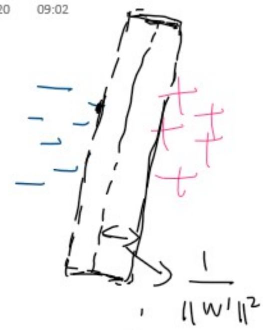
18 November 2020 09:02

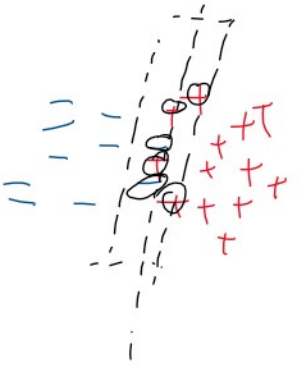


$\Rightarrow$

$$\min \|w'\|^2$$

$$\text{s.t. } \forall_i \ y_i(\overbrace{\langle w', x_i\rangle + w_0}^{f(x,w)}) \geq 1$$

$$\Uparrow$$



$$\min_{w} \|w'\|^2 + \sum_i \epsilon_i$$

$$\text{s.t. } \forall_i \ y_i(\underbrace{\langle w', x_i\rangle + w_0}_{f(x,w)}) + \epsilon_i \geq 1$$

$$\epsilon_i \geq 0$$

Cvx opt.

all local opt. are global opt.

$$l(\lambda, \lambda') := \min_{w,\epsilon} \|w'\|^2 + \sum_i \epsilon_i - \sum_i \lambda_i y_i(\langle w', x_i\rangle + w_0) + \lambda_i \epsilon_i - \lambda_i] - \sum_i \lambda'_i \epsilon_i$$

$$\max_{\lambda, \lambda'} l(\lambda, \lambda')$$

$$\begin{cases} \lambda \geq 0 \\ \lambda' \geq 0 \end{cases}$$

$$\Downarrow$$

$$2w' = \sum_i \lambda_i y_i x_i$$

$$\boxed{\begin{cases} w' = \dfrac{\sum_i \lambda_i y_i x_i}{2} \\ 1 = \lambda_i + \lambda'_i \\ \sum_i \lambda_i y_i = 0 \end{cases}}$$

(1) $\|w'\|^2 = \dfrac{\|\sum_i \lambda_i y_i x_i\|^2}{4}$

(2) $-\sum_i \lambda_i y_i \langle w', x_i\rangle = -\dfrac{\langle \sum_i \lambda_i y_i x_i, \sum_i \lambda_i y_i x_i\rangle}{2}$

(1)+(2) $= -\dfrac{\langle \sum_i \lambda_i y_i x_i, \sum_i \lambda_i y_i x_i\rangle}{4}$

$$① + ② = -\frac{\langle \sum_i \lambda_i y_i x_i, \sum_i \lambda_i y_i x_i \rangle}{4}$$

$$= -\frac{\tilde{\lambda}^T X^T X \tilde{\lambda}}{4}, \quad \tilde{\lambda} = [\lambda_1 y_1 \cdots \lambda_n y_n]$$

$$③ \quad \sum_i \epsilon_i - \sum_i \lambda_i \epsilon_i - \sum_i \lambda_i' \epsilon_i$$

$$= \sum_i \epsilon_i \underbrace{(1 - \lambda_i - \lambda_i')}_{0} = 0$$

$$④ \quad -\sum_i \lambda_i y_i w_0 = -w_0 \sum_i \lambda_i y_i = 0$$

$$⑤ \quad \sum_i \lambda_i$$

$$① + ② + ③ + ④ + ⑤ = -\frac{\tilde{\lambda}^T X^T X \tilde{\lambda}}{4} + \sum_i \lambda_i$$

$$\begin{cases} \max_{\lambda, \lambda'} -\frac{\tilde{\lambda}^T X^T X \tilde{\lambda}}{4} + \sum_i \lambda_i \\ \forall_i \ \lambda_i \geq 0 \quad \lambda_i' \geq 0 \\ \lambda_i + \lambda_i' = 1 \\ \sum_i \lambda_i y_i = 0 \end{cases} \qquad \begin{cases} \lambda_i' \geq 0 \\ \lambda_i + \lambda_i' = 1 \end{cases}$$

$$\Rightarrow \ 0 \leq \lambda_i \leq 1$$

$$\boxed{\begin{array}{l} \max_{\lambda} \ -\tilde{\lambda}^T X^T X \tilde{\lambda} + \sum_i \lambda_i \\ \text{s.t.} \ \lambda_i \in [0, 1], \ \sum_i \lambda_i y_i = 0 \end{array}} \Rightarrow \lambda_i$$

$$K := X^T X \in R^{n \times n}$$

$$K_{ij} = \langle x_i, x_j \rangle$$

$$\Rightarrow \ \underline{k(x_i, x_j)}$$

SM-SVM

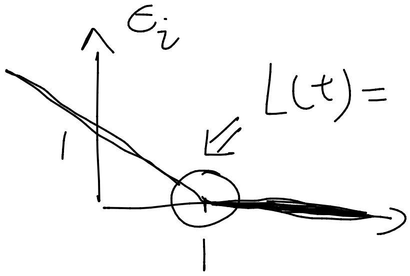
$$\min \ \|w'\|^2 + \boxed{\sum_i \epsilon_i}$$

$$\text{S.T.} \quad y_i f(x_i, w) + \epsilon_i \geq 1 \quad \forall i$$

$$\epsilon_i \geq 0$$

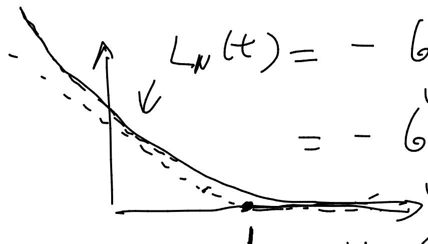
$$\text{if} \quad y \cdot f(x,w) \geq 1 \Rightarrow \epsilon_i = 0$$

$$\text{if} \quad y \cdot f(x,w) \leq 1 \Rightarrow \epsilon_i = 1 - y f(x)$$



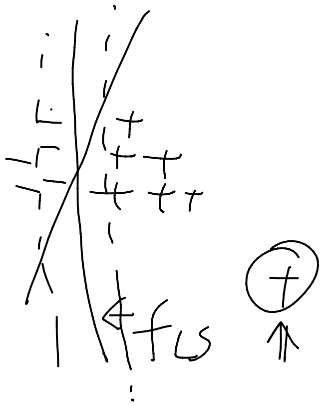
$$L(t) = \begin{cases} 0 & t \geq 1 \\ 1 - t, & t \leq 1 \end{cases}$$

$$\min_w \ \|w'\|^2 + \sum_i L(y_i f(x_i))$$



$$L_N(t) = -\log \boxed{\frac{1}{1+\exp(-t)}}$$

$$= -\log \sigma(-t)$$



$$\min_w \ \|w'\|^2 - \sum_i \log \sigma(-y_i f(x_i))$$

$$\max_w \ -\|w'\|^2 + \sum_i \log \sigma(-y_i f(x_i))$$

reg

HW $$\min_w \sum_i (y_i - f(x_i; w))^2$$

$$\text{S.T.} \quad \|w'\|^2 \leq C$$

$\hat{w}$ using λ, y_i, x_i

$\underline{\underline{\hat{w}}}$ using λ, y_i, x_i