

# IID and Factorization of $P(y_1 \dots y_n | x_1 \dots x_n)$

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$$\begin{aligned}
 & P(y_1, y_2 \dots y_n | x_1 \dots x_n) \\
 &= \frac{P(y_1 \dots y_n, x_1 \dots x_n)}{P(x_1 \dots x_n)} \quad \text{Bayes Rule} \\
 &= \frac{\prod_i P_i(y_i, x_i)}{P(x_1 \dots x_n)} \quad \text{① independence}
 \end{aligned}$$

$$(x_i, y_i) \perp (x_j, y_j) \quad \forall i \neq j$$

$$\Rightarrow x_i \perp x_j, \quad \forall i \neq j$$

$$\begin{aligned}
 & \text{Lemma: } (A, B) \perp (C, D) \\
 & \Rightarrow A \perp C
 \end{aligned}$$

$$\begin{aligned}
 P(A, C) &= \int P(A, B, C, D) dB dD \quad \text{independence} \\
 &= \int P(\underline{A}, B) P(\underline{C}, D) d\underline{B} d\underline{D} \quad \begin{array}{l} \text{sum} \quad \text{product} \\ = \text{prod} \quad \text{sum} \end{array} \\
 &= \left[ \int P(A, B) dB \right] \left[ \int P(C, D) dD \right] \\
 &= P(A) P(C)
 \end{aligned}$$

$$\text{Therefore } P(x_1 \dots x_n) = \prod_i P_i(x_i)$$

$$\Rightarrow \text{①} = \prod_i \frac{P_i(y_i, x_i)}{P_i(x_i)}$$

$$\begin{aligned}
 &= \prod_i P_i(y_i | x_i) \\
 &= \prod_i P_{\theta_i}(y_i | x_i) \\
 &= \prod_i \mathcal{N}_{y_i}(f(x_i), \sigma^2)
 \end{aligned}$$

identically distributed