Physics 405, Fall 2017
Problem Set 4

due Wednesday, October 4

1. **Feynman-Hellman and Beyond (20 pts)**: In class, we considered a small perturbation around a known Hamiltonian H_0:

$$H = H_0 + \epsilon H_1$$

where $\epsilon \ll 1$. If one has a family of Hamiltonia $H(\lambda)$ that depend on an external parameter λ, one can reinterpret this equation as the first two terms in a Taylor series expansion of $H(\lambda)$ about some λ_0.

   a) For the nth state $|n\rangle$ and eigenenergy E_n, use this reinterpretation to find a relation between $\frac{\partial E_n}{\partial \lambda}$ and $\langle n|\frac{\partial H}{\partial \lambda}|n\rangle$.

   b) In the special case that $H = H_0 + \lambda H_1$, λ not necessarily small, use second order perturbation theory to argue that for the ground state energy

$$\frac{\partial^2 E_{\mathrm{gs}}}{\partial \lambda^2} \leq 0 \ .$$

   c) For the 2×2 Hamiltonian $H = \mathrm{id} + a\sigma_x + b\sigma_y + c\sigma_z$, a, b, and c real numbers, verify (b) for the ground state energy $E_{\mathrm{gs}}(a, b, c)$. [The matrices σ_x, σ_y, and σ_z are often called Pauli matrices. See Chapter 4 in Griffiths for a definition.]

2. **Zeeman Effect (20 pts):** Consider the qualitative plot of energy versus magnetic field for the $n = 2$ energy levels of hydrogen shown in figure 1. In the small magnetic field limit, label each line by its quantum numbers j, m_j, and ℓ. In the large field limit, label each line by its quantum numbers m_s, m_ℓ, and ℓ. [[j is the total angular momentum quantum number and ℓ is the orbital angular momentum quantum number. The numbers m_j, m_s and m_ℓ are the corresponding z-components for j, ℓ, and s respectively.]]

3. **Hyperfine Structure (20 pts):** Griffiths provides a result for the hyperfine splitting in the hydrogen atom. We would like to generalize his result. Instead of a proton, consider a particle of mass m_1, charge q_1, and g-factor g_1. Instead of an electron, consider a particle of mass m_2, charge $-q_2$ and g-factor g_2.

   a. What is the corresponding hyperfine splitting for the ground state of this "ion" of total charge $q_1 - q_2$? [[Do not leave your answer in terms of the Bohr radius as Griffiths does.]]

   b. If we fix $m_1 + m_2 = M$ to be a constant, for what values of m_1 and m_2 is the hyperfine splitting maximized?

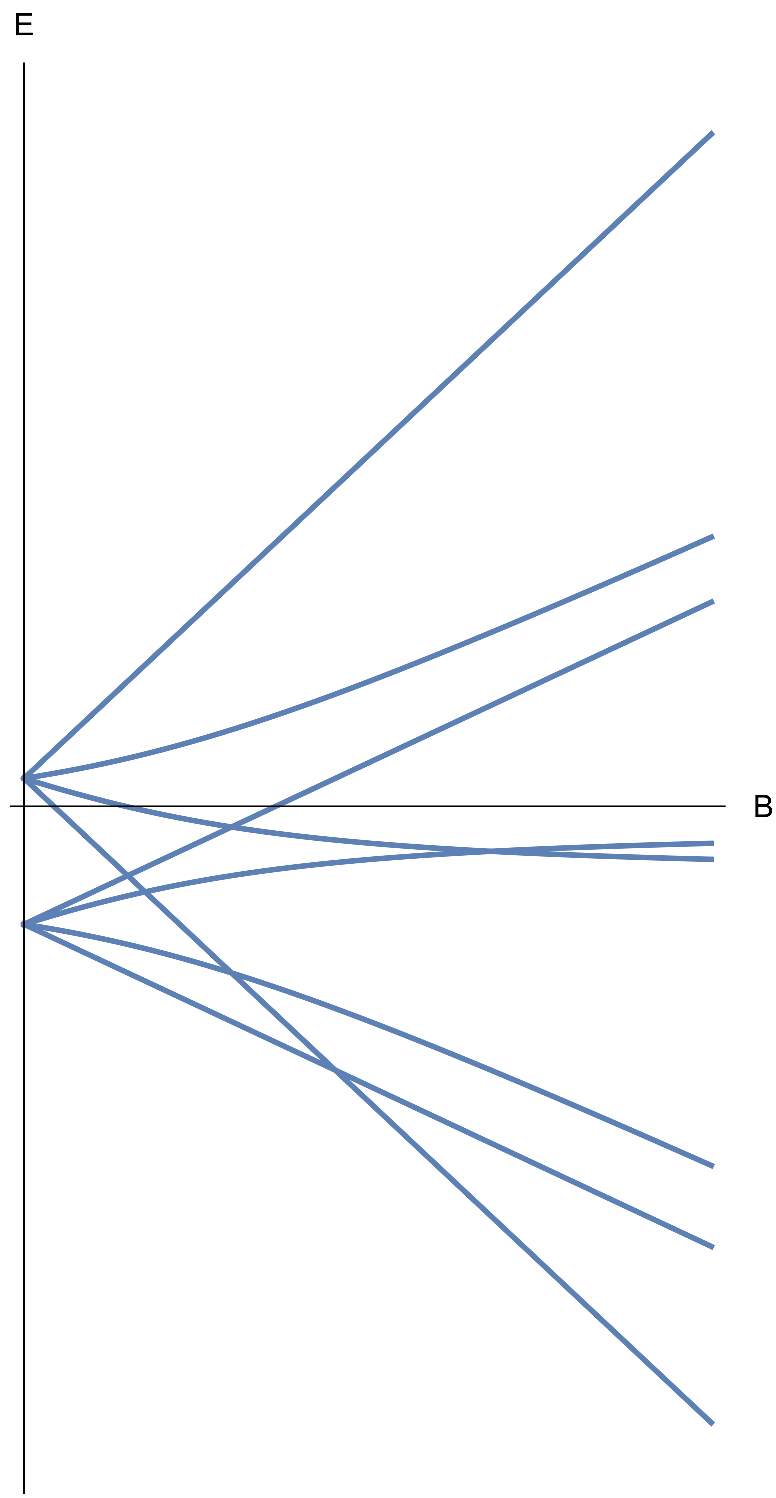

Figure 1: A picture for problem 2.