CSCI 2824 Written HW 4

Alex Garcia-Gonzalez

TOTAL POINTS

48 / 51

QUESTION 1

1 Logical Inference 8 / 8

✓ + **8 pts All Correct**

+ **3 pts** instantiations; existential first then universals
+ **4 pts** Full credit: Applying valid logical inferences & rules of inference
+ **1 pts** Final existential generalization
+ **2 pts** Half credit: applying the logical inferences & rules of inference
+ **0 pts** Incorrect Approach
+ **7 pts** Everything correct except order of instantiation

QUESTION 2

Prove or Disprove 9 pts

2.1 (a) average of integers 3 / 3

✓ + **3 pts All Correct: may be direct proof or contradiction**

+ **1 pts** Correct assumption to begin proof
+ **1 pts** demonstrating knowledge of how to progress through proof mathematically
+ **1 pts** correct conclusion of proof
+ **2 pts** Mostly right proof but lacking in one key area
+ **1 pts** Some correct work, but mostly incorrect
+ **0 pts** Incorrect/Blank

2.2 (b) parity 3 / 3

✓ + **3 pts All Correct, looking for a proof by contradiction but direct might be ok**

+ **1 pts** Correct assumption
+ **1 pts** Using mathematical justifications to deduce conclusion
+ **1 pts** correct overall form of proof
+ **0 pts** Incorrect/Blank

2.3 (c) irrational triangle 3 / 3

✓ + **3 pts All Correct**

+ **1 pts** recognizing need for a counterexample
+ **1 pts** providing a counterexample
+ **1 pts** using a correct counterexample
+ **0 pts** Incorrect - tried to prove an untrue statement
+ **0 pts** Incorrect for other reasons
+ **0 pts** No solution

QUESTION 3

3 Divisibility by 3 10 / 10

✓ + **10 pts All Correct**

+ **1 pts** Assuming N=100a+10b+c
+ **1 pts** Utilizing assumption that a+b+c=3n
+ **4 pts** Manipulating N=100a+10b+c in some way to allow substitution of "a+b+c=3n"
+ **2 pts** Showing how N must be divisible by 3
+ **2 pts** Concluding statement
+ **0 pts** Did not attempt/ Incorrect

QUESTION 4

Prove 10 pts

4.1 (a) Biconditional 3 / 6

+ **6 pts** All Correct

✓ + **3 pts Forward direction correct**

+ **3 pts** Backward direction correct
+ **1 pts** Correct assumption for forward direction
+ **1 pts** Correct supporting math for forward direction
+ **1 pts** Correct proof structure for forward direction
+ **1 pts** Correct assumption for backward direction
+ **1 pts** Correct supporting math for backward direction
+ **1 pts** Correct proof structure for backward

direction

+ **0 pts** Incorrect

- **2 pts** No proper explanations of the mathematical steps

+ **0 pts** No answer provided

+ **5.5 pts** Mentioned "contradiction" while actually "using contraposition"

+ **5 pts** Mislabeled forward direction as backward direction and vice-versa

4.2 (b) more parity 4 / 4

✓ + **4 pts** **All Correct**

+ **2 pts** Correct proof structure

+ **2 pts** Correct supporting math

+ **2 pts** Some correct work, one key error

+ **0 pts** Incorrect

+ **0 pts** Answer not provided

+ **3 pts** Mentioned contraposition but assumed n is an odd number

+ **3 pts** Minor error

QUESTION 5

5 Proof by Cases 8 / 8

✓ + **8 pts** **All Correct**

+ **2 pts** Correctly identifying the cases required

+ **1 pts** Case for (x >=8): replacing |x-8| with x-8

+ **1 pts** Case for (x<8): replacing |x-8| with 8-x

+ **2 pts** Justifying that expression is >= 8 with math in Case 1

+ **2 pts** Justifying that expression is >= 8 with math in Case 2

+ **0 pts** Incorrect/ Did not attempt

QUESTION 6

6 Style Points 6 / 6

✓ + **6 pts** **All correct**

+ **5 pts** Did not type the assignment - rest everything is good

+ **1 pts** Latexing assignment

+ **2 pts** Completing assignment

+ **1 pts** Name on assignment

+ **2 pts** Neatness

1. Use rules of inference to show that if $\forall x(P(x) \vee Q(x)), \forall x(\neg Q(x) \vee S(x)), \forall x(R(x) \longrightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

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1. $\forall x(P(x) \vee Q(x))$	Premise
2. $\forall x(\neg Q(x) \vee S(x))$	Premise
3. $\forall x(R(x) \rightarrow \neg S(x))$	Premise
4. $\exists x \neg P(x)$	Premise
5. $\neg P(c)$ (for some c)	Existential Instantiation (4)
6. $P(c) \vee Q(c)$	Universal Instantiation (1)
7. $\neg P(c) \rightarrow Q(c)$	RBI (6)
8. $Q(c)$	Modus Ponens (8, 10)
9. $\neg Q(c) \vee S(c)$	Universal Instantiation (2)
10. $Q(c) \rightarrow S(c)$	RBI (9)
11. $S(c)$	Modus Ponens (8, 10)
12. $R(c) \rightarrow \neg S(c)$	Universal Instantiation (3)
13. $S(c) \rightarrow \neg R(c)$	Contrapositive (12)
14. $\neg R(c)$	Modus Ponens (11, 13)
15. $\therefore \exists x \neg R(x)$	Existential Generalization (5-15)

1 Logical Inference 8 / 8

- ✓ + **8 pts** **All Correct**
- \+ **3 pts** instantiations; existential first then universals
- \+ **4 pts** Full credit: Applying valid logical inferences & rules of inference
- \+ **1 pts** Final existential generalization
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- \+ **0 pts** Incorrect Approach
- \+ **7 pts** Everything correct except order of instantiation

2. Prove or disprove the following claims. Be sure to indicate whether you are using a Direct Proof, a Contrapositive Proof, a Proof by Cases, or a Proof by Contradiction, or a Counterexample.

   (a) If the average of $a_1, a_2, ..., a_n$ is some number $\bar{a}$, then at least one of the real numbers $a_1, a_2, ..., a_n$ must be greater than or equal to $\bar{a}$.

   (b) If n is an integer and $3n + 2$ is even, then n is even. [Note: Do NOT use a contrapositive proof.]

   (c) If the lengths of two sides of a triangle are irrational, then the third side must be irrational also.

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a.

Proof by Contradiction:

S'Pose the average of (a1,a2,…,an) is greater than or equal to the average of these numbers

ā = (a1 + a2 + … + an) / (n)

Hypothesis:

a1 < ā
a2 < ā
… < ā
an < ā

Summing up these equations we can now add them
and compare them against *"ā = (a1 + a2 + ... + an) / (n)" or n+ā*

a1 + a2 + … + an < n+ā

By replacing ā in the equation above by the its value

a1+a2+…+an < a1+a2+…+an

Conclusion: This is not possible thus we have reached a contradiction.

The number cannot be strictly smaller than itself…

Therefore our hypothesis is wrong, the original statement is correct.

2.1 (a) average of integers 3 / 3

✓ + **3 pts** **All Correct: may be direct proof or contradiction**

+ **1 pts** Correct assumption to begin proof
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b.

If n is an integer and 3n + 2 is even, then n is even.

Using proof by contradiction:
S'pose 3n+2 is even and n is odd; Find a contradiction

n=2k+1
3n+2=3(2k+1)+2
=6k+3+2
=2(3k+2)+1

Proof:
Let n be an integer.
S'pose for proof by contradiction that 3n+2 is even and n is odd.
If n is odd by definition of odd integers n=2k+1 for some integer k. Then we must plug in to solve for proof.
So, 3n+2=3(2k+1)+2
=6k+3+2
=6k+5+(-1)+1
=6k+4+1
3n+2=2(3k+2)+1
3n+2

This contradicts our assumption that 3n+2 is even. Hence it is not the case that 3n+2 is even and n is odd. This 3n+2 is 2 times some integer plus 1. So by definition of odd integer 3n+2 is odd.

c.
Consider
The triangle ABC
Angle of (B) = 90
Angle of(AB)=sqrt (2)
Angle of(BX)=sqrt (2)
Then AC =2

Thus this proves the preceding statement to be false

2.2 (b) parity **3 / 3**

- ✓ **+ 3 pts** **All Correct, looking for a proof by contradiction but direct might be ok**
- **+ 1 pts** Correct assumption
- **+ 1 pts** Using mathematical justifications to deduce conclusion
- **+ 1 pts** correct overall form of proof
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This contradicts our assumption that 3n+2 is even. Hence it is not the case that 3n+2 is even and n is odd. This 3n+2 is 2 times some integer plus 1. So by definition of odd integer 3n+2 is odd.

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Consider
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Thus this proves the preceding statement to be false

2.3 (c) irrational triangle **3 / 3**

- ✓ **+ 3 pts** **All Correct**
- **+ 1 pts** recognizing need for a counterexample
- **+ 1 pts** providing a counterexample
- **+ 1 pts** using a correct counterexample
- **+ 0 pts** Incorrect - tried to prove an untrue statement
- **+ 0 pts** Incorrect for other reasons
- **+ 0 pts** No solution

3. The divisibility rule by 3 is a rule that a number N is divisible by 3 if the sum of the digits is divisible by 3. For example, is 132 divisible by 3? $1+3+2=6$ and 6 is a number divisible by 3. So 132 must also be divisible by 3.

Please prove this "rule" for three digit numbers:
Let the positive integer N have the form $N = 100a + 10b + c$ where $a + b + c = 3n$ and a, b, and c are digits with $a \neq 0$. Prove that N is divisible by 3.

Proof:

Let positive integer N have the form:

N=100a+10b+c

&

a+b+c=3n

Which means

a+b+c are divisible by 3 & DNE: 0

N=100a+10b+c

N=99a+a+9b+b+c

=99a+9b+a+b+c

=3(33a+3b)+(a+b+c)

=3(33a+3b)+3n

Conclusion Both "33a", "3n" & the sum are divisible by 3. According to divisible by 3 rule

(sum of digits divisible by 3 then numbers is divisible by 3)

Thus N=100a+10b+c is divisible by 3 we have proved this rule for three digit numbers divisibility.

3 Divisibility by 3 **10 / 10**

- ✓ **+ 10 pts** **All Correct**
- **+ 1 pts** Assuming N=100a+10b+c
- **+ 1 pts** Utilizing assumption that a+b+c=3n
- **+ 4 pts** Manipulating N=100a+10b+c in some way to allow substitution of "a+b+c=3n"
- **+ 2 pts** Showing how N must be divisible by 3
- **+ 2 pts** Concluding statement
- **+ 0 pts** Did not attempt/ Incorrect

4. Prove the following:

(a) Prove that n is even if and only if $n^2 - 6n + 5$ is odd.

(b) Prove that if $2n^2 + 3n + 1$ is even, then n is odd.

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a.

n^2 - 6n+5

= (n^2-5n-n+5)

= (n-5) (n-1)

If (N^2-6n-5) = even

either of them is even .

n-1 = even

n-5=even

n=1+even

n=5+even

Since odd + even = odd .... then n is going to be odd for both cases.

so if n^2-6n+5 is even, n is odd

if n^2-6n+5 is odd then (n-1)*(n-5) is odd

we know odd+odd=even always

thus, n is even is both side

hence n is even if and only if n^2-6n+5 is odd.

4.1 (a) Biconditional 3 / 6

- \+ **6 pts** All Correct
- ✓ + **3 pts** **Forward direction correct**
- \+ **3 pts** Backward direction correct
- \+ **1 pts** Correct assumption for forward direction
- \+ **1 pts** Correct supporting math for forward direction
- \+ **1 pts** Correct proof structure for forward direction
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- \+ **0 pts** Incorrect
- \- **2 pts** No proper explanations of the mathematical steps
- \+ **0 pts** No answer provided
- \+ **5.5 pts** Mentioned "contradiction" while actually "using contraposition"
- \+ **5 pts** Mislabeled forward direction as backward direction and vice-versa

b.

The given $2n^2+3n+1$ is even

Assuming n is even

let $n=2k$

lets plug and solve

$2n^2+3n+1 = 2(2k)^2 +3(2k)+1$

$=2(4k^2) + 6k+1$

$=8k^2+6k+1$

$=2(4k^2+3k)+1$

$=2m+1$

Conclusion: Since $2n^2+3n+1$ is odd, this is our contradiction, concluding (n is even) is false

n is odd

4.2 (b) more parity 4 / 4

✓ + **4 pts** **All Correct**

+ **2 pts** Correct proof structure
+ **2 pts** Correct supporting math
+ **2 pts** Some correct work, one key error
+ **0 pts** Incorrect
+ **0 pts** Answer not provided
+ **3 pts** Mentioned contraposition but assumed n is an odd number
+ **3 pts** Minor error

5. Use proof by cases to prove that $x + |x - 8| \geq 8$ for all real numbers x. [Hint: $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$.]

Case 1:
If x -8 < 0
then by moduluo property if x < 0 => |x| => -x
|x-8| =-(x-8)

from L.H.S. of equation above then

x-x+8 =>

8 >= 8

… NEXT

Case 2:

let |x-8|=0

x=8=0

x=8

then from case1 equation => 8+0 >= 8

=> 8 >= 8

Case 3:

if x-8 > 0

if x > 0 => |x| => x

so we have |x-8|=x-8

the using first equation we conclude:

=x+x-8

=2x-8

since n=8 >0

then n>8

thus 2n-8 always greater than 8

5 Proof by Cases 8 / 8

✓ + **8 pts** **All Correct**

+ **2 pts** Correctly identifying the cases required
+ **1 pts** Case for (x >=8): replacing |x-8| with x-8
+ **1 pts** Case for (x<8): replacing |x-8| with 8-x
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+ **2 pts** Justifying that expression is >= 8 with math in Case 2
+ **0 pts** Incorrect/ Did not attempt

CSCI 2824
Spring 2019

Write **clearly**:

Name:	Alex Garcia Gonzalez
Student ID:	104288519
Section number:	001 COX
Assignment:	HW4

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn one extra credit point per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

6 Style Points 6 / 6

- ✓ + **6 pts** **All correct**
- + **5 pts** Did not type the assignment - rest everything is good
- + **1 pts** Latexing assignment
- + **2 pts** Completing assignment
- + **1 pts** Name on assignment
- + **2 pts** Neatness