Angular Correlations and FRESCO

I. INTRODUCTION

The nature of the particle emission resulting from the decay $B \to C + c$ following the reaction $A(a,b)B$ is discussed in Sec. 10.7.4 of Satchler [1]. The double differential cross section for detecting b and c is given by Satchler, Eq. 10.126[1]:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}\Omega_c} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_b}\frac{W}{4\pi}. \tag{1}$$

The angular correlation function W given by Satchler Eqs. 10.127 and 10.130

$$W = \sum_{kq} t_{kq}(I_B)\, R_k\, C_{kq}^*, \tag{2}$$

where I_B and $t_{kq}(I_B)$ are the spin and polarization tensor of the nucleus B, R_k are the real radiation parameters, and C_{kq} are related to a the spherical harmonics:

$$C_{kq} = \left[\frac{4\pi}{2k+1}\right]^{1/2} Y_{kq}. \tag{3}$$

Following the convention used by FRESCO [2], we will choose the z axis to be along the incident beam. We then have $t_{kq}(I_B) = t_{kq}(\theta_B)$ and $C_{kq} = C_{kq}(\theta_c, \phi_c)$ where $\theta_B = \pi - \theta_b$ and θ_c are the usual polar angles in the c.m. system and ϕ_c is the azimuthal angle between the particles b and c.

II. RADIATION PARAMETERS

The particle radiation parameters R_k are discussed in Sec. 10.7.4.2 of Satchler. Assuming that $B \to C + c$ occurs with a single orbital angular momentum L and channel spin S,

$$R_k = (2I_B + 1)^{1/2}(2L+1)(-1)^{k+S-I_B}\langle LL00|k0\rangle\, W(LLI_BI_B; kS), \tag{4}$$

where $\langle LL00|k0\rangle$ is a Clebsch-Gordon coefficient and $W(LLI_BI_B; kS)$ is a Racah coefficient. Parity conservation implies that k must be even. For the case of $^{19}\text{Ne}^* \to {}^{15}\text{O}(1/2^-) + \alpha(0^+)$ we have $S = \frac{1}{2}$ and L is uniquely determined for a given J^π state in ^{19}Ne. For the general case, relative partial width amplitudes for the decay $B \to C + c$ must be specified.

III. POLARIZATION TENSORS

The polarization tensor $t_{kq}(I_B)$ describes the polarization of the final nucleus B; the precise definition is given in Secs. 10.3.2 and 10.3.3 of Satchler. The polarization tensors can be calculated with FRESCO by noting that the FRESCO scattering amplitudes $f_{m'M':mM}$ are equivalent to Satchler's transition matrix elements $T_{\beta\alpha}$ defined by his Eq. 9.2. Satchler's Eq. 10.32 reads

$$t_{kq}(I_B) = \frac{tr[\boldsymbol{T}\boldsymbol{T}^\dagger \tau_{kq}(I_B)]}{tr[\boldsymbol{T}\boldsymbol{T}^\dagger]}. \tag{5}$$

In the notation of Thompson (analogous to Eq. 3.33 of Ref. [2]) this formula becomes

$$
\begin{aligned}
t_{kq}(I_B) &= \frac{tr[\boldsymbol{f}\boldsymbol{f}^\dagger \tau_{kq}(I_B)]}{tr[\boldsymbol{f}\boldsymbol{f}^\dagger]} \\
&= \sqrt{2k+1}\, \frac{\displaystyle\sum_{m'M'mM} f_{m'M':mM}(\theta)^* f_{m'M'':mM}(\theta)\langle I_B M' kq | I_B M''\rangle}{\displaystyle\sum_{m'M'mM} |f_{m'M':mM}(\theta)|^2},
\end{aligned} \tag{6}
$$

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[1] The branching-ratio factor Γ_c/Γ in Eq. 10.126 has been suppressed.

where $M'' = M' + q$ is required for the Clebcsh-Gordon coefficient to be non-zero. Also note that the differential cross section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_b} = \frac{1}{(2I_A + 1)(2I_a + 1)} \sum_{m'M'mM} |f_{m'M':mM}(\theta)|^2. \tag{7}$$

The above equations can be used to calculate the angular correlation using the scattering amplitudes output by FRESCO. Note that the scattering amplitude file output by FRESCO is controlled with the LAMPL parameter; typically LAMPL=-2 is needed.

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[1] G. R. Satchler, *Direct Nuclear Reactions*, (Clarendon, Oxford, 1983).
[2] I. J.Thompson, Computer Physics Reports **7**, 167 (1988).