

Two Additional Questions

- We have proved the expected loss can be decomposed into **Irreducible Error**, **Bias** and **Variance**.
- Notice this result does not depend on the construction of f_{LS} .
- However, knowing f_{LS} is the prediction function of a least square regression, we can prove more specific results.

Question 1, unbiasedness of LS

- **Assuming**
- additive noise as described in the slides
- $\exists \mathbf{w}^*$, such that $g(\mathbf{x}) = f(\mathbf{x}; \mathbf{w}^*) = \langle \mathbf{w}^*, \mathbf{x} \rangle$,
- f_{LS} is the prediction function obtained by least squares regression.
- Show “Bias” is zero.

Question 2, specifying variance

- **Assuming**
- additive noise as described in the slides
- f_{LS} is the prediction function obtained by least squares regression.
- $\text{Var}[f_{LS}|x_0] = \langle \mathbf{h}, \mathbf{h} \rangle \sigma^2$, where $\mathbf{h} := \mathbf{x}_0^\top (\mathbf{X}\mathbf{X}^\top)^{-1} \mathbf{X}$, $\mathbf{h} \in R^n$.