IID and Factorization of $P(y_1 \cdots y_n | x_1 \cdots x_n)$

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$$P(y_1, y_2 \cdots y_n | x_1 \cdots x_n)$$

$$= \frac{P(y_1 \cdots y_n, x_1 \cdots x_n)}{P(x_1 \cdots x_n)} \quad \text{Bayes Rule}$$

$$= \frac{\prod_i P_i(y_i, x_i)}{P(x_1 \cdots x_n)} \quad (1) \quad \text{independance}$$

$$(x_i, y_i) \perp (x_j, y_j) \ \forall_{i \neq j}$$

$$\Rightarrow x_i \perp x_j, \ \forall_{i \neq j}$$

Lemma: $(A, B) \perp (C, D)$

$$\Rightarrow A \perp C$$

$$P(A, C) = \int P(A, B, C, D) \, dB \, dD \quad \text{independence}$$

$$= \int P(A, B) P(C, D) \, dB \, dD \quad \text{sum product} = \text{prod sum}$$

$$= \left[\int P(A, B) \, dB\right]\left[P(C, D) \, dD\right]$$

$$= P(A) P(C)$$

Therefor $P(x_1 \cdots x_n) = \prod_i P_i(x_i)$

$$\Rightarrow (1) = \prod_i \frac{P_i(y_i, x_i)}{P_i(x_i)}$$

$$= \prod_i P_i(y_i | x_i) \quad \text{identically distributed}$$

$$= \prod_i P_0(y_i | x_i)$$

$$= \prod_i \mathcal{N}_{y_i}(f(x_i), \sigma^2)$$