{N_1} = \left[\widehat{\mathbf{FG}}\right]_1' \to^P \mathcal{O}_1 \left[\mathbf{F}'\mathbf{J}_T \; \mathbf{G}_1'\mathbf{J}_T\right]$$

and

$$\text{eigenvectors of } \frac{\widehat{\mathbf{R}}_2'\widehat{\mathbf{R}}_2}{N_2} = \left[\widehat{\mathbf{FG}}\right]_2' \to^P \mathcal{O}_2 \left[\mathbf{F}'\mathbf{J}_T \; \mathbf{G}_2'\mathbf{J}_T\right]$$

- Note that $\mathbf{F}'\mathbf{J}_T$ are common!
- Exploit commonality

$\Rightarrow$ **Canonical correlation analysis**

Step 2: Separate $\mathbf{F}'\mathbf{J}_T$ and $\mathbf{G}_g'\mathbf{J}_T$ II

- Under commonality there exists $\mathbf{w}_1$ and $\mathbf{w}_2$ such that

$$\rho\left([\widehat{\mathbf{FG}}]_1\mathbf{w}_1, [\widehat{\mathbf{FG}}]_2\mathbf{w}_2\right) \to^P 1$$

- This correlation is known as canonical correlation
- Find $\mathbf{W}_1$ $(K_1 \times K_1)$ and $\mathbf{W}_2$ $(K_2 \times K_2)$ such that

$$\max_{\mathbf{W}_1,\mathbf{W}_2} \operatorname{tr}\left(\operatorname{diag}\left(\mathbf{W}_1'\left(\frac{[\widehat{\mathbf{FG}}]_1'[\widehat{\mathbf{FG}}]_2}{T}\right)\mathbf{W}_2\right)\right) \quad \text{subject to}$$

$$\mathbf{W}_1'\left(\frac{\widehat{\mathbf{FG}}_1'\widehat{\mathbf{FG}}_1}{T}\right)\mathbf{W}_1 = \mathbf{I} \quad \text{and} \quad \mathbf{W}_2'\left(\frac{\widehat{\mathbf{FG}}_2'\widehat{\mathbf{FG}}_2}{T}\right)\mathbf{W}_2 = \mathbf{I}$$

Step 2: Separate $\mathbf{F}'\mathbf{J}_T$ and $\mathbf{G}_g'\mathbf{J}_T$ III

- From canonical correlations, first K_c elements for $\mathbf{F}$ rest for $\mathbf{G}$

$$\mathbf{W}_g'[\widehat{\mathbf{FG}}]_g' = \left[\mathbf{W}_g^c\ \mathbf{W}_g^s\right]'[\widehat{\mathbf{FG}}]_g' = \left[\widehat{\mathbf{F}}_g'\ \widehat{\mathbf{G}}_g'\right]$$

- Separately identified $\mathbf{F}$ and $\mathbf{G}$
- **Consistency for $\mathbf{F}'\mathbf{J}_T$ and $\mathbf{G}_g'\mathbf{J}_T$**

$$\left[\widehat{\mathbf{F}}_g'\ \widehat{\mathbf{G}}_g'\right] \to^P \left[\mathcal{S}^{c\prime}\mathbf{F}'\mathbf{J}_T\ \mathcal{S}_g^{s\prime}\mathbf{G}_g'\mathbf{J}_T\right]$$

where

$$\mathcal{S}^c\mathcal{S}^{c\prime} = \Sigma_F^{-1} \quad \text{and} \quad \mathcal{S}_g^s\mathcal{S}_g^{s\prime} = \Sigma_{G_g}^{-1}$$

- Factors are normalized to have unit variance

Step 3: Estimate Loadings through Θ_g^c and Θ_g^s I

De-meaned & projected returns

$$\begin{aligned}\widehat{\mathbf{R}}_g &\equiv \mathbf{P}_g \mathbf{R}_g \mathbf{J}_T \\ &\approx \mathbf{X}_g \Theta_g^c \mathbf{F}' \mathbf{J}_T + \mathbf{X}_g \Theta_g^s \mathbf{G}_g' \mathbf{J}_T\end{aligned}$$

- Use consistent factor estimates from steps 1 & 2
  - $\widehat{\mathbf{F}}_g' \to^P \mathcal{S}^{c\prime} \mathbf{F}' \mathbf{J}_T$
  - $\widehat{\mathbf{G}}_g' \to^P \mathcal{S}_g^{s\prime} \mathbf{G}' \mathbf{J}_T$
- Now can estimate *factor loading functions*
  - $\mathbf{\Theta}_g^c$ for common factor loadings
  - $\mathbf{\Theta}_g^s$ for country specific factor loadings

Step 3: Estimate Loadings through Θ_g^c and Θ_g^s II

Solve quadratic program

$$\left(\widehat{\Theta}_g^c, \widehat{\Theta}_g^s\right) = \arg \min_{\left(\Theta_g^c, \Theta_g^s\right)} \left\| \widehat{\mathbf{R}}_g - \left(\mathbf{X}_g \Theta_g^c\right) \widehat{\mathbf{F}}'_g - \left(\mathbf{X}_g \Theta_g^s\right) \widehat{\mathbf{G}}'_g \right\|$$

- $\widehat{\Theta}_g^c \to^P \Theta_g^c \left(\mathcal{S}^{c\prime}\right)^{-1}$ where $\mathcal{S}^c \mathcal{S}^{c\prime} = \Sigma_F^{-1}$
- $\widehat{\Theta}_g^s \to^P \Theta_g^s \left(\mathcal{S}_g^{s\prime}\right)^{-1}$ where $\mathcal{S}_g^s \mathcal{S}_g^{s\prime} = \Sigma_{G_g}^{-1}$
- Solution: Closed Form
- Consistent estimators for
    - Factors: $\mathbf{F}$, $\mathbf{G}$
    - Factor loadings: $\mathbf{\Theta}_g^c$ and $\mathbf{\Theta}_g^s$

$\Rightarrow$ **Build portfolios**

Step 4a: Country-Specific Portfolio I

- Original motivation of international investing
- Estimated loadings

$$\widehat{\mathbf{B}}_g := \mathbf{X}_g \widehat{\boldsymbol{\Theta}}_g^c \quad \text{and} \quad \widehat{\mathbf{D}}_g := \mathbf{X}_g \widehat{\boldsymbol{\Theta}}_g^s$$

- Consider the portfolio weight

$$\underbrace{\widehat{\mathbf{w}}_g^b = \left[\widehat{\mathbf{B}}\ \widehat{\mathbf{D}}\right] \left(\left[\widehat{\mathbf{B}}\ \widehat{\mathbf{D}}\right]' \left[\widehat{\mathbf{B}}\ \widehat{\mathbf{D}}\right] \right)^{-1}}_{\text{somewhat like } (\widehat{\beta}'\widehat{\beta})^{-1}\widehat{\beta}' \text{ in FMB}}$$

- Portfolio weights capture
  - Country specific exposures only
  - Differences in risk premia across countries

Step 4a: Country-Specific Portfolio II

- Partition the weights into *common* and *country specific*

$$\widehat{\mathbf{w}}_g^{b\prime}\mathbf{R}_g = \begin{bmatrix} \widehat{\mathbf{w}}_g^{c\prime}\mathbf{R}_g \\ \widehat{\mathbf{w}}_g^{s\prime}\mathbf{R}_g \end{bmatrix} \rightarrow^P \begin{bmatrix} \mathcal{S}^{c\prime}\left(\lambda_g^c \mathbf{1}_T' + \mathbf{F}'\right) \\ \mathcal{S}_g^{s\prime}\left(\lambda_g^s \mathbf{1}_T' + \mathbf{G}_g'\right) \end{bmatrix}$$

- Exposure to **country 2 risk only**

$$\widehat{\mathbf{w}}_2^{s\prime}\mathbf{R}_2 \rightarrow^P \mathcal{S}_2^{s\prime}\left(\lambda_2^s \mathbf{1}_T' + \mathbf{G}_2'\right)$$

- Can be extended to multiple sources of country 2 risk
- This portfolio captures the core of international diversification
- Out-of-sample implementation

$$r_{T+1}^{cs} = \widehat{\mathbf{w}}_{2,T}'\mathbf{R}_{2,T+1}$$

- Multivariate Construction

Step 4b: Segmentation Portfolio I

- Explore $\boldsymbol{\lambda}_1^c \neq \boldsymbol{\lambda}_2^c$
- Could run Fama and MacBeth (1973) for each g

$$\overline{\mathbf{R}} = \widehat{\mathbf{B}}\boldsymbol{\lambda}_g^c + \widehat{\mathbf{D}}\boldsymbol{\lambda}_g^s + \mathbf{U}$$

- Estimate prices of risk $\widehat{\boldsymbol{\lambda}}_g^c$ (and $\widehat{\boldsymbol{\lambda}}_g^s$)
- Consistency only for $T \to \infty$
- T potentially small
- Project returns onto $\mathbf{X}\widehat{\Theta}_g^{c,s}$ each period
- Recall

$$\widehat{\mathbf{w}}_g^{b\prime}\mathbf{R}_g = \begin{bmatrix} \widehat{\mathbf{w}}_g^{c\prime}\mathbf{R}_g \\ \widehat{\mathbf{w}}_g^{s\prime}\mathbf{R}_g \end{bmatrix} \to^P \begin{bmatrix} \mathcal{S}^{c\prime}\left(\lambda_g^c \mathbf{1}_T' + \mathbf{F}'\right) \\ \mathcal{S}_g^{s\prime}\left(\lambda_g^s \mathbf{1}_T' + \mathbf{G}_g'\right) \end{bmatrix}$$

- Study $\widehat{\mathbf{w}}_1^c$ and $\widehat{\mathbf{w}}_2^c$ (common risks in *each* country)
- Problem **pollution by factor realization**

Step 4b: Segmentation Portfolio II

- Assume for simplicity that $K_c = 1$ (one common factor)

$$\widehat{\mathbf{w}}_2'\mathbf{r}_{2,t} - \widehat{\mathbf{w}}_1'\mathbf{r}_{1,t} \to^P (\lambda_2 + f_t) - (\lambda_1 + f_t) = \lambda_2 - \lambda_1 + (f_t - f_t)$$

- Run regression

$$\underbrace{\widehat{\mathbf{w}}_2^{c\prime}\mathbf{R}_2}_{1\times T} = \beta_0 + \beta_1 \left(\underbrace{\widehat{\mathbf{w}}_1^{c\prime}\mathbf{R}_1}_{1\times T} \right) + \mathbf{e}$$

- Intercept captures difference in risk premia, i.e.

$$\widehat{\beta}_0 \to^P (\lambda_2^c - \lambda_1^c) \quad \text{and} \quad \widehat{\beta}_1 \to^P 1$$

- $\widehat{\beta}_0 > 0$ implement (out-of-sample)

$$\mathbf{r}_{T+1}^{\Delta\lambda} = \underbrace{\widehat{\mathbf{w}}_{2,T}^{c\prime}\mathbf{r}_{2,T+1}}_{\text{common factor in country 2}} - \underbrace{\widehat{\beta}_1 \widehat{\mathbf{w}}_{1,T}^{c\prime}\mathbf{r}_{1,T+1}}_{\text{common factor in country 1}}$$

- ▸ Multivariate Construction

Simulation

- We validate our method, enabling investors in Country 1 to:
  - Diversify into Country 2-specific factors
  - Capitalize on differences in risk premia
- Simulation investigates: Impact of
  - Total number of estimated factors
  - Choice between common and country-specific factors

Calibration

- Return-generating process calibrated using:
  - USA (Country 1) and Canada (Country 2) returns
- Baseline specification:
  - Six systematic factors per country
  - Three common factors across both countries
  - Three country-specific factors
- Using the most recent 60-month window, we estimate:
  - Betas
  - Residual variance
  - Common and country-specific factors

Canonical Correlations for Different Numbers of Estimated Factors

Canonical Correlations	1	2	3	4	5	6	7
$K = 5$, Underestimation	0.99	0.94	0.77	0.23	0.07	-	-
$K = 6$, Correct	0.99	0.96	0.94	0.33	0.18	0.06	-
$K = 7$, Overestimation	0.99	0.97	0.94	0.41	0.27	0.15	0.05

- Average canonical correlations over 10,000 repetitions.
- Overestimation is safer than underestimation.

Sharpe Ratios of Segmentation and Country-Specific Portfolios

K_c (Common Factors)	1	2	3	4	5
K_s (Country-Specific Factors)	5	4	3	2	1
Panel A: Neither					
Segmentation Portfolio SR	-0.01	-0.02	-0.04	-0.05	-0.05
Country-Specific Portfolio SR	0.01	0.01	0.01	0.00	-0.01
Panel B: Country-Specific Premia Only					
Segmentation Portfolio SR	0.01	0.05	0.04	0.29	0.46
Country-Specific Portfolio SR	0.87	0.87	0.87	0.68	0.44
Panel C: Segmentation in Common Factor Premia Only					
Segmentation Portfolio SR	0.55	0.69	0.79	0.78	0.78
Country-Specific Portfolio SR	0.23	0.14	0.01	0.01	-0.02
Panel D: Both					
Segmentation Portfolio SR	0.71	0.86	0.96	1.06	1.13
Country-Specific Portfolio SR	0.92	0.90	0.88	0.68	0.44

- True K_c and K_s: reveals the state of the international market

Sharpe Ratios of Segmentation and Country-Specific Portfolios

K_c (Common Factors)	1	2	3	4	5
K_s (Country-Specific Factors)	5	4	3	2	1
Panel B: Country-Specific Premia Only					
Segmentation Portfolio SR	0.01	0.05	0.04	0.29	0.46
Country-Specific Portfolio SR	0.87	0.87	0.87	0.68	0.44
Panel C: Segmentation in Common Factor Premia Only					
Segmentation Portfolio SR	0.55	0.69	0.79	0.78	0.78
Country-Specific Portfolio SR	0.23	0.14	0.01	0.01	-0.02

- Overestimating K_c:
  Country-specific premia → Segmentation premia (Panel B)
- Overestimating K_s:
  Segmentation premia → Country-specific premia (Panel C)

Data

- Build on Jensen et al. (2023)
- Consider 27 international markets (usa, twn, tur, tha, swe, sgp, pol, nor, mex, kor, jpn, ita, ind, hkg, gbr, fra, esp, dnk, deu, can, bra, aut, chn, chl, bel, aus, arg)
- Drop smallest 20% of the firms in local currency for each country
- Impute missing characteristics Freyberger, Höppner, Neuhierl, and Weber (2024)
- Contrary to many other studies **we do not winsorize returns**
- Excess returns from US investor perspective, i.e. returns in USD

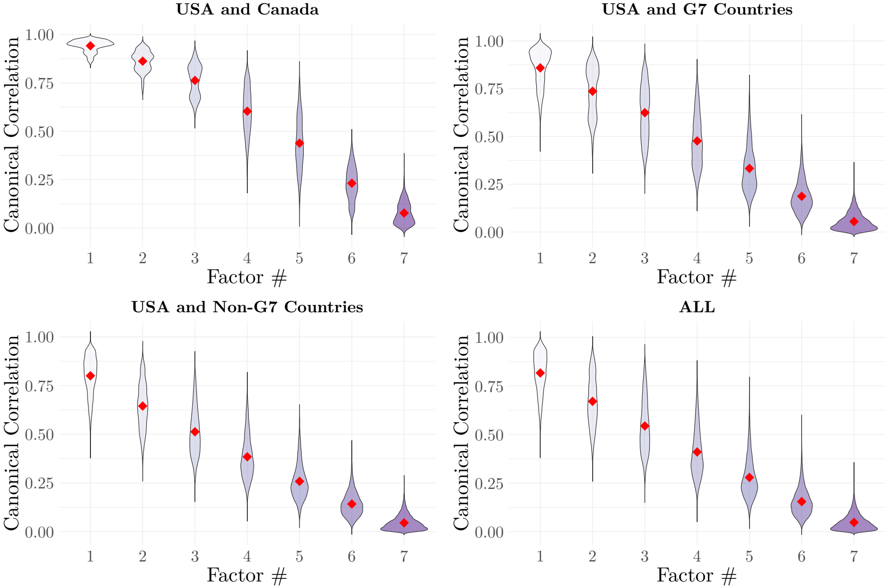
Country Overview

Country			# Firms	Market Cap. q_{25}	Market Cap. Avg.	Market Cap. q_{75}	Cross Section $N_{\min}$	Cross Section $\bar{N}$	Cross Section $N_{\max}$	Excess Return $\bar{R}$	Excess Return $\sigma(\bar{R})$
arg	Jul-2005	Dec-2022	95	21.55	907.08	479.11	46	54.88	69	0.18	0.38
aus	Feb-1986	Dec-2022	3721	8.93	650.58	125.35	117	842.87	1583	0.08	0.27
aut	Nov-1999	Dec-2022	153	44.49	1226.49	1226.17	44	61.02	79	0.08	0.20
bel	Jan-1991	Dec-2022	286	35.52	1590.81	585.27	44	103.57	133	0.07	0.16
bra	Dec-2006	Dec-2022	336	323.76	2419.44	2223.03	61	143.95	216	0.09	0.35
can	Jan-1983	Dec-2022	3191	4.02	505.00	81.45	179	751.27	1392	0.11	0.23
chl	Feb-1997	Dec-2022	229	153.78	1623.10	1414.77	52	111.80	144	0.31	1.08
chn	Jan-1995	Dec-2022	4883	335.38	1255.28	1074.47	40	1430.51	3793	0.12	0.32
deu	Jan-1987	Dec-2022	1774	6.96	1220.62	195.40	75	502.61	830	0.03	0.18
dnk	Jan-1994	Dec-2022	379	19.62	813.37	221.08	65	126.32	171	0.08	0.18
esp	May-1988	Dec-2022	417	175.73	5091.85	2978.30	36	115.99	180	0.05	0.21
fra	Jan-1987	Dec-2022	1814	19.91	2386.31	453.87	97	480.59	687	0.07	0.19
gbr	Feb-1986	Dec-2022	5548	12.77	1382.21	254.38	258	1225.09	1819	0.05	0.20
hkg	Apr-1987	Dec-2022	2872	54.34	1504.17	608.74	38	814.20	1963	0.10	0.29
ind	May-1989	Dec-2022	5096	11.94	658.76	194.24	41	1298.94	3299	0.15	0.35
ita	Jan-1987	Dec-2022	781	72.85	2296.99	1092.67	43	192.64	310	0.02	0.23
jpn	Jan-1987	Dec-2022	5648	30.43	945.28	369.38	1107	2537.70	3242	0.05	0.22
kor	Mar-1989	Dec-2022	3309	22.52	442.11	121.12	91	1012.31	2027	0.05	0.34
mex	Mar-2001	Dec-2022	195	207.32	3095.61	2506.78	56	82.48	105	0.12	0.21
nor	Jun-1993	Dec-2022	664	39.91	1100.65	563.90	54	151.54	269	0.10	0.26
pol	Mar-1996	Dec-2022	1176	11.26	349.57	126.11	51	345.52	684	0.08	0.29
sgp	Feb-1989	Dec-2022	1143	21.55	620.93	187.04	42	370.89	660	0.07	0.28
swe	Jan-1992	Dec-2022	1336	9.41	956.02	265.10	52	300.46	653	0.10	0.24
tha	Jan-1989	Dec-2022	1121	14.93	327.42	139.11	88	376.24	713	0.12	0.31
tur	Dec-1993	Dec-2022	606	30.43	834.56	436.42	50	232.79	419	0.23	0.47
twn	Jan-1989	Dec-2022	2549	35.50	461.45	248.47	46	846.08	1691	0.08	0.32
usa	Jan-1966	Dec-2022	25603	62.35	3010.13	1269.86	1579	4008.30	6448	0.07	0.20

Baseline Implementation

- All results are out-of-sample
- Rolling estimation window of 60 months
- $K = 7$, i.e. seven factors in total
- Use the test by Gonçalves et al. (2025) to test for the number of common factors
- Normalize all zero-investment portfolios' in-sample standard deviation to 20% annualized
- Rank normalize all characteristics to $[0, 1]$ each period

Canonical Correlations



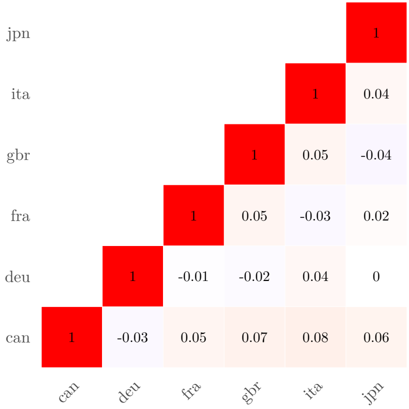
Country-Specific Portfolio: G7

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
can	14.53	20.48	3.99	0.71	50.59	-20.06	25.41	0.06
deu	20.53	17.76	6.08	1.16	44.21	-16.69	17.58	-0.02
fra	12.85	18.77	3.60	0.68	50.01	-14.98	25.88	-0.03
gbr	17.17	20.81	4.16	0.82	40.66	-19.28	19.52	-0.03
ita	16.76	20.40	4.33	0.82	47.34	-18.90	18.97	-0.02
jpn	9.48	19.43	2.76	0.49	41.83	-18.98	17.34	0.03

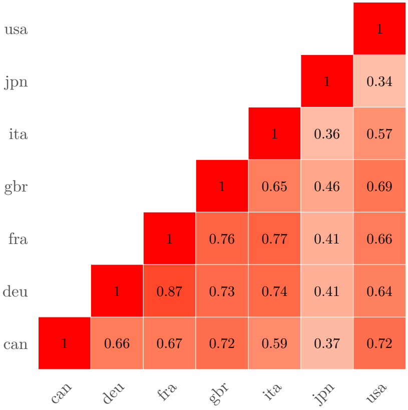
- Strong local factors among G7 factors
- Virtually uncorrelated with US market
- Much lower correlation than market correlation

Correlation: G7 Country-Specific Portfolios

Country Specific Factor Correlation



Correlation of Country Market Portfolios

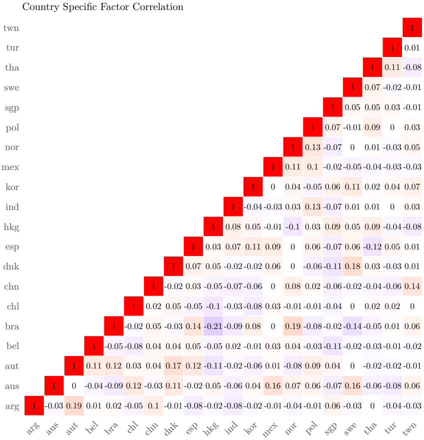


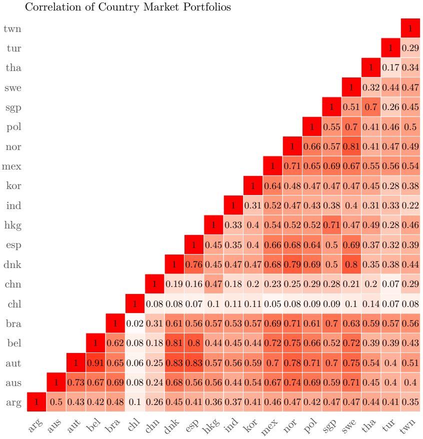
Country-Specific Portfolio: Non-G7 ($K_s = 3$)

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
arg	20.14	19.07	3.54	1.06	22.96	-10.47	23.99	0.02
aus	16.66	20.96	4.57	0.79	29.42	-16.23	40.68	0.05
aut	19.54	18.93	3.81	1.03	36.14	-13.09	18.61	-0.14
bel	27.49	20.93	7.23	1.31	33.22	-14.78	24.83	-0.08
bra	10.68	21.33	1.63	0.50	32.95	-17.79	18.59	0.10
chl	19.78	21.75	4.65	0.91	50.88	-14.36	27.19	-0.03
chn	17.14	21.38	3.26	0.80	47.28	-19.60	22.59	-0.04
dnk	26.71	22.06	5.87	1.21	29.91	-18.43	21.02	-0.02
esp	18.79	20.11	5.39	0.93	34.48	-16.84	24.38	0.05
hkg	21.87	20.40	6.94	1.07	32.45	-23.01	26.09	0.10
ind	18.01	20.02	4.06	0.90	40.30	-16.09	21.62	-0.07
kor	11.39	19.68	2.62	0.58	50.19	-15.89	25.05	0.02
mex	22.92	21.03	4.19	1.09	39.66	-21.93	20.86	0.03
nor	18.14	22.34	3.88	0.81	60.58	-22.76	25.27	-0.01
pol	19.54	21.67	3.64	0.90	37.27	-22.24	23.41	-0.07
sgp	24.22	20.84	5.40	1.16	34.21	-13.83	29.82	0.09
swe	20.18	22.10	4.56	0.91	40.78	-28.07	37.55	-0.03
tha	12.47	19.23	3.62	0.65	44.87	-25.35	25.65	-0.02
tur	22.67	20.81	4.71	1.09	32.10	-18.28	29.24	-0.03
twn	12.10	19.32	3.14	0.63	49.63	-12.60	17.14	0.02

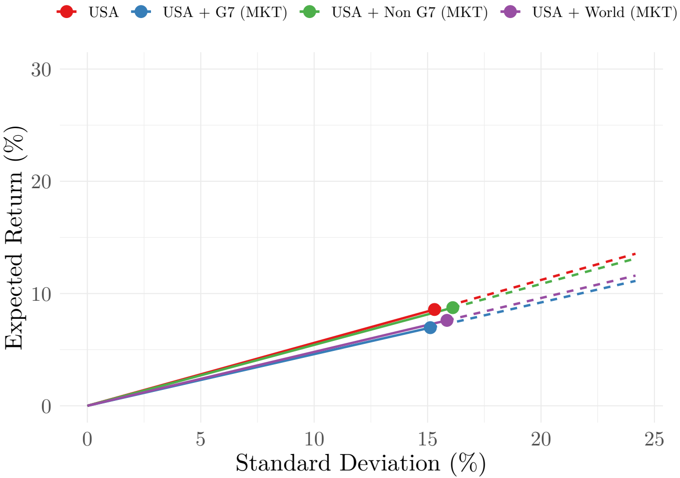
- Very low correlation with US market

Correlation: Non-G7 Country-Specific Portfolios



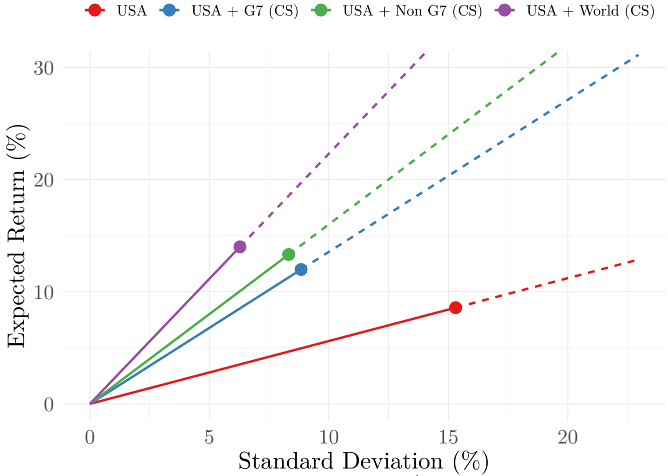


Revisiting Diversification Benefits I



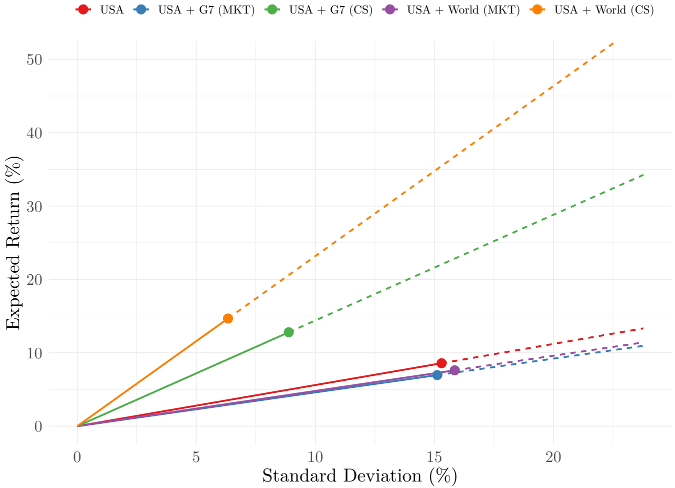
- USA + EW G7 / USA + EW Non G7 (1/2) each
- USA + World (1/3 each)

Revisiting Diversification Benefits II



- USA + CS G7 / USA + CS Non G7 (1/2) each
- USA + World (1/3 each)

Revisiting Diversification Benefits III



- More effective diversification through country specific factors

Diversification Benefits

Portfolio	Mean (%)	Standard Deviation (%)	Sharpe Ratio
USA	8.58	15.30	0.56
G7 (MKT)	5.34	16.31	0.33
Non G7 (MKT)	8.91	18.92	0.47
USA + G7 (MKT)	6.96	15.12	0.46
USA + Non G7 (MKT)	8.74	16.11	0.54
USA + World (MKT)	7.61	15.85	0.48
USA + G7 (CS)	11.98	8.84	1.36
USA + Non G7 (CS)	13.32	8.32	1.60
USA + World (CS)	14.01	6.28	2.23

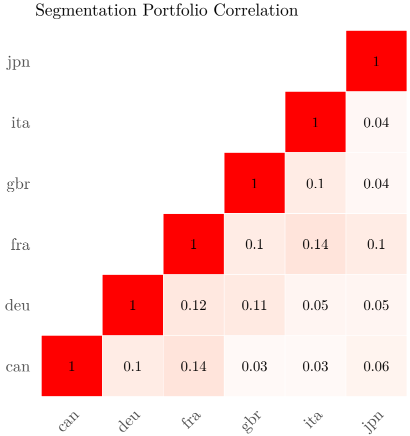
- Portfolios
  - 1/2 US + 1/2 (Non) G7 for either index or CS
  - 1/3 US + 1/3 G7 + 1/3 Non G7 for either index or CS
- Only moderate benefits from international index investing
- Strong diversification benefits from isolating country specific risk

Segmentation Portfolio: G7

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
can	15.75	23.43	4.17	0.67	43.44	-31.34	25.18	-0.02
deu	30.25	22.90	7.85	1.32	22.48	-16.02	22.23	-0.01
fra	23.16	23.04	5.13	1.01	42.92	-23.47	29.87	0.03
gbr	21.60	25.96	4.77	0.83	61.98	-60.43	19.95	-0.04
ita	23.94	22.63	5.42	1.06	49.48	-17.57	23.35	-0.08
jpn	26.25	22.45	5.48	1.17	54.24	-20.03	27.05	0.03

- Surprisingly large segmentation
- **Not an arbitrage**, maybe a good deal
- Structural breaks such as tariffs, capital flow suspensions, or martial law can occur even among developed countries
- "Placebo test" of NASDAQ vs. NYSE shows no evidence of segmentation
- Normally need large T to estimate $\widehat{\lambda} = \frac{1}{T}\sum_{t=1}^{T} f_t$, but here we estimate $\widehat{\lambda}$ from two sources and cancel f_t

Correlation: G7 Segmentation Portfolios



- Correlation of country specific and segmentation portfolio ≈ 0.21

Segmentation Portfolio: Non-G7 ($K_c = 3$)

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
arg	16.19	21.20	2.46	0.76	42.13	-15.57	34.76	0.05
aus	19.60	25.31	3.92	0.77	51.90	-29.09	74.09	0.02
aut	10.68	21.08	1.76	0.51	50.42	-19.86	21.48	0.11
bel	17.29	22.16	3.85	0.78	68.21	-16.21	25.67	0.08
bra	10.73	20.69	1.59	0.52	37.15	-11.12	16.66	0.15
chl	18.07	20.12	3.81	0.90	28.83	-18.69	25.73	-0.10
chn	25.03	23.55	4.51	1.06	36.17	-18.44	29.84	0.01
dnk	28.28	23.87	5.91	1.18	28.84	-21.08	26.53	0.07
esp	23.27	21.30	5.48	1.09	38.77	-19.41	19.22	0.02
hkg	16.12	21.33	4.24	0.76	38.91	-25.03	33.22	0.00
ind	19.04	24.94	3.44	0.76	47.56	-20.75	27.88	-0.02
kor	21.75	22.16	4.92	0.98	54.72	-19.10	32.48	0.02
mex	15.47	21.29	2.92	0.73	40.42	-13.81	17.29	0.03
nor	19.74	21.65	4.47	0.91	45.88	-18.09	22.47	-0.04
pol	21.24	24.48	3.49	0.87	32.26	-25.90	59.76	0.03
sgp	33.13	21.32	6.93	1.55	27.67	-18.46	29.27	0.03
swe	22.91	22.60	5.22	1.01	27.44	-22.34	21.99	-0.02
tha	21.81	24.41	4.07	0.89	48.48	-35.35	30.99	-0.03
tur	11.52	21.26	2.74	0.54	50.19	-14.29	25.45	0.02
twn	23.47	20.44	6.21	1.15	28.68	-18.87	23.68	-0.02

- Large differences in common factor risk

Correlation: Non-G7 Segmentation Portfolios

Segmentation Portfolio Correlation

	arg	aus	aut	bel	bra	chl	chn	dnk	esp	hkg	ind	kor	mex	nor	pol	sgp	swe	tha	tur	twn
twn																				1
tur																			1	0.1
tha																		1	-0.02	0.02
swe																	1	0.09	0.01	0.06
sgp																1	-0.04	-0.06	0	-0.05
pol															1	-0.01	0.15	0.14	0	0.07
nor														1	0.14	0.06	0.07	-0.02	0.08	-0.06
mex													1	-0.01	0.13	0.12	0.09	0.07	-0.01	0.16
kor												1	0.01	0.04	0.03	-0.02	-0.07	0.08	0.14	-0.02
ind											1	0.05	-0.02	-0.04	-0.08	0.07	0.06	0.08	0.07	0.05
hkg										1	-0.03	-0.13	-0.03	-0.02	0.04	0.06	-0.05	-0.06	0.05	0
esp									1	0.05	0.04	0.04	0.01	0.14	0.08	0.12	0.12	-0.11	0.01	0.01
dnk								1	0.06	-0.06	0.1	-0.03	0.03	0.14	0.08	0	0.12	-0.01	0	-0.08
chn							1	0.04	0	-0.02	-0.04	0.1	-0.14	0.09	-0.07	0.12	-0.1	-0.02	0.09	0.03
chl						1	-0.03	-0.02	0.08	0.08	0.05	0.01	-0.11	0.04	-0.02	0.07	0.1	0.11	0.05	0.04
bra					1	-0.09	0.08	-0.07	-0.06	0.04	0.06	0.08	-0.09	0.18	0	0.02	-0.13	-0.18	0.12	-0.19
bel				1	-0.13	0.12	0.07	0.17	0.09	0.07	0	-0.08	0.13	0.04	0.05	0.12	0.07	-0.02	-0.07	0.05
aut			1	0.25	0.04	0.1	0.02	0.22	0.05	0.03	0.01	-0.04	0.04	0.03	0.13	0.01	0.22	0.22	0.02	-0.08
aus		1	0.07	0.01	-0.06	0.08	-0.02	-0.01	0.08	-0.02	0.08	0.05	0.02	0.07	0.02	0.09	0.04	0	0.05	0.01
arg	1	-0.06	0.03	-0.08	0.22	0.09	-0.06	-0.04	-0.07	0.09	0.07	-0.02	0.02	0.08	0.13	-0.11	-0.05	0.01	-0.09	0.09

Correlation of Country Market Portfolios

	arg	aus	aut	bel	bra	chl	chn	dnk	esp	hkg	ind	kor	mex	nor	pol	sgp	swe	tha	tur	twn
twn																				1
tur																			1	0.29
tha																		1	0.17	0.34
swe																	1	0.32	0.44	0.47
sgp																1	0.51	0.7	0.26	0.45
pol															1	0.55	0.7	0.41	0.46	0.5
nor														1	0.66	0.57	0.81	0.41	0.47	0.49
mex													1	0.71	0.65	0.69	0.67	0.55	0.56	0.54
kor												1	0.64	0.48	0.47	0.47	0.47	0.45	0.28	0.38
ind											1	0.31	0.52	0.47	0.43	0.38	0.4	0.31	0.33	0.22
hkg										1	0.33	0.4	0.54	0.52	0.52	0.71	0.47	0.49	0.28	0.46
esp									1	0.45	0.35	0.4	0.66	0.68	0.64	0.5	0.69	0.37	0.32	0.39
dnk								1	0.76	0.45	0.47	0.47	0.68	0.79	0.69	0.5	0.8	0.35	0.38	0.44
chn							1	0.19	0.16	0.47	0.18	0.2	0.23	0.25	0.29	0.28	0.21	0.2	0.07	0.29
chl						1	0.08	0.08	0.07	0.1	0.11	0.11	0.05	0.08	0.09	0.09	0.1	0.14	0.07	0.08
bra					1	0.02	0.31	0.61	0.56	0.57	0.53	0.57	0.69	0.71	0.61	0.7	0.63	0.59	0.57	0.56
bel				1	0.62	0.08	0.18	0.81	0.8	0.44	0.45	0.44	0.72	0.75	0.66	0.52	0.72	0.39	0.39	0.43
aut			1	0.91	0.65	0.06	0.25	0.83	0.83	0.57	0.56	0.59	0.7	0.78	0.71	0.7	0.75	0.54	0.4	0.51
aus		1	0.73	0.67	0.69	0.08	0.24	0.68	0.56	0.56	0.44	0.54	0.67	0.74	0.69	0.59	0.71	0.45	0.4	0.4
arg	1	0.5	0.43	0.42	0.48	0.1	0.26	0.45	0.41	0.36	0.37	0.41	0.46	0.47	0.42	0.47	0.47	0.44	0.41	0.35

- Correlation of country specific and segmentation portfolio ≈ 0.16

Intuition: Segmentation Portfolio – G7

Panel A: G7 Countries as Country 2

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown (%)	Worst Month (%)	Best Month (%)	ρ_{US}	Pr[US Long]
can	4.09	6.67	3.70	0.61	15.04	-7.41	8.89	-0.09	0.34
deu	12.75	11.54	6.50	1.10	22.50	-13.01	16.16	0.09	0.76
fra	10.16	10.20	5.73	1.00	15.23	-7.81	14.35	0.15	0.78
gbr	6.00	9.39	3.26	0.64	33.85	-8.74	8.95	-0.03	0.69
ita	8.60	11.72	3.57	0.73	34.65	-12.77	15.25	0.22	0.80
jpn	6.75	11.07	3.11	0.61	28.10	-9.31	10.61	0.20	0.67

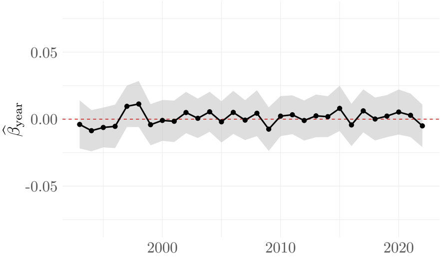
- Implementation for the single factor case
- Tends to be long the US and short the “other country”
- Single factor is likely understating the results, because Gonçalves et al. (2025) rejects a single common factor most periods (for all pairs)

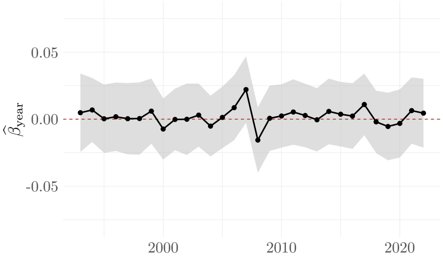
Intuition: Segmentation Portfolio – Non-G7

Panel B: Non-G7 Countries as Country 2

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown (%)	Worst Month (%)	Best Month (%)	ρ_{US}	Pr[US Long]
arg	5.99	13.36	1.52	0.45	36.68	-12.26	12.04	0.04	0.64
aus	6.42	12.20	2.90	0.53	30.84	-13.16	12.74	0.02	0.53
aut	8.28	8.26	4.88	1.00	10.29	-5.16	7.61	0.08	0.67
bel	11.16	11.10	4.95	1.01	24.18	-7.78	13.63	0.13	0.72
bra	5.56	9.15	2.59	0.61	12.38	-7.19	7.21	-0.11	0.54
chl	6.69	11.39	2.26	0.59	28.63	-10.38	11.82	-0.18	0.52
chn	9.89	14.65	3.16	0.67	29.41	-10.25	13.97	0.15	0.65
dnk	5.78	10.34	3.21	0.56	18.73	-7.82	9.65	-0.14	0.53
esp	15.48	11.59	6.77	1.33	17.09	-9.74	14.05	0.16	0.83
hkg	5.63	12.57	2.09	0.45	36.20	-11.48	15.60	0.07	0.62
ind	10.53	12.40	4.13	0.85	24.08	-10.57	13.35	-0.08	0.59
kor	9.30	12.01	4.25	0.77	24.86	-12.75	12.97	0.05	0.77
mex	6.70	8.88	3.37	0.75	17.20	-7.45	6.95	0.05	0.43
nor	6.39	8.35	3.46	0.77	25.26	-11.47	8.90	-0.07	0.36
pol	5.39	9.70	2.79	0.56	22.77	-7.38	10.16	0.03	0.65
sgp	10.61	10.49	4.97	1.01	20.43	-7.72	10.52	0.24	0.76
swe	5.24	10.79	2.48	0.49	34.25	-9.69	10.19	0.05	0.61
tha	9.38	12.40	3.85	0.76	39.03	-12.02	10.66	0.20	0.74
tur	10.28	13.07	3.61	0.79	31.45	-15.93	10.75	-0.09	0.53
twn	7.87	11.65	3.60	0.68	24.52	-9.82	11.84	-0.06	0.51

Further Analysis





- Results are robust to
  - Different base currencies, GBP and JPY
  - Diversification does not break during US downturns
  - Currency risk factors (carry, dollar factor)
  - No extreme weights

Conclusion

- Develop latent cross-country factor model
- Feasible estimation for fixed T and large N
- New angle on cross-country investing with larger benefits
- International version of α-portfolio (▸ details) largely confirms the US findings of Kim, Korajczyk, and Neuhierl (2021)
- Surprisingly large differences in risk prices
- **Thank you for your comments**

References I

Andreou, E., P. Gagliardini, E. Ghysels, and M. Rubin (2019). Inference in group factor models with an application to mixed-frequency data. *Econometrica 87*(4), 1267–1305.

Bae, J. W., R. Elkamhi, and M. Simutin (2019). The best of both worlds: Accessing emerging economies via developed markets. *The Journal of Finance 74*(5), 2579–2617.

Bekaert, G. and C. R. Harvey (1995). Time-varying world market integration. *the Journal of Finance 50*(2), 403–444.

Bekaert, G., C. R. Harvey, C. T. Lundblad, and S. Siegel (2011). What segments equity markets? *The Review of Financial Studies 24*(12), 3841–3890.

Carrieri, F., V. Errunza, and K. Hogan (2007). Characterizing world market integration through time. *Journal of Financial and Quantitative Analysis 42*(4), 915–940.

Chaieb, I., H. Langlois, and O. Scaillet (2021). Factors and risk premia in individual international stock returns. *Journal of Financial Economics 141*(2), 669–692.

Chamberlain, G. and M. Rothschild (1983). Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica*, 1281–1304.

Cho, D. C., C. S. Eun, and L. W. Senbet (1986). International arbitrage pricing theory: An empirical investigation. *The Journal of Finance 41*(2), 313–329.

Connor, G., M. Hagmann, and O. Linton (2012). Efficient semiparametric estimation of the Fama–French model and extensions. *Econometrica 80*(2), 713–754.

Connor, G. and R. A. Korajczyk (1986). Performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of Financial Economics 15*(3), 373–394.

Connor, G. and R. A. Korajczyk (1988). Risk and return in an equilibrium apt: Application of a new test methodology. *Journal of Financial Economics 21*(2), 255–289.

De Santis, G. and B. Gerard (1997). International asset pricing and portfolio diversification with time-varying risk. *The Journal of Finance 52*(5), 1881–1912.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

References II

Fama, E. F. and K. R. French (2017). International tests of a five-factor asset pricing model. *Journal of financial Economics 123*(3), 441–463.

Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–636.

Fan, J., Y. Liao, and W. Wang (2016). Projected principal component analysis in factor models. *Annals of Statistics 44*, 219–254.

Ferson, W. E., A. F. Siegel, and J. L. Wang (2025). Factor model comparisons with conditioning information. *Journal of Financial and Quantitative Analysis 60*(3), 1401–1426.

Freyberger, J., B. Höppner, A. Neuhierl, and M. Weber (2024). Missing data in asset pricing panels. *The Review of Financial Studies*, hhae003.

Gonçalves, S., J. Koh, and B. Perron (2025). Bootstrap inference for group factor models. *Journal of Financial Econometrics 23*(2), nbae020.

Griffin, J. M. (2002). Are the fama and french factors global or country specific? *The Review of Financial Studies 15*(3), 783–803.

Grubel, H. G. (1968). Internationally diversified portfolios: welfare gains and capital flows. *The American economic review 58*(5), 1299–1314.

Harvey, C. R. (1991). The world price of covariance risk. *The Journal of Finance 46*(1), 111–157.

Jensen, T. I., B. Kelly, and L. H. Pedersen (2023). Is there a replication crisis in finance? *The Journal of Finance 78*(5), 2465–2518.

Karolyi, G. A. and R. M. Stulz (2003). Are financial assets priced locally or globally? *Handbook of the Economics of Finance 1*, 975–1020.

Kelly, B., S. Pruitt, and Y. Su (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics 134*, 501–534.

Kim, S., R. A. Korajczyk, and A. Neuhierl (2021). Arbitrage Portfolios. *Review of Financial Studies 34*(6), 2813–2856.

Patton, A. J. and B. M. Weller (2022). Risk price variation: The missing half of empirical asset pricing. *The Review of Financial Studies 35*(11), 5127–5184.

Pukthuanthong, K. and R. Roll (2009). Global market integration: An alternative measure and its application. *Journal of Financial Economics 94*(2), 214–232.

Quinn, D. P. and H.-J. Voth (2008). A century of global equity market correlations. *American Economic Review 98*(2), 535–540.

Solnik, B. H. (1974). Why not diversify internationally rather than domestically? *Financial analysts journal 30*(4), 48–54.

Verdelhan, A. (2018). The share of systematic variation in bilateral exchange rates. *The Journal of Finance 73*(1), 375–418.

Zaffaroni, P. (2025). Factor models for conditional asset pricing. *Journal of Political Economy*. Forthcoming.

Country-Specific Portfolio: Non-G7 ($K_s = 1$)

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
arg	20.49	20.03	3.49	1.02	23.66	-11.44	24.86	0.01
aus	16.68	20.87	4.73	0.80	26.83	-14.60	44.65	0.06
aut	19.96	19.47	3.69	1.02	31.63	-19.53	20.73	-0.12
bel	27.90	19.45	7.62	1.43	25.50	-21.40	18.70	-0.06
bra	17.80	20.61	2.56	0.86	31.55	-19.10	18.11	-0.05
chl	19.99	21.78	4.41	0.92	53.23	-19.16	26.02	-0.01
chn	14.83	22.02	2.87	0.67	49.72	-19.55	26.68	-0.05
dnk	26.31	22.96	5.83	1.15	26.77	-19.47	21.01	-0.08
esp	20.29	22.49	5.69	0.90	34.16	-18.76	26.48	0.03
hkg	19.22	19.37	5.58	0.99	42.81	-25.11	24.13	0.08
ind	18.67	20.76	4.21	0.90	35.76	-21.18	20.99	-0.01
kor	15.10	18.80	4.04	0.80	34.64	-16.25	19.79	-0.01
mex	20.88	20.89	3.99	1.00	42.02	-15.15	18.20	0.08
nor	19.49	21.67	4.12	0.90	53.49	-23.85	19.65	-0.02
pol	24.26	21.26	4.82	1.14	33.57	-19.87	25.95	0.00
sgp	23.31	21.24	5.17	1.10	31.07	-27.04	25.62	0.08
swe	22.20	22.87	5.28	0.97	58.37	-14.87	34.46	-0.01
tha	11.96	20.52	3.51	0.58	31.96	-31.20	30.07	-0.02
tur	20.29	19.99	4.24	1.01	31.52	-13.87	30.88	-0.07
twn	12.50	18.49	3.30	0.68	36.75	-12.65	18.49	0.01

- Very low correlation with US market

Segmentation Portfolio: Non-G7 ($K_c = 1$)

Country	Mean (%)	Standard Deviation (%)	t	Sharpe Ratio	Maximum Drawdown	Worst Month	Best Month	ρ_{US}
arg	10.85	23.08	1.67	0.47	53.32	-14.34	38.04	0.09
aus	10.86	26.62	2.60	0.41	59.75	-20.99	67.66	-0.02
aut	13.24	21.41	2.45	0.62	40.33	-16.89	23.13	0.18
bel	10.57	21.86	2.20	0.48	57.85	-25.30	24.92	0.06
bra	-0.81	23.18	-0.11	-0.03	48.88	-19.07	15.65	-0.02
chl	15.42	19.38	3.39	0.80	55.44	-14.43	19.08	0.05
chn	4.93	23.93	1.03	0.21	55.97	-19.93	41.88	0.06
dnk	10.75	20.50	2.40	0.52	53.77	-12.62	20.02	-0.05
esp	10.67	21.05	2.58	0.51	45.99	-18.29	17.80	0.03
hkg	2.61	20.92	0.60	0.12	67.01	-26.01	21.24	0.09
ind	18.14	21.46	3.88	0.85	43.11	-20.03	24.82	-0.14
kor	7.88	21.75	1.83	0.36	58.47	-22.64	23.55	-0.03
mex	4.54	20.06	0.87	0.23	52.68	-12.30	16.35	0.02
nor	5.55	21.63	1.30	0.26	66.79	-16.93	18.31	0.00
pol	7.77	19.50	1.98	0.40	46.79	-25.70	20.13	0.17
sgp	16.19	23.07	3.44	0.70	48.76	-26.98	35.76	0.19
swe	7.13	20.23	1.76	0.35	61.25	-34.35	14.39	-0.01
tha	6.89	21.74	1.73	0.32	65.94	-38.16	23.34	0.00
tur	12.34	22.08	2.81	0.56	48.59	-20.24	25.68	0.15
twn	8.96	20.54	2.72	0.44	35.25	-19.23	17.16	0.00

- Large differences in common factor risk

Closed form Solution for Θ_g^c and Θ_g^s

- Hence, the solution $\left(\widehat{\Theta}_g^c, \widehat{\Theta}_g^s\right)$ will be determined by

$$\operatorname{vec}\left(\left[\widehat{\Theta}_g^c \; \widehat{\Theta}_g^s\right]\right) = \left(\mathcal{X}'\mathcal{X}\right)^{-1} \mathcal{X}'\operatorname{vec}\left(\widehat{\mathbf{R}}_g\right),$$

  where $\mathcal{X} = \left(\left[\widehat{\mathbf{F}} \; \widehat{\mathbf{G}}\right] \otimes \mathbf{X}_g\right)$.

- Return to Step3

Multivariate Country-specific Portfolio

- Estimate the country-specific risk premia and its variance

$$\widehat{\lambda}_2^s = \frac{\widehat{\mathbf{w}}_2^{s\prime}\mathbf{R}_2\mathbf{1}_T}{T}$$

$$\operatorname{Var}\left(\widehat{\lambda}_2^s\right) = \frac{\widehat{\mathbf{w}}_2^{s\prime}\mathbf{R}_2\mathbf{R}_2^{\prime}\widehat{\mathbf{w}}_2^s}{T-1} - \widehat{\lambda}_2^s\widehat{\lambda}_2^{s\prime}$$

- Then, we hold

$$\left(\operatorname{Var}\left(\widehat{\lambda}_2^s\right)^{-1}\widehat{\lambda}_2^s\right)^{\prime}\left(\widehat{\mathbf{w}}_2^{s\prime}\mathbf{r}_{2,T+1}\right)$$

- ▸ Return to Univariate Construction

Multivariate Segmentation Portfolio

- Run vector regression

$$\underbrace{\widehat{\mathbf{w}}_2^{c\prime}\mathbf{R}_2}_{K^c\times T} = \beta_0 + \beta_1\left(\underbrace{\widehat{\mathbf{w}}_1^{c\prime}\mathbf{R}_1}_{K^c\times T}\right) + \mathbf{e}$$

- Intercept captures difference in risk premia, i.e.

$$\beta_0 \to^P \mathcal{S}^{c\prime}(\lambda_2^c - \lambda_1^c)$$

- After fitting the regression, we hold

$$\left(\mathrm{Var}\left(\widehat{\beta}_0\right)^{-1}\widehat{\beta}_0\right)'\left(\widehat{\mathbf{w}}_2^{c\prime}\mathbf{r}_{2,T+1} - \widehat{\beta}_1\widehat{\mathbf{w}}_1^{c\prime}\mathbf{r}_{1,T+1}\right)$$

- Return to Univariate Construction

Alpha Portfolio

- Allow for non-zero alpha

$$R_{git} = \alpha_{ig} + \beta_{ig} f_t + \delta_{ig} g_t + e_{itg}, \quad \alpha_{ig} = \mathbf{x}'_{ig} \Theta^a_g + \gamma^a_{ig}$$

Solve constrained regression

$$\widehat{\Theta}^a_g = \arg\min_{\Theta^a_g} \| \overline{\mathbf{R}}_g - \mathbf{X}_g \Theta^a_g \|,$$

$$\text{subject to } \left[\widehat{\Theta}^c_g \; \widehat{\Theta}^s_g \right]' \mathbf{X}'_g \mathbf{X}_g \Theta^a_g = \mathbf{0}_{K_g \times 1}$$

- $\widehat{\Theta}^a_g \to^P \Theta^a_g$
- Construct the portfolio in an out-of-sample manner: $\mathbf{w}^a_{g,T} = \frac{1}{N} \mathbf{X}_g \widehat{\Theta}^a_{g,T}$

$$\mathbf{w}^{a\prime}_{g,T} \mathbf{r}_{g,T+1} \to^P \lim_{N_g} \frac{1}{N_g} \Theta^{a\prime}_g \mathbf{X}'_g \mathbf{X}_g \Theta^a_g \geq 0$$

back

Segmentation Portfolio - Relation to Currencies

- Regression on changes in exchange rate

$$r_{i,t}^{\Delta\lambda} = \alpha_i + \beta \times \Delta(\text{Exchange Rate vs. USD})_{it} + u_{it}$$

$\widehat{\beta} \approx -0.052, \quad t_{\widehat{\beta}} \approx -1.4, \quad R^2 < 0.01$

- Regression on currency risk factors (Verdelhan (2018))

$$r_i^{\Delta\lambda} = \alpha + \beta_1 \times \text{Carry}_t + \beta_2 \times \text{Dollar}_t + u_t$$

$\widehat{\beta}_1 \approx -0.043, \quad t_{\widehat{\beta}_1} \approx -1.1, \quad \widehat{\beta}_2 \approx -0.0042, \quad t_{\widehat{\beta}_2} \approx -0.08, \quad R^2 < 0.01$

- **Ongoing work**
  - No strong time trends
  - NB Patton and Weller (2022) find λ-cluster even for the US