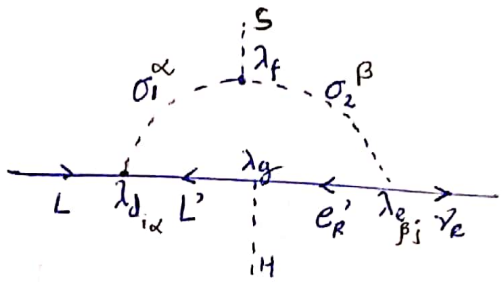
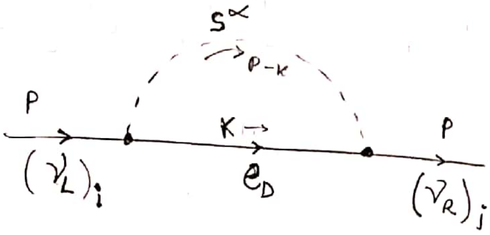
Like

Zee model :



M

$$\mathcal{L} \supset h_d \bar{L}'_L L_i \sigma_1^\alpha + h_g H^\dagger \bar{e}_R' l_L' + h_e^{\alpha i} \bar{\nu}_{R_i} e_R \sigma_2^{*\alpha} + K_s^{\alpha\beta} S^4 \sigma_1^\alpha \sigma_2^\beta \quad (1)$$

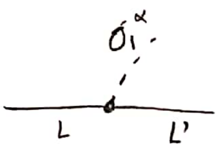


$$M_{ij}^\nu = i\int \frac{d^4k}{(2\pi)^4} (-i \tilde{h}_d^{T\, i\alpha})\, i S_f(k) (-i \tilde{h}_e^{j\alpha})\, i\Delta(p-k)$$

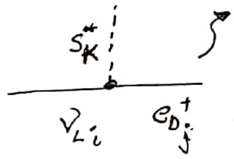
$$= -i \frac{\tilde{h}_d^{i\alpha} \tilde{h}_e^{j\alpha}}{16\pi^2} \int \frac{d^4k}{\pi^2} \frac{\not{k} + m_{e_D}}{(k^2 - m_{e_D}^2)\left[(p-k)^2 - m_{S^\alpha}^2\right]}$$

$p \to 0$

$$M_{ij}^\nu = i^2 \frac{\tilde{h}_d^{i\alpha} \tilde{h}_e^{j\alpha}}{16\pi^2} \quad m_{e_D} B_0(0; m_{e_D}^2, m_{S_\alpha}^2) \quad \text{ver tesis}$$



$\Rightarrow$



$$-i\, U^L_{L_{j1}} U^{\nu\,-1}_{ib}\, h_d^{ab}\, Z^s_{K_{3+\alpha}} P_L \quad - i\, h_e^{ab} \sum_{a=1,2} Z_{K_{3+\alpha}} U^{\nu\,-1}_{3+b}\, U^R_{j1} P_R$$

$$\sum_{a=1,2}$$

$$= -\frac{\tilde{h}_d^{i\alpha} \tilde{h}_e^{j\alpha}}{16\pi^2} m_{e_D} \left[\frac{m_{e_D}^2 Ln(m_{e_D}^2) - m_{S_\alpha}^2 Ln(m_{S_\alpha}^2)}{m_{e_D}^2 - m_{S_\alpha}^2}\right] \quad (2)$$

Zee original model
$$\begin{pmatrix} S_1^\pm \\ S_2^\pm \end{pmatrix} = \begin{pmatrix} c\varphi & -s\varphi \\ s\varphi & c\varphi \end{pmatrix} \begin{pmatrix} \sigma_1^\pm \\ \sigma_2^\pm \end{pmatrix} \to S_1^\pm = c\varphi\, \sigma_1^\pm - s\varphi\, \sigma_2^\pm$$

$$\sigma_1^\pm = c\varphi\, S_1^\pm + s\varphi\, S_2^\pm$$
$$\sigma_2^\pm = -s\varphi\, S_1 + c\varphi\, S_2^\pm$$

$$-\mathcal{L} \supset f_{\alpha\beta} \bar{L}^c_\alpha L_\beta \sigma_1^+$$
$$f \bar{L}^c L (c\varphi S_1^+ + s\varphi S_2^+)$$

$$M_{ij}^\nu = -\frac{f_{i\beta}(c\varphi)\quad h_{\beta j}(-s\varphi)}{16\pi^2} m_{e^\beta} \frac{\left[m_{e\beta}^2 Ln(m_{e_\beta}^2) - m_{S_1}^2 Ln(m_{S_1}^2)\right]}{m_{e_\beta}^2 - m_{S_1}^2} + \left\{ \begin{matrix} S_1 \to S_2 \\ c\varphi \to +s\varphi \\ s\varphi \to +c\varphi \end{matrix} \right\}$$

Por diagramas con S_1^+ y S_2^+

$$= + \frac{\text{sen}(2\varphi)/2}{16\pi^2} f^T_{i\beta} h_{\beta j}\, m_{e_\beta} \frac{[\qquad]}{m_{e_\beta}^2 - m_{S_1}^2} + \{\quad - \quad\}$$

$$\sim \frac{m_e^2 Ln(m_e^2) - m_{S_1}^2 Ln(m_{S_1}^2)}{m_e^2 - m_{S_1}^2} - \frac{m_e^2 Ln(m_e^2) - m_{S_2}^2 Ln(m_{S_2}^2)}{m_e^2 - m_{S_2}^2} + \dots$$

$$\sim Ln(m_e^2) + \frac{m_{S_1}^2 Ln(m_e^2) - m_{S_1}^2 Ln(m_{S_1}^2)}{m_e^2 - m_{S_1}^2}$$

$$2 Ln(m_e^2) \quad \frac{m_{S_1}^2 Ln(m_e^2/m_{S_1}^2)}{m_e^2 - m_{S_1}^2} - \quad m_{1S} \to m_{S_2}$$

$$m_{e\beta} \ll m_{S_1}^2 \Rightarrow \sim \left[\frac{m_e^2 \mathrm{Ln}(m_e^2) - m_{S_1}^2 \mathrm{Ln}(m_{S_1}^2)}{\cancel{m_{e\beta}^2}^{\to 0} - m_{S_1}^2}\right] + \left[\frac{m_e^2 \mathrm{Ln}(m_e^2) - m_{S_2}^2 \mathrm{Ln}(m_{S_2}^2)}{-m_{S_2}^2}\right]$$

$$\left[-\cancel{\frac{m_e^2}{m_{S_1}^2}\mathrm{Ln}(m_e^2)} + \mathrm{Ln}(m_{S_1}^2)\right] + \left[\cancel{\frac{m_e^2}{m_{S_2}^2}\mathrm{Ln}(m_e^2)} - \mathrm{Ln}(m_{S_2}^2)\right]$$

$$M^\nu_{ij} = -\frac{1}{16\pi^2} f^T_{i\beta}\, h_{\beta j}\, m_{e\beta}\left[-\frac{\mathrm{Sen}(2\varphi)}{2}\mathrm{Ln}(m_{S_1}^2) + \frac{\mathrm{Sen}(2\varphi)}{2}\mathrm{Ln}(m_{S_2}^2)\right]$$

error de $-1/2$.

$$\boxed{M^\nu_{ij} = -\frac{K}{2} f^T_{i\beta}\,[m_{e\beta\beta}]\,h_{\beta j} \quad \therefore \quad K = \frac{\mathrm{Sen}(2\varphi)}{16\pi^2}\mathrm{Ln}(m_{S_2}^2/m_{S_1}^2)}$$ (3)

En $U(1)_B$ model:

$$\begin{pmatrix} S_1^\pm \\ S_2^\pm \end{pmatrix} = \begin{pmatrix} c\varphi & -s\varphi \\ s\varphi & c\varphi \end{pmatrix}\begin{pmatrix} \sigma_1^\pm \\ \sigma_2^\pm \end{pmatrix}$$

$$\mathcal{L} \supset \begin{pmatrix} S_1^\alpha & S_2^\beta \end{pmatrix}\begin{pmatrix} \frac{\lambda_6^\alpha}{2} + \mu_1^\alpha & \lambda_f^\alpha \frac{v_x}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\lambda^{T\beta} v_x & \frac{\lambda_7^\beta}{2} + \mu_2 \end{pmatrix}\begin{pmatrix} S_1^{\alpha *} \\ S_2^{\beta *} \end{pmatrix}$$

$$\sim \begin{pmatrix} S_1^1 & S_1^2 & S_2^1 & S_2^2 \end{pmatrix}\left(\begin{array}{cc|cc} \frac{\lambda_6^1}{2}+\mu_1^1 & \lambda_f^1 \frac{v_x}{\sqrt{2}} & & \\ \lambda_6^2+\mu_1^2 & \lambda_f \frac{v_x^2}{\sqrt{2}} & \multicolumn{2}{c}{0} \\ \hline & & \frac{\lambda_6^2}{2}+\mu_1^2 & \lambda_f^2/\sqrt{2} \\ \multicolumn{2}{c|}{0} & \frac{\lambda_f^2+\mu_1^2}{\sqrt{2}} & \frac{\lambda_7^2}{2}+\mu_2 \end{array}\right)\begin{pmatrix} S_1^1 \\ S_1^2 \\ S_2^1 \\ S_2^2 \end{pmatrix}^*$$

$\alpha = \beta = 1, 2$.

No mixing between S_1, S_2.

$\longrightarrow$ Diagonal by blocks

$$\begin{pmatrix} S_1^1 \\ S_1^2 \end{pmatrix} = \begin{pmatrix} c\varphi_1 & -s\varphi \\ s\varphi & c\varphi \end{pmatrix}\begin{pmatrix} \sigma_1^1 \\ \sigma_1^2 \end{pmatrix}$$

$$\begin{pmatrix} S_2^1 \\ S_2^2 \end{pmatrix} = \begin{pmatrix} c\beta & -s\beta \\ s\beta & c\beta \end{pmatrix}\begin{pmatrix} \sigma_2^1 \\ \sigma_2^2 \end{pmatrix}$$

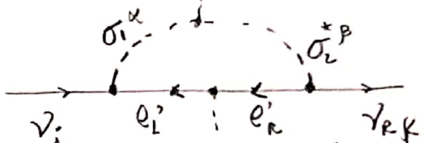
$$\mathcal{L} \supset (S_1^\alpha\ S_2^\beta)\, M'^2 \begin{pmatrix} S_1^\alpha \\ S_2^\beta \end{pmatrix}$$

$$\underbrace{(\ )U^\dagger}_{\chi}\,\underbrace{U M'^2 V}_{M_D}\,\underbrace{V^\dagger(\ )}_{\chi^\dagger}$$

En general.

$M^\nu_{i_1} = -$ $\qquad \mathcal{L} \supset h_d^{\alpha i'} L_L L_i \sigma_1^\alpha + h_e^{\beta j} \nu_{R_K} e'_R \sigma_2^{*\beta} + h.c$

$h_d^{\alpha i}(e'_L \nu_j)\begin{pmatrix}\nu_i\\ e_i\end{pmatrix}\sigma_{1+}^{\alpha} \dots \qquad \Rightarrow h_d^{\alpha i} e'_L \nu_i \sigma_1^\alpha + h_e^{\beta K} e'_R \nu_{R_K} \sigma_2^{*\beta} + h.c$



$$\sigma_i^\pm = \sum_{j=1}^4 Z_{ji} H_j^\pm \quad \Rightarrow \quad \mathcal{L} \supset h_d^{\alpha i} e'_L \nu_i (Z^1_{j\alpha} H_j^\pm) + h_e^{\beta K} e'_R \nu_{R_K}(Z^2_{j\beta} H_j^\pm) + h.c$$

$$= (h_d)^{\alpha i} e'^+_L \nu_i^+ (Z^1_{j,\alpha+1} H_j^{\pm *}) + h_e^{\beta K} e'_R \nu_{R_K} (Z^2_{j,\beta+3} H_j^{\pm *}) + h.c$$

$$\Rightarrow \tilde h_d = (h_d^T)^{\alpha i} Z^1_{j\alpha} \quad \wedge \quad \tilde h_e^{\beta K} = h_e^{\beta K} Z^2_{j\beta}$$

$$M^\nu_{iK} = -\frac{[(h_d^T)^{\alpha i} Z^1_{j\alpha}]\,[h_e^{\beta K} Z^2_{j\beta}]}{16\pi^2} \sum_{\ell=1}^2 m_{e'_\ell}\left[\frac{m_{e'_\ell}^2 Ln(m_{e'_\ell}^2) - m_{H_j^\pm}^2 Ln(m_{H_j^\pm}^2)}{(m_{e_\ell}^2 - m_{H_j}^2)}\right] \times V_{LD_{\ell 1}} U_{RD_{\ell 1}} \qquad \ell = 1,2$$

$$= -\sum_{\alpha=1,2} \frac{(h_d)^{\alpha i}}{16\pi^2} Z^1_{j\alpha} \sum_{\beta=1,2}^{2} h_e^{\beta K} Z^2_{j\beta} \sum_{\ell=1}^2 V_{LD,\ell 1} U_{RD,1\ell}\, m_{e'_\ell} \left[\rule{3cm}{0.4pt}\right]$$

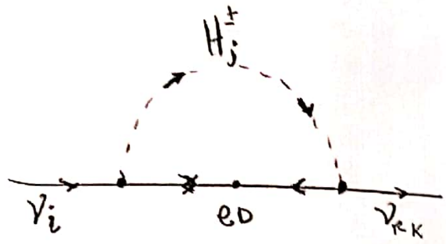
$$M^\nu_{iK} = \sum_{\alpha=1,2}\sum_{\beta=1,2} \frac{(h_d)^{\alpha i}}{16\pi^2} \sum_j Z^1_{j\alpha} Z^2_{j\beta} \sum_{\ell=1}^2 V^L_{\ell 1} U^R_{1\ell}\, m_{e'_\ell} \left[\frac{\ell, j}{\dots}\right] h_e^{\beta K}$$

$$\sum_\alpha\sum_\beta \frac{(h_d)^{\alpha i}}{16\pi^2}\sum_j \Lambda^{\alpha\beta}_j h_e^{\beta K} \simeq \underset{2\times3}{(h_d)^{\alpha i}}\, \underset{2\times2}{\Lambda^{\alpha\beta}}\, \underset{2\times3}{h_e^{\beta K}} \propto \underline{(h_d^T)^{i\alpha}\Lambda^{\alpha\beta} h_e^{\beta K}}$$

$$\boxed{\Lambda^{\alpha\beta} = \frac{1}{16\pi^2}\sum_{j=1}^4 \underbrace{Z^1_{j,\alpha+1}}_{=2,3} \underbrace{Z^2_{j,\beta+3}}_{=4,5} \sum_{\ell=1}^2 \underbrace{V^L_{\ell 1}}_{\delta_{\ell 1}} \underbrace{U^R_{\ell 1}}_{U_{1\ell}}\, m_{e_\ell} \left[\frac{m_{e_\ell}^2 Ln(m_{e_\ell}^2) - m_{H_j}^2 Ln(m_{H_j}^2)}{(m_{e_\ell}^2 - m_{H_j}^2)}\right]}$$

Zee: original
$$\begin{cases} Z_{11}Z_{12}\, Ln(m_{H_1}^2)' + Z_{21}Z_{22}\, Ln(m_{H_2}^2) \\ -c_\varphi s_\varphi\, Ln(m_{H_1}^2) + c_\varphi s_\varphi\, Ln(m_{H_2}^2) \sim \frac{Sen(2\varphi)}{2} Ln(m_{H_2}^2/m_{H_1}^2) \end{cases}$$

Aclaración :



$$M^{\nu}_{ik} = -\Big[(h_d)^{\alpha i}_{\underset{1,2}{j}} \, Z_{j\underset{2,3}{\alpha}}\Big]\Big[\underset{1,2}{h_e^{\beta k}} \, Z_{j\underset{4,5}{\beta}}\Big]\sum_{\ell} \Omega_{\ell j}$$

$$= -\Big[(h_d)^{\alpha i} \, Z_{j,\alpha+1}\Big]\Big[(h_e)^{\alpha k} \, Z_{j,\alpha+3}\Big]\sum_{\ell} \Omega_{\ell j}$$

$$\boxed{-M^{\nu}_{ik} = +\underset{2\times3}{(h_d)^{\alpha i}} \; \underset{2\times2}{\Lambda^{\alpha}} \; \underset{2\times3}{(h_e)^{\alpha k}}} \quad (1)$$

$$\boxed{\underset{\alpha=\{1,2\}}{\Lambda^{\alpha}} = \frac{1}{16\pi^2}\sum_{j=1}^{4} Z_{j,\alpha+1} Z_{j,\alpha+3} \sum_{\ell=1}^{2} V^{L}_{\ell 1} U^{R}_{\ell 1} (m_{eD})_{\ell} \left[\frac{m_{eD\ell}^2 \, \mathrm{Ln}(m_{eD\ell}^2) - m_{H_j}^2 \, \mathrm{Ln}(m_{H_j}^2)}{m_{eD\ell}^2 - m_{H_j}^2}\right]} \quad (2)$$

$$-\underset{3\times3}{M^{\nu}_{ik}} = \begin{pmatrix} h_d^{1i} & h_d^{2i} \end{pmatrix}_{1\times2} \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^2 \end{pmatrix}_{2\times2} \begin{pmatrix} h_e^{1k} \\ h_e^{2k} \end{pmatrix}_{2\times1} = \Big[h_d^{1i}\Lambda^1 h_e^{1k} + h_d^{2i}\Lambda^2 h_e^{2k}\Big]_{1\times1} \quad (3)$$

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$$-M_{ik} = (U^{PMNS})_{i\ell}(m_\nu)_{\ell\ell} \underset{\delta_{\ell k}}{V_{\ell k}} = U_{ik}(m_\nu)_k \quad (4)$$

$$(3)=(4) \Rightarrow \underbrace{\underset{3}{h_d^{1i}}\Lambda^1 \underset{2}{h_e^{1k}} + \underset{3}{h_d^{2i}}\Lambda^2 \underset{2}{h_e^{2k}}}_{10 \text{ parameters}} = \underbrace{U_{ik} \, m_{\nu k}}_{6 \text{ ec for NH}} \quad ; \forall \, i,k = \{1,2,3\}$$