Scattering Amplitudes

The Rutherford amplitude for pure Coulomb scattering (with no $e^{2i\sigma_0}$ factor) is

$$F_c(\theta) = -\frac{\eta}{2k} \; \frac{\exp(-2i\eta \ln(\sin\theta/2))}{\sin^2\theta/2} \tag{1}$$

The Legendre coefficients for the scattering to the projectile state J'_p and target state J'_t from initial projectile state J_p and target state J_t are given by

$$\begin{aligned} A^{L'}_{m'M';mM} \quad &= \sum_{L,J,J',J_T} \langle L0J_pm|Jm\rangle\langle JmJ_tM|J_TM_T\rangle \\ &\quad \langle L'M_{L'}J'_pm'|J'M_{L'}+m'\rangle\langle J'M_{L'}+m'J'_tM'|J_TM_T\rangle \\ &\quad \frac{4\pi}{k}\sqrt{\frac{k'}{\mu'}\frac{\mu}{k}}e^{i(\sigma_L-\sigma_0)}e^{i(\sigma'_{L'}-\sigma'_0)} \\ &\quad \left(\frac{i}{2}\right)\left[\delta_{\alpha,\alpha'} - S^{J_T}_{\alpha,\alpha'}\right]\sqrt{\frac{2L+1}{4\pi}}\;Y_c(L',M_{L'}) \end{aligned} \tag{2}$$

where $Y_c(L,M)$ is the coefficient of $P_L^{|M|}(\cos\theta)e^{iM\phi}$ in $Y_L^M(\theta,\phi)$, $\sigma_L = \arg\Gamma(1+L+i\eta)$ is the Coulomb phase shift, α' refers to the primed values $L'J'_pJ'_tk'\mu'$ etc., and α refers to the unprimed values $LJ_pJ_tk\mu$.

For each outgoing channel J'_p, J'_t, we may then calculate the angular-dependent scattering amplitudes

$$f_{m'M':mM}(\theta) = \delta_{J_p,J'_p}\delta_{J_t,J'_t}F_c(\theta) + \sum_{L'} A^{L'}_{m'M':mM}P_{L'}^{m'+M'-m-M}(\cos\theta) \tag{3}$$

in terms of which the differential cross section is

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{(2J_p+1)(2J_t+1)}\sum_{m'M'mM}\left|f_{m'M':mM}(\theta)\right|^2. \tag{4}$$

Improvements?

Can we avoid the excessive recalculation of the second two Clebsch-Gordan coefficients in eq.(2)? Do they really have to be recalculated for every $m'M';mM'$ and α,α'?