A Reformulation of the Equalization Estimand

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1 Jackson and Lundberg estimand

Lundberg favors *randomly* giving the disadvantaged group the population distribution of treatment, and Jackson favors *randomly* giving them the treatment distribution of the advantaged group. But for the discussion below, their differences do not matter. Nor do the difference between binary and continuous treatments. For now, the expressions below only applies exactly to the Jackson version, but the logic is exactly the same for the Lundberg version.

observed mean: $E(Y) = E(Y_0) + E(D \cdot \tau) = E(Y_0) + Cov(D, \tau) + E(D)E(\tau)$

simulated mean under equalization: $E(\tilde{Y}) = E(Y_0) + Cov(\tilde{D}, \tau) + E(\tilde{D})E(\tau)$

reduction in disparity: $Cov(\tilde{D}, \tau) - Cov(D, \tau) - [E(\tilde{D}) - E(D)]E(\tau)$

And $\tilde{D}$ is *randomly* drawn from whatever the target distribution is. Hence $\tilde{D}$ is independent of all variables, so $Cov(\tilde{D}, \tau) = 0$, and $E(\tilde{Y}) = E(Y_0) + E(\tilde{D})E(\tau)$. There is a conceptual problem with this estimand. From the expression of the reduction in disparity, it is clear that the intervention changes not only the marginal distribution of D, but also its correlation with τ, which reflects the extent of self-selection based on treatment effects among the disadvantaged. In other words, if there is self-selection in treatment effects (eg. those who will get higher returns from being treated are more likely to be treated), then the reduction in disparity will partly be due to the self-selection. Even if the advantaged and disadvantaged groups actually have the exact same distribution of treatment, this version of the estimand will still produce a nonzero reduction in disparity as long as self-selection exists. In fact, if there is positive self-selection, the estimated reduction in disparity will be downward biased relative to the reduction in disparity due to equalization of treatment distribution. The root of this conceptual problem is that in this version, the hypothetical intervention equalizes treatment both across groups and within-groups by randomizing the treatment distribution.

(side note 1: On a non-causal level, this should also be what the entire Blinder–Oaxaca-Kitagawa tradition is about.)

(side note 2: In a previous note, I have shown the equivalence between the Jackson estimand in the notation above and that in his own notation. It's very intuitive that they are the same, but formally, given the same set of assumptions, the nonparametric estimators derived from his notation is the same as those derived using the new notation.)

2 Reformulating the estimand

The observed mean can always be written, without any assumption (except for things like the existence of the first and second moments), as $E(Y) = E(Y_0) + E(D \cdot \tau) = E(Y_0) +$

$Cov(D, \tau) + E(D)E(\tau)$. Therefore, it naturally follows that there can be a four-way causal decomposition of group differences. That is, any mean group difference can be decomposed into (1) the difference in mean potential outcome under control; (2) the difference in self-selection in effect; (3) the difference in treatment expectation; (4) the difference in ATE. Estimation is also straightforward. One can use any model, from an OLS to Causal Forests, to predict the individual-level treatment effects and the potential outcomes under control and then using the predictions obtain the moments estimates involved in the decomposition formula. Inference should also be analytically routine, as long as the methods used for the first-step predictions produce asymptotically normal predictions, such as OLS and Causal Forests.

If it is desirable to have a more succinct two-way decomposition, there may be two alternatives to the current versions of the equalization estimand.

2.1 Equalization of treatment distribution

This new version only equalizes the marginal distribution of treatment distribution, i.e., it gives the disadvantaged group the distribution of treatment of the advantaged group, but not randomly or in other ways that change the other three components of the decomposition formula.

simulated mean under equalization: $E(\dot{Y}) = E(Y_0) + Cov(D, \tau) + E(\dot{D})E(\tau)$

reduction in disparity: $[E(\dot{D}) - E(D)]E(\tau)$

Therefore, the conceptual problem no longer exists: if the two groups share the same treatment distribution, this estimand will make the reduction in disparity zero. Estimation is simply about $E(\tau)$, and the researcher can use any estimator of ATE. And thanks to the MTE framework, the IV estimation of this version can be easily implemented as well.

2.2 Equalization of treatment regime

Another new version of the estimand is not only about equalizing the marginal distribution of the treatment, but also the entire treatment regime, to the extent that it is related to the decomposition formula.

simulated mean under equalization: $E(\overline{Y}) = E(Y_0) + Cov(\overline{D}, \overline{\tau}) + E(\overline{D})E(\tau)$

reduction in disparity: $Cov(\overline{D}, \overline{\tau}) - Cov(D, \tau) - [E(\overline{D}) - E(D)]E(\tau)$

Here, $Cov(\overline{D}, \overline{\tau})$ represents the covariance between D and τ in the target group. Thus, in the hypothetical intervention, the selection regime is also equalized, the disadvantaged group is made to have the same expectation of treatment as well as the same level of self-selection. This version will make sense if part of the group difference is attributable to differential decision-making patterns in entering treatment and the researcher wants to evaluate the joint impact of differential decision-making and the distribution of treatment on the gap. And this version equalizes the "entire" treatment regime in the sense that both parameters involving D in the four-way decomposition are changed. And different from the Jackson version, the changes for the two parameters are parallel.

3 Conditional equalization

For conditional equalization, there is another vector of variable $\boldsymbol{Q}$, which is the conditioning set. In order to accommodate $\boldsymbol{Q}$, we expand the four-way decomposition to five-way decomposition.

$$
\begin{aligned}
&E(Y) \\
&= E(Y_0) + E(D \cdot \tau) \\
&= E(Y_0) + Cov(D, \tau) + E(D)E(\tau) \\
&= E(Y_0) + E[Cov(D, \tau|\boldsymbol{Q})] + Cov[E(D|\boldsymbol{Q}), E(\tau|\boldsymbol{Q})] + E[E(D|\boldsymbol{Q})]E(\tau)
\end{aligned}
$$

The last equality follows from the law of total covariance. In the Jackson version of conditional equalization,

$$
\begin{aligned}
&E(\tilde{Y}) \\
&= E(Y_0) + E[Cov(\tilde{D}, \tau|\boldsymbol{Q})] + Cov[E(\tilde{D}|\boldsymbol{Q}), E(\tau|\boldsymbol{Q})] + E(\tilde{D})E(\tau) \\
&= E(Y_0) + Cov[E(\tilde{D}|\boldsymbol{Q}), E(\tau|\boldsymbol{Q})] + E[E(\tilde{D}|\boldsymbol{Q})]E(\tau)
\end{aligned}
$$

The last equality holds because $\tilde{D}$ is randomly drawn from the target group conditional on $\boldsymbol{Q}$, so the conditional covariance between $\tilde{D}$ and τ is zero. Therefore, under the Jackson framework, the conditional covariance, as opposed to the marginal covariance, is set to zero, in addition to changing $E(D|\boldsymbol{Q})$. This amounts to eliminating residual self-selection after $\boldsymbol{Q}$ is accounted for. And the conceptual problem analogously exists, namely, even if the conditional distribution of D is the same for both groups across all $\boldsymbol{Q}$ value, $E(\tilde{Y})$ will still be different from $E(Y)$ as long as there is residual self-selection.

Analogously to the unconditional case, we also proposed two alternative equalization estimands. The first is to leave $E[Cov(D, \tau|\boldsymbol{Q})]$ as is, preserving the residual self-selection and focusing on the impact of differential $E(D|\boldsymbol{Q})$. The second is to replace $E[Cov(D, \tau|\boldsymbol{Q})]$ with the corresponding expectation of covariance in the target group, hence equalizing the residual decision-making process in addition to equalizing the conditional expectation of treatment.