Additional Matrix and Vector Operations and Graphics 101

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These are tricks not frequently used, but making good use of them (or avoid abusing them) may help you write good code.

Index Elements Except ...

- If you want to index elements in your vectors except specific elements, you can do

```
a <- c(1,2,3,4)
a[-3] # all elements in a without the 3rd one

[1] 1 2 4
```

Index Rows/Cols Except ...

- You can also use it to index rows in a matrix except ...

```
A <- matrix(1:9, nrow = 3)
A[-3,] # all rows in A except the third row
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
# all rows in A except the 2nd and 3rd row
A[c(-2,-3),]
[1] 1 4 7
```

Concatenate Vectors

- You can concatenate two vectors to create a bigger vector.

```
a <- 1:4
b <- 5:8
ab <- c(a, b)
ab
[1] 1 2 3 4 5 6 7 8
```

Concatenate Matrices

- You can concatenate two matrices in R, by rows or by columns.
- `cbind`, concatenate two matrices by columns

```
a <- matrix(1:4, nrow = 2)
b <- matrix(5:8, nrow = 2)
a
     [,1] [,2]
[1,]    1    3
[2,]    2    4
b
     [,1] [,2]
[1,]    5    7
[2,]    6    8
ab <- cbind(a, b)
ab
     [,1] [,2] [,3] [,4]
[1,]    1    3    5    7
[2,]    2    4    6    8
```

Concatenate Matrices

- `rbind`, concatenate two matrices by rows

```
a <- matrix(1:4, nrow = 2)
b <- matrix(5:8, nrow = 2)
a
     [,1] [,2]
[1,]    1    3
[2,]    2    4
b
     [,1] [,2]
[1,]    5    7
[2,]    6    8
ab <- rbind(a, b)
ab
     [,1] [,2]
[1,]    1    3
[2,]    2    4
[3,]    5    7
[4,]    6    8
```

Insert New Elements in a Vector

```
a <- 1:4
# inserting a new element 2.5 after 2.
a <- c(a[1:2], 2.5 ,a[3:4])
a
[1] 1.0 2.0 2.5 3.0 4.0
```

- Once a vector is created by R, you cannot change its size.
- The above code does NOT modify the original vector `a`. Instead, it created a new vector by calling `c(a[1:2], 2.5 ,a[3:4])`, and reassigned it to the original variable `a`.
- This operation is called "insertion by reassignment", requires **additional memory allocations**, thus is time-consuming when carried out repeatedly.

Delete Elements in a Vector

```
a <- 1:4
# delete the third and fourth element.
a <- a[c(-3,-4)]
a
[1] 1 2
```

Similar to vector insertion, you are not modifying the original vector. You are simply creating a new vector by `a[c(-3,-4)]` and assigned it to `a`.

Delete Rows/Columns in a Matrix

```
A <- matrix(1:9, nrows = 3)
A
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
# delete the second row.
A <- A[-2, ]
A
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    3    6    9
```

Tetris

- Eliminate the row(s) that are filled with 1s.

```
A <- matrix(c(0, 0, 0, 0,
+               1, 0, 1, 0,
+               1, 0, 1, 1,
+               1, 1, 1, 1), nrow = 4, byrow = T)
A
     [,1] [,2] [,3] [,4]
[1,]    0    0    0    0
[2,]    1    0    1    0
[3,]    1    0    1    1
[4,]    1    1    1    1
A <- A[rowSums(A) != 4,]
     [,1] [,2] [,3] [,4]
[1,]    0    0    0    0
[2,]    1    0    1    0
[3,]    1    0    1    1

A
```

Recycling

- In vector ops, R requires two vectors to be **the same length**.
- If two vectors do not have the same length, R automatically repeats the shorter vector to match the length of the longer one.

```
a <- 1:4
b <- 1:8

a + b #a is repeated twice
# the same as c(1:4,1:4) + 1:8
[1]  2  4  6  8  6  8 10 12
```

Recycling

- If the longer vector is not a multiple of the shorter vector, the shorter vector will repeat until it is long enough.

```
a <- 1:3
b <- 1:4
a + b
# the same as c(1:3, 1) + 1:4
[1] 2 4 6 5
Warning message: In a + b : longer object length
is not a multiple of shorter object length
```

Recycling

- Recycling does NOT works on matrices

```
A <- matrix(1:4, nrow = 2)
B <- matrix(1:2, nrow = 2)

A
B

A+B
Error in A + B : non-conformable arrays
```

- In NumPy or MATLAB, there is a feature called "broadcasting", which allows recycling in matrices.

Recycling

- However, the following code works

```
A <- matrix(1:4, nrow = 2)
B <- matrix(1:2, nrow = 2, ncol = 2)
A
     [,1] [,2]
[1,]    1    3
[2,]    2    4
B
     [,1] [,2]
[1,]    1    1
[2,]    2    2

A+B
         [,1] [,2]
[1,]    2    4
[2,]    4    6
```

The matrix function automatically repeat the vector `1:2` twice to create a 2 by 2 matrix.

Recycling

In other words,

```
B <- matrix(1:2, nrow = 2, ncol = 2)
```

is the same as

```
B <- matrix(c(1:2, 1:2), nrow = 2, ncol = 2)
```

When creating a matrix, if not enough elements are provided to fill the matrix, the data is automatically recycled.

`apply` Function to Rows/Cols of Matrix

- If you have a function, and want to apply this function to all rows/columns to a matrix, you can use `apply` function.

```
# create a 100 by 2 random matrix, filled with samples
# from the standard normal distribution.
# Each row of x is a random point in 2D space.
x <- matrix(rnorm(2*100), nrow = 100)
# checking if a point vector is in the unit circle.
is_in_circle <- function(p){
    if( sqrt(sum(p^2)) <1 ){
        return(T)
    }else{
        return(F)
    }
}
```

- How do I apply `is_in_circle` to all the rows in `x`?

`apply` Function to Rows/Cols of Matrix

`apply(m,dim,f)`: applies `f` to rows (when `dim = 1`) or columns (when `dim = 2`) to matrix `m`.

```
apply(x,1,is_in_circle)
[1] FALSE TRUE TRUE FALSE TRUE FALSE FALSE ...
```

`apply` Function to Rows/Cols of Matrix

- Your function can also output a vector:

```
# project a point p on the circle.
project_on_circle <- function(p){
return(p / sqrt(sum(p^2)))
}
```

- `apply` will run your function on rows/columns of `m` and stack the outcome vectors **by column**.

```
px <- apply(x,1,project_on_circle)
px
          [,1]       [,2]      [,3]       [,4]      [,5]
[1,]  0.1483527 -0.9574072 0.9174074 -0.9469062 0.6920671
[2,] -0.9889345 -0.2887412 0.3979494 -0.3215100 0.7218332
...
```

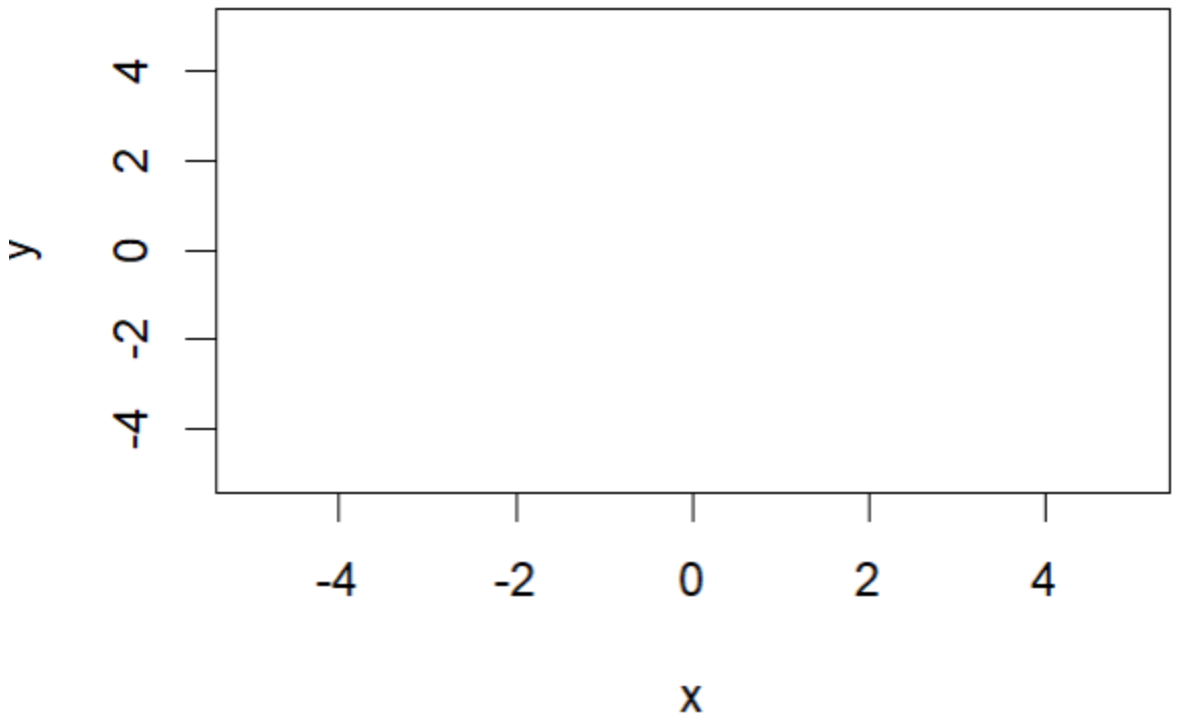
Graphics 101 in R

- R has a powerful graphics features, which allows users to visualize data easily.
    - Hint: You can type `demo("graphics")` in the command line to see demo code/plots.
- You may have already seen some of the graphics function, so I will be brief.

Create a plot: `plot`

`plot` function is usually used to create an empty canvas, ready for further plotting actions.

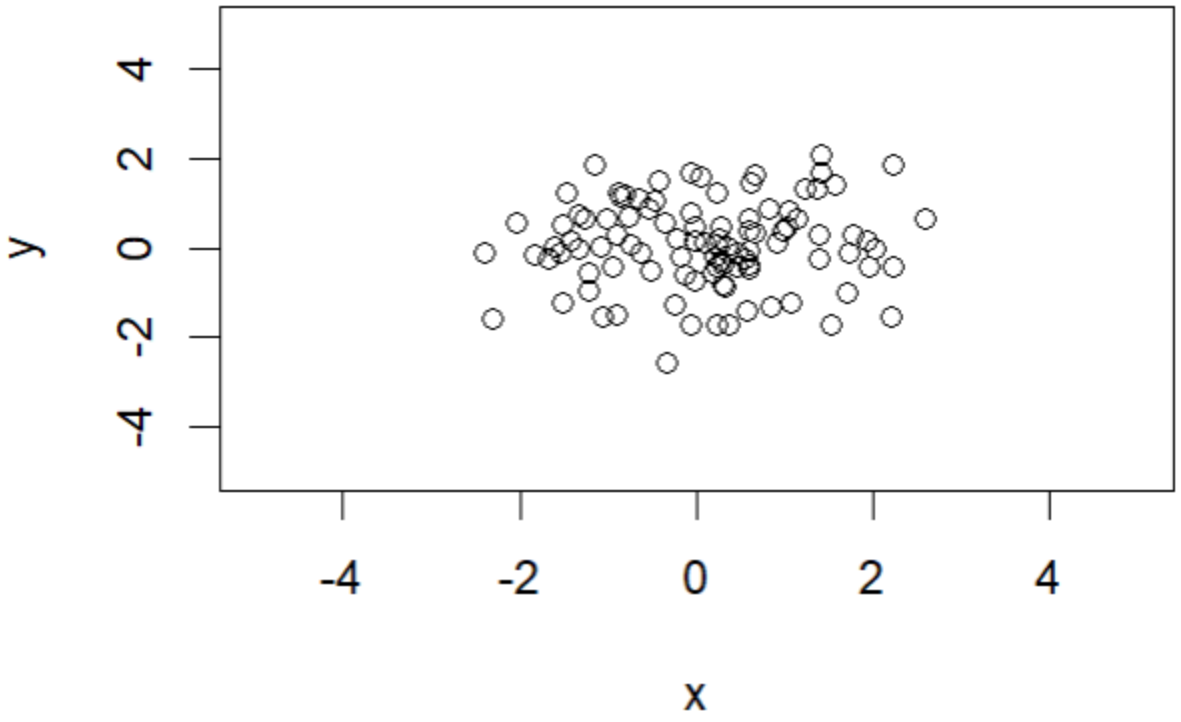
```
# create an empty plot, with x axis ranges from -5 to 5
# y axis ranges from -5 to 5.
plot(c(-5,5),c(-5,5), type = "n", xlab = "x", ylab = "y")
```



Visualize Data Points

`points` can be used to draw data points on a 2D plot.

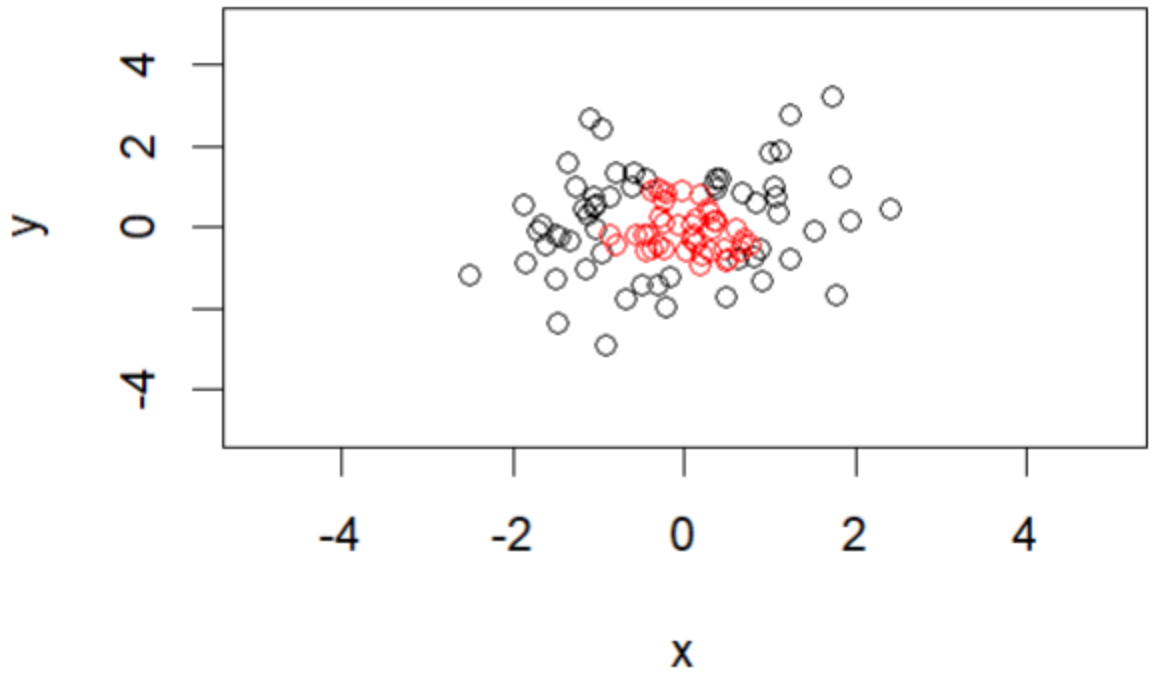
```
x <- rnorm(100)
y <- rnorm(100)
# draw points on the current plot. The first dimension
# of points are stored in a vector x and the second
# dimension are stored in a vector y.
points(x,y)
```



Visualize Data Points

You can change the the color of plotted points

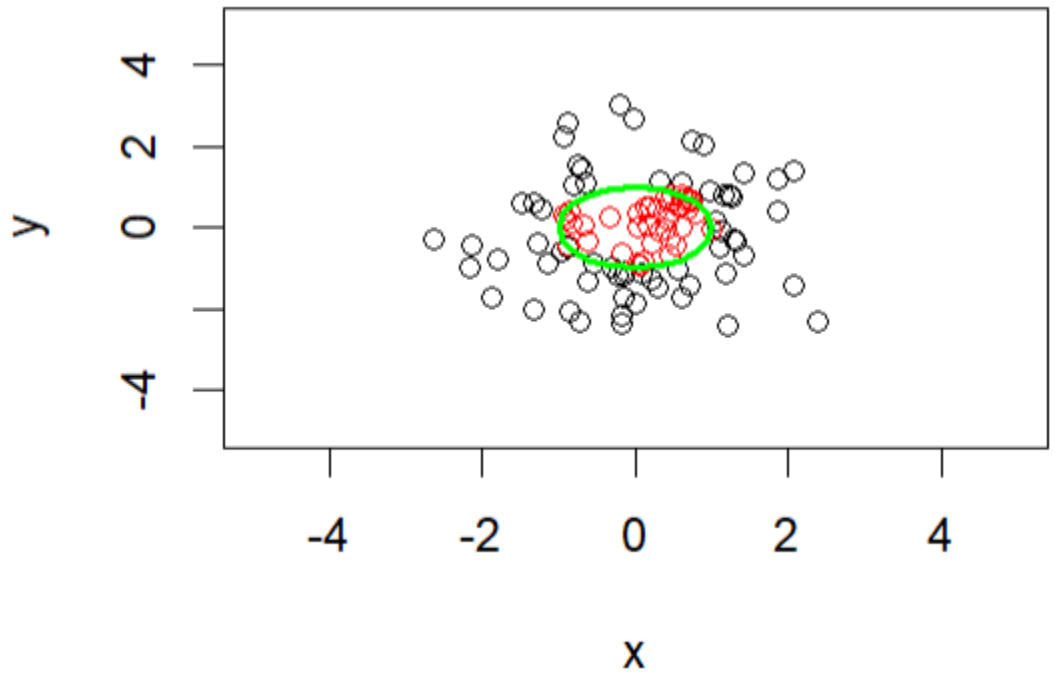
```
p <- cbind(x,y)
in_circ <- t(apply(p,1, is_in_circle))
# plot all points in the unit circle in red
points(p[in_circ,1],p[in_circ,2],col="red")
```



Draw Lines

You can draw line in a 2D plot using `lines` function:

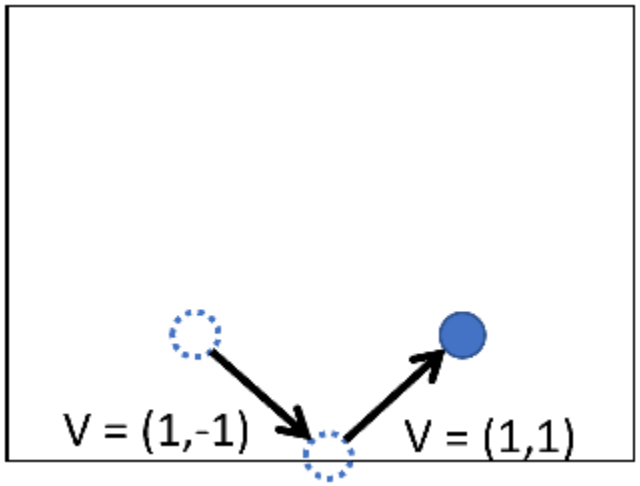
```
# generate points on a circle
circ <- cbind(cos(0:20*pi/10), sin(0:20*pi/10))
# draw lines by connecting these points
lines(circ[,1],circ[,2], col = "green", lwd = 2)
```



Homework

0. Read lecture slides on visualizations carefully.
1. Create an empty plot, whose x-axis and y-axis range from -5 to 5.
2. Initialize two vectors:
    - $x = (0, 0)$ and $v = (.23456, .12345)$.
    - Draw the vector x as a **red** dot in the plot.
    - Add plot title: "point simulation, press ESC to stop."

Homework

3. Now let us write a function that updates the point `x`'s location based on the velocity vector `v`.
    - If the position of `x` is slightly out of the box, the point will be **bounced back**.
    - 

Homework

3. Write a function `update(x,v)` that takes two vectors `x` and `v` as inputs and returns the updated `x` and `v`.
    - First, the function checks if x is outside of the boundary.
        - If so, it updates the velocity vector by "reflecting it". See the picture in the previous slide.
    - Then, update `x <- x + v`
    - Finally, `return(list(x,v))`

- Hint: `x[2] > 5 || x[2] < -5` is true if x is above the ceiling or is below the floor.

Homework (submit)

4. Your code should look like this:

```
update <- function(x,v){
    # TODO: Check the boundary collision, and update v.

    x <- x + v #update x
    return(list(x,v))
}

x <- c(0,0) #the initial position of x
v <- c(.23456,.12345) #the initial velocity of x

while(T){
    #TODO: create a new plot and draw x's current location

    xv <- update(x,v) # update x and v
    # getting new x and v from the returned list.
    # list indexing uses double brackets [[]]
    x<- xv[[1]]
    v<- xv[[2]]

    # wait a bit to allow RStudio plot the picture.
    Sys.sleep(.1)
}
```

Homework (submit)

- Your plot should look like this. https://github.com/anewgithubname/MATH10017/raw/main/labs/lab14_1.mp4

Homework (Challenge)

5. Now, let us add more points to the simulation.
    - Copy your previous code to a new R file.
    - Replacing the old vectors `x` and `v` with two 50×2 matrices `x` and `v`.
        - Elements in `x` are samples drawn from a uniform distribution $U(-1, 1)$.
        - Elements in `v` are samples drawn from a uniform distribution $U(-.5, .5)$.
    - Now `x` contains 50 points in a 2D space while the corresponding rows of `v` are velocity vectors for `x`.
6. Rewrite `update` function **using vectorized code** so that it can update 50 points' locations at once.

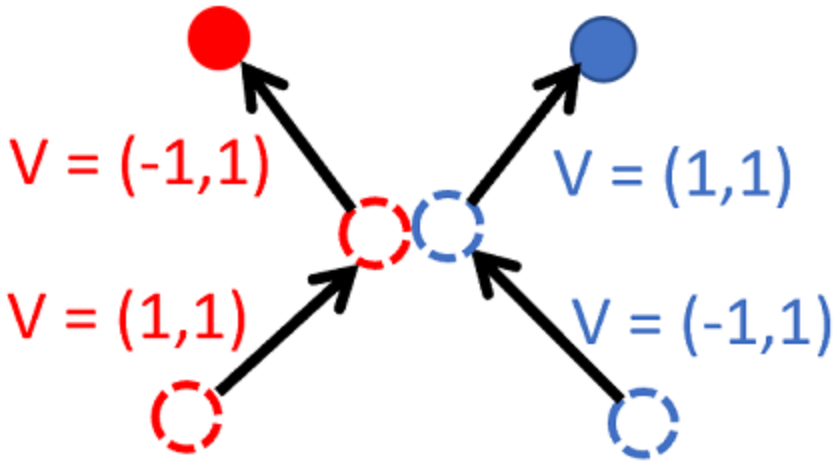
Homework (Challenge)

- Hint: `(x[,2] > 5 | x[,2] < -5)` will produce a **logical vector** that is `TRUE` for points which are above the ceiling or below the floor.

Homework (Challenge)

- Your plot should look like this: https://github.com/anewgithubname/MATH10017/raw/main/labs/lab14_2.mp4

Homework (difficult Challenge)

- Now consider each point is a radius 0.2 tennis ball.
- Rewrite your `update` function so it checks for the collision of balls.
  - If two balls collide, they will exchange momentum.
  - 

  - Consider using the `pdist` functions we wrote last week for checking ball collisions.