

# Equity Portfolio Management: Principles and Applications

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# 1 Prologue

## 1.1 Retiring Wall Street Strategist's Amazing Investment Advice

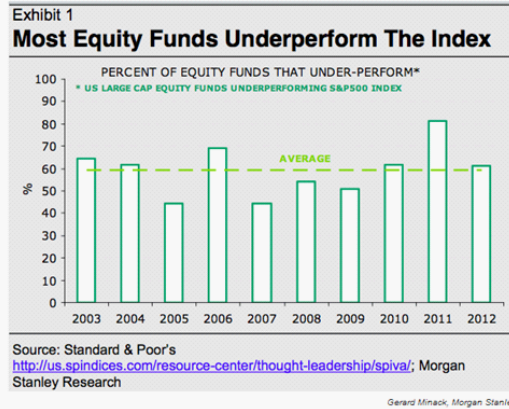
Gerard Minack, a Retiring Wall Street Strategist Gives  
Amazing Investment Advice Just Before He Quits

[www.businessinsider.com/best-investment-advice-2013-5](http://www.businessinsider.com/best-investment-advice-2013-5)

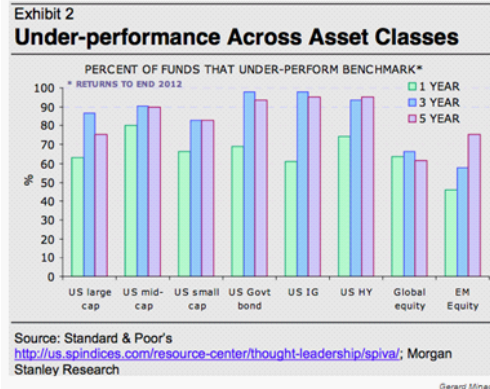
- Don't try to pick stocks
- Don't try to time the market
- Just invest in a portfolio of low-cost, tax-efficient index funds



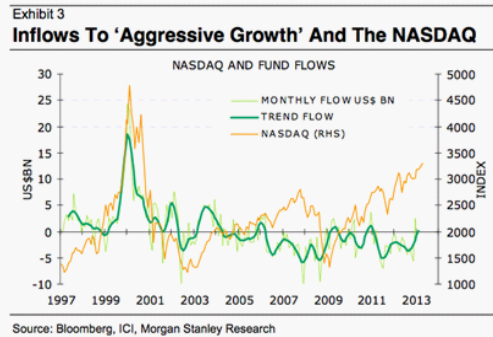
1. On average, 60% of funds lag the market.



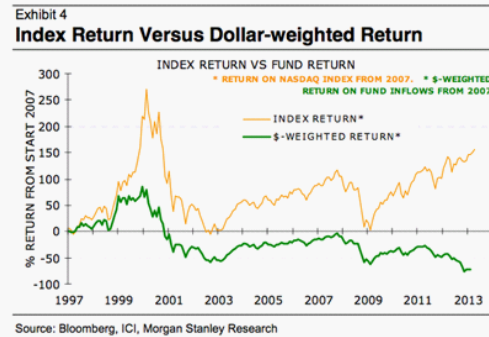
2. Over longer periods, the percentage of funds that lag the market is much higher:



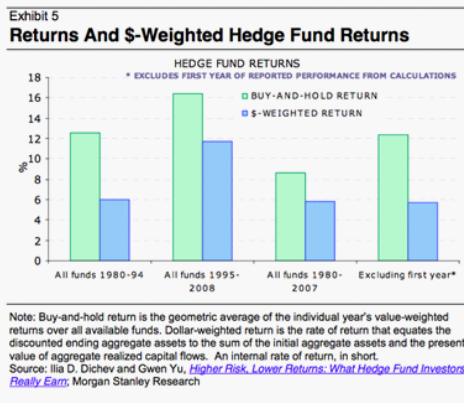
3. Investors are terrible at market timing. They put money into the market after stocks have done well for a long time, and they pull money out when stocks have collapsed.



4. Because investors are lousy at market timing, their actual returns are far worse than the market returns or the average fund returns. They sell before funds do well and buy before they do badly.



5. Hedge funds are terrible, too (mainly because of the super-high fees).



## 1.2 The Efficient Market Hypothesis (EMH)

### 1.2.1 Description and Illustration

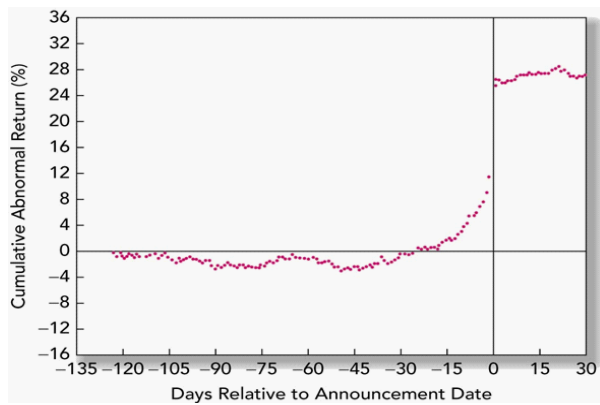
From Stewart/Piros/Heisler (2010), Running Money, p.188. Emphasis added.

- The EMH states that security prices reflect all available information. The process through which securities reflect information is **market competition**. Any information, either data or the means of analyzing data, which investors find that helps predict future security prices, will be used by investors today. ...
- The result is that prices move to a new equilibrium level where investors can expect to earn a return

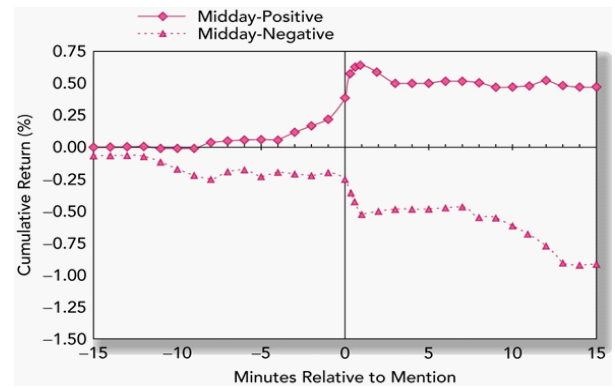
commensurate with the risk taken. On average, informed investors are expected to profit, increasing their influence in the price-setting process, while uninformed investors are expected to lose, decreasing their influence in the price-setting process.

- This leads to a market where **prices react immediately to new information** due to the behavior of informed traders and where profitable strategies are exploited to the point where the inefficiency is removed. If the arrival of new information is unpredictable, then security price changes will also be unpredictable.

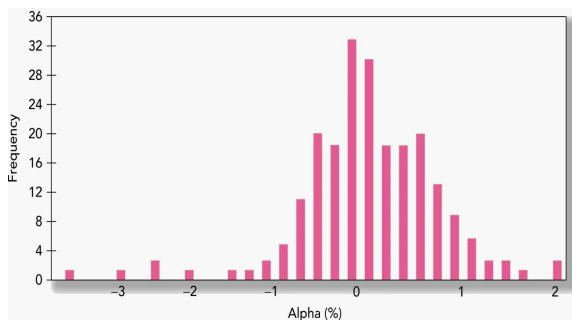
Charts in this section are taken from Bodie/Kane/Marcus's popular textbook, Investments (8th ed.)



Takeover example: Stock price reactions of target companies.

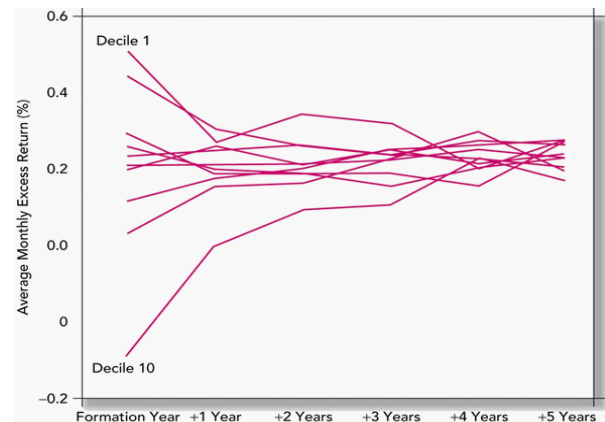


Stock price reaction to CNBC midday reports.



Mutual fund alpha vs. SP500.

Mean=0. Mean alpha < 0 vs. better benchmarks.



Mutual fund performance does not persist, except for the worst.

### Performance of Mutual Funds Based on Three-Index Model

Type of Fund (Wiesenberger Classification)	Number of Funds	Alpha (%)	t-Statistic for Alpha
Equity funds			
Maximum capital gain	12	-4.59	-1.87
Growth	33	-1.55	-1.23
Growth and income	40	-0.68	-1.65
Balanced funds	31	-1.27	-2.73

Note: The three-index model calculates the alpha of each fund as the intercept of the following regression:

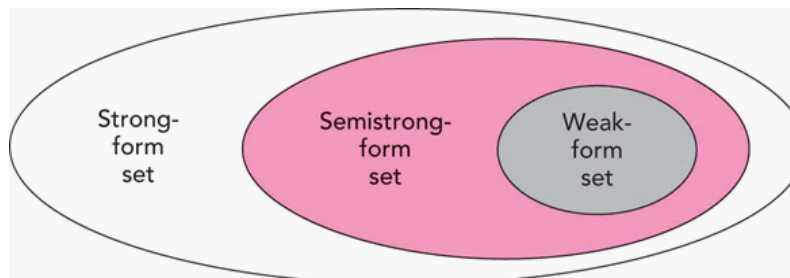
$$r - r_f = \alpha + \beta_M(r_M - r_f) + \beta_S(r_S - r_f) + \beta_D(r_D - r_f) + e$$

where  $r$  is the return on the fund,  $r_f$  is the risk-free rate,  $r_M$  is the return on the S&P 500 index,  $r_S$  is the return on a non-S&P small-stock index,  $r_D$  is the return on a bond index,  $e$  is the fund's residual return, and the betas measure the sensitivity of fund returns to the various indexes.

Source: E. J. Elton, M. J. Gruber, S. Das, and M. Hlavka, "Efficiency with Costly Information: A Reinterpretation of Evidence from Managed Portfolios," *Review of Financial Studies* 6 (1993), pp. 1-22.

On average, mutual funds underperform their benchmarks.

The familiar taxonomy of weak-form, semistrong-form, and strong-form efficiency based on the information set.

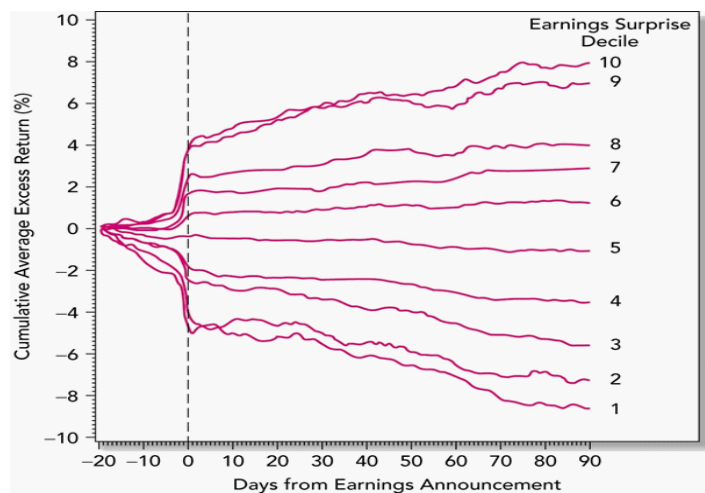


### 1.2.2 Implications

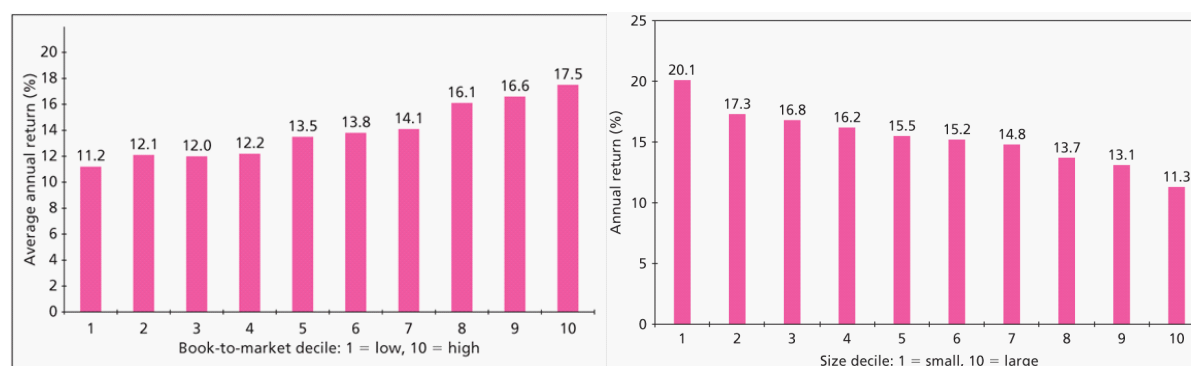
- To earn excess profits, an investor has to either trade on information before the market, or have and trade on information that the market doesn't have, in the form of either proprietary data or a proprietary way of analyzing the data. This means **holding a view at odds with current expectations**.
- Market efficiency is not all or nothing. Different markets can display different degrees of efficiency, and the efficiency of a market can vary over time.
- EMH does not say that no investors earn a profit. What it implies is that, on average, investors should expect to earn the appropriate **risk-adjusted return**.

## 1.2.3 Anomalies

Empirical regularities that look inconsistent with EMH.



### Post-Earnings Announcement Drift



Value effect (1926-2007)

Size effect (1926-2007).

## 1.2.4 Remarks

- SPH, p.189: “All tests of market efficiency are **joint tests** of both efficiency and the expectations model.”
  - In order to test the EMH, economists need to create a model (like CAPM, for example) for how the market actually works—a task that has yet to be successfully completed.
  - This point is extremely important. We will come back to this point later.
- “**Liquidity risk premium**” (high return premiums for illiquid assets) is likely to be very important in practice.



- “Grossman-Stiglitz puzzle”
  - If markets are efficient, (and everybody know this), then nobody will engage in costly research to find undervalued stocks. But if no one does research, markets will not be efficient. So there’s no way to have an efficient market where all investors believe the market is efficient.
  - Brokers and active money managers believe that market is inefficient. But the harder they work to find undervalued stocks, the more efficient the market becomes.
  - As the brokerage industry does its job better and better, the less useful their services will be to individual investors.

- EMH is not dogmatic. Although the markets are competitive, there are still profits to be made. The greatest value of EMH is the **healthy skepticism** it suggests when you evaluate claims of a successful active strategy (stock picking, market timing).

## 1.3 Cliff Asness on Efficient Markets

Cliff Asness (Founder of AQR Capital Management) Interviews (Morningstar; June 20, 2014).

- Why I'm a Cynic of Active Stock-Picking

[www.morningstar.com/cover/videocenter](http://www.morningstar.com/cover/videocenter)

[.aspx?id=651661&lineup=mutualfunds](http://www.morningstar.com/cover/videocenter.aspx?id=651661&lineup=mutualfunds)

- Splitting the Middle on Market Efficiency

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From a Morningstar editor's summary:

We have proponents of the EMH on one side saying that even if the market isn't perfectly efficient, it mostly is, and that investors shouldn't try and beat the market. While skeptics see behavioral and other factors in the marketplace that introduce potential significant distortion and believe there is more room for active management.

Asness sees the truth as lying somewhere in between. His take is that the efficient-market camp is mostly correct, but that the behaviorists make some strong points, too.

He sees the existence and persistence of value (cheaper assets outperforming more expensive assets) and momentum factors across time periods, asset classes, and geographies as a sign that the market is in fact not completely efficient.

He pointed to the dot-com bubbles as one of the clearest examples of the market working in an irrational manner. ...

So what does this mean for investors? Asness stressed that even though there are inefficiencies in the marketplace, it is very hard to both find them and exploit them over time. He said there is very much a case for not holding the index, as long as managers have a healthy respect for the power of mostly efficient markets. And given how hard it is to beat the market, Asness says that self-directed individual investors are far better off holding a portfolio of index funds than trying to actively pick stocks.

## 1.4 The Science of Investing:

### A Morningstar Article

[news.morningstar.com/articlenet/article.aspx?id=608389](http://news.morningstar.com/articlenet/article.aspx?id=608389)

- ... I'd go as far as to say **evidence-driven investing** is the best approach for the majority of investors, because it's based on an efficient learning strategy. Many investors pick a terrible learning strategy: personal experience. ...
- ... Scientific investing, at its best, is about engaging the data honestly, with the intention of learning something new, hopefully something discordant with previously held beliefs.

- ... What are the fruits of science as it pertains to investing? There's a lot of nonsense, but also a great deal of sense: **Factors are important, and most investors should focus on investing in them.**
- Factor Investing ... It's really not as esoteric as it sounds. You've heard of style investing—small cap versus large cap, or value versus growth. If you've ever tilted to a particular style, you've engaged in factor investing. Style investing is a kind of factor investing, dealing with only two factors: size (large-small) and value (value-growth).
- A working definition of a factor is **an attribute of an asset that both explains and produces excess returns.**

- Factor investing can be thought of as buying these return-generating attributes rather than buying asset classes or picking stocks.
- Examples of these attributes:
  - CAPM: Beta on Market Factors
  - Fama and French Model: Betas on Market, Size, and Value Factors
  - Recent studies have shown Momentum, Quality, Low Volatility, etc. are significant factors.
- Though it's been around for decades, factor investing has only in the past decade gained adherents. Recent converts include the **Government Pension Fund of Norway**, the biggest pension fund in Eu-

rope, and **CalPERS**, the biggest public pension fund in the United States. They've seen the light after realizing that the active managers they were richly compensating were simply offering factor risks and factor-based strategies under the guise of skill.

- An implication of factor-based investing is that what was once legitimately deemed “alpha” – excess returns attributable to skill – has morphed into “beta” (or a factor) once researchers identify a simple strategy that replicates the alpha.



## 1.5 Regulation Fair Disclosure (Reg.FD)



<http://www.sec.gov/answers/regfd.htm>

- Companies are now required to give fair disclosure of events. (2000-)
  - All information must be uniformly distributed and available through public data resources.
- ⇒ Investors can no longer get company news ahead of the rest of the market through informal, one-on-one conversations with company executives.

## 2 “Efficient Markets Today”

### 2.1 John Cochrane’s Perspective: Joint Hypothesis, Second Revolution, and Factor/Style Investing

- Stock prices reflect both firm fundamentals (expected stream of future cash-flows) and discount rates = expected returns (risk premiums in particular). We used to ask if the stock market incorporates the information about **firm fundamentals** efficiently, *holding the discount rates constant*.

- We have realized that changes in **discount rates** (**risk premiums** in particular) have large effects on stock prices. Risk premiums are driven by macroeconomic forces. [⇒ **Second Revolution**]
  - We are yet to understand how risk premiums are determined by systematic/macroeconomic forces.
- Since our models of expected returns (discount rates) remain poor, we cannot tell whether “alphas” come from mispricing (market inefficiency) or model misspecification (poor benchmark model).  
[⇒ **“Joint Hypothesis” problem**].

Prof. Fama’s (one of the 2013 Nobel Laureates) explains:

“The joint hypothesis problem says that you can’t test market efficiency without a model of market equilibrium.

But the reverse is also true. You can't test models of market equilibrium without market efficiency because most models of market equilibrium start with the presumption that markets are efficient. They start with a strong version of that hypothesis, that everybody has all relevant information. Tests of market efficiency are tests of some model of market equilibrium and vice versa. The two are joined at the hip."

"Once I pointed that out, it was clear that the random walk model was kind of irrelevant. You could have prices not following random walks because the model of market equilibrium could generate expected returns that had some predictable time-varying patterns to them. So the whole nature of the game changed."

- Significant “alphas” can be perfectly consistent with market efficiency if “alphas” capture risk premiums associated with some unrecognized “betas” (sensitivities to some less-known sources of systematic risk) that the benchmark model fails to capture.
- We should be able to earn return premiums from those unrecognized betas (or possible mispricings) with **Factor/Style Investing**.

⇒ The “style” choice is now the main, hard, and only portfolio problem.

⇒ The alpha/beta, style/selection, active/passive, skill/index paradigm has become meaningless.

## 2.2 Introduction to Factor/Style Investing: Fama-French Analysis

### 2.2.1 Size Effects and Value Effects: Fama-French Data Analysis

Size (Market Capitalization) and B/M ratio capture the cross-section of equity returns well.

- B of the B/M is the book value of equity (on the firm's balance sheet).
- M of the B/M is the market value of equity (market capitalization = # of shares  $\times$  share price).

Stocks with high B/M ratios are called Value stocks.

Stocks with low B/M ratios are called Growth stocks.

Useful data source:

[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Let's calculate average monthly returns (value weighted portfolio returns in excess of T-bill returns; %) for 25 US portfolios formed on Size and BE/ME ( $5 \times 5$ ).

**Construction:** The portfolios, which are constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year  $t$  are the NYSE market equity quintiles at the end of June of  $t$ . BE/ME for June of year  $t$  is the book equity for the last fiscal year end in  $t-1$  divided by ME for December of  $t-1$ . The BE/ME breakpoints are NYSE quintiles.

**Stocks:** The portfolios for July of year  $t$  to June of  $t+1$  include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of  $t-1$  and June of  $t$ , and (positive) book equity data for  $t-1$ .

Let's use the sample from 1971.07 to 2014.04 for the following analysis. (Nasdaq started in 1971.02. A lot of developments in financial markets in 1970s.)

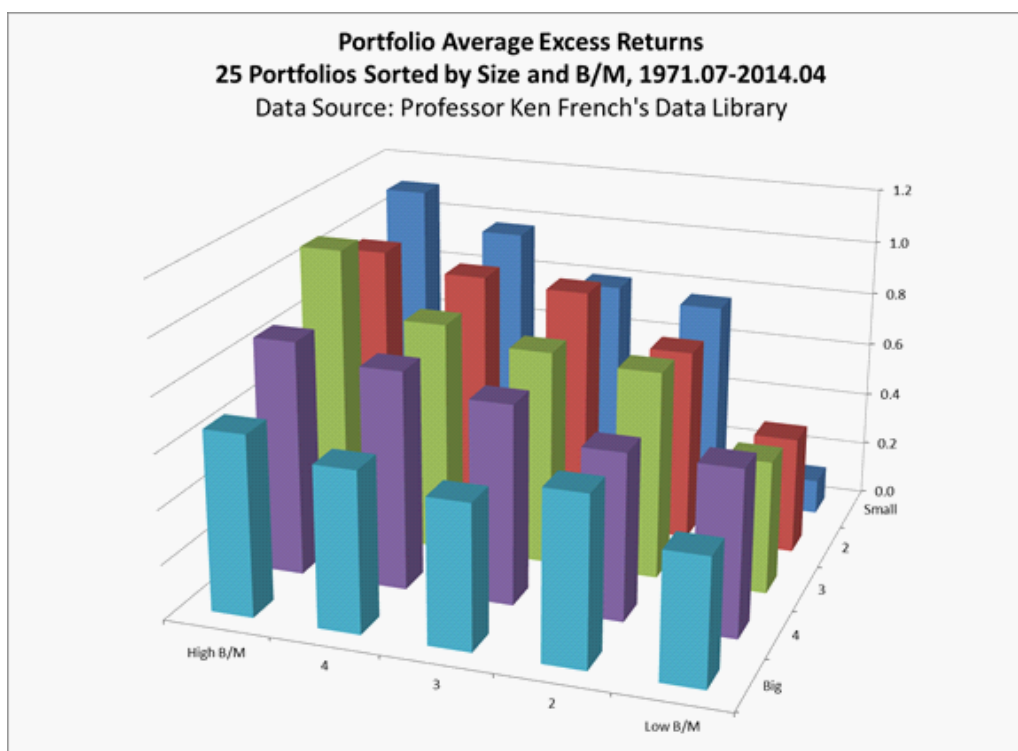
1. Calculate average excess returns (total returns in excess of the risk-free rate) of the 25 portfolios. Tabulate the results into a  $5 \times 5$  format so that we can compare average portfolio returns along the Size dimension and the B/M dimension.
2. Estimate market betas of the 25 portfolios.  $= Slope()$  function is useful. To see if betas explain the cross-sectional variation in average excess returns, plot average excess returns against market betas.



The sample period for this calculation is from 1971.07 to 2014.04.

	Low B/M	2	3	4	High B/M	High-Low
Small	0.127	0.766	0.806	0.977	1.109	0.982
2	0.430	0.712	0.896	0.915	0.973	0.543
3	0.486	0.763	0.790	0.849	1.084	0.598
4	0.607	0.611	0.731	0.802	0.862	0.255
Big	0.461	0.616	0.529	0.589	0.663	0.202
Small-Big	-0.334	0.150	0.277	0.388	0.447	

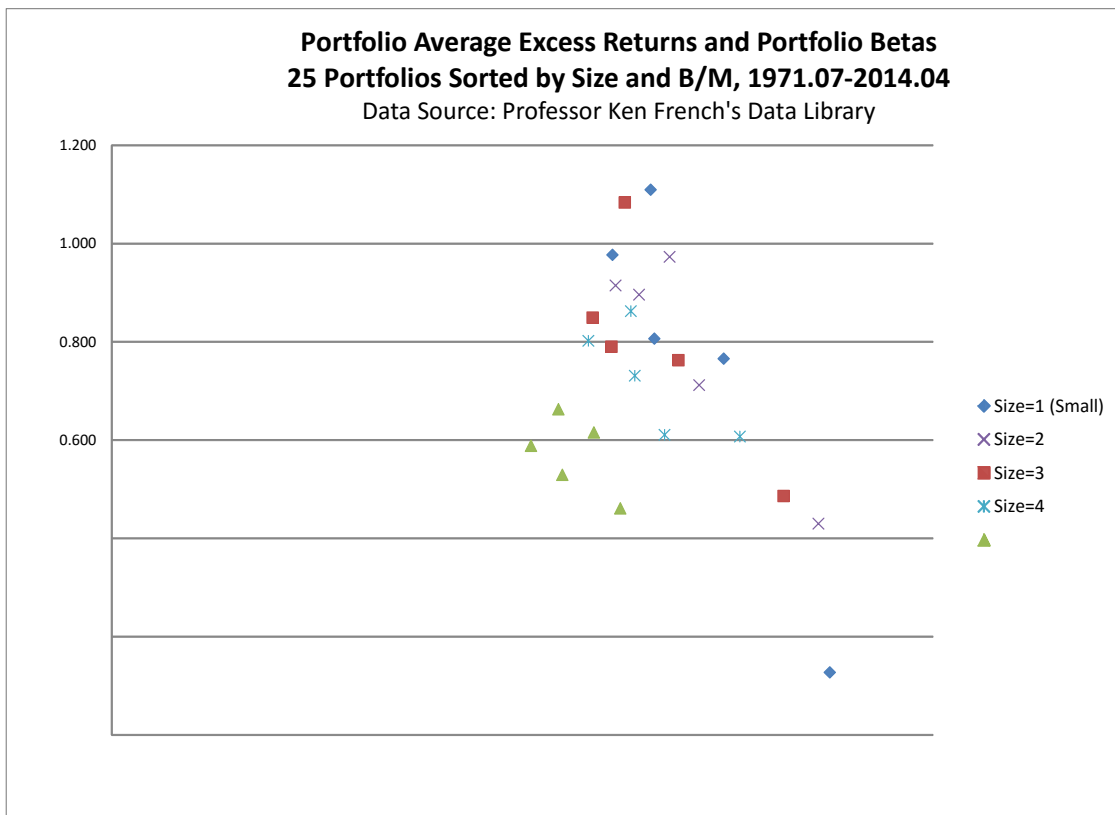
Average monthly excess returns (%)



Similar results hold in Asia Pacific ex Japan, Japanese, and European markets.

## Observations

- Within each Size group, stocks with high B/M (Value stocks) tend to have higher average returns than those with low B/M (Growth stocks).
- Within each B/M group, small stocks tend to have higher average returns than big stocks.
- The CAPM Betas fail to explain these differences in portfolio returns. The following chart plots average returns of the 25 portfolios against their market betas.



- Under the CAPM, average excess returns should be positively related to market betas (the Security Market Line). However, the relation between the 25 portfolio average excess returns and their market betas are very weak (or even negative).

## 2.2.2 Fama-French Factor Model

Professors Fama and French propose a model to augment the CAPM with two additional factors:

$$r_i - r_f = \alpha_i + \underbrace{\beta_{mkt,i}(r_M - r_f)}_{\text{market factor}} + \underbrace{\beta_{size,i}(SMB)}_{\text{size factor}} + \underbrace{\beta_{value,i}(HML)}_{\text{value factor}} + \varepsilon_i$$

where the Fama/French factors are constructed using the 6 value-weight portfolios formed on size and B/M.

	Median ME	
70th BE/ME percentile	Small Value	Big Value
	Small Neutral	Big Neutral
30th BE/ME percentile	Small Growth	Big Growth

- These portfolios are formed at the end of June in each year (annual rebalancing).

- SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios:

$$SMB = \frac{\text{Small Value} + \text{Small Neut.} + \text{Small Growth}}{3} - \frac{\text{Big Value} + \text{Big Neut.} + \text{Big Growth}}{3}$$

SMB captures the size effect.

- HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios:

$$HML = \frac{\text{Small Value} + \text{Big Value}}{2} - \frac{\text{Small Growth} + \text{Big Growth}}{2}$$

HML captures the value effect.

- We often add the momentum factor (recent winners minus recent losers) as the fourth factor. (Also called the Fama-French-Carhart 4 factor model.)

$$\begin{aligned}
 r_i - r_f = & \alpha_i + \beta_{mkt,i}(r_M - r_f) \\
 & + \beta_{size,i}(SMB) + \beta_{value,i}(HML) \\
 & + \beta_{mom,i} \underset{\text{momentum factor}}{(MOM)} + \varepsilon_i
 \end{aligned}$$

The model fares much better than the CAPM in explaining average excess portfolio returns. The following plots average portfolio excess returns against predicted excess return by a 4 factor model,

$$\begin{aligned}
 E[r_i] - r_f = & \hat{\beta}_{mkt,i}(E[r_M] - r_f) + \hat{\beta}_{size,i}E[SMB] \\
 & + \hat{\beta}_{value,i}E[HML] + \hat{\beta}_{mom,i}E[MOM].
 \end{aligned}$$



### 2.2.3 Performance Analysis with a Fama-French Factor Model: Vanguard Active Mutual Funds

The Fama-French 3 (or 4) factor model has been widely used in analyzing risk-adjusted portfolio performance.

- The alpha ( $\alpha$ ) from the Fama-French 3 (or 4) factor model measures the portfolio's expected excess return after adjusting for the 3 (or 4) factors.
- The **Information Ratio (IR)**:

$$IR_i = \frac{\alpha_i}{\sigma[\varepsilon_i]} \text{ where } \sigma[\varepsilon_i] \text{ is the st.dev. of } \varepsilon_i.$$

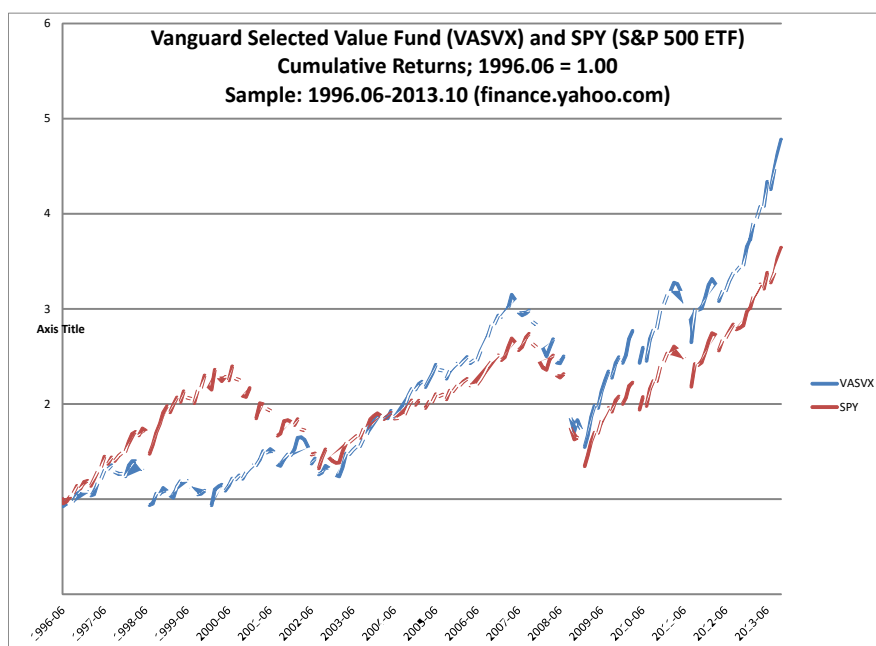
is a relevant measure of a manager's skill.

- People often calculate IR casually, like  $\frac{E[r_i - r_M]}{\sigma[r_i - r_M]}$ .
- Before-fee IRs (annualized) typically fall close to the following distribution.

Percentile	Information Ratio
90	1.0
75	0.5
50	0.0
25	-0.5
10	-1.0



**Vanguard Selected Value (VASVX):** *VASVX* has achieved respectable performance, and currently it has a Morningstar 5-star rating (among top 10%).



Fund Overview	
Category:	Mid-Cap Value
Fund Family:	Vanguard
Net Assets:	7.02B
Year-to-Date Return:	33.79%
Yield:	1.57%
Morningstar Rating:	★★★★★
Fund Inception Date:	Feb 15, 1996

#### Mid-Cap Value

[\[View Category Definition\]](#)

			Size
			Medium
			Small
Value	Blend	Growth	Investment Valuation

Does the strong performance of *VASVX* reflect a superior stock selection skill?

- A Quick Summary (07/1996-10/2013: 208 months):

	VASVX	Mkt-Rf	Tracking Err
Average (monthly)	0.653	0.540	0.114
Volatility (monthly)	5.040	4.765	3.167
Sharpe (annualized)	0.449	0.392	0.125
Beta	0.839	1.000	
Treynor Ratio	0.779	0.540	
M^2 (% per month)	0.083		

- “Tracking error” here is simply the difference between *VASVX* and the market.
- The Sharpe ratio of the tracking error is 0.125. People often call this ratio the fund’s IR (a casual/crude calculation).

- vs. Fama/French 3 Factor Model

SUMMARY OUTPUT				
<i>Regression Statistics</i>				
Multiple R	0.879023			
R Square	0.772682			
Adjusted R	0.769339			
Standard E	2.420737			
Observatio	208			
<i>Coefficients</i>				
		<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	-0.0402	0.170558	-0.2357	0.813901
Mkt-Rf	0.918294	0.036963	24.84338	1.31E-63
SMB	0.100232	0.050775	1.974033	0.049728
HML	0.595581	0.052869	11.26521	3.34E-23

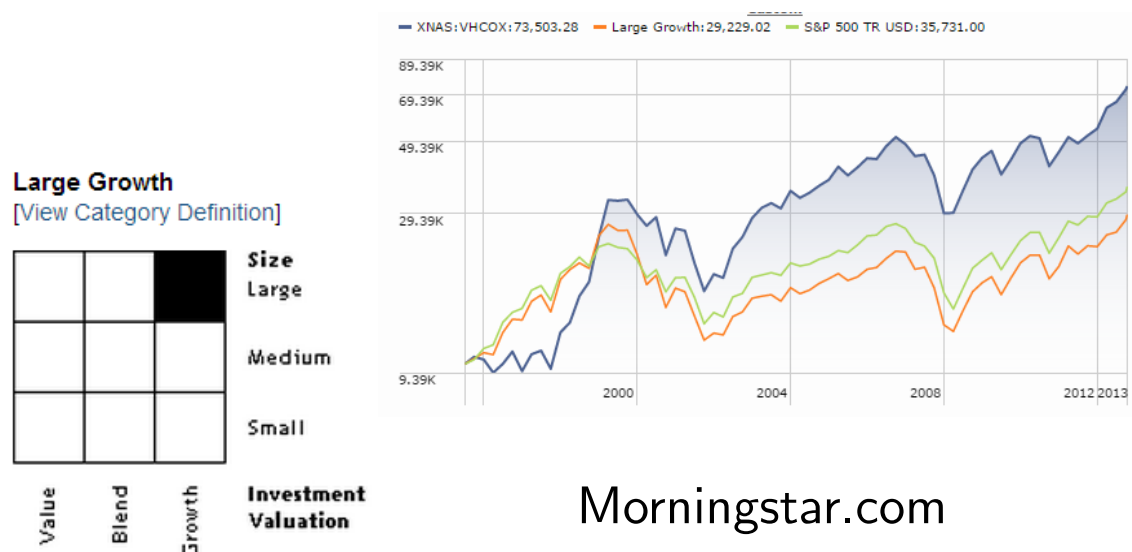
- *VASVX* has a significant positive exposure to *HML*. (Not surprising because the fund pursues a value strategy.)
- After adjusting for the risk of *HML* and *SMB*, *VASVX*'s alpha becomes essentially zero.
- The Information Ratio based on the 3 factor model:

$$IR \text{ (annualized)} = \frac{-0.040}{2.421} \times \sqrt{12} = -0.058$$

- We do not find evidence that *VASVX* (Vanguard Selected Value) reflects significant stock-selection skills. Rather, its excellent performance can be ascribed to its large exposure to the HML (value) factor (and to the SMB factor to a lesser extent).
- *VASVX* is still an excellent fund as it has taken a large exposure to the value factor (to harvest the value premium), while keeping the market beta to around 0.92.

## Vanguard Capital Opportunity (VHCOX): VHCOX

is another Morningstar 5-star rated fund with an excellent track record.



- A Quick Summary (07/1996-10/2013: 208 months):

	VHCOX	Mkt-Rf	Tracking Err
Average (monthly)	0.950	0.540	0.410
Volatility (monthly)	6.306	4.765	3.109
Sharpe (annualized)	0.522	0.392	0.457
Beta	1.163	1.000	
Treynor Ratio	0.817	0.540	
M^2 (% per month)	0.236		

- The crude IR measure looks quite high (0.46).

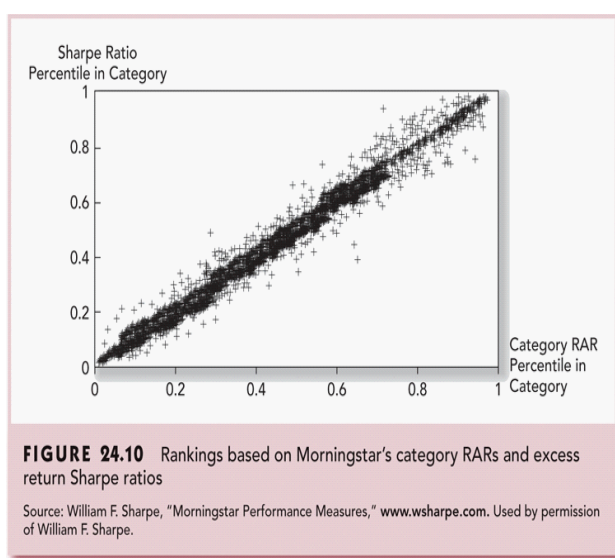
- vs. Fama/French 3 Factor Model

SUMMARY OUTPUT				
<i>Regression Statistics</i>				
Multiple R	0.918807			
R Square	0.844207			
Adjusted R Square	0.841916			
Standard Error	2.507152			
Observations	208			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	0.3414	0.176647	1.93267	0.054661
Mkt-Rf	1.054033	0.038283	27.53275	1.24E-70
SMB	0.390344	0.052588	7.422736	3.05E-12
HML	-0.20127	0.054756	-3.67582	0.000303

- $VHCOX$  has a negative exposure to HML and a positive exposure to SMB.  $VHCOX$  is a “growth” (not a value) fund with exposure to small stocks.
  - Alpha (0.34;  $t = 1.93$ ) looks quite significant.
  - $IR = 0.47$  (annualized)
- We find some evidence for the superior stock picking skills (about top quartile) of  $VHCOX$  after accounting for the 3 FF factors.

## 2.2.4 (Digression) Morningstar's Ratings

- Morningstar computes fund returns as well as a risk measure based primarily on fund performance in its worst years.
- The risk-adjusted performance is ranked across funds in a style group and stars are awarded.
- **Star ratings are highly correlated with Sharpe ratio rankings (within each style).**



## 2.3 Beyond the Fama-French Factor Model

### 2.3.1 Documented Alpha Opportunities

After accounting for Fama-French factors, we still observe positive [negative] “alphas” for:

- firms with lower [higher] *accruals* (e.g. reported income minus operating cash flows);
- firms with lower [higher] *volatility* (including both lower market beta and lower idiosyncratic risk);
- firms with less [more] *net equity issuances* (more repurchases);
- firms with more conservative [aggressive] *asset growth* (e.g. via capital investments);



- firms with stronger [weaker] *profitability* (e.g. ROE);
- etc. (among others)

**Remark:** These “alphas” may represent risk premiums for unknown “beta” risks or the effects of “mispricing.” We cannot distinguish between the two (and we do not need to).

Factor/style investing is useful to capture these effects (either by exploiting mispricing or by harvesting additional risk premiums).

## 2.3.2 Examples: “Non-Organic” Growth

Universe: Russell 1000 index (large cap firms); Period: 02/10/2007 - (due to my subscription)

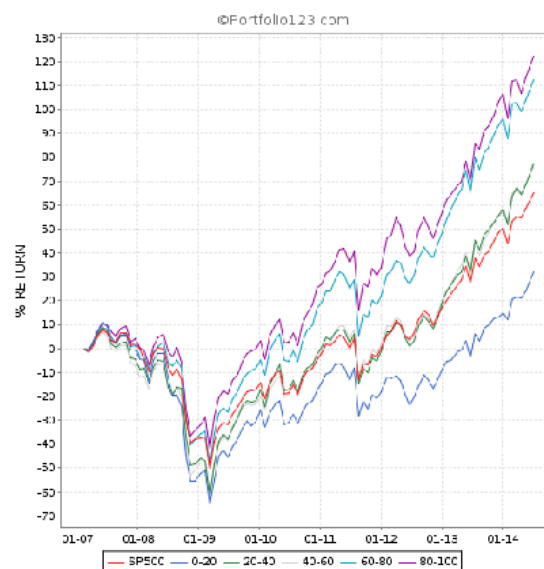
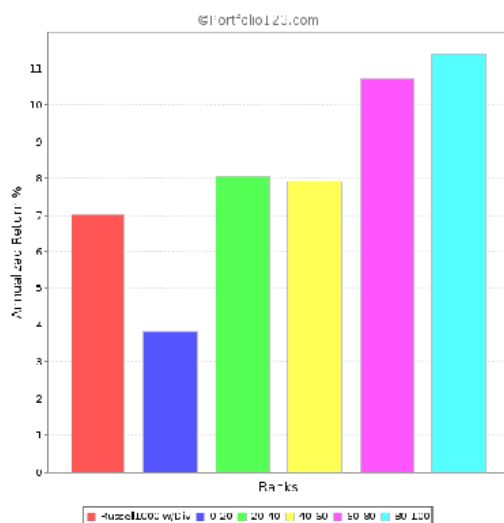
- (Operating Cash Flows - Capex)/Total Assets

Historical Performance by Ranks

Period: 02/10/07 - 07/05/14  
Rebalance Frequency: 4 Weeks  
Ranking Method: Percentile NAS Neutral  
Slippage (% of trade amount): 0.0  
Transaction Type: Long  
Universe: Russell 1000 (NEW)  
Benchmark: Russell 1000 w/Div

Number of Buckets: 5  
Minimum Price: 5.0  
Sector: ALL

[Change Settings](#) [Download](#)



- Firms w/ large Capex (beyond Op. CFs) tend to have poor stock return performance.

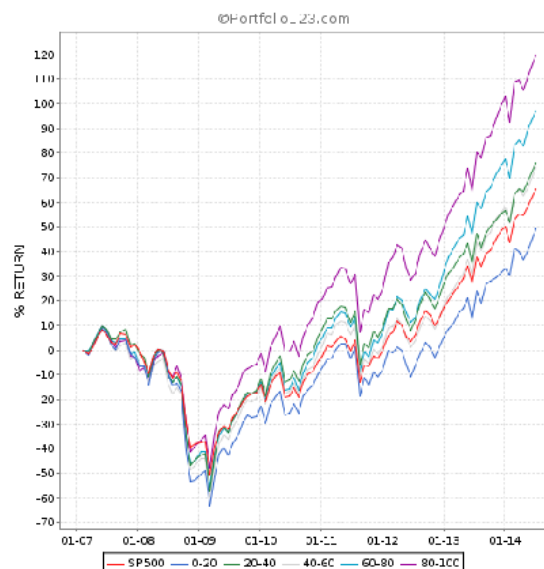
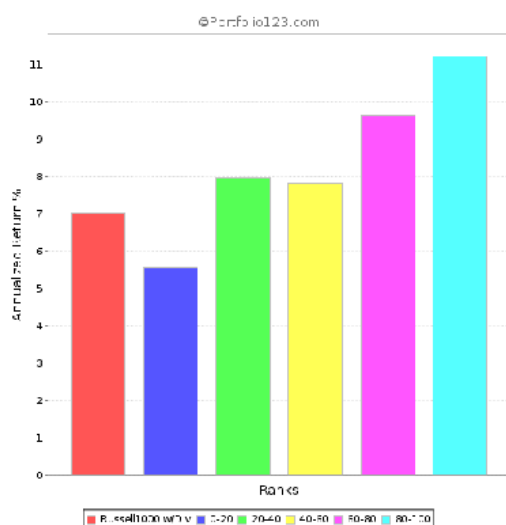
- $(\text{Debt Reductions} + \text{Equity Repurchases} - \text{Debt Issuances} - \text{Equity Issuances}) / \text{Total Assets}$

#### Historical Performance by Ranks

Period: 02/10/07 - 07/05/14  
 Rebalance Frequency: 4 Weeks  
 Ranking Method: Percentile N/A's: Neutral  
 Slippage (% of trade amount): 0.0  
 Transaction Type: Long  
 Universe: Prussell 1000 (NEW)  
 Benchmark: Russell 1000 w/Div

Number of Buckets: 5  
 Minimum Price: 5.0  
 Sector: -- ALL --

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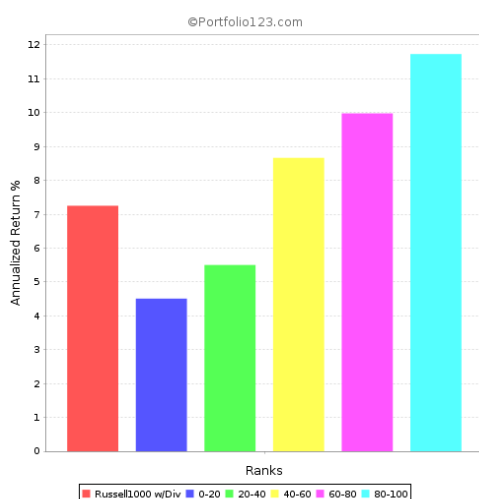


- Firms w/ large net ext. financing tend to have poor stock return performance.
- These examples suggest that firms with “non-organic growth” (externally financed growth) to have poor stock return performance down the road.

## 2.3.3 Another Example: An Industry Selection Strategy

- Prediction: At the industry level, returns on invested assets (profitability) should be close to the industry cost of capital (expected return) in equilibrium.
- Industry Level Profitability:  $(\text{Revenues} - \text{COGS}) / \text{Total Assets}$  (at the industry level)

Historical Performance by Ranks				
Period	02/10/07 - 12/01/14	Number of Buckets	5	
Rebalance Frequency	4 Weeks	Minimum Price	5.0	
Ranking Method	Percentile NAs Neutral	Sector	-- ALL --	
Slippage (% of trade amount)	0.0			
Transaction Type	Long			
Universe	Russell 1000 (NEW)			
Benchmark	Russell1000 w/Div			
<a href="#">Change Settings</a> <a href="#">Download</a>				



## 2.4 “Buffett’s Alpha”

### 2.4.1 Background: Warren Buffett

	Buffett Performance			Overall stock market performance
	Berkshire Hathaway	Public U.S. stocks (from 13F filings)	Private holdings	
Sample	1976-2011	1980-2011	1984-2011	1976-2011
Beta	0.67	0.77	0.28	1.00
Average excess return	19.0%	11.8%	9.6%	6.1%
Total Volatility	24.8%	17.2%	22.3%	15.8%
Idiosyncratic Volatility	22.4%	12.0%	21.8%	0.0%
Sharpe ratio	0.76	0.69	0.43	0.39
Information ratio	0.66	0.56	0.36	0.00
Leverage	1.64	1.00	1.00	1.00
Sub period excess returns:				
1976-1980	42.1%	31.4%		7.8%
1981-1985	28.6%	20.9%	18.5%	4.3%
1986-1990	17.3%	12.5%	9.7%	5.4%
1991-1995	29.7%	18.8%	22.9%	12.0%
1996-2000	14.9%	12.0%	8.8%	11.8%
2001-2005	3.2%	2.2%	1.7%	1.6%
2006-2011	3.3%	3.0%	2.3%	0.8%



“Whether we’re talking about socks or stocks, I like buying high quality merchandise when it’s marked down...” – Berkshire Hathaway, Annual Report, 2008

- Warren Buffett has been characterized as the world's greatest investor.
- \$1 invested in *BRK* in November 1976 would be worth more than \$1,500 at year-end 2011
- BRK achieved an average annual excess return of 19% above the risk-free rate, compared to a 6.1% excess return of general stock market.
- A Sharpe ratio at 0.76 is higher than any stock or mutual fund with a history of more than 30 years and in the 99th percentile of all stocks with a least 10 year history.

## 2.4.2 A Study by Frazzini/Kabiller/Pedersen

<https://copy.com/8TM4BQHUSCSd>

(An Economist article) <https://copy.com/aCKeENPpVnjI>

	Berkshire stock 1976 - 2011			13F portfolio 1980 - 2011		
Alpha	<b>12.1%</b> (3.19)	<b>9.2%</b> (2.42)	6.3% (1.58)	<b>5.3%</b> (2.53)	3.5% (1.65)	0.3% (0.12)
MKT	<b>0.84</b> (11.65)	<b>0.83</b> (11.70)	<b>0.95</b> (10.98)	<b>0.86</b> (21.55)	<b>0.86</b> (21.91)	<b>0.98</b> (20.99)
SMB	<b>-0.32</b> (-3.05)	<b>-0.32</b> (-3.13)	-0.15 (-1.15)	<b>-0.18</b> (-3.14)	<b>-0.18</b> (-3.22)	0.00 (0.02)
HML	<b>0.63</b> (5.35)	<b>0.38</b> (2.79)	<b>0.46</b> (3.28)	<b>0.39</b> (6.12)	<b>0.24</b> (3.26)	<b>0.31</b> (4.24)
UMD	0.06 (0.90)	-0.03 (-0.40)	-0.05 (-0.71)	-0.02 (-0.55)	<b>-0.08</b> (-1.98)	<b>-0.10</b> (-2.66)
BAB		<b>0.37</b> (3.61)	<b>0.29</b> (2.67)		<b>0.22</b> (4.05)	<b>0.15</b> (2.58)
QMJ			<b>0.43</b> (2.34)			<b>0.44</b> (4.55)
R2 bar	0.25	0.27	0.28	0.57	0.58	0.60

From the paper's Table 4.

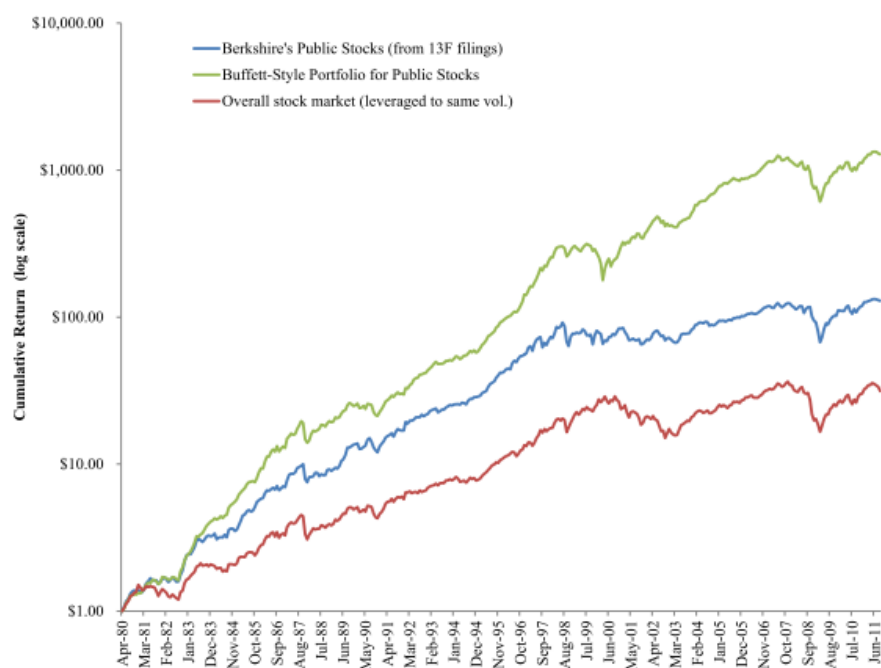
Note: UMD (Up Minus Down) is the momentum factor (MOM).

- “Betting-Against-Beta ( $BAB$ )” and “Quality Minus Junk ( $QMJ$ )” factors can explain a great deal of Buffett’s alpha.
  - $QMJ$  factor combines information about firms’ Profitability, Growth, Safety, and Payout.
  - $BAB$  factor works because many investors have leverage aversion. They tend to overpay for stocks that have higher betas.
- Buffett’s  $\alpha$  is not reliably different from zero when we add these two factors to the Fama-French-Carhart 4 factor model.
  - A large portion of Buffett’s  $\alpha$  can be attributed to the portfolio’s exposures to  $HML$ ,  $BAB$ , and  $QMJ$  factors.



- Buffett also employs financial leverage very effectively.
- A systematic portfolio formed on these factors would have dominated Buffett's portfolio performance, though portfolio compositions can be very different.

**Panel A: Berkshire's Public Stocks and Buffett-Style Portfolio**



From the paper's Figure 3

## 2.5 Back to the Second Revolution:

### Drivers of Stock Returns

#### 2.5.1 Return Decomposition

We can express stock returns as follows:

$$\begin{aligned} \text{Return} = & \text{Expected return} \\ & + \text{Effects of an increase in expected CF growth} \\ & - \text{Effects of an increase in expected return} \end{aligned}$$

Example: Gordon Growth model:

$$P_0 = \frac{D_0 (1 + g)}{\bar{r} - g}; \quad \bar{r} - g > 0.$$

- An increase in  $g$  (expected growth) has a large positive effect on the stock return.

$$\frac{1}{P_0} \frac{\partial P_0}{\partial g} = \frac{(1 + \bar{r})}{(\bar{r} - g)(1 + g)} \gg 0$$

- Stock prices change a lot at earnings announcements.

- An increase in  $\bar{r}$  (expected return=discount rate) has a large negative effect on the stock return.

$$\frac{1}{P_0} \frac{\partial P_0}{\partial r} = -\frac{1}{(\bar{r} - g)} \ll 0$$

Stock prices reflect our expectations (forecasts, predictions) that change with the arrival of information (news).  
e.g. Earnings announcements, macroeconomic announcements, etc.

$$\frac{\Delta P}{P_0} \approx \frac{(1 + \bar{r})}{(\bar{r} - g)(1 + g)} \Delta g - \frac{1}{(\bar{r} - g)} \Delta \bar{r}$$

A high unexpected stock return indicates

- a good news about the stock's fundamentals (expected cash-flow growth), or/and
- a decline in the stock's expected return (discount rate).

Market Efficiency implies that the effects of news are unpredictable, because investors use all available information to predict these effects in advance.

For example:

- A firm's reported earnings exceeded analysts' forecasts. (A positive earnings surprise.)

⇒ A good news about the firm's fundamentals, driving up the stock price.

- The Fed announced to increase the target Fed Funds rate more than the market had anticipated.

⇒ An increase in expected return (discount rate) drives stock prices down.

The monetary policy change (an increase in the FF rate) exerts contractionary effects on expected CF growth of many firms.

⇒ A decrease in expected CF growth drives stock prices down.

- Suppose that unemployment rate was much higher than anticipated. How would this news affect stock prices?

## 2.5.2 Excess Volatility and the Second Revolution

Stock returns are much more volatile than the effects of firm fundamental changes.

- Professor Robert Shiller's (another 2013 Nobel Laureate) "Excess Volatility Puzzle" means that stock returns are too volatile to be explained by the effects of news about firm fundamentals.

Excess volatility indicates that expected returns change over time (due to investors' trading behavior)

- Investors' risk preferences may change over time. They are more risk averse (and hence require higher expected returns) in recessions than in economic booms.

- Booms: Risk premiums are low, discount rates are low, and stock prices are high.
- Recessions: Risk premiums are high, discount rates are high, and stock prices are low.
- Investors also care about various sources of risk, such as market illiquidity, funding risk, etc. Changes in these conditions tend to affect expected returns.

Even if the market is efficient, we may still be able to predict stock returns when expected returns vary over time and across stocks (beyond what the CAPM or other benchmark models predict.)  $\Rightarrow$  Prof. Cochrane called this “Second Revolution.”

## 2.5.3 Measures of Investors' Risk Perception

- *VIX* (S&P500 option implied volatility) tends to increase [decrease] when investors become more bearish [bullish].



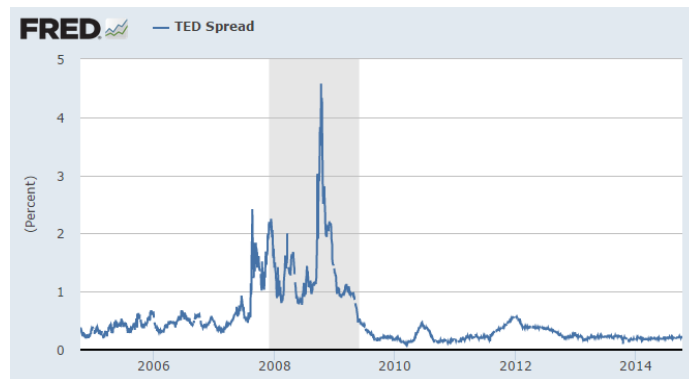
Past 10 years



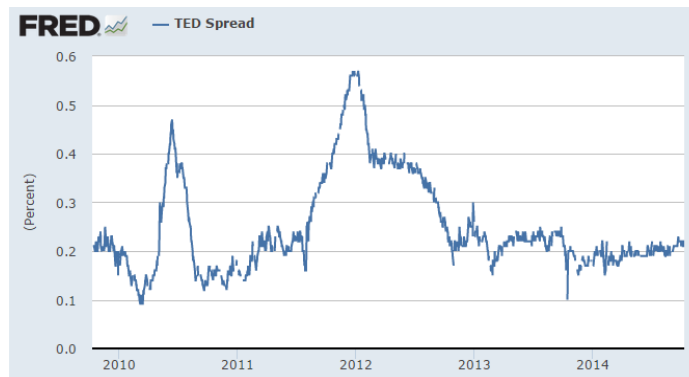
Past 5 years



- TED spread (3-month LIBOR minus 3 month T-bill rates) captures the funding risk. An increase in TED spread implies more difficult funding (e.g. for portfolio leveraging).



Past 10 years



Past 5 years

- Bond credit spread (yield spread between Baa Corporate Bonds and 10-year T-bonds). A measure of investors' fear for default risk.



Past 10 years



Past 5 years

## **3 “Five Principles to Hold Onto”**

### **3.1 Even When Your Boss Says the Opposite – Siegel/Waring/Scanlan (2009)**

This article argues for the importance of separating between “alpha” and “beta.”

In appearance, their argument looks contradictory to Cochrane’s argument. How can we reconcile them?

In fact, Cochrane and Siegel/Waring/Scanlan use the “Alpha” terminology to mean different things.

### 3.1.1 Different Definitions of “Alpha”

**“Alpha” by Cochrane:** For Cochrane, alpha is a measure of “model mispricing.” For example, take a look at the following expression

$$E[r_i] = \underbrace{\alpha}_{\text{“alpha”}} + \underbrace{\beta_{1,i}\lambda_1 + \beta_{2,i}\lambda_2 + \cdots + \beta_{k,i}\lambda_k}_{\text{exp. ret. from a } k\text{-factor benchmark model}},$$

where  $\lambda_1, \dots, \lambda_k$  are factor risk premiums.

- $\alpha$  is the “pricing error” of the benchmark model.
- $\alpha$  may indicate either the market irrationality (when the benchmark model is correct) or the model deficiency (even when the market is efficient). The Joint Hypothesis problem.  $\rightarrow$  “The Alpha/Beta separation is meaningless.”

**“Alpha” by Siegel/Waring/Scanlan:** For Siegel/Waring/Scanlan, alpha is the active portfolio return:

$$r_{portfolio} = r_{benchmark} + r_{active} \text{ “alpha”}$$

- $r_{benchmark}$  is the return on a passive strategy buying and holding a benchmark index (typically a market index).  $r_{active}$  is the active portion of  $r_{portfolio}$ .
- Any portfolios have the benchmark component and the active portfolio component (=deviation from the benchmark).

$$\begin{bmatrix} \vdots \\ w_{portfolio,i} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ w_{benchmark,i} \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ w_{active,i} \\ \vdots \end{bmatrix}$$

where  $\sum_{i=1}^n w_{portfolio,i} = 1$ ,  $\sum_{i=1}^n w_{benchmark,i} = 1$ , and  $\sum_{i=1}^n w_{active,i} = 0$ .

- The active portfolio is a zero-cost long-short portfolio.

Assets	Portfolio	Benchmark	Active
Appli	40	40	0
Bank Ameria	0	30	−30
Cock Cola	40	20	+20
Dismey	20	10	+10
Total	100	100	0

- Siegel/Waring/Scanlan’s “alpha” is the return on the active portfolio.

$$r_{\text{active}}^{\text{“alpha”}} = \sum_{i=1}^n w_{\text{active},i} \times r_i.$$

- $r_{\text{active}} > 0$  (positive “alpha” in Siegel/Waring/Scanlan’s terminology) means that the portfolio return exceeded the benchmark return. This does not necessarily imply the market’s “mispricing” (to which Cochrane applies the “alpha” terminology).

- The Information Ratio ( $IR$ ) is often calculated casually as

$$IR = \frac{E[r_{active}]}{St.Dev.[r_{active}]}$$

- This casual definition does not depend on a particular benchmark model (e.g. CAPM, Fama-French factor model). This is practically appealing. (We do not need to agree on the benchmark model.)
- It is the Reward-to-Risk ratio (e.g. Sharpe ratio) for the active return.

An important insight is that the active portfolio (the alpha seeking part) is a **zero-sum game**.

- When we aggregate all competitors' portfolios, we arrive at the benchmark portfolio.
- If our portfolio return exceeds the benchmark return, someone's portfolio return must be lower return than the benchmark.
- With fierce competition in the market, winning in the zero-sum game consistently is extremely difficult.



## 3.2 Active Bets in Portfolios

Siegel/Waring/Scanlan's "alpha" comes from "active bets" (deviations from the benchmark portfolio).

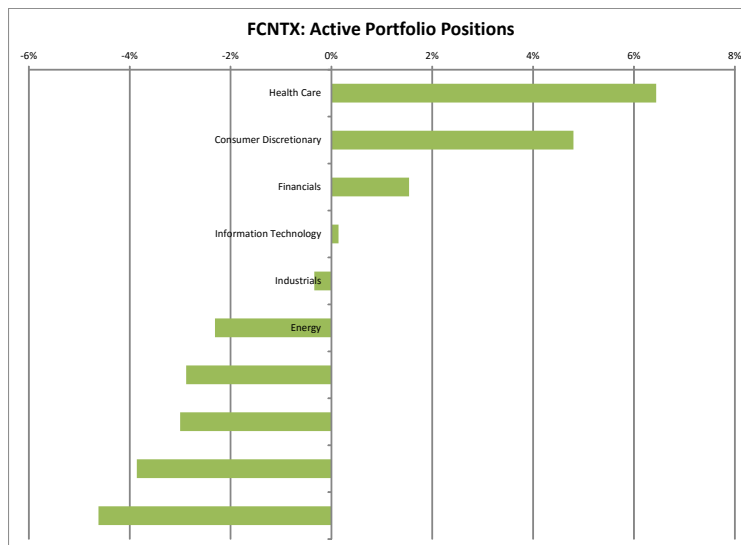
### 3.2.1 Fidelity Mutual Funds Examples

Let's look at sector allocations of Fidelity's three major large-cap mutual funds. (Sources: [fundresearch.fidelity.com](http://fundresearch.fidelity.com))

**Fidelity Contrafund (*FCNTX*):** *FCNTX* is a gigantic mutual fund with net assets of \$108.47 billion (as of 9/30/2014). Net expense ratio is 0.67%. Turnover rate is 47%. Morningstar rating: 4 stars.

AS OF 9/30/2014

	Portfolio Weight	S&P 500	
Information Technology	26.10%	19.66%	
Consumer Discretionary	16.50%	11.70%	
Financials	16.48%	16.34%	
Health Care	15.43%	13.89%	
Consumer Staples	6.67%	9.55%	
Industrials	6.42%	10.28%	
Energy	5.08%	9.70%	
Materials	3.11%	3.45%	
Telecommunication Services	0.12%	2.43%	
Utilities	0.00%	3.00%	
Other	0.00%	0.00%	



At the sector level, *FCNTX* places:



- active bets on Information Technology (26.10% – 19.66%  $\doteq$  +6.4%), Consumer Discretionary (+4.8%), and Financials (+1.5%); and

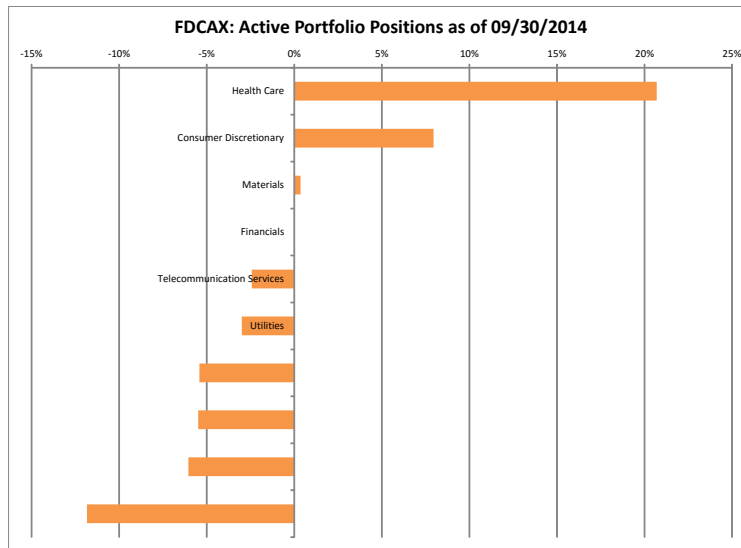
- active bets against Utilities (−4.6%), Telecommunication Services (−3.9%), Materials (−3.0%), Energy (−2.9%) and Industrials (−2.3%).

## **Fidelity Capital Appreciation Fund (*FDCAX*):** *FDCAX*

is another large mutual fund with net assets of \$8.4 billion (as of 9/30/2014). Net expense ratio is 0.79%. Turnover rate is 110%. Morningstar rating: 4 stars.

AS OF 9/30/2014

	Portfolio Weight	S&P 500	
Health Care	34.59%	13.89%	
Consumer Discretionary	19.65%	11.70%	
Financials	16.33%	16.34%	
Information Technology	7.82%	19.66%	
Industrials	4.86%	10.28%	
Energy	4.21%	9.70%	
Materials	3.81%	3.45%	
Consumer Staples	3.50%	9.55%	
Telecommunication Services	0.00%	2.43%	
Utilities	0.00%	3.00%	
Other	0.00%	0.00%	



At the sector level, *FDCAX* places:

- active bets on Healthcare (+20.7%) and Consumer Discretionary (8.0%); and
- active bets against Information Technology (−11.8%), Consumer Staples (−6.1%), Energy (−5.5%), Industrials (−5.4%), Utilities (−3.0%), and Telecommunication Services (−2.4%).
- *FDCAX* makes more active sector bets than *FCNTX*.

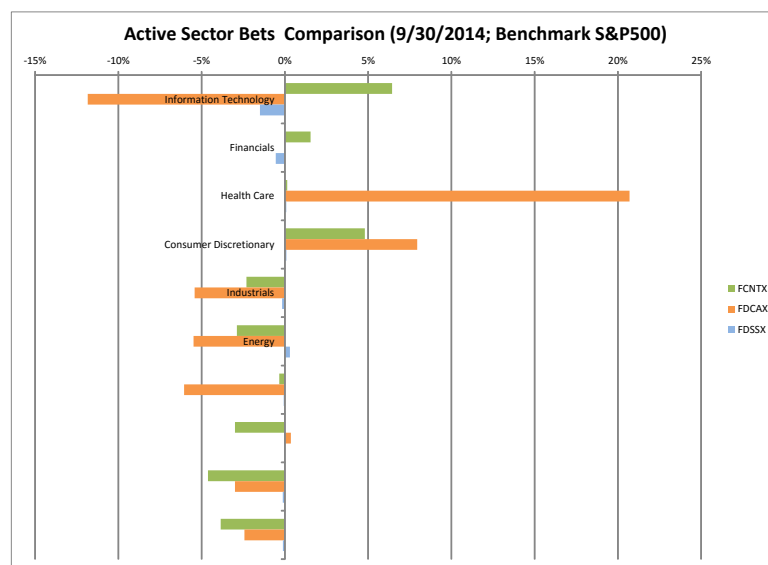
## Fidelity Stock Selector All Cap Fund (*FDSSX*):

*FDSSX* is another large mutual fund with net assets of \$5.1 billion (as of 9/30/2014). Net expense ratio is 0.71%. Turnover rate is 11%. Morningstar rating: 3 stars.



*FDSSX*'s sector allocation does not deviate much from that of the benchmark (S&P500).

- *FDSSX* does not appear to seek active sector bets. Its focus is on picking stocks within each sector, without making large sector bets.
- On the other hand, *FCNTX* and *FDCAX* employ sizable sector bets.



- *FDSSX* has much lower turnover rates (only 11%) than *FCNTX* or *FDCAX*.
- Is *FDSSX* a “Closet Indexer”?

### 3.2.2 Active Shares and “Closet Indexing”

We can relate the Siegel/Waring/Scanlan’s discussion to a recent topic discussed in a FT article, *Active Fund Managers are Closet Index Huggers*” (March 12, 2014).

<https://copy.com/173U43AA64VK>

- “Active Share” =  $\frac{1}{2} \sum_{i=1}^n |w_{active,i}|$

**Figure 1.** A hypothetical active share calculation

Consider a ten-security benchmark index and a portfolio that invests in five of those securities. For simplicity, we assume both are equally weighted. The active share of such a portfolio would be 50%.

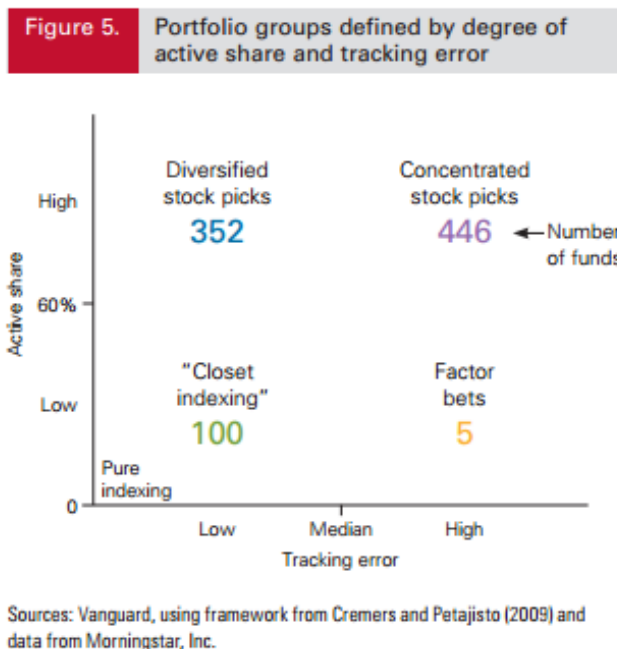
Security	Portfolio weighting	Benchmark weighting	Active share
1	20%	10%	5%
2	20%	10%	5%
3	20%	10%	5%
4	20%	10%	5%
5	20%	10%	5%
6	—	10%	5%
7	—	10%	5%
8	—	10%	5%
9	—	10%	5%
10	—	10%	5%
			50%

Note: These results are hypothetical and do not represent any particular mutual fund.

Source: Vanguard.

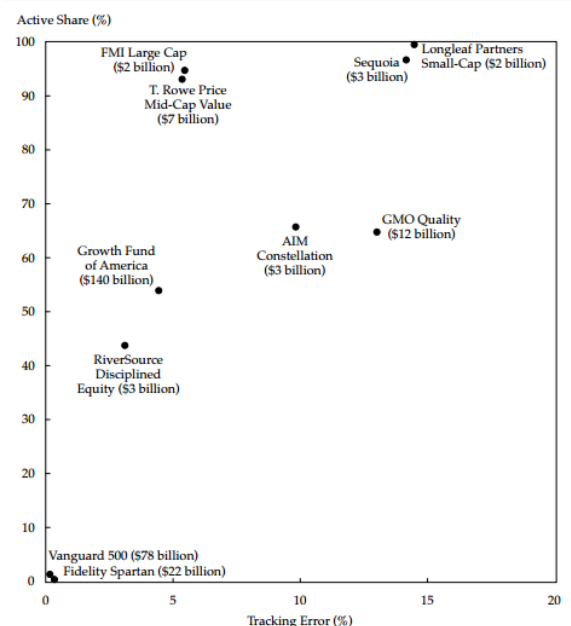
Active Share calculation example

- $r_{active}$  is the deviation of the portfolio return from the benchmark return, and closely related to the Tracking Error. Tracking Error is usually measured by the standard deviation of  $r_{active}$ ,  $St.Dev.(r_{active})$ .



Source: Vanguard

**Figure 2.** Examples of Funds in Each Category, 2009



From Petajisto (2013)



- Active funds have struggled, despite high fees, in recent years.

b. Average measures of active management and expense ratio by active management group

Evaluation period (December 1, 2001–December 31, 2005)

Performance period (December 1, 2006–December 31, 2011)

	Evaluation period				Performance period		
	Concentrated	Diversified	"Closet indexing"		Concentrated	Diversified	"Closet indexing"
Active share	87.62%	77.98%	51.91%	Excess return	−0.77%	−0.42%	−1.22%
Concentration	33.84%	27.48%	26.69%	Tracking error	6.44%	4.68%	3.36%
Style drift	22.30%	13.82%	10.29%	Information ratio	−0.12	−0.09	−0.36
Excess return	2.96%	0.11%	−0.67%				
Tracking error	9.84%	4.88%	3.50%				
Expense ratio	1.37%	1.18%	0.99%				
Information ratio	0.30	0.02	−0.19				

Notes: Expense ratios are based on the five-year average from the evaluation period. Portfolios classified as diversified outperformed for the three years ended December 2003 and then underperformed for the subsequent two years, leading to slight outperformance over the evaluation period.

Sources: Vanguard calculations, using data from Morningstar, Inc.

Source: Vanguard

- Here a portfolio's Information Ratio (IR) is calculated as  $\frac{Average(r_{active})}{St.Dev.(r_{active})}$  (a casual way).
- Many "active" funds charge high fees though they barely vary from the index. (*Closet Index Huggers.*)

### 3.2.3 What Active Bets Does GMVP Imply?

GMVP (Global Minimum Variance Portfolio) involves an **active bet against the market beta** as demonstrated below:

GMVP weight

$$w_{GMVP} = (\mathbf{1}'\Sigma^{-1}\mathbf{1})^{-1}\Sigma^{-1}\mathbf{1} = k\Sigma^{-1}\mathbf{1}$$

with  $k \equiv (\mathbf{1}'\Sigma^{-1}\mathbf{1})^{-1}$ .

Let  $w_{mkt}$  be the market benchmark weight. Then,

$$\begin{aligned} w_{mkt} &= \Sigma^{-1} \cdot \Sigma \cdot w_{mkt} \\ &= \Sigma^{-1} \cdot \underbrace{[Cov(r_i, r_{mkt})]}_{\text{N vector of covariances}} \\ &= \Sigma^{-1} \cdot \underbrace{\beta}_{\text{N vector of betas}} \times \sigma_{mkt}^2 \end{aligned}$$

where  $\sigma_{mkt}^2 \equiv \mathbf{1}'\Sigma w_{mkt}$  is the return variance of the market benchmark.

The active portfolio position of the GMVP is

$$\begin{aligned} w_{GMVP} - w_{mkt} &= k\Sigma^{-1}\mathbf{1} - \sigma_{mkt}^2\Sigma^{-1}\beta \\ &= \sigma_{mkt}^2\Sigma^{-1}[c\mathbf{1} - \beta] \\ &\propto \Sigma^{-1} \underbrace{[c\mathbf{1} - \beta]}_{\text{a bet against beta}} \end{aligned}$$

where  $c \equiv \frac{k}{\sigma_{mkt}^2}$  is a scalar constant.

### 3.2.4 “Portable Alpha”

Taking Siegel/Waring/Scanlan’s point further, any portfolios can be viewed as

$$\text{portfolio} = \text{index fund} + \text{long/short hedge fund}$$

- An investor can use index funds and/or index derivatives (e.g. S&P500 futures, total return swaps) to achieve her/his desired beta exposure.
- For example, the investor can short index futures to make the portfolio “market neutral” to extract its “pure alpha.” This process is known as “**Portable Alpha.**”

- Portable Alpha would probably be better named “*Portable Beta*” since the investor typically uses index derivatives (e.g. futures, swaps) to salt and pepper the portfolio to taste with beta exposures.
- Siegel/Waring/Scanlan.

### Example

Suppose the excess return of a \$1MM portfolio is described as

$$r_p - r_f = \alpha_p + 0.88(r_{SP} - r_f) + \varepsilon_p,$$

where  $r_{SP}$  is the return on the S&P500 index. The portfolio's beta (with respect to the S&P500 index) is 0.88.

In this case, we can short  $\$0.88MM$  worth of S&P500 futures to create a market-neutral pure alpha-seeking portfolio.

	Excess Return
Portfolio ( $\$1\ MM$ )	$\alpha_p + 0.88 (r_{SP} - r_f) + \varepsilon_p,$
Short S&P500 futures (notional $-\$0.88MM$ )	$-0.88 (r_{SP} - r_f)$
Combined	$\alpha_p + \varepsilon_p$

- The futures requires zero cash transaction today (though you need to put some cash collateral called margins).
- The combined portfolio has no exposure to S&P500. It is called “market neutral.”
- Expected return on the combined portfolio is  $\alpha_p$ . This portfolio allows one to extract the “pure alpha.”

The IR of the portfolio is

$$IR = \frac{\alpha_p}{St.Dev.(\varepsilon_p)}.$$

- In this case, IR and the Sharpe ratio are the same (because the portfolio is neutral to the benchmark.).

Alternatively, we can long \$1.12MM worth of S&P500 futures to make the portfolio's beta to 2.0 (if we want to have such high market exposure), for example.

	Excess Return
Portfolio (\$1 MM)	$\alpha_p + 0.88 (r_{SP} - r_f) + \varepsilon_p,$
Long S&P500 futures (notional \$1.12MM)	$+1.12 (r_{SP} - r_f)$
Combined	$\alpha_p + 2.0 (r_{SP} - r_f) + \varepsilon_p$

## 4 Active Portfolio Management



Recall

$$r_{portfolio} = r_{benchmark} + r_{active}$$

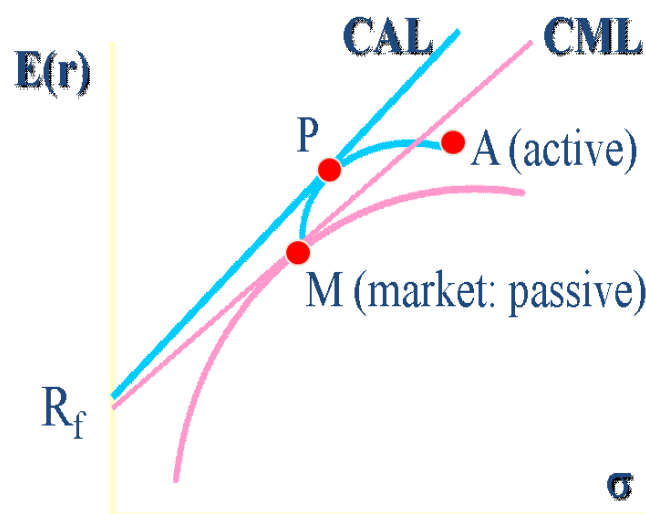
- Factor/Style Investing strategies involve Active Portfolios (implicit bets on specific factors/styles).
- Factor/Style Investing strategies = **Rule-based Active Strategies**

⇒ Understanding the Active Portfolio Management is important for factor/style investing.

## 4.1 The Treynor-Black Model

### 4.1.1 Main Insights

- A classic (1973) model to combine optimally an actively managed portfolio of stocks with a passively managed market index portfolio – assuming a single index (CAPM) model.
- The idea is summarized by the following diagram.





The maximal Sharpe ratio of the optimally combined portfolio (active + passive), denoted by  $SR_P$ , satisfies:

$$\max SR_P^2 = SR_M^2 + IR_A^2$$

we can achieve  $\Rightarrow SR_P > SR_M$  when  $IR_A \neq 0$ .

- $SR_M$  is the Sharpe ratio of the market index:

$$SR_M = \frac{E(r_M) - r_f}{\sigma_M}.$$

- $IR_A$  is the Information Ratio of the active portfolio:

$$IR_A = \frac{\alpha_A}{\sigma_{\varepsilon_A}}.$$

### **4.1.2 The Theory/Recipe**

Jack Treynor and Fischer Black (1973), “How to use security analysis to improve portfolio selection,” *Journal of Business*.

#### **Assumptions**

- Single index model using a market index (the passive portfolio) as the single factor (benchmark).
- Analysts have some ability to find a few undervalued securities.
- Portfolio managers can estimate the expected return and risk, and the abnormal performance (alpha) for the actively-managed portfolio.

- Portfolio managers can estimate the expected risk and return parameters for a broad market (passively managed) portfolio.

## Single Index Model example

- The single index model (market model)

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + \varepsilon_i.$$

implies

$$E[r_i] - r_f = \alpha_i + \beta_i(E[r_M] - r_f)$$

- “Consensus” return forecast is  $\beta_i(E[r_M] - r_f)$
- Your fundamental analysis makes additional prediction of  $\alpha_i$ .

## Fundamental Analysis and Alpha

- Macroeconomic analysis is used to estimate the risk premium and risk of the market index.
  - The market-driven expected return is conditional on information common to all securities.
  - Recall that, under the CAPM, the market risk premium depends on both the investors' risk aversion and expected (perceived) variance of the market.
- We also use statistical analysis to estimate the beta coefficients of all securities and their residual variances,  $\sigma_{\varepsilon_i}^2 = Var(\varepsilon_i)$ .

- Using various security-valuation models, we analyze security specific expected return forecasts,  $\alpha_i$ .
- In the actively managed portfolio  $A$ , optimal weights should be proportional to

$$w_i^A \propto \left[ \frac{\alpha_i}{\sigma_{\varepsilon_i}^2} \right]$$

where  $\sigma_{\varepsilon_i}^2$  is the idiosyncratic variance of security  $i$ .

- This weight is similar to the optimal weight in the standard mean-variance portfolio optimization (with zero correlation),

$$w_i \propto \left[ \frac{E(r_i) - r_f}{\sigma_i^2} \right]$$

## The Actively Managed Portfolio

- We can define the “alpha” of the actively managed portfolio  $A$  by

$$\alpha_A = \sum_i w_i^A \alpha_i.$$

- The active portfolio’s “Beta” is

$$\beta_A = \sum_i w_i^A \beta_i.$$

- We can also define the idiosyncratic variance of the actively managed portfolio  $A$  by

$$\sigma_{\varepsilon_A}^2 = \sum_i \left(w_i^A\right)^2 \sigma_{\varepsilon_i}^2$$

The **Information ratio (IR)** of the actively managed portfolio

$$IR_A = \frac{\alpha_A}{\sigma_{\varepsilon_A}}.$$

- Similar to Sharpe ratio but the information ratio (IR) focuses on the unsystematic component of returns.

### **Optimal Allocation b/w the Active and Passive**

- If the Active portfolio has beta of one ( $\beta_A = 1$ ), the optimal position in the active portfolio is

$$w_A^o = \frac{\left[ \frac{\alpha_A}{\sigma_{\varepsilon_A}^2} \right]}{\left[ \frac{E(r_M) - r_f}{\sigma_M^2} \right]} = \frac{\left[ \frac{IR_A}{\sigma_{\varepsilon_A}} \right]}{\left[ \frac{SR_M}{\sigma_M} \right]}$$

where  $SR_M$  is the Sharpe ratio of the passive portfolio (market index).

- If  $\beta_A \neq 1$ , we make the following adjustment to the optimal weight on the active portfolio.

$$w_A^* = \frac{w_A^o}{1 + (1 - \beta_A)w_A^o}$$

- The optimal weight on the passive benchmark is  $1 - w_A^*$ .

1. Initial position of security $i$ in the active portfolio	$w_i^0 = \frac{\alpha_i}{\sigma^2(e_i)}$	<b>TABLE 27.1</b> Construction and properties of the optimal risky portfolio
2. Scaled initial positions	$w_i = \frac{w_i^0}{\sum_{i=1}^n \alpha_i}$	
3. Alpha of the active portfolio	$\alpha_A = \sum_{i=1}^n w_i \alpha_i$	
4. Residual variance of the active portfolio	$\sigma^2(e_A) = \sum_{i=1}^n w_i^2 \sigma^2(e_i)$	
5. Initial position in the active portfolio	$w_A^0 = \frac{\alpha_A}{E(R_M) - \sigma_M^2}$	
6. Beta of the active portfolio	$\beta_A = \sum_{i=1}^n w_i \beta_i$	
7. Adjusted (for beta) position in the active portfolio	$w_A^* = \frac{w_A^0}{1 + (1 - \beta_A)w_A^0}$	
8. Final weights in passive portfolio and in security $i$	$w_M^* = 1 - w_A^*$ ; $w_i^* = w_A^* w_i$	
9. The beta of the optimal risky portfolio and its risk premium	$\beta_P = w_M^* + w_A^* \beta_A = 1 - w_A^* (1 - \beta_A)$ $E(R_P) = \beta_P E(R_M) + w_A^* \alpha_A$	
10. The variance of the optimal risky portfolio	$\sigma_P^2 = \beta_P^2 \sigma_M^2 + [w_A^* \sigma(e_A)]^2$	
11. Sharpe ratio of the risky portfolio	$S_P^2 = S_M^2 + \sum_{i=1}^n \left( \frac{\alpha_i}{\sigma(e_i)} \right)^2$	



### 4.1.3 Implementation Issues

Similar to Mean-Variance Optimization.

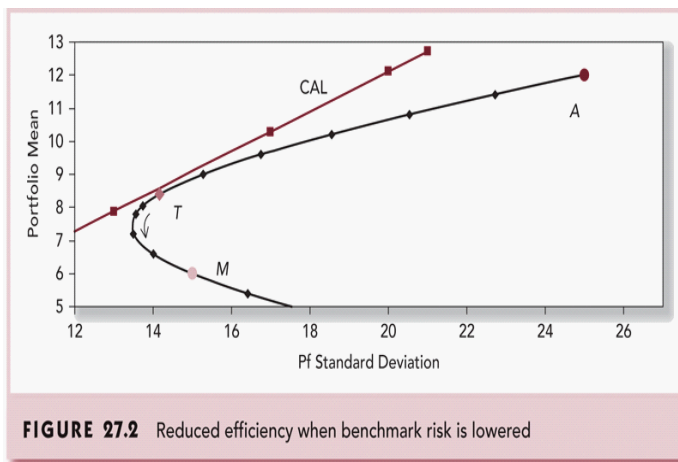
- The implementation of the Treynor-Black framework is difficult, because our ex ante  $\alpha$  forecasts tend to be very poor (noisy).
  - “Garbage in, garbage out.” Poor out-of-sample performance.

Tracking Error Constraints often help.

- Tracking error is defined as: Tracking Error = Portfolio Return – Benchmark Return

$$\begin{aligned} T_E &= r_P - r_M \\ &= \alpha_i + (\beta_i - 1)(r_M - r_f) + \varepsilon_i. \end{aligned}$$

- The portfolio manager must be mindful of the standard deviation of the tracking error (also called the benchmark risk).
- A constraint on the tracking error imposes additional cost, i.e., it may reduce the efficiency of the portfolio.



- This chart is magnified to highlight the effect of the tracking error constraint.
- $A$  is the active portfolio.  $M$  is the benchmark (passive) portfolio.

- The optimal combination is shrunk toward the benchmark  $M$  to mitigate the effects of possible estimation errors.
- **Black-Litterman** framework incorporates the level of confidence in our alpha forecasts.
  - Black-Litterman model is quite sophisticated. Some training in statistics (Bayesian analysis, mixed estimation, generalized least squares) will be helpful to understand the model. Please see the “Asset Allocation” handout for a brief introduction.
- Remember

$$\max SR_P^2 = SR_M^2 + IR_A^2.$$

This can be generalized to multi-factor settings.

## 4.2 The Fundamental Law of Active Management à la Grinold (1989)

$$IR \simeq IC \times \sqrt{BR}$$

Percentile	Information Ratio
90	1.0
75	0.5
50	0.0
25	-0.5
10	-1.0

- $IC$  stands for **Information Coefficient**, which represents your “skill” of security selection.
  - $IC$  is the correlation between your forecast alphas and subsequent firm specific returns.
- $BR$  stands for “breadth”, which is the number of independent bets per period.

## 4.2.1 Understanding IC's

- A simplified example of firm-specific return

$$\text{specific return } r = \begin{cases} +1 & \text{with probability 0.5} \\ -1 & \text{with probability 0.5} \end{cases}$$

- Your task is to forecast the specific return.

$$\text{your forecast } f = \begin{cases} +1 & \text{with probability 0.5} \\ -1 & \text{with probability 0.5} \end{cases}$$

- $IC$  measures your skill of correctly predicting the specific return.

$IC$  is the **correlation** between  $r$  and  $f$ . That is

$$IC = 2\theta - 1$$

where  $\theta = M/N$  is the fraction of correct calls (the “hit ratio”).

Alternatively,

$$\theta = \frac{1 + IC}{2}.$$

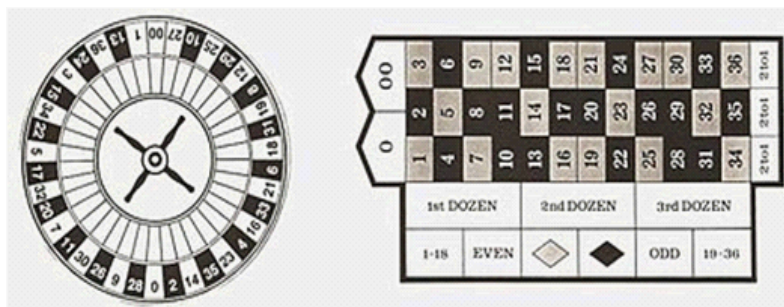
Skill	$IC$	$\theta$
Fantastic	0.10	55.0%
Quite Good	0.05	52.5%
Average	0.00	50.0%

- The following illustrates a great skill ( $IC = 0.087$ ).



- We tend to significantly over-estimate available levels of skill.

## 4.2.2 A Roulette Example: Analysis for the Casino



- Consider an American roulette wheel, with numbers from 1-36, plus 0 and 00. There are 38 numbers in all. We are the casino.
- Bet on Odds. This pays off if the ball lands on an odd number from 1, ... 35. (18 numbers). Payout: one to one.
- Assume that gamblers bet \$1 million on such bets on this wheel over the course of a year. The casino has, therefore, \$1 million at risk.

- $IC = 2\theta - 1 = 0.05264$
- Consider the following three cases.
  1. One \$1 million bet.
  2. Ten thousand \$100 bets.

### One \$1 million bet ( $IC = 0.05264$ )

- The casino wins with probability  $\frac{20}{38} = 52.632\%$

Casino	Probability	Payoff
Wins	$\frac{20}{38} = 52.632\%$	+1 million
Loses	$\frac{18}{38} = 47.368\%$	−1 million

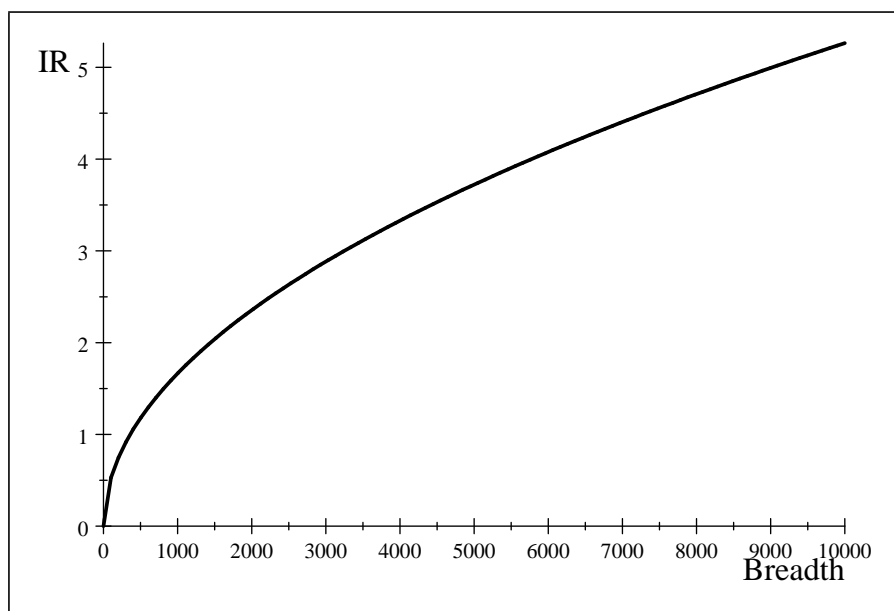
Expected value of the payoff is  $52.632\% - 47.368\% \doteq +5.26\%$  of the bet to the casino ( $-5.26\%$  to the player).

- $BR = 1$ . (one bet)
- $IR = \frac{0.05264}{0.99861} \doteq 0.05271 \approx IC \times \sqrt{BR}$ .



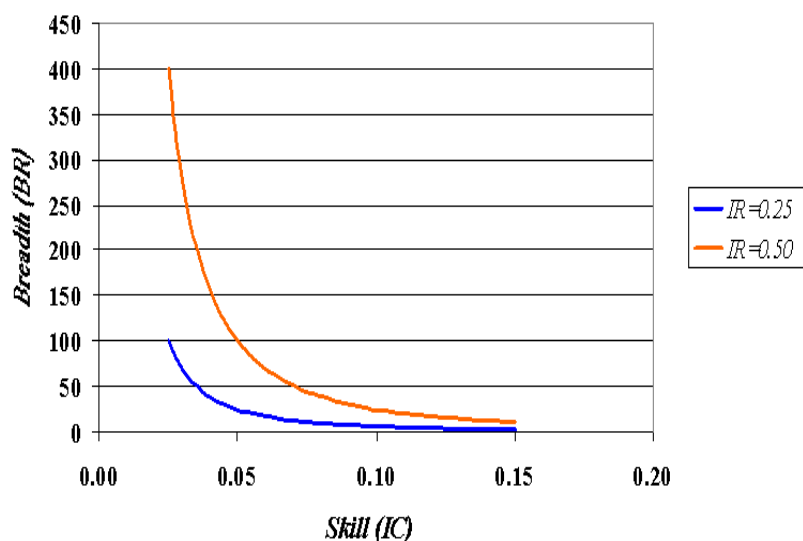
## Ten thousand \$100 bets ( $IC = 0.05264$ )

- $BR = 10,000$ .
- $IR = \frac{5.264}{0.99861} = 5.2713 \approx IC \times \sqrt{BR}$ .



$$IC = 0.05264$$

### 4.2.3 Breadth-Skill Trade-off



- In practice, IC is very low and very difficult to increase (due to fierce competition in the market).
- With technological advance, increasing the number of bets is the key for success.
- **Factor/Style Investing utilizes a larger breadth than conventional stock picking.**

## Example: Stock Selection vs. Market Timing

- A stock selection team follows 400 stocks, with quarterly rebalances based on new information.  $BR = 400 \times 4 = 1,600$  per year.
- The team correctly calls stocks about 51% of the time.  $IC = 2fr - 1 = 0.02$ . (Slightly above average skill)
- This leads to IR of  $0.02 \times \sqrt{1600} = 0.80$ . Great performance.
  - Moderate skill  $\Rightarrow$  Great performance.

- A market timer makes 4 independent calls per year.  
 $BR = 4$ .
- Correct 55% of the time.  $IC = 2fr - 1 = 0.10$ .  
(Fabulous skill)
- This leads to IR of  $0.10 \times \sqrt{4} = 0.20$ . The performance is just above average.
  - Fantastic skill  $\Rightarrow$  Moderate performance.
  - Lower IR doesn't mean that the market timer will not have some great years.

## 4.2.4 Implementation Efficiency

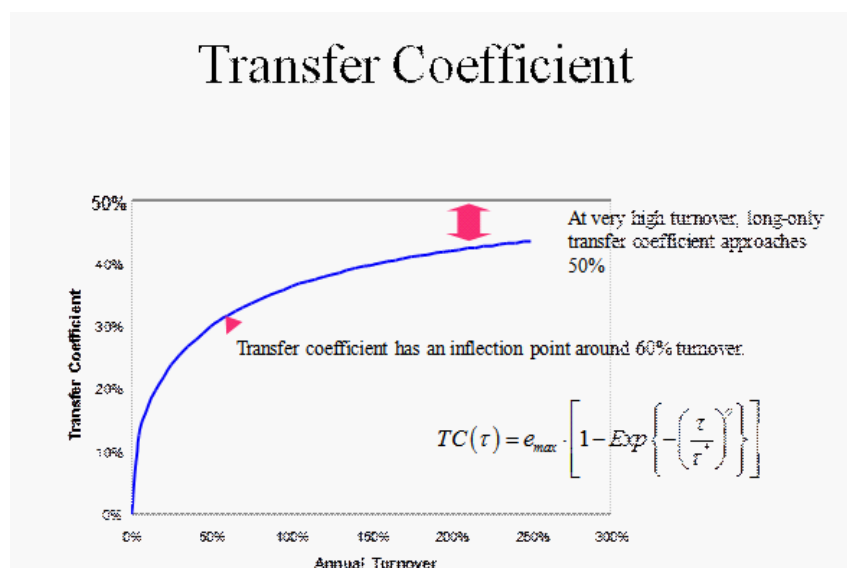
- In practice, constraints and transaction costs keep us from the optimal trade-off between return and risk and hence our actual portfolio  $P$  deviates from the optimal portfolio  $Q$ .
- What is the IR of the actual portfolio  $P$ ?

$$IR_P = IC \times \sqrt{BR} \times TC$$

where  $TC$  is called the **Transfer Coefficient**.

- $TC = \rho_{PQ}$  is the correlation of the returns of the portfolio  $P$  and the idealized portfolio  $Q$ .
  - $TC$  measures the efficiency of our implementation, and has a maximum value of 1.

- $TC$  modifies the fundamental law to account for the effects of constraints and costs.
- Long-only constraint has a significant impact on portfolios. This constraint alone can decrease the transfer coefficient from 1 to 0.5.
- In fact,  $TC \approx 0.3$  is typical for a long-only fund.
- $TC$  tends to increase with Turnover (trading). But higher turnover implies higher transaction costs.



(In the chart, you can ignore the formula for  $TC(\tau)$  or the inflection point.)

- In the presence of transaction costs, we often need to constrain our turnover (by rebalancing less frequently than desired), which lowers the  $TC$  and hence  $IR_P$ .

#### Example

- Long-only stock selection fund with  $IC = 0.05$ ,  
 $BR = 400 \times 4$ ,  $TC = 0.3$ .

$$IR_{SP} = 0.05\sqrt{1600} \times 0.3 = 0.60$$

- Long-only asset allocation fund with  $IC = 0.10$ ,  
 $BR = 6 \times 12$ ,  $TC = 0.3$ .

$$IR_{AA} = 0.10\sqrt{72} \times 0.3 = 0.25$$

- Suppose we convert the asset allocation fund to a global macro hedge fund, which utilizes futures contracts (both long and short positions with low trading costs).  $TC : 0.3 \rightarrow 0.8$ .

$$IR_{GM} = 0.10\sqrt{72} \times 0.8 = 0.68$$

- This is related to the “Portable Alpha” strategy described above.



## 4.3 Value Strategies (Examples)

### 4.3.1 The Gordon Constant Growth Model

$$P_0 = \frac{D_1}{E[r] - g}$$

where  $D_1$  is the dividend forecast and  $g$  is the dividend growth rate.  $E[r]$  is the stock's expected return.

This implies:

$$E[r] = \frac{D_1}{P_0} + g$$

$$\frac{D_1}{P_0} = E[r] - g.$$

A high  $\frac{D_1}{P_0}$  signals

- a high expected return,
  - possibly due to a high risk premium (a rational story). The “true” risk premium of a stock may be higher than what existing theories can predict.
  - possibly due to mispricing (#1): Under-valued stocks have expected returns (discount rates) that are too high. Repricing (price correction) will lower their expected returns and hence increase their stock prices.
  - possibly due to mispricing (#2): Stocks may be under-valued due to under-estimated growth. As the growth estimates are revised upward, their prices will increase.

→ Returns of these stocks are expected to be higher.

- or, a (truly) low dividend growth: a high  $\frac{D_1}{P_0}$  may simply imply a low growth.

To the extent that high  $\frac{D_1}{P_0}$  ratios reflect high expected returns, the  $\frac{D_1}{P_0}$  ratio is correlated with future returns.

- We expect a positive IC for the  $\frac{D_1}{P_0}$  ratio.
- To the extent that this is true, buying stocks with high  $\frac{D_1}{P_0}$  ratios and shorting stocks with low  $\frac{D_1}{P_0}$  ratios should generate positive alphas and  $IRs$ .
- Caveats: By buying high  $\frac{D_1}{P_0}$  stocks, we may end up buying stocks with truly low growth potential without high expected returns.

### 4.3.2 More General Valuation Models

There are many valuation models (e.g. Residual Income Valuation models). Let  $V$  be the “fundamental value” of a stock.

- The value  $V$  depends on the book value, earnings, cash-flows, R&D, etc. and the benchmark cost of capital (benchmark expected return).

$$V = f(B, E, CF, RD, \dots; \text{benchmark ER}).$$

- We can then use the ratio  $\frac{V}{P}$  to find under-valued (or over-valued) stocks.
  - High  $\frac{V}{P}$  stocks are likely to have high expected returns.

- Low  $\frac{V}{P}$  stocks are likely to have low expected returns.
- Because  $V$  is linearly homogenous in observed fundamental quantities such as  $B$ ,  $E$ ,  $CF$ , etc., we can approximate  $\frac{V}{P}$  as

$$\frac{V}{P} \approx \text{const} + \frac{\partial V}{\partial B} \frac{B}{P} + \frac{\partial V}{\partial E} \frac{E}{P} + \frac{\partial V}{\partial CF} \frac{CF}{P} + \frac{\partial V}{\partial RD} \frac{RD}{P} + \dots$$

with  $\frac{\partial V}{\partial B}, \frac{\partial V}{\partial E}, \frac{\partial V}{\partial CF}, \frac{\partial V}{\partial RD} > 0$ , meaning that  $\frac{B}{P}, \frac{E}{P}, \frac{CF}{P}, \frac{RD}{P}$ , etc. are potentially useful to identify stocks with high expected returns (under-valued stocks).

- $\frac{B}{P}$  (book to market) is known to predict stock returns well.  $\frac{E}{P}, \frac{CF}{P}, \frac{RD}{P}$  etc. may also exhibit incremental return predictability.

### 4.3.3 Mean Reversion of Valuation Ratios

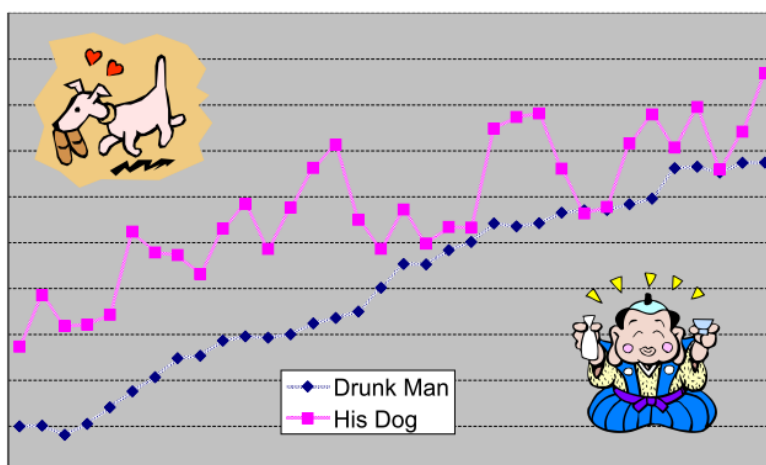
#### A Drunk Man and His Dog

A famous example for co-integration.



- A drunk man is random walking. It is very difficult to forecast his next steps.
- Around the drunk man, his dog is walking randomly.
- Can we predict the direction to which the dog moves next? How?
- This is how I would explain “value signals” to my daughter: Drunk man  $\Rightarrow$  fundamentals; his dog  $\Rightarrow$  equity value.

A Drunk Man and His Dog

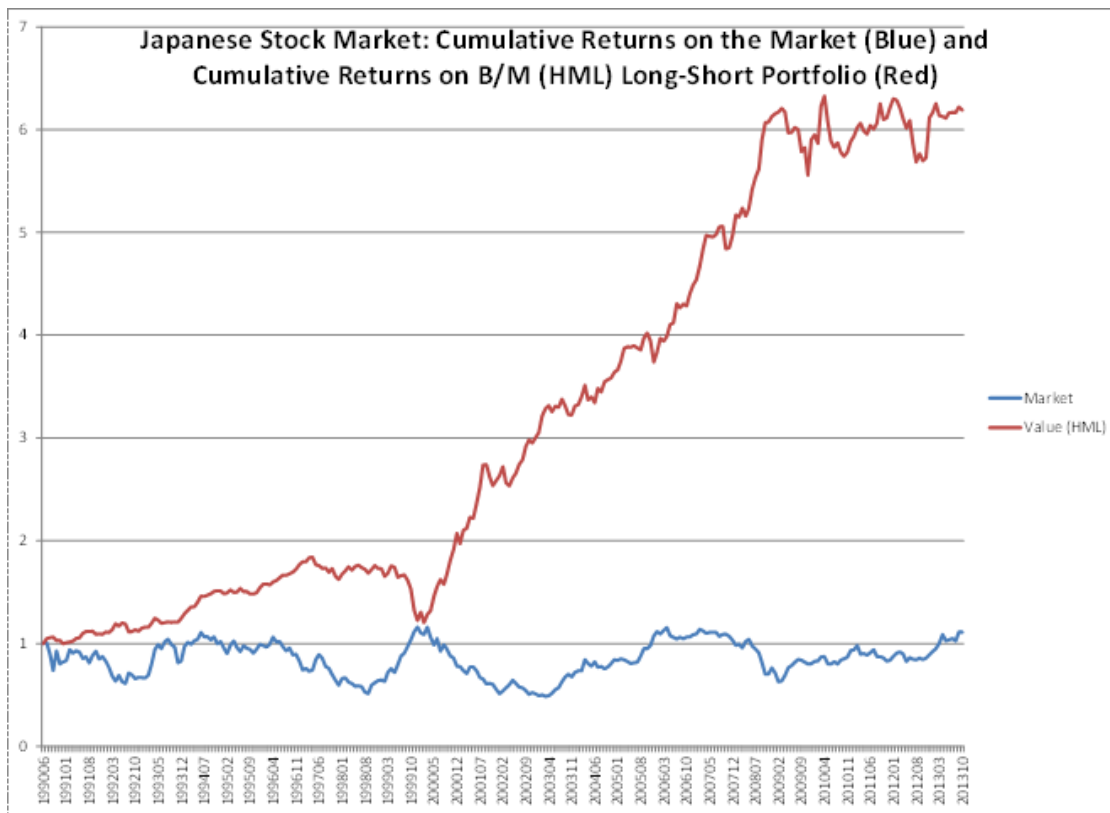


- A high valuation ratio should predict a decline in the numerator or an increase in the denominator.

#### **4.3.4 An Example – Japanese Market**

The graph compares 2 strategies:

- Investing \$1 (or 100 yen) in the market (Blue),
- Investing the same amount in the risk-free asset, long a portfolio of high B/M stocks, and short a portfolio of low B/M stocks (Red).



Value Strategies have worked strongly in Japan (except late 1990s), even though the stock market has been very sluggish over the past 20+ years.



### 4.3.5 An Experiment: Country Selection with Dividend Yields

#### Investment universe and data

- MSCI Country Indexes for 18 developed countries (including the US).
- Standard (Large Cap + Mid Cap) Index data available from 12/1969.

#### Variables Construction

- Trailing 1-year dividend yield (to avoid the effects of seasonality) is

$$DYld_t = \frac{GROSS_t}{GROSS_{t-12}} - \frac{PRICE_t}{PRICE_{t-12}},$$

where *GROSS* is the cumulative gross total return index (with dividends reinvested), and *PRICE* is the price index.

- Trailing 1-year dividend growth rate is

$$DGrwth_t = \frac{DYld_t}{DYld_{t-12}} \times \frac{PRICE_{t-12}}{PRICE_{t-24}}.$$

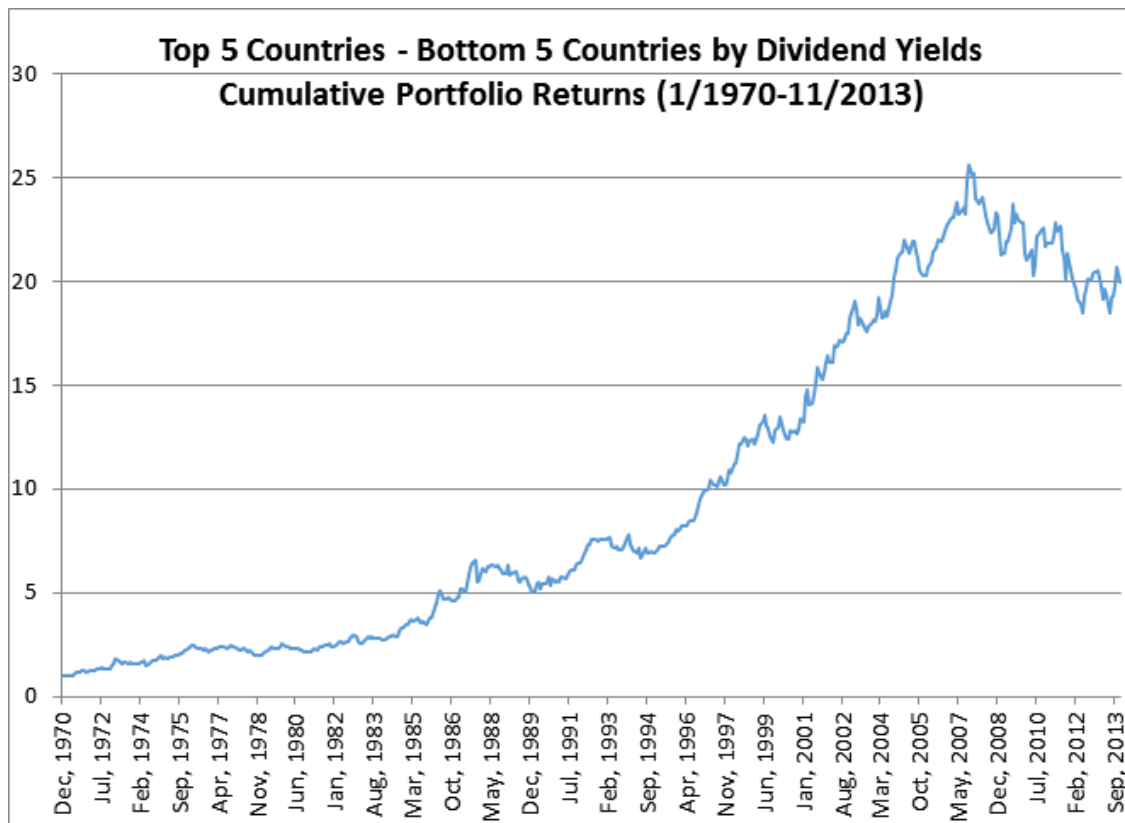
**1st Strategy: Top 5 Minus Bottom 5** Every month, we construct a long-short portfolio as follows:

- A long position in a portfolio of 5 countries (equally-weighted) with the highest dividend yields.
- A short position in a portfolio of 5 countries (equally-weighted) with the lowest dividend yields.

The following figure shows long positions (5 countries) in blue and short positions (5 countries) in red at the end of June in each year.

	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
AUSTRALIA	-1	-1	-1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
AUSTRIA	-1	0	1	0	0	-1	-1	-1	0	-1	-1	0	-1	0	0	-1	-1	0	0	0	-1	-1
BELGIUM	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
CANADA	0	0	-1	1	0	0	0	0	0	0	0	-1	1	0	0	-1	0	0	0	0	0	0
DENMARK	1	1	1	0	1	1	1	0	0	0	1	0	0	-1	-1	-1	0	0	0	-1	-1	-1
FRANCE	0	1	1	0	1	0	1	1	1	0	0	1	0	0	0	0	-1	-1	-1	0	0	0
GERMANY	0	0	0	1	0	0	0	0	-1	0	1	0	0	-1	0	0	-1	-1	-1	0	0	0
HK	1	0	-1	-1	0	0	0	0	-1	1	0	-1	0	1	1	0	1	1	1	1	1	1
ITALY	-1	-1	0	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	0	0	0	-1	0	0	0	0	0
JAPAN	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
NETHERLANDS	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
NORWAY	0	-1	0	-1	-1	-1	0	0	1	0	-1	0	0	0	0	0	0	0	0	-1	-1	-1
SINGAPORE	0	0	0	-1	-1	0	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1
SPAIN	-1	0	0	0	-1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
SWEDEN	0	0	0	1	0	0	0	0	0	0	1	0	0	-1	-1	1	0	0	0	-1	0	0
SWITZERLAND	-1	-1	-1	0	0	-1	-1	-1	-1	-1	-1	0	-1	0	-1	0	0	-1	-1	0	0	0
UK	1	0	0	0	1	1	1	1	0	1	0	1	0	1	0	1	0	0	0	0	0	1
USA	0	-1	-1	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	1	0	0	0
	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	
AUSTRALIA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
AUSTRIA	-1	-1	-1	-1	-1	0	-1	0	1	0	0	0	-1	-1	-1	-1	-1	0	0	-1	0	
BELGIUM	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	-1	-1	0	0	0	
CANADA	0	0	0	0	0	0	0	0	-1	0	-1	-1	-1	-1	-1	-1	0	0	-1	-1	-1	
DENMARK	-1	-1	-1	0	0	-1	-1	0	0	-1	0	0	0	0	0	-1	-1	-1	-1	-1	-1	
FRANCE	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	0	0	0	1	0	1	
GERMANY	0	0	0	-1	-1	0	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	
HK	1	1	0	1	1	0	1	1	0	1	1	1	0	0	0	0	1	0	-1	0	-1	
ITALY	0	0	-1	0	0	0	0	0	1	1	1	0	1	1	1	1	0	0	1	0	0	
JAPAN	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
NETHERLANDS	1	1	1	1	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	
NORWAY	-1	0	0	0	0	-1	0	1	1	1	0	1	1	1	1	1	-1	1	1	1	1	
SINGAPORE	-1	-1	-1	-1	-1	-1	1	-1	0	0	0	0	0	0	1	1	1	1	0	1	0	
SPAIN	1	1	1	1	1	1	0	0	-1	0	0	0	0	1	1	1	1	1	1	1	1	
SWEDEN	0	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	1	
SWITZERLAND	0	0	0	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	0	-1	0	0	0	0	0	
UK	0	0	1	0	1	1	1	1	0	0	1	0	0	1	0	0	1	1	0	1	0	
USA	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	

The combined performance:



- Annualized Sharpe ratio = 0.67.
- Excellent performance until around 2007.
- Terrible performance since the Global Financial Crisis period.

## **2nd Strategy: Top 5 Minus Bottom 5 with Growth**

**Control** Every month, we construct a long-short portfolio as follows:

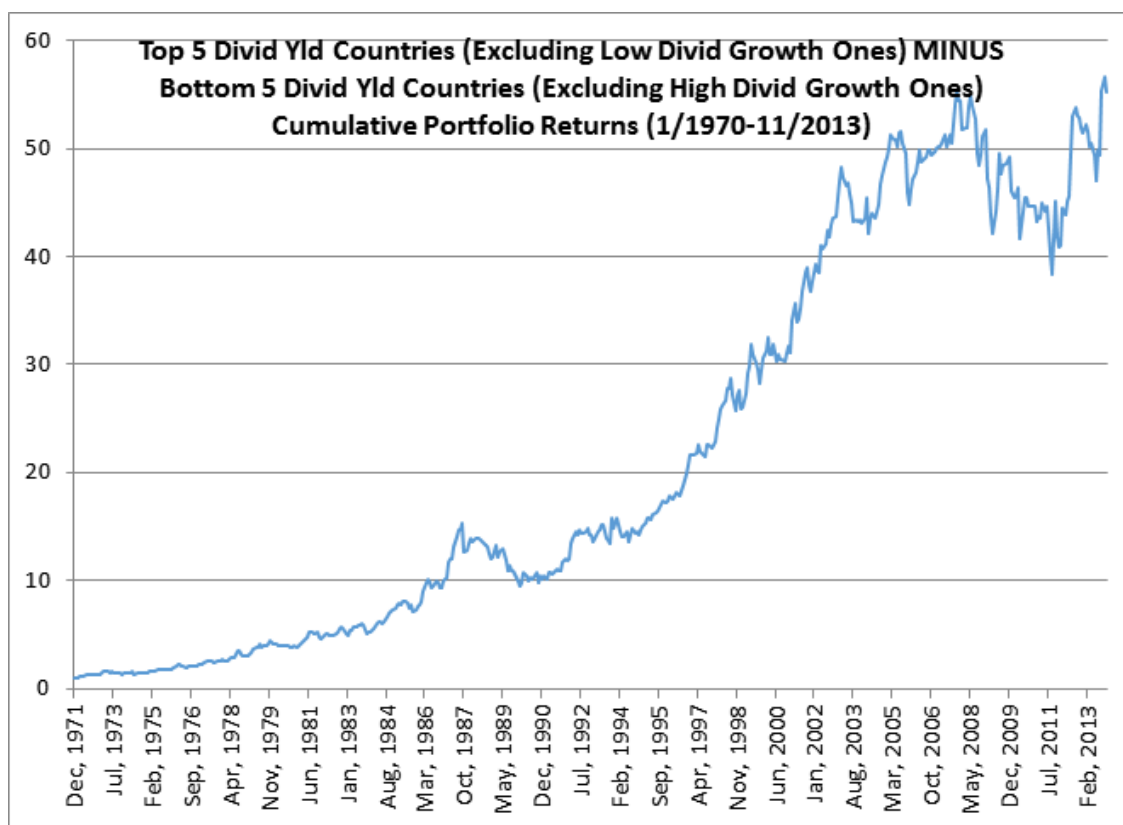
- Select 5 countries with the highest dividend yields, but exclude any countries that fall in the bottom 1/3 of dividend growth. → Construct an equally-weighted portfolio and take a long position.
- Select 5 countries with the highest dividend yields, but exclude any countries that fall in the top 1/3 of dividend growth. → Construct an equally-weighted portfolio and take a short position.

Basically, we prefer countries with high dividend yields except those with very low dividend growth.

The following figure shows long positions ( $\leq 5$  countries) in blue and short positions ( $\leq 5$  countries or less) in red at the end of June in each year.

	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	
AUSTRALIA	0	-1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
AUSTRIA	0	1	0	0	-1	0	-1	0	-1	-1	0	-1	0	0	-1	-1	0	0	0	-1	0	-1	
BELGIUM	1	1	0	1	0	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	
CANADA	0	-1	1	0	0	0	0	0	0	0	-1	1	0	0	-1	0	0	0	0	0	0	0	
DENMARK	1	1	0	1	1	1	0	0	0	1	0	0	-1	-1	-1	0	0	0	-1	-1	-1	0	
FRANCE	1	1	0	1	0	0	1	1	0	0	1	0	0	0	0	-1	0	-1	0	0	0	0	
GERMANY	0	0	1	0	0	0	0	-1	0	1	0	0	-1	0	0	0	-1	-1	0	0	0	0	
HK	0	-1	0	0	0	0	0	-1	1	0	0	0	0	1	0	1	1	0	1	1	1	1	
ITALY	-1	0	-1	-1	0	-1	0	0	-1	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	
JAPAN	1	0	-1	-1	0	-1	0	-1	-1	0	-1	-1	0	-1	0	-1	-1	-1	-1	0	-1	0	
NETHERLANDS	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	0	0	1	0	1	1	
NORWAY	-1	0	-1	-1	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	-1	-1	-1	-1	
SINGAPORE	0	0	-1	0	0	-1	0	-1	0	0	-1	0	-1	-1	-1	0	-1	-1	-1	0	-1	0	
SPAIN	0	0	0	-1	0	0	1	1	0	1	0	0	1	0	1	0	1	0	1	0	1	1	
SWEDEN	0	0	1	0	0	0	0	0	0	1	0	0	-1	-1	1	0	0	0	-1	0	0	0	
SWITZERLAND	-1	0	0	0	-1	-1	0	0	-1	-1	0	-1	0	-1	0	0	-1	-1	0	0	0	0	
UK	0	0	0	1	0	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1	0	
USA	-1	-1	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	1	0	0	0	0	
	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013		
AUSTRALIA	1	0	1	0	0	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1			
AUSTRIA	-1	-1	0	-1	0	-1	0	1	0	0	0	0	-1	-1	0	-1	0	0	-1	0			
BELGIUM	1	1	0	0	1	0	1	1	0	0	0	1	0	0	0	-1	0	0	0	0			
CANADA	0	0	0	0	0	0	0	-1	0	-1	-1	-1	-1	0	0	0	0	-1	-1	-1			
DENMARK	-1	0	0	0	-1	-1	0	0	-1	0	0	0	0	-1	-1	-1	-1	0	0	0			
FRANCE	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	0	0	0	1	0	0			
GERMANY	0	0	-1	-1	0	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0			
HK	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0	1	0	-1	0	-1			
ITALY	0	-1	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0			
JAPAN	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	0	-1	0	-1	0	0	-1	-1	-1	0			
NETHERLANDS	1	1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0			
NORWAY	0	0	0	0	-1	0	1	1	1	0	1	1	0	1	1	-1	1	1	1	1			
SINGAPORE	0	-1	-1	-1	-1	1	-1	0	0	0	0	0	0	1	1	1	1	0	1	0			
SPAIN	0	1	1	1	1	0	0	-1	0	0	0	0	1	1	1	1	0	1	0	0			
SWEDEN	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	1			
SWITZERLAND	0	0	-1	-1	-1	-1	0	0	-1	0	-1	-1	-1	0	-1	0	0	0	0	0			
UK	0	1	0	1	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0			
USA	0	0	0	0	0	0	-1	-1	0	-1	-1	-1	-1	-1	-1	0	-1	-1	0	0			

The combined performance:



- Annualized Sharpe ratio = 0.72. More volatile, but stronger overall performance.
- Dividend growth control appears to help enhance the portfolio performance (at the cost of increased volatility due to weaker diversification).

## 4.4 Piotroski's Value Strategies

Accounting-based fundamentals help enhance value strategies.

- Some value stocks are cheap for good reasons – poor operating performance, poor growth prospects, poor financial health, distressed, etc.
- We would like to buy value stocks with strong fundamentals.

### 4.4.1 Fundamental Scores (F\_Scores)

F\_Scores: How many of the following 9 items do the firms satisfy?



**Profitability** Ability to generate funds internally.

1. *ROA*: return on assets for the last fiscal year (FY1)  
is positive  
$$= \text{net income before extraordinary items} / \text{total assets} > 0$$
2. *CFO*: cash from operations for the last fiscal year  
(FY1) is positive  
$$= \text{CFO} / \text{total assets} > 0$$
3. *DROA*:  $\text{ROA for FY1} > \text{ROA for the FY2}$   
(FY2 is the fiscal year preceding FY1)

4. *ACCRUAL*:  $\text{CFO}/\text{total assets} > \text{ROA}$

- Earnings driven by positive accrual adjustment, i.e.  $\text{profits} > \text{CFO}$ , is a bad signal, suggesting earnings management.

**Financial Leverage/Liquidity**    Capital structure & ability to service debt

1. *DLEVER*: long-term debt to assets ratio for FY1  $<$  long-term debt to assets ratio for FY2
2. *DLIQUID*: Current Ratio for FY1  $>$  Current Ratio for FY2
3. *EQ\_OFFER*: average shares outstanding for FY1  
■ average number of shares outstanding for FY2.

**Operating Efficiency** Trends in profit margin & asset turnover

1. *DMARGIN*: Gross Profit Margin for FY1  $>$  Gross Profit Margin for FY2.
2. *DTURN*: Asset Turnover for FY1  $>$  Asset Turnover for FY2

#### **4.4.2 F\_Scores and Value Strategy Performance**

Focus on portfolios of firms within top 1/5 of B/M. Filtering firms based on fundamentals help enhance the value strategy.

**Table 4: One-Year Market-Adjusted Buy-and-Hold Returns to a Value Investment Strategy Based on Fundamental Signals by Size Partition**

	Small Firms			Medium Firms			Large Firms		
	Mean	Median	n	Mean	Median	n	Mean	Median	n
All Firms	0.091	-0.077	<u>8302</u>	0.008	-0.059	<u>3906</u>	0.003	-0.028	<u>1835</u>
<b>F_SCORE</b>									
0	0.000	-0.076	32	-0.146	-0.235	17	-0.120	-0.047	8
1	-0.104	-0.227	234	-0.083	-0.228	79	-0.136	-0.073	26
2	-0.016	-0.171	582	-0.045	-0.131	218	0.031	-0.076	59
3	0.003	-0.168	1028	-0.049	-0.108	429	-0.036	-0.068	161
4	0.058	-0.116	1419	-0.024	-0.104	687	-0.002	-0.023	356
5	0.079	-0.075	1590	0.028	-0.060	808	-0.004	-0.031	389
6	0.183	-0.030	1438	0.029	-0.041	736	0.012	-0.004	405
7	0.182	0.005	1084	0.027	-0.028	540	0.028	-0.015	270
8	0.170	0.001	671	0.081	0.024	312	0.012	-0.041	132
9	0.204	-0.017	224	0.068	0.032	80	0.059	-0.045	29
<b>0 &amp; 1 Low Score</b>	-0.091	-0.209	266	-0.094	-0.232	96	-0.132	-0.066	34
<b>8 &amp; 9 High Score</b>	0.179	-0.007	895	0.079	0.024	392	0.020	-0.045	161
High-All	0.088	0.070	—	0.071	0.083	—	0.017	-0.017	—
t-statistic/(p-value)	2.456	(0.000)	—	2.870	(0.000)	—	0.872	(0.203)	—
High-Low	0.270	0.202	—	0.173	0.256	—	0.152	0.021	—
t-statistic/(p-value)	4.709	(0.000)	—	2.870	(0.000)	—	1.884	(0.224)	—

## 4.5 PEG Ratio

[http://en.wikipedia.org/wiki/PEG\\_ratio](http://en.wikipedia.org/wiki/PEG_ratio)

<http://www.fool.com/pegulator/pegulator.htm>

### **DEFINITION OF 'PRICE/EARNINGS TO GROWTH - PEG RATIO'**

A stock's price-to-earnings ratio divided by the growth rate of its earnings for a specified time period. The price/earnings to growth (PEG) ratio is used to determine a stock's value while taking the company's earnings growth into account, and is considered to provide a more complete picture than the P/E ratio. While a high P/E ratio may make a stock look like a good buy, factoring in the company's growth rate to get the stock's PEG ratio can tell a different story. The lower the PEG ratio, the more the stock may be undervalued given its earnings performance. The calculation is as follows:

**$$\text{P/E ratio} \div \text{Annual EPS Growth}$$**

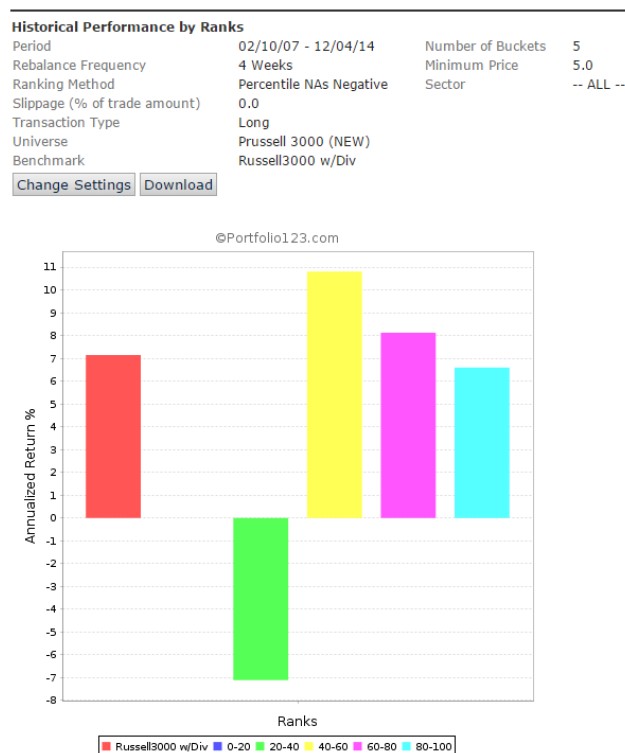
### **INVESTOPEDIA EXPLAINS 'PRICE/EARNINGS TO GROWTH - PEG RATIO'**

The PEG ratio that indicates an over or underpriced stock varies by industry and by company type, though a broad rule of thumb is that a PEG ratio below one is desirable. Also, the accuracy of the PEG ratio depends on the inputs used. Using historical growth rates, for example, may provide an inaccurate PEG ratio if future growth rates are expected to deviate from historical growth rates. To distinguish between calculation methods using future growth and historical growth, the terms "forward PEG" and "trailing PEG" are sometimes used.

<http://www.investopedia.com/terms/p/pegratio.asp>

$$PEG = (P/E) \div \text{Annual EPS Growth}$$

Investors prefer lower PEG ratio. In the following charts, the PEG ratio decreases to the right. The light blue corresponds to the group with the lowest PEG ratios. The red bar to the left is the Russell 3000 index.



- The relation between PEG and average stock returns is non-monotonic. The evidence for the PEG's return predictability has been weak.

## 4.6 Possibly Better Alternatives to PEG

### 4.6.1 E/P + Growth

- PEG places a bet against  $(P/E) / Growth$ . That is, a PEG approach prefers stocks with low  $(P/E) / G = P / (E \times G)$  ratios (and hence the name "PEG").
- This is equivalent to an approach that prefers stocks with high  $G \times (E/P)$ , i.e., E/P "Times" Growth. (E/P is the earnings yield.)

However, a simple Dividend Discount Model (Gordon Growth Model) implies that

$$E[R] = (D/P) + G.$$

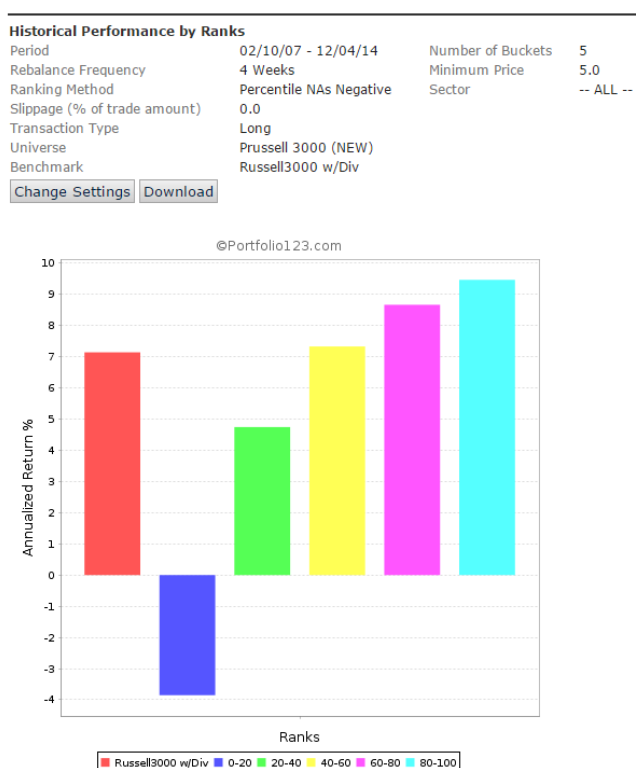
Notice that the Growth rate ( $G$ ) is “added,” rather than “multiplied.”

- Motivated by this simple theoretical result, we can try capturing  $E[R]$  by  $E/P + Growth$ . ( $E/P$  "Plus" Growth).
- To estimate the “Growth” component, I combined two methods:
  1. The mean of analysts’ long-term EPS growth forecasts.
  2. Sustainable Growth Rate = Retention rate  $\times$



ROE (both retention rate and ROE are from trailing 12 months)

As you can see in the following chart, this  $E/P + Growth$  is able to separate groups with high stock returns with those with low stock returns.



- The relation between portfolio returns and  $E/P +$

*Growth* is quite monotonic and increasing.

- We can also see that the portfolio of stocks with very low  $E/P + Growth$  have negative average returns in the sample.

This empirical evidence is consistent with the simple Dividend Discount Model (Gordon Growth Model). That is, this approach is “theoretically sound” and “empirically relevant.”

#### **4.6.2 Generic Value Strategies with Industry Control**

A Simple and heuristic approach: How about a combination of

- $B/P$  (B is the BPS in most recent quarter) Minus Industry Average
- $E/P$  (E is the analysts' EPS forecast)
- $CF/P$  (CF is the operating cash-flows per share in the trailing 12 months)

These ratios are compared within each industry. That is, **we subtract the industry averages.**

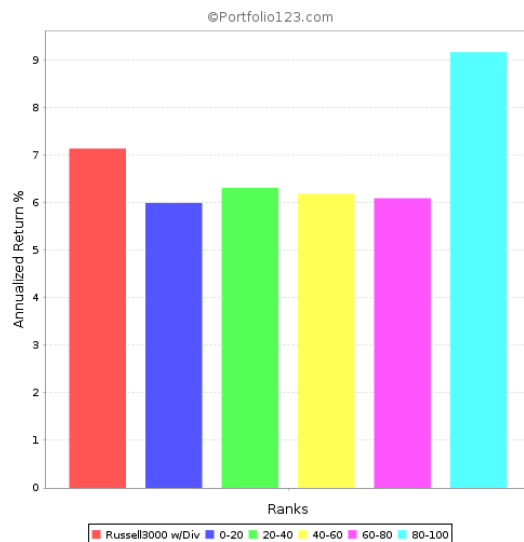
Notice that value metrics are more effective in capturing the difference in average stock returns when growth rates do not vary much.

- Growth rates vary a lot across industries.
- Growth rates vary little within each industry.

I sort stocks into 5 portfolios.

1. Stocks are ranked according to each of the three criteria.
2. I then calculate the average of the three ranks for each stock.
3. I then sort stocks into 5 groups by the average rank in the Russell 3000 universe.

Historical Performance by Ranks			
Period	02/10/07 - 12/04/14	Number of Buckets	5
Rebalance Frequency	4 Weeks	Minimum Price	5.0
Ranking Method	Percentile NAs Neutral	Sector	-- ALL --
Slippage (% of trade amount)	0.0		
Transaction Type	Long		
Universe	Prussell 3000 (NEW)		
Benchmark	Russell3000 w/Div		
<a href="#">Change Settings</a>		<a href="#">Download</a>	



Stocks in the top 1/5 have higher average returns than others in this sample.  $\Rightarrow$  Generic value metrics are useful return predictors especially within industries.

## **4.7 Fundamental Indexing**

### **4.7.1 Market Cap. Weighting vs. Fundamental Value Weighting**

Fundamental Indexing [e.g. Arnott, Hsu, Moore (2005)] is gaining growing popularity in the industry.

Instead of weighing by market capitalization, a fundamental index uses fundamental values to assign weights.

Examples of useful fundamental values include:

- Book equity values
- Sales revenues or gross profits
- Operating cash flows
- Payouts (Dividends and Repurchases)

## An Illustrative Example

Assets	Market Cap.		Fundamentals		Active	$\frac{FV}{MV}$
Appli	500	50.0%	150	30.0%	−20.0%	0.30
Bk.Ameria	175	17.5%	250	50.0%	+37.5%	1.43
Cock Cola	175	17.5%	50	10.0%	−7.5%	0.29
Dismey	150	15.0%	50	10.0%	−5.0%	0.33
Total	1,000	100%	500	100%	0.0%	0.50

- Fundamental Indexing assigns a greater weight on Bank Ameria and lower weights on the others than the market cap. weighting benchmark.
- That is, its active position bets on Bank Ameria (that has a high FV/MV ratio) and bets against Appli, Cock Cola, and Dismey (that have low FV/MV ratios).

## 4.7.2 Active Value Bets Embedded in Fundamental Indexing

Fundamental Indexing involves an active value bet, as demonstrated below:

Fundamental index weight of stock  $i$  ( $i = 1, \dots, N$ )

$$w_i^{FV} = \frac{FV_i}{\sum_{j=1}^N FV_j}$$

Market benchmark weight (market cap weighting)

$$w_i^{MV} = \frac{MV_i}{\sum_{j=1}^N MV_j}$$

The active portfolio position of the fundamental index is

$$\begin{aligned} w_i^{FV} - w_i^{MV} &= \frac{FV_i}{\sum_{j=1}^N FV_j} - \frac{MV_i}{\sum_{j=1}^N MV_j} \\ &= \left( \frac{\sum_{j=1}^N MV_j}{\sum_{j=1}^N FV_j} \right) \underbrace{\left( \frac{MV_i}{\sum_{j=1}^N MV_j} \right)}_{\text{market cap weighting}} \underbrace{\left( \frac{FV_i}{MV_i} - \frac{\sum_{j=1}^N FV_j}{\sum_{j=1}^N MV_j} \right)}_{\text{a bet on FV/MV}} \end{aligned}$$

- Fundamental Indexing involves a rule-based bet on firms with high  $FV/MV$ s, and against firms with low  $FV/MV$ s.



## 5 Portfolio Management Process

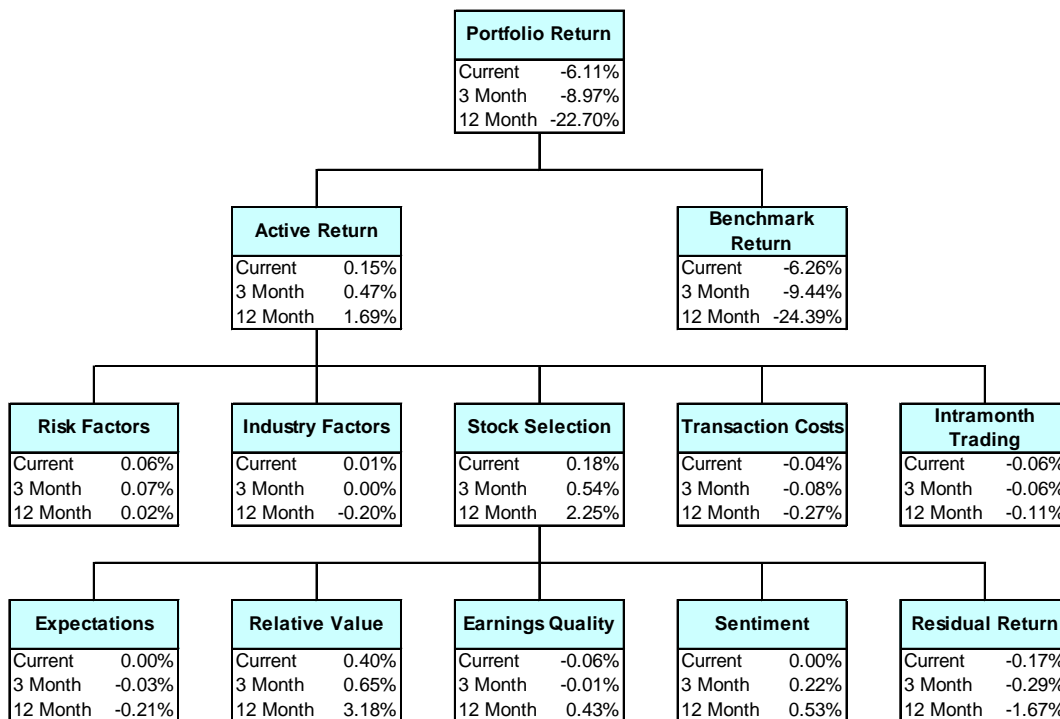
- The distance between “science of investments” and industry practice has actually narrowed in recent years.
  - Growth of professional designation and continuous education contributed by the CFA Institute, GARP, etc.
- More and more professional investors realize the importance of Factor/Style Investing. Factor/Style investing allows us to exploit possible mispricing or/and to harvest factor risk premiums efficiently.

- Factor/Style Investing typically involves an active portfolio (of the factors/styles used). Recall

$$r_{portfolio} = r_{benchmark} + r_{active},$$

where  $r_{active}$  can be attributed to various sources.

## Portfolio Performance Attribution Example





# A No Arbitrage Pricing Rules

## A.1 Arbitrage: An Example

### A.1.1 Question

Consider 3 diversified risky portfolios (no idiosyncratic risk) and a risk-free asset with the following return processes:

$$r_f = 0.03,$$

$$r_A = 0.06 + 0\tilde{F}_1 + 2\tilde{F}_2,$$

$$r_B = 0.08 + 2\tilde{F}_1 + 1\tilde{F}_2,$$

$$r_C = 0.14 + 4\tilde{F}_1 + 4\tilde{F}_2,$$

where  $\tilde{F}_1$  and  $\tilde{F}_2$  (common/systematic risk factors) are random variables with mean zero. Is there an arbitrage opportunity?

### A.1.2 Solution

Construct a risk-free (zero-beta) portfolio

$$w_A + w_B + w_C = 1$$

$$2w_B + 4w_C = 0$$

$$2w_A + w_B + 4w_C = 0$$

The solution to the system of equations is:

$$w_A = \frac{1}{2}; \quad w_B = 1; \quad w_C = -\frac{1}{2}.$$

Consequently, the following portfolio (denoted by  $Z$ ) is “risk-free”

$$r_Z = \frac{1}{2}r_A + r_B - \frac{1}{2}r_C.$$

Expected return of this “risk-free” portfolio is

$$E(r_Z) = \frac{1}{2} \times 0.06 + 1 \times 0.08 - \frac{1}{2} \times 0.14 = 0.04,$$

which is greater than the risk free rate  $r_f = 0.03$ .

Therefore, an arbitrage is possible by

- Taking a long position in the zero beta portfolio  $Z$  (4%), and
- Taking a short position in the risk-free asset (3%).

This strategy exploits the return difference ( $4\% - 3\% = 1\%$ ) with no cash outlay and no risk.

## A.2 No Arbitrage $\Rightarrow$ Linear Pricing Rule

### A.2.1 Question

Assume that the rates of return on assets A, B, and C are given by:

$$\tilde{r}_A = \mu_A + \beta_A \tilde{F}_1 + \gamma_A \tilde{F}_2,$$

$$\tilde{r}_B = \mu_B + \beta_B \tilde{F}_1 + \gamma_2 \tilde{F}_2,$$

$$\tilde{r}_C = \mu_C + \beta_C \tilde{F}_1 + \gamma_2 \tilde{F}_2.$$

$\tilde{F}_1$  and  $\tilde{F}_2$  are random variables (common/systematic risk factors) with mean zero. We can see that  $\mu_i = E[\tilde{r}_i]$  for  $i = A, B, C$ . Let  $r_f$  be the risk-free rate of return. What condition(s) must parameters  $\{\mu_i, \beta_i, \gamma_i\}_{i=A,B,C}$  and  $r_f$  satisfy to preclude arbitrage opportunities?

### A.2.2 Solution

Construct a risk-free (zero-beta) portfolio  $w = (w_A, w_B, w_C)'$  such that

$$w_A + w_B + w_C = 1$$

$$w_A\beta_A + w_B\beta_B + w_C\beta_C = 0$$

$$w_A\gamma_A + w_B\gamma_B + w_C\gamma_C = 0$$

To preclude arbitrage, this zero-beta portfolio must have an expected return equal to  $r_f$ . Hence

$$w_A\mu_A + w_B\mu_B + w_C\mu_C = r_f$$

or with  $w_A + w_B + w_C = 1$ ,

$$w_A(\mu_A - r_f) + w_B(\mu_B - r_f) + w_C(\mu_C - r_f) = 0$$



We can summarize these conditions as

$$\underbrace{\begin{bmatrix} \mu_A - r_f & \mu_B - r_f & \mu_C - r_f \\ \beta_A & \beta_B & \beta_C \\ \gamma_A & \gamma_B & \gamma_C \end{bmatrix}}_X \begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since  $w \neq 0$ , the matrix  $X$  must be singular. (One can show that no arbitrage exists if and only if  $\det(X) = 0$ ).

That is, a row of  $X$  is a linear combination of the other 2 rows. Thus, to preclude arbitrage, there must exist  $\lambda_1$  and  $\lambda_2$  such that

$$\begin{bmatrix} \mu_A - r_f \\ \mu_B - r_f \\ \mu_C - r_f \end{bmatrix} = \begin{bmatrix} \beta_A \\ \beta_B \\ \beta_C \end{bmatrix} \lambda_1 + \begin{bmatrix} \gamma_A \\ \gamma_B \\ \gamma_C \end{bmatrix} \lambda_2.$$

That is, we obtain a no-arbitrage pricing rule

$$E[r_i] - r_f = \beta_i \lambda_1 + \gamma_i \lambda_2; \quad i = A, B, C$$

We can extend this to a larger number of assets.

## A.3 Arbitrage Pricing Theory (APT):

### Main Result

The absence of arbitrage implies a multi-factor linear relationship between expected portfolio returns and factor risk premiums.

- Factor Model for well diversified portfolios:

$$r_i = E[r_i] + \beta_{i1}\tilde{F}_1 + \cdots + \beta_{iK}\tilde{F}_K$$

$$\text{where } E[\tilde{F}_1] = \cdots = E[\tilde{F}_K] = 0$$

NO ARBITRAGE implies

- APT pricing relation

$$E[r_i] = r_f + \beta_{i1}\lambda_1 + \cdots + \beta_{iK}\lambda_K.$$

where  $\lambda_1, \dots, \lambda_K$  are **factor risk premiums**.

- When idiosyncratic risk is present (e.g. individual stocks), we can still view this as an approximate pricing relation:

$$E[r_i] \approx r_f + \beta_{i1}\lambda_1 + \dots + \beta_{iK}\lambda_K.$$

- APT is more general in that it gets to an expected return and beta relationship without the assumption of the market portfolio.
  - APT employs fewer restrictive assumptions than the CAPM.
  - APT can accommodate multiple factors. However, APT does NOT tell what factors we should use.

### A.3.1 Practice Question 1

Calculate the risk-free rate and the two factor risk premiums from the following factor model:

$$r_A = 0.15 + 2\tilde{F}_1 + 2\tilde{F}_2,$$

$$r_B = 0.12 + 5\tilde{F}_1 - 1\tilde{F}_2,$$

where  $E[\tilde{F}_1] = E[\tilde{F}_2] = 0$ . The risk free rate is 0.05.

Both  $A$  and  $B$  are well diversified portfolios.

[Answer]  $\lambda_1 = 0.02, \lambda_2 = 0.03$ .

### A.3.2 Practice Question (CFA)

Assume that both X and Y are well-diversified portfolios and the risk-free rate is 8%.

Portfolio	$E(r)$	$\beta$
X	16%	1.00
Y	12%	0.25

In this situation, you would conclude that portfolios X and Y:

1. Are in equilibrium.
2. Offer an arbitrage opportunity.
3. Are both fairly priced.

## A.4 Practical Insights of the APT

APT is build on the premise that idiosyncratic risk is “small” and systematic risk is “large.”

$$\tilde{r}_i = E [\tilde{r}_i] + \beta_{i1}\tilde{F}_1 + \cdots + \beta_{iK}\tilde{F}_K + \varepsilon_i$$

Approximate pricing formula (K-factor model) in the presence of idiosyncratic risk:

$$\hat{E} [\tilde{r}_i] \approx r_f + \beta_{i1}\lambda_1 + \cdots + \beta_{iK}\lambda_K.$$

Let  $\alpha_i = E [\tilde{r}_i] - \hat{E} [\tilde{r}_i]$  be the “pricing error” of the K factor model, and set  $\alpha = (\alpha_1, \dots, \alpha_N)'$ .

### A.4.1 Important result

$$\alpha' \Omega^{-1} \alpha = SR_{\max}^2 - SR_{Factor}^2$$

where  $SR_{\max}$  is the maximum Sharpe ratio attainable in the economy and  $SR_{Factor}$  is the maximum Sharpe ratio in the factor space.  $\Omega$  is the covariance matrix of the idiosyncratic returns,  $\varepsilon_i$ ,  $i = 1, \dots, N$ .

In the mean-variance world, the no arbitrage condition can be restated as “there is no portfolio with infinite Sharpe ratio.” That is, APT requires that  $\alpha' \Omega^{-1} \alpha$  is bounded above even when  $N \rightarrow \infty$ .

$\sqrt{\alpha' \Omega^{-1} \alpha}$  is the maximum Information Ratio (IR) when we knew  $\alpha$  and formed a portfolio  $w_{active} \propto \Omega^{-1} \alpha$ . (This is a long/short portfolio formed on  $\alpha_i$  which can

be either positive or negative.) Therefore, we can restate the result as

$$SR_{\max}^2 = SR_{F_{actor}}^2 + IR_{\max}^2.$$

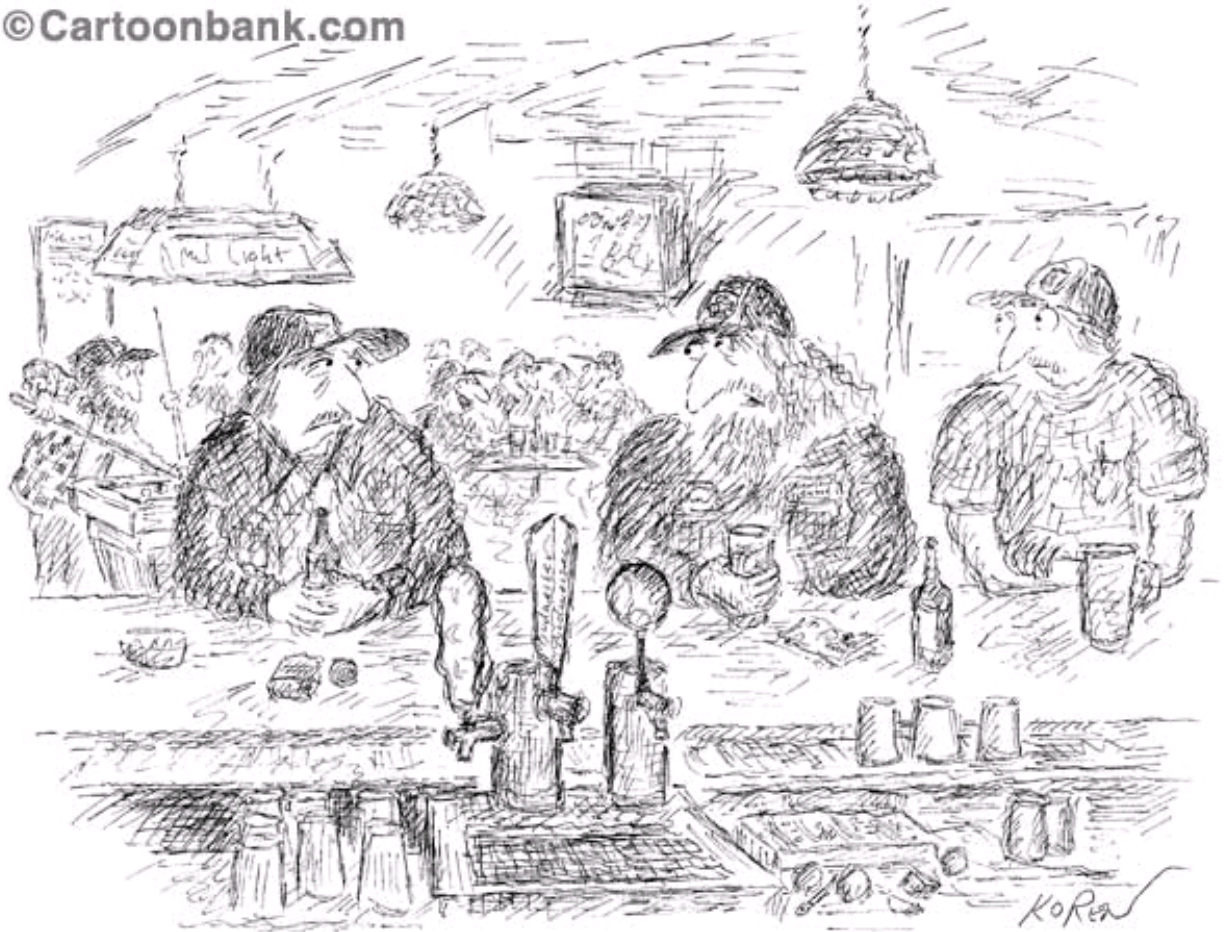
- $IR_{\max}$  is the Information Ratio of the optimal active portfolio:  $w_{active} \propto \Omega^{-1}\alpha$

$$IR_{\max} = \frac{\alpha_{active}}{\sigma_{\varepsilon_{active}}}, \text{ where}$$

$$\alpha_{active} = w'_{active}\alpha \text{ and } \sigma_{\varepsilon_{active}}^2 = w'_{active}\Omega w_{active}.$$

- APT does not preclude Active management. Under APT, active managers can earn alphas relative to multi-factor benchmark models. APT merely requires that the manager's Information Ratio can not increase without bound (even when  $N \rightarrow \infty$ ).





*"Are you just pissing and moaning, or can you verify what you're saying with data?"*