Two Additional Questions

- We have proved the expected loss can be decomposed into **Irreducible Error**, **Bias** and **Variance**.

- Notice this result does not depend on the choice of f_{LS}.

- However, knowing f_{LS} is the prediction function of a least square regression, we can prove more specific results.

Question 1, unbiasedness of LS

- **Assuming**
- additive noise as described in the slides
- $\exists\, w^*$, such that $g(\boldsymbol{x}) = f(\boldsymbol{x}; \boldsymbol{w}^*) = \langle \boldsymbol{w}^*, \boldsymbol{x} \rangle$,
- f_{LS} is the prediction function obtained by least squares regression.

- Show "Bias" is zero.

Question 2, specifying variance

- **Assuming**
- additive noise as described in the slides
- f_{LS} is the prediction function obtained by least squares regression.

- $\mathrm{Var}[f_{\mathrm{LS}}|x_0] = \langle \boldsymbol{h}, \boldsymbol{h} \rangle \, \sigma^2, \text{where } \boldsymbol{h} := \boldsymbol{x}_0^\top (\boldsymbol{X}\boldsymbol{X}^\top)^{-1}\boldsymbol{X}, \boldsymbol{h} \in R^n.$