Causal Disparity Analysis

Ang Yu

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Takeaway messages

1. Causal disparity analysis (CDA) has a similar structure to causal mediation analysis.
2. But these two are different species.
3. Causal disparity analysis is a hot topic, feel free to consider writing your final paper on it.

What questions do we ask in CDA?

- How much, and in what ways, do racial differences in medical care contribute to racial disparities in health?

  race $\rightarrow$ medical care $\rightarrow$ health

- In which ways does occupational sorting contribute to the gender wage gap?

  gender $\rightarrow$ occupation $\rightarrow$ wage

- What are the roles of educational attainment in the relationship between social class origin and destination?

  origin $\rightarrow$ education $\rightarrow$ destination

- Does racial differences in incarceration explain the racial disparity in income?

  race $\rightarrow$ incarceration $\rightarrow$ income

The common structure of these questions

- Three temporally ordered variables

  group $\rightarrow$ treatment $\rightarrow$ outcome
- Goal of CDA:

  Quantify the mechanisms by which a treatment variable causally explains a descriptive group disparity
- Why do we care about *descriptive* group disparity?
- Why do we care about *causal* explanation of group disparities?

Wait, isn't this a mediation class?

- In causal mediation analysis (CMA), we also have a three variable structure

  treatment → mediator → outcome

- But, CMA explains causal effect of treatment, not *descriptive* group disparity, e.g.,

$$\mathrm{E}\left(Y^{a} - Y^{a^*}\right) = \mathrm{E}\left(Y^{aM^a} - Y^{aM^{a*}}\right) + \mathrm{E}\left(Y^{aM^{a*}} - Y^{a^* M^{a*}}\right).$$

  Randomized interventional analogues:

$$\mathrm{E}\left(Y^{aG^a} - Y^{a^* G^{a*}}\right) = \mathrm{E}\left(Y^{aG^a} - Y^{aG^{a*}}\right) + \mathrm{E}\left(Y^{aG^{a*}} - Y^{a^* G^{a*}}\right).$$

How can we do CDA (causal disparity analysis)?

1. Kitagawa-Blinder-Oaxaca decomposition (Kitagawa, 1955; Blinder, 1973; Oaxaca, 1973)
2. Random equalization decomposition (VanderWeele and Robinson, 2014; Jackson and VanderWeele, 2018; Jackson, 2022; Lundberg, 2022)
3. Self-promotion: Yu and Elwert (2023)

KBO

Group: $G \in \{a, b\}$; treatment: D; outcome: Y; baseline covariates: X

Group-specific linear regression:

$$Y = \alpha_g + \beta_g D + \gamma_g^T X + \epsilon.$$

KBO decomposition:

$$\mathrm{E}_a(Y) - \mathrm{E}_b(Y) = \underbrace{\alpha_a - \alpha_b}_{\text{intercept}} + \underbrace{\beta_b[\mathrm{E}_a(D) - \mathrm{E}_b(D)]}_{\text{characteristic}} + \underbrace{\mathrm{E}_a(D)[\beta_a - \beta_b]}_{\text{slope}}$$
$$+ \gamma_b^T[\mathrm{E}_a(X) - \mathrm{E}_b(X)] + \mathrm{E}_a(X)^T[\gamma_a - \gamma_b].$$

Problem?

Random equalization decomposition

Two versions: Jackson's and Lundberg's

$\mathrm{E}_g\left(Y^{R(D|g')}\right)$: post-intervention mean outcome for group g after each member of group g is counterfactually given a treatment value randomly drawn from group g'.

The Jackson version:

$$\mathrm{E}_a(Y) - \mathrm{E}_b(Y) = \underbrace{\mathrm{E}_b\left(Y^{R(D|a)}\right) - \mathrm{E}_b(Y)}_{\text{change in disparity}} + \underbrace{\mathrm{E}_a(Y) - \mathrm{E}_b\left(Y^{R(D|a)}\right)}_{\text{remaining disparity}}.$$

This corresponds to Propositions 1 and 4 in Jackson and VanderWeele (2018). Let's verify this.

Random equalization decomposition

$\mathrm{E}_g\left(Y^{R(D)}\right)$: post-intervention mean outcome for group g after each member of group g is counterfactually given a treatment value randomly drawn from the whole population.

The Lundberg version:

$$\mathrm{E}_a(Y)-\mathrm{E}_b(Y)=\underbrace{\mathrm{E}_a(Y)-\mathrm{E}_b(Y)-\left[\mathrm{E}_a\left(Y^{R(D)}\right)-\mathrm{E}_b\left(Y^{R(D)}\right)\right]}_{\text{change in disparity}}+\underbrace{\mathrm{E}_a\left(Y^{R(D)}\right)-\mathrm{E}_b\left(Y^{R(D)}\right)}_{\text{remaining disparity}}$$

Recall equation (3) in Lundberg (2022), p.15

This corresponds to Intervention c in Figure 6 in Lundberg (2022).

What's good about random equalization decompositions?

Do they have any problems, though?

Problem with random equalization decomposition

We want a decomposition that can capture the contribution of differential prevalence of D between the two groups.

Does change in disparity work? Not really...

Null criterion: a valid measure of the contribution of differential prevalence should should be zero when there is no group difference in treatment prevalence, i.e., $\mathrm{E}_a(D) = \mathrm{E}_b(D)$.

Change in disparity in Jackson's version $\mathrm{E}_b(\tau)[\mathrm{E}_a(D) - \mathrm{E}_b(D)] - \mathrm{Cov}_b(D, \tau)$.

(the self-equalization paradox)

Change in disparity in Lundberg's version

$\mathrm{E}(\tau)[\mathrm{E}_a(D) - \mathrm{E}_b(D)] - [p_a - p_b][\mathrm{E}_a(D) - \mathrm{E}_b(D)][\mathrm{E}_a(\tau) - \mathrm{E}_b(\tau)] + \mathrm{Cov}_a(D, \tau) - \mathrm{Cov}_b(D, \tau)$.

A new decomposition

Notation: Y^0: baseline potential outcome; $\tau = Y^1 - Y^0$: treatment effect. $D \in \{0, 1\}$

$$
\begin{aligned}
& \mathrm{E}_a(Y) - \mathrm{E}_b(Y) \\
= & \underbrace{\mathrm{E}_a\left(Y^0\right) - \mathrm{E}_b\left(Y^0\right)}_{\text{baseline}} + \underbrace{\mathrm{E}_b(\tau)[\mathrm{E}_a(D) - \mathrm{E}_b(D)]}_{\text{prevalence}} \\
& + \underbrace{\mathrm{E}_a(D)[\mathrm{E}_a(\tau) - \mathrm{E}_b(\tau)]}_{\text{effect}} + \underbrace{\mathrm{Cov}_a(D, \tau) - \mathrm{Cov}_b(D, \tau)}_{\text{selection}}.
\end{aligned}
$$

Example:

- baseline: racial disparity in outcome unrelated to college attendance
- prevalence: differential college attendance rates
- effect: differential average returns to college attendance
- selection: differential selection into college based on individual-level returns from college

A new decomposition

Nice things about this:

1. Nonparametric causal decomposition of group disparities
2. Four-way
3. Novel selection component (related to effectiveness, fairness)
4. Satisfies the null criterion
5. Has interventional interpretation (prescriptive)
6. Nonparametric estimation with theoretically guaranteed good properties

Interventional and graphical representations

Two-step intervention: within-group randomization first, between-group equalization second

$$\begin{aligned} \mathrm{E}_a(Y) - \mathrm{E}_b(Y) - \left[\mathrm{E}_a\left(Y^{R(D|a)}\right) - \mathrm{E}_b\left(Y^{R(D|b)}\right)\right] &= \text{selection} \\ \mathrm{E}_b\left(Y^{R(D|a)}\right) - \mathrm{E}_b\left(Y^{R(D|b)}\right) &= \text{prevalence} \\ \mathrm{E}_a\left(Y^{R(D|a)}\right) - \mathrm{E}_b\left(Y^{R(D|a)}\right) &= \text{baseline} + \text{effect} \end{aligned}$$

Why we can avoid violating the null criterion: we separate equalization from randomization

Graph on board

Conditional decomposition

Natural question arising from the interventional/prescriptive interpretation: what if we are not satisfied with the marginal interventions?

Conditional decomposition comes to our rescue

Conditional decomposition

$$
\begin{aligned}
\mathrm{E}_a(Y) - \mathrm{E}_b(Y) = {} & \underbrace{\mathrm{E}_a\left(Y^0\right) - \mathrm{E}_b\left(Y^0\right)}_{\text{baseline}} \\
& + \underbrace{\int \left[\mathrm{E}_a(D \mid q) - \mathrm{E}_b(D \mid q)\right] \mathrm{E}_b(\tau \mid q) f_b(q) \mathrm{d}q}_{\text{conditional prevalence}} \\
& + \underbrace{\int \left[\mathrm{E}_a(\tau \mid q) - \mathrm{E}_b(\tau \mid q)\right] \mathrm{E}_a(D \mid q) f_a(q) \mathrm{d}q}_{\text{conditional effect}} \\
& + \underbrace{\mathrm{E}_a[\mathrm{Cov}_a(D, \tau \mid Q)] - \mathrm{E}_b[\mathrm{Cov}_b(D, \tau \mid Q)]}_{\text{conditional selection}} \\
& + \underbrace{\int \mathrm{E}_a(D \mid q)\, \mathrm{E}_b(\tau \mid q) [f_a(q) - f_b(q)] \mathrm{d}q}_{Q \text{ distribution}} .
\end{aligned}
$$

Conditional decomposition

Interventional interpretation: within-group conditional randomization first, between-group conditional equalization second

$$
\begin{aligned}
\mathrm{E}_a(Y) - \mathrm{E}_b(Y) - \left[\mathrm{E}_a\left(Y^{R(D|a,Q)}\right) - \mathrm{E}_b\left(Y^{R(D|b,Q)}\right)\right] &= \text{conditional selection} \\
\mathrm{E}_b\left(Y^{R(D|a,Q)}\right) - \mathrm{E}_b\left(Y^{R(D|b,Q)}\right) &= \text{conditional prevalence} \\
\mathrm{E}_a\left(Y^{R(D|a,Q)}\right) - \mathrm{E}_b\left(Y^{R(D|a,Q)}\right) &= \text{baseline} + \text{conditional effect} \\
&\quad + Q \text{ distribution}.
\end{aligned}
$$

Example: Q: prior achievement

Why/when would we want a conditional equalization instead of a marginal equalization?

Conditional decomposition

Proposition 2 in Jackson and VanderWeele (2018):

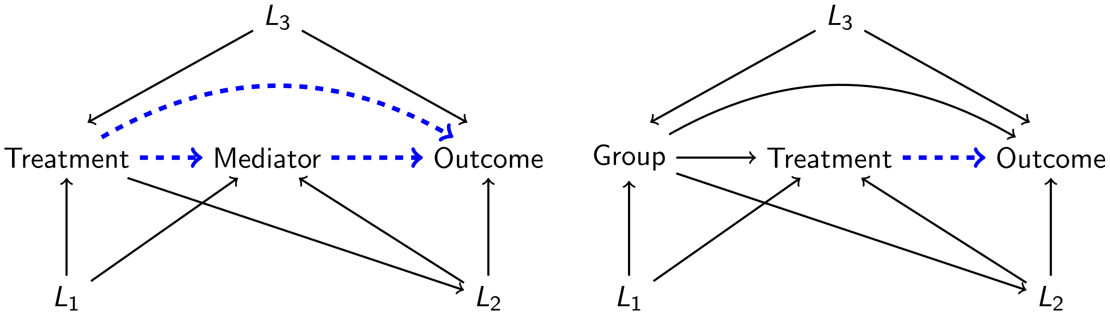
$$\mathrm{E}_a(Y) - \mathrm{E}_b(Y) = \underbrace{\mathrm{E}_b\left(Y^{R(D|a,Q)}\right) - \mathrm{E}_b(Y)}_{\text{change in disparity}} + \underbrace{\mathrm{E}_a(Y) - \mathrm{E}_b\left(Y^{R(D|a,Q)}\right)}_{\text{remaining disparity}}.$$

Similar for Lundberg's version (Intervention d in Figure 6)

Violate a conditional null criterion

Now let's talk about identification

Mediation? Random equalization decompositions? Yu and Elwert?



(a) Causal mediation (b) Group disparity

A little bit of estimation

Regression, weighting, or a mixture of the two (can be strengthened by data-adaptive techniques like machine learning)

Generally, regression estimators are derived by plugging in parametric functions.

For example, we may assume $\mathrm{E}_g(Y \mid D, X) = \alpha_g + \beta_g D + \gamma_g X$.

$$\begin{aligned}
&\mathrm{E}_b\left(Y^{R(D|a)}\right) \\
&= \mathrm{E}_b\left(Y^0\right)[1 - \mathrm{E}_a(D)] + \mathrm{E}_b\left(Y^1\right)\mathrm{E}_a(D) \\
&= \mathrm{E}_b\left[\mathrm{E}_b(Y \mid D = 0, X)\right][1 - \mathrm{E}_a(D)] + \mathrm{E}_b\left[\mathrm{E}_b(Y \mid D = 1, X)\right]\mathrm{E}_a(D) \\
&= [\alpha_b + \gamma_b \mathrm{E}_b(X)][1 - \mathrm{E}_a(D)] + [\alpha_b + \beta_b + \gamma_b \mathrm{E}_b(X)]\mathrm{E}_a(D) \\
&= \alpha_b + \gamma_b \mathrm{E}_b(X) + \beta_b \mathrm{E}_a(D).
\end{aligned}$$

$$\text{Prevalence component: } \mathrm{E}_b\left(Y^{R(D|a)}\right) - \mathrm{E}_b\left(Y^{R(D|b)}\right) = \beta_b[\mathrm{E}_a(D) - \mathrm{E}_b(D)].$$

Estimation

Weighting estimators are generally derived by using tilting functions to "tilt" the factual distribution to an interventional distribution. For $\mathrm{E}_b\left(Y^{R(D|a)}\right)$, we can use the weight

$$\frac{f_a(D)}{f_b(D \mid X)}.$$

For $\mathrm{E}_b\left(Y^{R(D|a,Q)}\right)$, we can use the weight

$$\frac{f_a(D \mid Q)}{f_b(D \mid X, Q)}.$$

Yu and Elwert (2023) develop a mixture of regression and weighting that is implemented using ML. It has every nice property you can ask for: flexible, efficient, robust.

Summary

1. Causal disparity analysis is related to but different from causal mediation analysis.
2. There are various approaches to do CDA. I think ours is the winner...
3. In R, you can use the "cdgd" package.