Computing Lab

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MVN Random Sample Generation

- Generating samples from an MVN with arbitrary mean μ and Σ , using **only univariate Normal sample generator.**
- **Hint**: After a coordinate transform, MVN become product of univariate normal distributions.
- The question is, given univariate samples, can you convert them to multivariate normal samples?
- Bonus question (hard): generate samples from MVN using only a uniform sample generator.
- Hint: check out <u>Inverse transform sampling</u>.

MVN Confidence Region

- You probably heard "3- σ " rule: Given a univariate normal, $N_{\chi}(\mu,\sigma)$, $P(\mu-3\sigma \le \chi \le \mu+3\sigma) \approx 0.997$. Let us extend this to 2D MVNs.
- Pick a $\mu \in \mathbb{R}^2$ and $\Sigma \in \mathbb{R}^{2 \times 2}$ that is positive definite.
- Visualize the following level set:
- $S = \{x \in R^2 | (x \mu)^T \Sigma^{-1} (x \mu) = 6\}$
- Let P be an MVN: N_x (μ , σ), prove $P(x \in V) \approx 0.95$, where V is the subspace bounded by the level set S.

MVN Confidence Region

• Hints:

- If x is an MVN random variable with mean μ and covariance Σ , $(x \mu)^{\mathsf{T}} \Sigma^{-1} (x \mu)$ is a Chi-square random variable, with degree of freedom equals to the dimensionality of x.
- The CDF of Chi-square with various degree of freedom can be found here: https://en.wikipedia.org/wiki/Chi-squared_distribution#Table_of_%CF%872_values_vs_p-values

MVN Confidence Region

479/10000 datapoints outside of S

