Computing Lab

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MVN Random Sample Generation

- Generating samples from an MVN with arbitrary mean $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, using **only univariate Normal sample generator.**
- **Hint**: After a coordinate transform, MVN become product of univariate normal distributions.
- The question is, given univariate samples, can you convert them to multivariate normal samples?

- *Bonus question (hard): generate samples from MVN using **only a uniform sample generator**.*
- **Hint**: check out Inverse transform sampling.

MVN Confidence Region

- You probably heard "3-σ" rule: Given a univariate normal, $N_x\ (\mu, \sigma), P(\mu - 3\sigma \leq x \leq \mu + 3\sigma) \approx 0.997$. Let us extend this to 2D MVNs.

- Pick a $\boldsymbol{\mu} \in R^2$ and $\boldsymbol{\Sigma} \in R^{2\times 2}$ that is positive definite.
- Visualize the following level set:
- $S = \{\boldsymbol{x} \in R^2 | (\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu}) = 6\}$

- Let P be an MVN: $N_x\ (\boldsymbol{\mu}, \boldsymbol{\sigma})$, prove $P(\boldsymbol{x} \in V) \approx 0.95$, where V is the subspace bounded by the level set S.

MVN Confidence Region

- **Hints:**
- If $\boldsymbol{x}$ is an MVN random variable with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$, $(\boldsymbol{x}-\boldsymbol{\mu})^{\top}\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})$ is a Chi-square random variable, with degree of freedom equals to the dimensionality of $\boldsymbol{x}$.

- The CDF of Chi-square with various degree of freedom can be found here: https://en.wikipedia.org/wiki/Chi-squared_distribution#Table_of_%CF%872_values_vs_p-values

MVN Confidence Region