C++ IN QUANTITATIVE FINANCE FINAL PROJECT

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In accordance with the Honor Code, I certify that my answers here are my own work, and I did not make my solutions available to anyone else.

PROJECT DESCRIPTION

The aim of this project was to write a C++ program to price a down-and-in barrier put option using Monte Carlo simulations.

ASSUMPTIONS

To calculate path dependent option, first we need to assume underlying asset motion. We assume that underlying (stock) follows geometric Brownian motion which is described by $dS_t = \mu S_t dt + \sigma S_t dW_t$,
Using Ito's lemma to solve this equation we obtain $logS_t = logS_0 + \left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t$.
Where wiener process can be rewritten as $W_T = \sqrt{T}x$, where $x \sim N(0,1)$. This leads to the discretized form for small time steps Δt: $logS_{t+\Delta t} = logS_t + \left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}x$. This discretized form is used to simulate the evolution of the underlying asset's price path over time in Monte Carlo simulations. For basic case of simulation number of timestep interval is set to 100 and time to maturity is 0,75. This means that each time step is approximately two days

OPTION CHARACTERISTICS

The assigned put option was activated by the knock-in barrier.

Parameters of options are following:

- Initial price of underlying 100
- Strike price 110
- Volatility 25%
- Risk free rate 5%
- Time to maturity 0.75

CODE

Project file structure is following:

```
.
├── ./includes
│       ├── ./includes/gen_rand.cpp
│       ├── ./includes/gen_rand.h
│       ├── ./includes/helpers.cpp
│       ├── ./includes/helpers.h
│       ├── ./includes/options.cpp
│       └── ./includes/options.h
└── ./main.cpp
```

Below is the description of each code file in the project:

Code file	Description
`main.cpp`	The main program file that executes the core functionality.
`options.cpp`	Contains the implementation of the class `Option`.
`helpers.cpp`	Includes additional utility functions used throughout the code.
`gen_rand.cpp`	Implements functions for generating random variables with a normal distribution.

File `options.cpp` contains class `Option` which has following data members:

Type	Datatype	Name	Description
private	`double`	`strike`	Strike price
	`double`	`spot`	Spot price
	`double`	`vol`	Annualized volatility
	`double`	`r`	Risk-free rate
	`double`	`expiry`	Time to expiry in years
	`unsigned int`	`n_intervals`	Number of intervals for simulation
	`unsigned int`	`n_sim`	Number of simulations
	`vector<double>`	`this_path`	Path of underlying from single simulation
	`vector<double>`	`this_payout`	Payout of the option for each simulation
	`double`	`price`	Price of the option (initial is -1 which indicates that none simulations was run)
Public	`enum`	`OptionType`	Grup of constants `call` or `put` indicating option type
	`enum`	`OptionStyle`	Grup of constants `european` or `barrier` indicating option style

Type	Datatype	Name	Description
	enum	BarrierType	Grup of constants `down_in`, `down_out`, `up_in`, `up_out` indicating option type

Methods of class Option are following:

Return type	Name	Arguments	Description
	Option		Constructs an `Option` object with default values as defined in homework assignment
	Option	• _strike • _spot • _vol • _r • _expiry • _n_intervals	Constructs an `Option` object by overwriting the default constructor with user-defined parameters.
void	print_details		Prints details of option object.
void	generate_path		Generates single path with `n_intervals` defined in the constructor.
void	generate_path	• n	Generates a single path with n intervals, currently used for pricing European options where `n = 1` (not path-dependent).
void	print_path		Prints last simulated path.
double	get_payout	• type • end_price	Calculates payout given `OptionType type` and end price.
bool	is_barrier_touched	• barrier_price • type	Check if barrier is touched given barrier price and `BarrierType type`.
double	get_price	• n_sim • tyoe • style	Calculates price of European option given number of simulations, `OptionType type` and `OptionStyle style`.
double	get_price	• n_sim • tyoe • style • b_tyoe • barrier_price	Calculates price of European /Barrier option given number of simulations, `OptionType type`, `OptionStyle style`, `BarrierType b_type` and barrier price.
double	get_price		Returns price of option if pricing simulations were made.
double	get_stddev		Returns standard deviation of the option price if pricing simulations were made.
void	print_barrier_sensitivity	• base_barrier • range	Calculates and prints the option price sensitivity to changes in the barrier price.

Return type	Name	Arguments	Description
		• type • style • b_type	

It's important to note that the barrier price is not defined as a separate member of the `Option` class, but rather as an argument in the `get_price` method. This design choice differs from what was suggested in the assignment description. I found it more efficient to analyze the option's price sensitivity to different barrier prices by running the method with varying barrier price values, rather than creating a new `Option` object for each experiment or or modifying the option's data members.

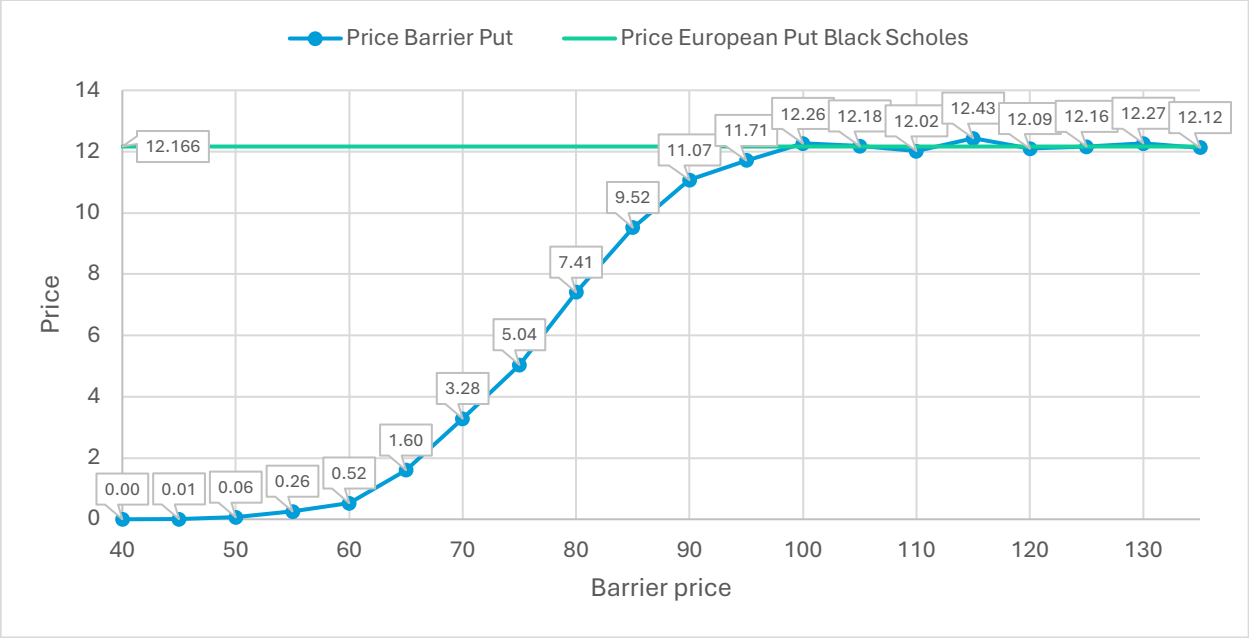
Additional functions from helpers.cpp and gen_rand.cpp are:

Return type	Name	Arguments	Description
double	generate_rand_norm		Generates a random number from the standard normal distribution using the modified Box-Muller method.
double	get_max	• a • b	Calculates the maximum value between two given numbers.
double	get_arr_mean	• arr	Calculates the mean of the elements in the given vector.
double	get_arr_stddev	• arr	Calculates the standard deviation of the elements in the given vector.
double	get_arr_stddev	• arr • mean	Calculates the standard deviation of the elements in the given vector, with the mean provided.

RESULTS

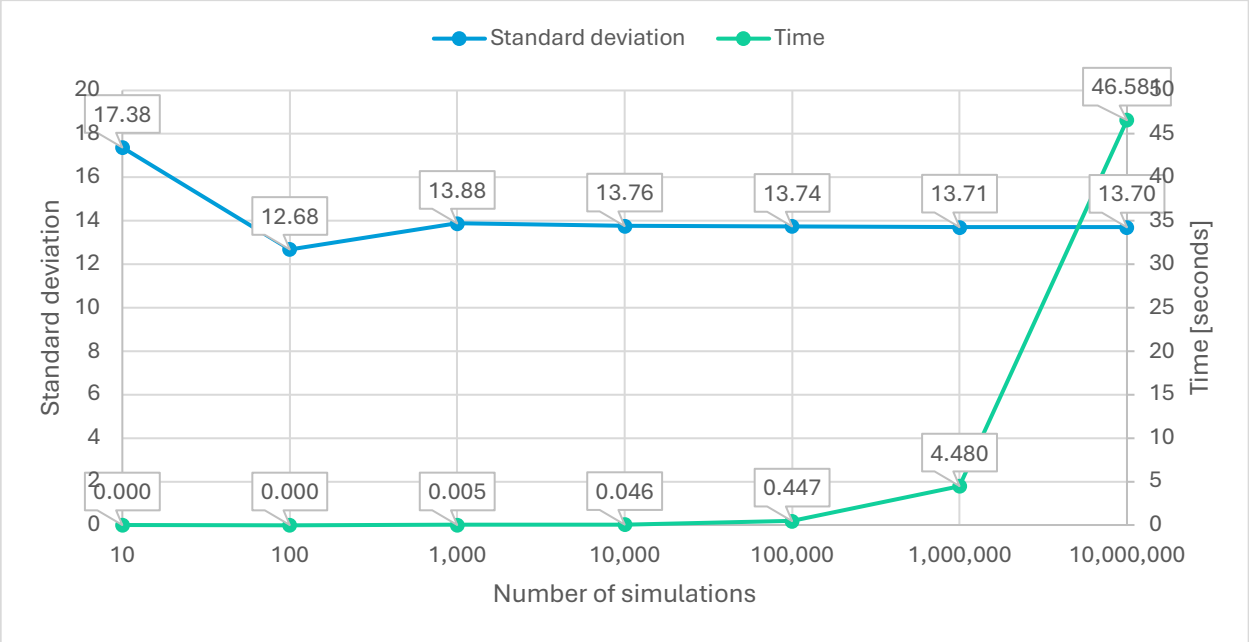
The price of a down-and-in put option with a barrier price of 90 is 11.0687. When the barrier price is set equal to the strike price, the option effectively becomes a regular European put option, as it will only generate profit if the underlying asset reaches the strike price. Similarly, setting the barrier price above the initial price transforms the option into a standard European put, as the barrier is immediately activated.

The graph below illustrates the sensitivity of the option price to changes in the barrier price.



As shown, setting the barrier price above the initial price of 100 has no impact on the option price since the barrier is already activated. However, as the barrier price decreases, the option becomes less likely to be activated. When it is activated, the profit potential increases because the option is only triggered when the underlying price drops significantly, aligning with the put option's purpose. Once the barrier price is set below 40, the expected profit becomes zero, as the option is unlikely to be triggered.

The standard deviation of option prices in the baseline scenario (10,000 simulations) was 13.76, which is unsatisfactory. Such a high standard deviation results in a wide confidence interval for the valuation, making the results less reliable and almost insignificant. While increasing the number of simulations reduces the standard deviation slightly, even with 10,000,000 simulations, the standard deviation remains unsatisfactory.



SUMMARY

Barrier Price Impact:

- Setting the barrier above the initial price converts the option to a European put.
- Lowering the barrier reduces activation likelihood but increases profit potential when triggered.

Simulation Results:

- Initial standard deviation was 13.76 for 10,000 simulations, leading to wide confidence intervals.
- Increasing simulations to 10,000,000 slightly reduced the standard deviation but remained unsatisfactory.

The next steps for this project could involve expanding the analysis to cover all types of barrier options, including down-and-up knock-in and knock-out options. Additionally, to improve the accuracy of valuations, variance reduction techniques such as antithetic variates should be tested and implemented.