Physics

1. (c) Volume swept by the molecule in time t=Area × Velocity × t

$=6\ a^2 v_{rel} t$

No of collisions in time t $=6\ a^2 v_{rel} t n$

Time between consecutive collisions $=1/6na^2 v_{rel} t$

So, Mean free path= $v/6a^2 v_{rel} n$

But $v_{rel}= \sqrt{2}\ v$ (same as that of spherical molecule)

So, Mean free path = $1/6\sqrt{2}a^2 n$

2. (a) Resistance of a wire = l/kA

Total resistance of system = (l+3l+5l+...49l)/kA + (2l+4l+...50l)/2kA

=l×(25×38)/kA

Temperature difference across 15^{th} B-wire$=T \times \frac{\frac{15l}{kA}}{\frac{l}{kA}\times 25\times 38}$

=3T/190

3. (c) Let the length of original pipe = l

" " open organ pipe $=l_1$

To find: v/4l

$v/2l_1 = 480$; $v/4(l-l_1) =160$

On solving, v/4l=480/5=96 Hz.

4. (a) Isothermal expansion, isobaric compression, Isochoric(T increases)

5. (b) $T=PV/nR=4\cos\theta\times 3\sin\theta\times 10^3/2R=3\sin 2\theta\times 10^3/R$

Therefore, maximum temperature = 3000/R

6. (a) Each stone raises water level by 15 cm^2

Each wooden block " " 10 cm^2

$V_{in}=V_0 \times \rho = 0.8 \times 12.5 = 10\ cm^3$

Total volume displaced by two wooden blocks and one stone=15+20=35 cm^3

Therefore, no. of stones required= [200/35] +1=6

7. (b) Energy of oscillation= ½ $K_{eq} A^2$

$$E = \frac{1}{2}(K/10)x^2$$

$$1000E = \frac{1}{2} nK\, 4x^2 \quad \Rightarrow \quad n = 1000/40 = 25$$

8. (c) Time for boy to hit the top of building=$(2h/g)^{1/2}$

Area = m(g+a)/(tensile strength)

For minimum area, 'a' should be minimum, so that he just reaches the top of building within 5 seconds.

$50 = \frac{1}{2} a\, 5^2 \quad \Rightarrow \quad a=4$

$A=60(10+4)/(1.4 \times 10^8)= 6\ mm^2$

9. (b) Velocity= $\dfrac{KZ/n - KZ/(n+1)}{KZ/(n+1)} = 1/n$

$$\text{Angular momentum} = \frac{(n+1)h/2\lambda - nh/2\lambda}{Nh/2\lambda} = 1/n$$

10. (a) i= ¾ A= ¾ 60=45^0

For minimum deviation, r=A/2= 30^0, $n_1=1$

By Snell's law, $n_1 \sin i = n_2 \sin r$

- $n_2 = 2^{1/2}$

$n=(\mu_r\varepsilon_r)^{1/2} \Rightarrow n^2= \mu_r\varepsilon_r$

So, $\mu_r=\mu/\mu_{vacuum} = n^2/\varepsilon_r = 2/5 = 0.4$

Then, $\mu= \mu_r \times \mu_{vacuum}=0.4 \times 1.257 \times 10^{-6} = 5.028 \times 10^{-7}$ H/m.

11. (c) Reason: Energy of wave travels perpendicular to wave front.

12. (c) A/R = (-dN/dt)/ (-dA/dt) = $\frac{-\frac{d}{dt}(N_0 e^{-\lambda t})}{-\frac{d}{dt}(N_0 e^{-\lambda t})}$ = $1/\lambda$

13.

14. (c) for maximum angle θ, $mg(1-\cos\theta)=ma\sin\theta$

$$2g\sin^2(\theta/2)=2\sin(\theta/2)\cos(\theta/2)a$$

$$\tan(\theta/2)=a/g$$

$$\theta = 2\tan^{-1}(a/g)$$

15. (b) Applying LCAM,

M.I of the hand at an angle θ with axis

$$\int dm\, r^2 = \int_0^L \frac{M}{L} dx\, x^2 \sin^2\theta = ML^2\sin^2\theta/3$$

Therefore, $(MR^2/2)\,\omega_1=\left(\frac{MR^2}{2}+\frac{2ML^2\sin^2 45}{30}\right)\omega_2$

$$\omega_2=(15/16)\,\omega_1$$

16. (b) $R=u^2\sin(2\theta)/g=20m$

Distance from point of projection=10 m

17. (c) $m_1r_1=m_2r_2$

$F_1=F_2$ -------------(1) => $m_1r_1\,\omega_1^2=m_2r_2\,\omega_2^2$

$\omega_1=\omega_2$--------(2)

$m_1v_1^2/r_1=m_2v_2^2/r_2$; $m_2=2m_1$; $r_2=r_1/2$

$v_1/v_2=2$, $(v_1/v_2)^2=4$

$P_1=m_1v_1$, $P_2=m_2v_2= 2m_1\, v_{1/2} = P_1$---------------(3)

$L=mvr$ => L_1 not equal to L_2

$T=2\pi/\omega$ => $T_1=T_2$

18. (c) Avg power= $\frac{\text{Total work done by external agent}}{\text{Total time taken}}$

Time, t=length/v

Total work done= $mgl/2 + (1/2)mv^2$

c = $\lambda lgl/2 + (1/2)\lambda lv^2$

$P_{avg} = \lambda lglv/2l + (1/2)\lambda lv^2$

$= \lambda glv/2 + (1/2)\lambda v^2$

19. (b) Maximum percentage error = 4+1+2+2 = 9%

As 8<9, answer is option b

20. (b) $dI=dmr^2=kr(2\pi)rdr\, r^2 = 2\pi k(r^4)dr$

$I=(2\pi k/5)R^5$

$d\tau= \mu mgr = \mu gkr2\pi rdr\, r= 2\pi k\mu gr^3$

$\tau=\int d\tau = 2\pi k\mu gR^4/4$

$\tau=I\alpha \Rightarrow \alpha= 5\mu g/4R = 5/4R$

$t= \omega/\alpha = 4R$

$\omega/5$

21) c)2×10^8

$I_E=q/t=N\times e/t=10^{10}\times1.6\times10^{-19}/10^{-6}=1.6mA$

$\beta=I_E/I_B=49$, $I_E=I_B+I_C=1.6$

$1.6-I_B=49I_B \Rightarrow I_B=1.6/50=0.032mA$

$IB=e\times N_{base}/t \Rightarrow N_{base}=I_B\times t/e=0.032\times10^{-91}/(0.6\times10^{-19})=2\times10^8$ electrons

22) a)A_m/A_c

$C_m(t)= [A_c+A_m\sin\omega_m t]\sin\omega_c t = A_c\sin\omega_c t+A_m\sin\omega_m t\sin\omega_c t = A_c\sin\omega_c+(A_m/2)\cos(\omega_c-\omega_m)t - (A_m/2)\cos(\omega_c+\omega_m)t$

So, $K=A_c$ $M= - A_m/2$ $L=A_m/2$

Hence, $(L-M)/K=A_m/A_c$.

23) c)125/6

f_e=5cm

for clear vision ⇒v_e=-D=-25cm

$1/v_e-1/u_e=1/f_e$

$(1/-25)-(1/u_e)=1/5$

u_e=-25/6cm

For objective, $1/v_e-1/u_e=1/f_e$ u_o = -50/47cm f_o=1cm

v_o = 1-(47/50) ⇒v_o=100/6cm

Separation between lenses=v_o+u_o=1256cm.

24) b)9.000

$B_5^{12} \rightarrow C_6^{12} + \beta_-$ Qvalue=[mB_5^{12}-mC_6^{12}]c^2

$B_5^{12} \rightarrow C_6^{12} + \beta_- \rightarrow C_6^{12}$ =[12.014-12]931.5

=13.041 MeV.

So, KEβ=13.041-4.041=9 MeV.

25) The Potential is independent of position,So we will take the centre of the shell instead of the given point so as to simplify calculations

There is no change in potential due to the charges on the surface as it is a scalar quantity. So difference is the potential due to 5nC at the centre ie,

$9\times10^{9}\times10^{4}\times10^{-9}/2=45V.$

26) $9\times10^{9}\times30\times10^{-4}\times4\times10^{-4}/(9\times10^{-4})=f\times10^{-2}$

f=120 flaps/min.

27) $E=(1/2)Bl\omega^2=iR$

$1/2\times2.5\times10^{-3}\times1/2\times\omega^2=25\times1$

$\omega^2=4\times10^6$

$\omega=2\times10^3 rad/s.$

28) b $\phi=LI$ $\frac{I_0}{2}=I_0e^{-RtL}$

$=\mu_0 ndI\pi r^2$, dI is current per unit length ln2=Rt/L

$=\mu_0\pi r^2(ndI)$ t=Lln2/R

$\phi=\mu_0(\pi/l)r^2I \Rightarrow LI$

$L=\mu_0\ \pi\ r^2/l$

$R=\rho l/A=\rho.2\Pi.r/(l.d)$

$t=\mu_0.\pi.r^2.\ln2\ l\ d/(l\times\rho2\pi r)$

$t=\mu_0\ln2dr/2\rho.$

29) b) K^2,K,1

30) a) no change in velocity.

Chemistry

35. l<2, n+l=7.

For l=1, n=6 ->p subshell

6p6

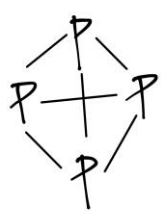
For l=0, n=7 ->s subshell
7s2

Max. no. of electrons = 6+2=8

36. Atomic no. 34 is Se. Se is in the chalcogen family (group 16). So it’s properties are similar to S(Atomic no. 16).

38.c. $OF_2 < H_2O < Cl_2O$
Since F is highly electronegative, F holds the electrons close to it, F-O bond is shifted away from central atom. Cl is bulky atom and repulsion occurs opening up the bond angle.

42. b is correct. Stability order: White<Red<Black.
White phosphorus is least stable due to its tetrahedral structure which creates strain(60 degree angle P-O-P bond).

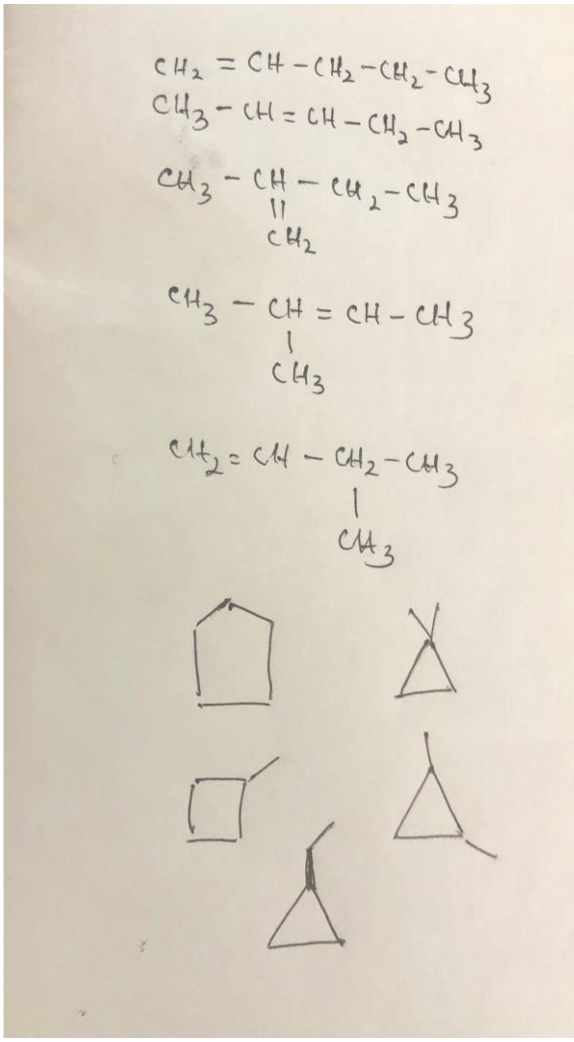


43. d. $\Delta G=\Delta G°-RT \ln K$

At equilibrium, $\Delta G° = -RT \ln K$

$\Delta G=0$.
44. d. There are 10 isomers.

52)



45. c. Alkanes cannot be further reduced and are very unreactive to such reagents.

49) Answer **C**

2 years => 2 x 365 days
= 730 days
~ 3 x 242 days

$[Gd]_{t=730} / [Gd]_{t=0} = (½)^3$

$= ⅛$

~ 12%

51) Answer **C**

(Memory Based)

52)

55) Answer **C**

Physical adsorption usually requires no significant activation energy.

56) Answer **D**

$P_c = a/27b^2$

$V_c = 3b$

$T_c = 8a/27Rb$

$B = V_c/3 = 0.123/3\ Lmol^{-1} = 0.041\ Lmol^{-1}$

The following reaction is always true.

$T_c < T_b < T_i$

=> Boyle’s Temperature is greater than 180 K

Also, a gas cannot be liquified regardless of pressure at temperatures above T_c.

57) Answer **B**

(PCl_5(s) and **$_2O_5$ (s))**

58) Answer **D**

H_3BO_3 is a Lewis Acid. Due to icomplete octet of Boron, it acts like a Bronsted Base which can accept proton. Due to this slightly basic nature, it cannot be used for acid - base titration againt NaOH.

Maths

61)

$$\angle I_1AI_2 = \angle I_1AX + \angle XAI_2 = \frac{\angle BAX}{2} + \frac{\angle CAX}{2} = \frac{\angle A}{2}$$

is a constant not depending on X, so by $[AI_1I_2] = 12(AI_1)(AI_2)\sin\angle I_1AI_2$ it suffices to minimize $(AI_1)(AI_2)$. Let $a = BC$, $b = AC$, $c = AB$, and $\alpha = \angle AXB$. Remark that

$$\angle AI_1B = 180^\circ - (\angle I_1AB + \angle I_1BA) = 180^\circ - 12(180^\circ - \alpha) = 90^\circ + \alpha 2.$$

Applying the Law of Sines to $\triangle ABI_1$ gives

$$\frac{AI_1}{AB} = \frac{\sin\angle ABI_1}{\sin\angle AI_1B} \qquad \Rightarrow \qquad AI_1 = \frac{c\sin\frac{B}{2}}{\cos\frac{\alpha}{2}}.$$

Analogously one can derive $AI_2 = b\sin\frac{C}{2}\sin\frac{\alpha}{2}$, and so

$$[AI_1I_2] = \frac{bc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}} = \frac{bc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\sin\alpha} \geq bc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2},$$

with equality when $\alpha = 90^\circ$, that is, when X is the foot of the perpendicular from A to $\overline{BC}$. In this case the desired area is $bc\sin A2\sin B2\sin C2$. To make this feasible to compute, note that

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{b^2+c^2-a^2}{2bc}}{2}} = \sqrt{(a-b+c)(a+b-c)4bc}.$$

Applying similar logic to $\sin B2$ and $\sin C2$ and simplifying yields a final answer of bcsin $\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = bc\cdot(a-b+c)(b-c+a)(c-a+b)8abc$
$= (30-32+34)(32-34+30)(34-30+32)8\cdot 32 = 126.$

62)Given that the shown face is black, the other face is black if and only if the card is the black card. If the black card is drawn, a black face is shown with probability 1. The total probability of seeing a black face is 1/2; the total probability of drawing the black card is 1/3. By Bayes' theorem, the conditional probability of having drawn the black card, given that a black face is showing, is

$$\frac{1 \times (1/3)}{(½)} = 2/3$$

63)$$\frac{(\ (n,1) - (n,3) + (n,5) - (n,7)....)\text{^}2}{2\text{^}(n/2)} = \frac{e\text{^}(in\boldsymbol{\pi}/4) - e\text{^}(-in\boldsymbol{\pi}/4)}{2i} = \sin(n\boldsymbol{\pi}/4)$$

$$\frac{(\ 1 - (n,2) + (n,4) - (n,6).....)\text{^}2}{2\text{^}(n/2)} = \frac{e\text{^}(in\boldsymbol{\pi}/4) + e\text{^}(-in\boldsymbol{\pi}/4)}{2} = \cos(n\boldsymbol{\pi}/4)$$

Therefore,

((n,1) - (n,3) + (n,5) - (n,7)....)^2 + (1 - (n,2) + (n,4) - (n,6).....)^2 = (sin(n**π**/4))^2+(cos(n**π**/4))^2

2^n

= 1!

The limit as n tends to infinity is therefore 1 too.

64) $e^{2x}(\,ydx\ +\ dy)$ - 6$e^{x}\ x^{2}dx$+ $(dy\ -\ ydx)$ = 0

$(e^{2x}-1)$ydx - 6$e^{x}\ x^{2}dx$ = 0

Dividing throughout with $e^{x}$and rearranging,

(e^{x}+e^{-x})dy + (e^{x}-e^{-x})ydx = $6x^{2}dx$

d((e^{x}+e^{-x})y) = 6$x^{2}dx$

Solving with the given initial condition gives

y(1) = $2e/(e^{2}+1)$

B

65.

Let $A(x, y)$ be the required locus.

First reflect $A(x, y)$ about $x + y = 0$, we get

$$B = (-y,\ -x)$$

Now reflect $A(x, y)$ about $x - 9y = 0$, we get $C = (x', y')$

$$\frac{y - y'}{x - x'} = -9$$

Also,

$$\frac{x - 9y}{\sqrt{1 + 9^2}} = -\frac{x' - 9y'}{\sqrt{1 + 9^2}}$$

$$x + x' = 9(y + y')$$

Solving (1) and (2),

$$C(x', y') = \left(\frac{40x + 9y}{41}, \frac{9x - 40y}{41}\right)$$

Now BC contains P, then

$$\frac{-x - b}{-y - a} = \frac{-x - \dfrac{9x - 40y}{41}}{-y - \dfrac{40x + 9y}{41}}$$

$$\frac{x + b}{y + a} = \frac{50x - 40y}{40x + 50y}$$

$$(x + b)(4x + 5y) = (y + a)(5x - 4y)$$

Hence, the locus is

$$4x^2 + 4y^2 - (5a - 4b)x + (4a + 5b)y = 0$$

66. The curve is ax^2+by^2+2hxy=1

And the line lx+my+n=0 intersects the curve at points p and q.

Given that the circle through PQ as diameter passes through origin.

Using homogenisation,

ax^2+by^2+2hxy-$\frac{(lx+my\blacksquare)2}{n2}$=0

coe of x^2+coe of y^2=0

a-$\frac{l^2}{n^2}$+b-$\frac{m^2}{n^2}$=0

or $n^2(a+b)$=l^2+m^2

67.

Let A ≡ (–a, 0) and B ≡ (a, 0) be two fixed points.

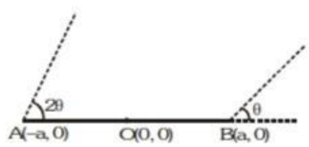
Let one line which rotates about B an angle θ with the x-axis at any time t and at that time the second line which rotates about A make an angle 2θ with x-axis.

Now equation of line through B and A are respectively

$$y - 0 = \tan\theta(x - a) \quad \ldots\ldots (i)$$

and

$$y - 0 = \tan 2\theta(x + a) \quad \ldots\ldots (ii)$$



From (ii), $y = \frac{2\tan\theta}{1-\tan^2\theta}(x+a)$

$$= \left\{\frac{\frac{2y}{(x-a)}}{1-\frac{y^2}{(x-a)^2}}\right\}(x+a) \quad \text{(from (i))}$$

$\Rightarrow$ $y = \frac{2y(x-a)(x+a)}{(x-a)^2-y^2}$ $\Rightarrow$ $(x - a)^2 - y^2 = 2(x^2 - a^2)$

or $x^2 + y^2 + 2ax - 3a^2 = 0$ which is the required locus.

68.

Consider a fixed face diagonal. It is parallel to corresponding one on the opposite face. It intersects the other diagonal of the same face and one diagonal on each of the four neighbouring faces. The other five face diagonals form five pairs of skew lines with the fixed one. By symmetry, each of the twelve faces diagonals. The total count of pair of skew lines is 12 × 5 = 60 but this must be halved since each pair is counted twice.

69.

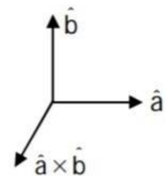
Let $\vec{c}$ makes an angle β from $\hat{a}\times\hat{b}$ then $\cos^2\theta + \cos^2\theta + \cos^2\beta = 1$

$2\cos^2\theta = 1 - \cos^2\beta$

$0 \le 1 - \cos^2\beta \le 1$

$0 \le 2\cos^2\theta \le 1 \Rightarrow 0 \le \cos^2\theta \le \frac{1}{2} \Rightarrow -\frac{1}{\sqrt{2}} \le \cos\theta \le \frac{1}{\sqrt{2}}$

$\theta \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$



70.

A = (2, −1, 1), B = (1, 2, 3), C = (3, 1, 2)

$\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{OC} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\overrightarrow{OA}\times\overrightarrow{OC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = -3\hat{i} - \hat{j} + 5\hat{k}$$

$\therefore$ Equation of plane OAC: $3x + y - 5z = d$

Put (0, 0, 0), d = 0

$\therefore$ Plane is $3x + y - 5z = 0$

Shortest distance $= \frac{|3\times1 + 2 - 5\times3|}{\sqrt{9+1+25}} = \frac{10}{\sqrt{35}} = 2\sqrt{\frac{5}{7}}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & -1 \\ 1 & 1 & 2 \end{vmatrix} = 1(8+1) - 1(3) + 1(1-4) = 9 - 3 - 3 = 3$$

71.

$$\left((\vec{a}\times\vec{b})\times\vec{a}\right)\left((\vec{b}\times\vec{a})\times\vec{b}\right) = \left((\vec{a}\cdot\vec{a})\vec{b} - (\vec{b}\cdot\vec{a})\vec{a}\right)\left((\vec{b}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{b})\vec{b}\right)$$

$$= \left(\vec{a}^2\,\vec{b}^2\right)\vec{a}\cdot\vec{b} - \vec{a}^2\cdot\vec{b}^2\left(\vec{a}\cdot\vec{b}\right) - \left(\vec{a}\cdot\vec{b}\right)\vec{a}^2\cdot\vec{b}^2 + \left(\vec{a}\cdot\vec{b}\right)^3 = -\left[\left(\vec{a}\cdot\vec{b}\right)\vec{a}^2\cdot\vec{b}^2 - \left(\vec{a}\cdot\vec{b}\right)^3\right]$$

$$= -\left(\vec{a}\cdot\vec{b}\right)\left[\vec{a}^2\cdot\vec{b}^2 - \left(\vec{a}\cdot\vec{b}\right)^2\right] = -\left(\vec{a}\cdot\vec{b}\right)\left|\vec{a}\times\vec{b}\right|^2$$

72.

73)

Let $AB = c$, $BC = a$, $AC = b$, $PA = x$, $PB = y$, and $PC = z$.

So by the Law of Cosines, we have: $x^2 = z^2 + b^2 - 2bz\cos\theta$
$y^2 = x^2 + c^2 - 2cx\cos\theta$
$z^2 = y^2 + a^2 - 2ay\cos\theta$ Adding these equations and rearranging, we have:

$$a^2 + b^2 + c^2 = (2bz + 2cx + 2ay)\cos\theta \qquad (1)$$

Now $[CAP] + [ABP] + [BCP] = [ABC] = \sqrt{(21)(8)(7)(6)} = 84$, by Heron's formula.

Now the area of a triangle, $[A] = \frac{mn\sin\beta}{2}$, where m and n are sides on either side of an angle, β. So, $[CAP] = bz\sin\theta\frac{}{2=\frac{cx\sin\theta}{2}=\frac{ay\sin\theta}{2}}$ Adding these equations yields: $[ABC] = 84 = (bz + cx + ay)\sin\theta\frac{}{2\Rightarrow 168=(bz+cx+ay)\sin\theta \qquad (2)}$ Dividing (2) by (1), we have: $168\frac{}{a^2+b^2+c^2=\frac{(bz+cx+ay)\sin\theta}{(2bz+2cx+2ay)\cos\theta}\Rightarrow\tan\theta=\frac{336}{a^2+b^2+c^2}=\frac{336}{14^2+15^2+13^2}=\frac{336}{590}=\frac{168}{295}}$

Thus, $m + n = 168 + 295 = 463$

74)

Let Q be the tangency point on AC, and R on BC. By the Two Tangent Theorem, $AP = AQ = 23$, $BP = BR = 27$, and $CQ = CR = x$. Using $rs = A$, where $s = \frac{27\cdot 2+23\cdot 2+x\cdot 2}{2} = 50 + x$, we get $(21)(50+x) = A$. By Heron's formula, $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(50+x)(x)(23)(27)}$. Equating and squaring both sides,

$$\begin{aligned}
[21(50+x)]^2 &= (50+x)(x)(621) \\
441(50+x) &= 621x \\
180x = 441\cdot 50 \implies x &= \frac{245}{2}
\end{aligned}$$

We want the perimeter, which is $2s = 2\left(50 + \frac{245}{2}\right) = 345$.

75. y^2-2hxy-4x+5h^2=0

$(y-h)^2 = 4(x-h^2)$

Hence eqn of locus is x=y^2

Point (a,a) lies on it where a$\neq 0$

$a^2 = a \Rightarrow a = 1$

The point is (1,1)

$m_{nor} = -2$

76) only c produces meaningful result due to domain restrictions

77)Since they are in A.P
2/(c+a)=1/(b+c) + 1/(a+b)
2(b^2 + (a+c)b +ac)=(c+a)^2 +2b(c+a)
2b^2=a^2+c^2
Which means that a^2,b^2,c^2 are in A.P

78)Given the roots are in A.P. so let a-d, a, a+d be the roots

From equation, sum of roots = 1

Sum of two roots taken at a time = +b

Product of two roots = -c

∴ (a-d)+ (a)+ (a+d) = 1

=) 3a = 1

=) a = 13

Also, (a-d)a+ a(a+d)+ (a-d)(a+d) = b

=) a2 – ad + a2 + ad + a2 – d2 = b

=) 3a2 – d2 = b

=) 3x19 – d2 = b

=) d2 = b - 13

Now, since d is a real number,

13−b>0=)b<13

Also,

(a-d)(a)(a+d) = -c

=) a(a2 – d2) = -c

=) 13∗(19−d2)=−c

=) c=d23−127

=) c=d2−127

=) d2=3c+19

Again, 3c+19>0

c>−127>0

∴ Only option (c) b= -1 , c =1 could be true .

Correct Answer: 9

79)tr(A)=tr(AB−BA)=tr(AB)−tr(BA)=tr(AB)−tr(AB)=0.
Thus tr(A)=0tr(A)=0 and it follows from the Cayley-Hamilton theorem that
A^2=−det(A)I
A^2=A(AB−BA)=A^2B−ABA
A^2=(AB−BA)A=ABA−BA^2
Adding these two, we have

2A^2=A^2B−BA^2=(∗)(−det(A)I)B−B(−det(A)I)=−det(A)B+det(A)B=O

As a result, we obtain A2=O

ans:B

80) $\int_0^{ln(3)} \blacksquare (e^x + e^{-x})/2\ dx\ -\ \int_1^{5/3} \blacksquare \ln\left(x^2 - \sqrt{x^2-1}\right) dx$

$\int_0^{ln(3)} \blacksquare (e^x + e^{-x})/2\ dx\ +\int_1^{5/3} \blacksquare \ln\left(x^2 + \sqrt{x^2-1}\right) dx$

$\int_a^b \blacksquare f(x) dx\ +\ \int_{f(a)}^{f(b)} \blacksquare f^{-1}(x) dx$ = bf(b) - af(a)

Therefore ans: $\frac{5}{3}$ln(3) = ⅓ ln(243)

B

81) $\lim_{n\to\infty}(6^n + 5^n)^{1/n}$

$\lim_{n\to\infty}6(1 + (\frac{5}{6})■^n)^{1/n}$

$\lim_{n\to\infty}6\{(1 + (\frac{5}{6})■^n)\}^{(6/5)■^n*(5/6)■^n*1/n}$

$\lim_{n\to\infty}6(e)^{(5/6)■^n*1/n}$

6

C

82) y = tan^{-1}((1 - log(x^2))/(1 + log(x^2))) + tan^{-1}((3 + log(x^2))/(1 - 3log(x^2)))

Apply $tan^{-1}(x)$+$tan^{-1}(y)$ = $tan^{-1}(\frac{x+y}{1-xy})$

y = $tan^{-1}(-2)$

Therefore ans: 0

C

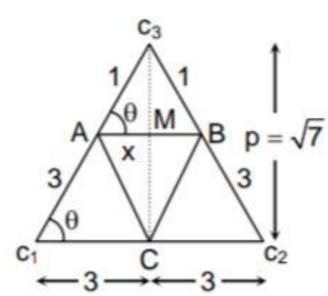
83.

$\cos\theta = \frac{3}{4} = \frac{x}{1}$

$\Rightarrow x = \frac{3}{4} \quad \Rightarrow AB = \frac{3}{2}$

$p = \sqrt{4^2 - 3^2} = \sqrt{7}$ and $c_3M = \frac{\sqrt{7}}{4}$

$\Rightarrow CM = \sqrt{7} - \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{4}$



84.

Let the circle be $(x - h)^2 + (y - k)^2 = r^2$ where r is variable. Its intersection with $x^2 - y^2 = 9a^2$ is obtained by putting $y^2 = x^2 - 9a^2$.

$$x^2 + x^2 - 9a^2 - 2hx + h^2 + k^2 - r^2 = 2k\sqrt{(x^2 - 9a^2)}$$

or $\quad [2x^2 - 2hx + (h^2 + k^2 - r^2 - 9a^2)]^2 = 4k^2(x^2 - 9a^2)$

or $\quad 4x^4 - 8hx^3 + \ldots.. = 0$

$\therefore$ Above gives the abscissas of the four points of intersection.

$$\therefore \quad \Sigma x_1 = \frac{8h}{4} = 2h$$

$x_1 + x_2 + x_3 + x_4 = 2h$

Similarly $\quad y_1 + y_2 + y_3 + y_4 = 2k.$

Now if (α, β) be the centroid of ΔPQR, then $3\alpha = x_1 + x_2 + x_3$, $3\beta = y_1 + y_2 + y_3$

$\therefore \quad x_4 = 2h - 3\alpha,\ y_4 = 2k - 3\beta$

But (x_4, y_4) lies on $x^2 - y^2 = 9a^2$

$\therefore \quad (2h - 3\alpha)^2 + (2k - 3\beta)^2 = 9a^2$

Hence the locus of centroid (α, β) is $(2h - 3x)^2 + (2k - 3y)^2 = 9a^2$

$$\text{or} \quad \left(x - \frac{2h}{3}\right)^2 + \left(y - \frac{2k}{3}\right)^2 = a^2$$

85)The problem would have been really trivial if the ants walked through each other instead of colliding and reversing. But practically, these scenarios are identical even now - only the identities of the ants are switched. The maximum time taken would be when the endpoint ants move towards the centre before the first collision.

86)For this type of question, we need to consider only the internal arrangement within the M and 2As.

M and 2As can be rearranged as AMA, AAM, or MAA.

So, the probability that M will feature between the 2As is 1/3.

Now, let us think why we need to consider only the M and 2As.

Let us start by considering a set of words where the M and 2 As are placed at positions 2, 3 and 5.

The other three letters have to be in slots 1, 4 and 6

Three letters can be placed in three different slots in 3! = 6 ways.

Now with __ M A __ A __ there are 6 different words.

With __ A M __ A __ there are 6 different words.

With __ A A __ M __ there are 6 different words.

For each selection of the positions for A,A and M, exactly one-third of words will have M between the two A's.

This is why only the internal arrangement between A, A and M matters.

So, probability of M being between 2 As is 1/3.

Answer choice (a).

Correct Answer : 1/3

87.

$$p^3(1-p)^2 . C_3^5 = 144/625$$
$$p^3(1-p)^2 = 72/3125$$

This equation has two solutions between 0 & 1, namely ⅖ and 0.78.

88) $\int \frac{e^{cot(x)}}{sin^2 x}$(2ln(cosec x) + sin(2x)) dx

$\int$ $2e^{cot(x)}$(ln(cosec x)$cosec^2 x$ + cotx) dx

-$\int$ $2e^{cot(x)}$(ln(cosec x)(-$cosec^2 x$) +(- cotx)) dx

-$2e^{cot(x)}$ln(cosec x) + c

A

89) $\lim_{x\to 0} \frac{tanxcos(sinx) - sin(sinx)}{tanxcos(tanx) - sin(tanx)}$

$$\lim_{x\to 0}\frac{sin(x-sinx)}{sin(x-tanx)}$$

$$\lim_{x\to 0}\frac{(x-sinx)}{(x-tanx)}$$

Use expansion for sin and tan.

Ans: -½

90.

$p \to (p \vee q)$ and $q \to (p \to q)$ both are tautologies, while formulae
$\sim p \wedge (p \vee q)$ and $\sim p \wedge \sim q$ are not

.