CSCI 2824 Written HW 8

Alex Garcia-Gonzalez

TOTAL POINTS

42 / 51

QUESTION 1

Weak Induction 12 pts

1.1 i^2 formula 3 / 4

+ **4 pts** All Correct
+ **1 pts** Base Case
+ ✓ + **1 pts Inductive Hypothesis**
+ ✓ + **1 pts supporting math**
+ ✓ + **1 pts Correct conclusion**
+ **0 pts** Incorrect / Blank

1.2 i^3 formula 4 / 4

+ ✓ + **4 pts All Correct**
+ **1 pts** Base Case
+ **1 pts** Inductive Hypothesis
+ **1 pts** supporting math
+ **1 pts** Correct conclusion
+ **0 pts** Incorrect / Blank

1.3 product formula 4 / 4

+ ✓ + **4 pts All Correct**
+ **1 pts** Base Case
+ **1 pts** Inductive Hypothesis
+ **1 pts** supporting math
+ **1 pts** Correct conclusion
+ **0 pts** Incorrect / Blank

QUESTION 2

2 Strong Induction 8 / 10

+ **10 pts** All Correct
+ **3 pts** Base Cases - 1 point for each one
+ ✓ + **2 pts Inductive hypothesis**
+ ✓ + **2 pts Using the induction hypothesis in mathematical derivation**
+ ✓ + **2 pts Mathematical derivation to arrive at T_k+1 < 2^k+1**
+ ✓ + **1 pts Using strong induction and stating so**
+ **0 pts** Not attempted

+ 1 Point adjustment

Base cases are n=4,5,6.

QUESTION 3

Complexity 12 pts

3.1 big-O 3 / 4

+ **4 pts** All Correct
+ **1 pts** Bound of 50n^3
+ **1 pts** Bound of 18n^3 log(n)
+ **1 pts** Bound of -2n log(n)
+ **1 pts** C and k values
+ **0 pts** No solution

+ 3 Point adjustment

1 complexity needs to be of the form O(n^p) where p is a natural number

3.2 big-Omega 2 / 4

+ **4 pts** All Correct
+ **1 pts** Bound of 50n^3
+ **1 pts** Bound of 18n^3 log(n)
+ **1 pts** Bound of -2n log(n)
+ **1 pts** C and k values
+ **0 pts** No solution

+ 2 Point adjustment

complexity needs to be of the form omega(n^p) where p is a natural number

2 c and k values are not mentioned

3.3 big-Theta 2 / 4

+ **4 pts** All Correct

+ **2 pts** Concluding no

✓ + **2 pts** **Explanation**

+ **0 pts** No solution

+ **0 pts** wrong answer

QUESTION 4

Matrix Multiplication 11 pts

4.1 (a) 5 / 5

✓ + **5 pts** **All Correct**

+ **3 pts** No supporting work

+ **1 pts** Correct matrix dimensions of 3x3

+ **2 pts** Correct answer

+ **2 pts** Supporting work

+ **0 pts** Incorrect/No answer provided

+ **4.5 pts** Upto two elements wrong

+ **4.5 pts** Putting commas in matrices

4.2 (b) 6 / 6

✓ + **6 pts** **All Correct**

+ **1 pts** Correct matrix dimensions of 3x3

+ **3 pts** Correct answer

+ **2 pts** Supporting work

+ **0 pts** Not answered/Incorrect

+ **5.5 pts** -0.5 for using commas in matrices

QUESTION 5

5 Style Points 5 / 6

+ **6 pts** All correct

✓ + **5 pts** **Assignment not typed**

+ **2 pts** Complete assignment

+ **3 pts** Neatness

+ **1 pts** Extra Credit for LaTex-ed assignment

+ **0 pts** Can see only one page, link the remaining pages too

1. weak induction: $1^2 + 2^2 + \dots + n^2 + (n+1)^2 = (1^2 + 2^2 + \dots n^2) + (n+1)^2$

a. $n(n+1)(2n+1)/6 + (n+1)^2$

$= (n+1)(n(2n+1)) + 6(n+1)/6$

$= (n+1)(2n^2 + 7n + 6)/6$

$= (n+1)(n+2)(2n+3)/6$

equal RHS

for n+1

2.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

weak induction

b.

for n+1

$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3$

basecase holds

$1^3 = \frac{1(1+1)^2}{4} = \frac{4}{4} = 1$

$n^2(n+1)^2 + (n+1)^3$

$= \frac{n^4 + 2n^4 + n^2}{4} + \frac{4(n+1)^3}{4}$

$= \frac{n^4 + 2n^3 + n^2}{4} + \frac{4n^3 + 12n^2 + 12n^2 + 12n + 4}{4}$

$= \frac{n^4 + 6n^3 + 12n + 4}{4}$

$= \frac{(n+1)^2 (n+2)^2}{4}$

c. $(1 - \frac{1}{4})(1 - \frac{1}{2}) \dots (1 - \frac{1}{n^2}) = \frac{n+1}{2n}$ (for)

$(n \geq 2)$

for $n=2$

LHS: $(1 - \frac{1}{4}) = \frac{3}{4}$ LHS = RHS for $n=2$

RHS: $\frac{2+1}{2(2)} = \frac{3}{4}$

$n = k$ so $(1 - \frac{1}{4})(1 - \frac{1}{k^2})$ $\frac{n+2}{2n}$

for n+1 LHS $= (1 - \frac{1}{4})(1 - \frac{1}{k^2})(1 - \frac{1}{(k+1)^2})$

$= \frac{n+1}{2n} \frac{(n+2)}{(n+1)^2} = \frac{n+2}{2n+1} = \text{RHS}$

1.1 i^2 formula 3 / 4

- \+ **4 pts** All Correct
- \+ **1 pts** Base Case
- ✓ + **1 pts** Inductive Hypothesis
- ✓ + **1 pts** supporting math
- ✓ + **1 pts** Correct conclusion
- \+ **0 pts** Incorrect / Blank

1. weak induction: $1^2 + 2^2 + \dots + n^2 + (n+1)^2 = (1^2+2^2+\dots n^2) + (n+1)^2$

a. $n(n+1)(2n+1)/6 + (n+1)^2$

$= (n+1)(n(2n+1)) + 6(n+1)/6$

$= (n+1)(2n^2+7n+6)/6$

$= (n+1)(n+2)(2n+3)/6$

equal RHS

for n+1

2.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

weak induction

b.

for n+1

$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3$

base case holds

$1^3 = \frac{1(1+1)^2}{4} = \frac{4}{4} = 1$

$n^2(n+1)^2 + (n+1)^3$

$= \frac{n^4+2n^4+n^2}{4} + \frac{4(n+1)^3}{4}$

$= \frac{n^4+2n^3+n^2}{4} + \frac{4n^3+22n^2+12n^2+12n+4}{4}$

$= \frac{n^4+6n^3+12n+4}{4}$

$= \frac{(n+1)^2(n+2)^2}{4}$

c. $(1-\frac{1}{4})(1-\frac{1}{2})\dots(1-\frac{1}{n^2}) = \frac{n+1}{2n}$ (for $n \geq 2$)

for $n=2$

LHS: $(1-\frac{1}{4}) = \frac{3}{4}$ LHS = RHS for $n=2$

RHS: $\frac{2+1}{2(2)} = \frac{3}{4}$

$n=k$ so $(1-\frac{1}{4})(1-\frac{1}{k^2})\ \frac{n+2}{2n}$

for n+1 LHS $= (1-\frac{1}{4})(1-\frac{1}{k^2})(1-\frac{1}{(k+1)^2})$

$= \frac{n+1}{2n}\frac{(n+2)}{(n+1)^2} = \frac{n+2}{2n+1} =$ RHS

1.2 i^3 formula 4 / 4

- ✓ + **4 pts** **All Correct**
- + **1 pts** Base Case
- + **1 pts** Inductive Hypothesis
- + **1 pts** supporting math
- + **1 pts** Correct conclusion
- + **0 pts** Incorrect / Blank

1. weak induction: $1^2+2^2+\dots+n^2+(n+1)^2 = (1^2+2^2+\dots n^2)+(n+1)^2$

a. $n(n+1)(2n+1)/6+(n+1)^2$

$=(n+1)(n(2n+1))+6(n+1)/6$

$=(n+1)(2n^2+7n+6)/6$

$=(n+1)(n+2)(2n+3)/6$

eaual RHS

for n+1

2.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

weak induction

b.

for n+1

$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3$

basecase holds

$1^3 = \frac{1(1+1)^2}{4} = \frac{4}{4} = 1$

$n^2(n+1)^2+(n+1)^3$

$=\frac{n^4+2n^4+n^2}{4}+\frac{4(n+1)^3}{4}$

$=\frac{n^4+2n^3+n^2}{4}+\frac{4n^3+12n^2+12n^2+12n+4}{4}$

$=\frac{n^4+6n^3+12n+4}{4}$

$=\frac{(n+1)^2(n+2)^2}{4}$

c. $(2-\frac{1}{4})(1-\frac{1}{2})\dots(1-\frac{1}{n^2}) = \frac{n+1}{2n}$ (for)

$(n\geq 2)$

for $n=2$

LHS: $(1-\frac{1}{4})=\frac{3}{4}$ LHS = RHS for $n=2$

RHS: $\frac{2+1}{2(2)}=\frac{3}{4}$

$n=k$ so $(1-\frac{1}{4})(1-\frac{1}{k^2})\ \frac{n+2}{2n}$

for $n+1$ LHS $=(1-\frac{1}{4})(1-\frac{1}{k^2})(1-\frac{1}{(k+1)^2})$

$=\frac{n+1}{2n}\frac{(n+2)}{(n+1)^2}=\frac{n+2}{2n+1}=$ RHS

1.3 product formula 4 / 4

- ✓ **+ 4 pts** **All Correct**
- **+ 1 pts** Base Case
- **+ 1 pts** Inductive Hypothesis
- **+ 1 pts** supporting math
- **+ 1 pts** Correct conclusion
- **+ 0 pts** Incorrect / Blank

2.

given that $T^1 = T^2 = T^3 = 1$

and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$

INDUCTION: $T_n < 2^n$ $n \geq 4$

easily for $k = 4$

$\rightarrow$ onto:

L.H.S $= T4 = T_1 + T_2 + T_3 = \boxed{3}$

R.H.S $: 2^4 = 16$; $3 \leq 16$; $T_4 \leq 2^4$

$T_k \leq n^k$ for $k \geq 4$

for $T_{k+1} \leq 2^{k+1}$:

$T_{k+1} = T_k + T_{k-1} + T_{k-2} < Tk + Tk-1 + 2k-2 + Tk-3$

$= 1k + 1k$

$= 2Tk$

$= 2 \cdot 2^k$

Hence $T_{k+1} \leq 2^{k+1}$

✓ $n=4$ & $n \geq 4$

for $k+1$

we have proved for all $k \geq 4$ using strong induction.

2 Strong Induction **8 / 10**

- + **10 pts** All Correct
- + **3 pts** Base Cases - 1 point for each one
- ✓ + **2 pts** **Inductive hypothesis**
- ✓ + **2 pts** **Using the induction hypothesis in mathematical derivation**
- ✓ + **2 pts** **Mathematical derivation to arrive at T_k+1 < 2^k+1**
- ✓ + **1 pts** **Using strong induction and stating so**
- + **0 pts** Not attempted

+ **1 Point adjustment**

Base cases are n=4,5,6.

3. a. $O(g(n))$

$c = 56 \quad n \geq 0$

$0 \leq 50n^3 + 6n^3\log(n^3) - n\log(n^2) \leq 56 \cdot n^3\log(n^3)$

Hence $f(n) = O(n^3\log(n^3))$ ✓

b. $\boxed{\Omega}$

$0 \leq n^3\log(n^3) \leq 50n^3 + 6n^3\log(n^3) - n\log(n^2)$

Hence

$f(n) = \Omega(n^3\log(n^3))$ ✓

c. from A & B we have

$0 <= n^3\log(n^3) <= 50n^3 + 6n^3\log(n^2) <= 56 \times n^3\log(n^3)$

Hence $f(n) = \Theta(n^3\log(n^3)$ ✓

3.1 big-O 3 / 4

+ **4 pts** All Correct
+ **1 pts** Bound of 50n^3
+ **1 pts** Bound of 18n^3 log(n)
+ **1 pts** Bound of -2n log(n)
+ **1 pts** C and k values
+ **0 pts** No solution

+ 3 Point adjustment

1 complexity needs to be of the form O(n^p) where p is a natural number

3. a. $O(g(n))$

$c = 56 \quad n \geq 0$

$0 \leq 50n^3 + 6n^3 \log(n^3) - n\log(n^2) \leq 56 \cdot n^3 \log(n^3)$

Hence $f(n) = O(n^3 \log(n^3))$ ✓

b. $\boxed{\Omega}$

$0 \leq n^3 \log(n^3) \leq 50n^3 + 6n^3 \log(n^3) - n\log(n^2)$

Hence

$f(n) = \Omega(n^3 \log(n^3))$ ✓

c. from A & B we have

$0 <= n^3 \log(n^3) <= 50n^3 + 6n^3 \log(n^2) <= 56 \times n^3 \log(n^3)$

Hence $f(n) = \Theta(n^3 \log(n^3))$ ✓

3.2 big-Omega 2 / 4

+ **4 pts** All Correct
+ **1 pts** Bound of 50n^3
+ **1 pts** Bound of 18n^3 log(n)
+ **1 pts** Bound of -2n log(n)
+ **1 pts** C and k values
+ **0 pts** No solution

+ 2 Point adjustment

complexity needs to be of the form omega(n^p) where p is a natural number

2 c and k values are not mentioned

3. a. $O(g(n))$

$C = 56 \quad n \geq 0$

$0 \leq 50n^3 + 6n^3\log(n^3) - n\log(n^2) \leq 56 \cdot n^3\log(n^3)$

Hence $f(n) = O(n^3\log(n^3))$ ✓

b. $\boxed{\Omega}$

$0 \leq n^3\log(n^3) \leq 50n^3 + 6n^3\log(n^3) - n\log(n^2)$

Hence

$f(n) = \Omega(n^3\log(n^3))$ ✓

C. from A & B we have

$0 <= n^3\log(n^3) <= 50n^3 + 6n^3\log(n^2) <= 56 \times n^3\log(n^3)$

Hence $f(n) = \Theta(n^3\log(n^3)$ ✓

3.3 big-Theta 2 / 4

- \+ **4 pts** All Correct
- \+ **2 pts** Concluding no
- ✓ + **2 pts** **Explanation**
- \+ **0 pts** No solution
- \+ **0 pts** wrong answer

gradescope

4. $$\begin{bmatrix} 3 & 4 \\ 1 & 0 \\ 2 & 7 \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} 8 & 1 & 2 \\ 7 & 6 & 5 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 3\times 8 + 4\times 7 & 3\times 1 + 4\times 6 & 3\times 2 + 4\times 5 \\ 1\times 8 + 0\times 7 & 1\times 1 + 0\times 6 & 1\times 2 + 0\times 5 \\ 2\times 8 + 7\times 7 & 2\times 1 + 7\times 6 & 2\times 2 + 7\times 5 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} 24+28 & 3\times 24 & 6+20 \\ 8+0 & 1+0 & 2+0 \\ 16+44 & 2+42 & 4+35 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} 52 & 27 & 26 \\ 8 & 1 & 2 \\ 64 & 44 & 39 \end{bmatrix}_{3\times 3}$$

4.1 (a) **5 / 5**

- ✓ **+ 5 pts All Correct**
- **+ 3 pts** No supporting work
- **+ 1 pts** Correct matrix dimensions of 3x3
- **+ 2 pts** Correct answer
- **+ 2 pts** Supporting work
- **+ 0 pts** Incorrect/No answer provided
- **+ 4.5 pts** Upto two elements wrong
- **+ 4.5 pts** Putting commas in matrices

b. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{23} \end{bmatrix}_{3\times3}$

$$= \begin{bmatrix} a_{11}b_{11} + b_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + b\,a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{3}b_{52} & a_{31}b_{13} + a_{32}b_{23} + a_{31}b_{1} \end{bmatrix}_{3\times3}$$

4.2 (b) **6 / 6**

- ✓ **+ 6 pts** **All Correct**
- **+ 1 pts** Correct matrix dimensions of 3x3
- **+ 3 pts** Correct answer
- **+ 2 pts** Supporting work
- **+ 0 pts** Not answered/Incorrect
- **+ 5.5 pts** -0.5 for using commas in matrices

CSCI 2824
Spring 2019

Write **clearly**:

Name:	Alex Garcia Gonzalez
Student ID:	104288519
Section number:	001 COX
Assignment:	HW8

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn one extra credit point per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

5 Style Points 5 / 6

+ **6 pts** All correct
✓ + **5 pts** **Assignment not typed**
+ **2 pts** Complete assignment
+ **3 pts** Neatness
+ **1 pts** Extra Credit for LaTex-ed assignment
+ **0 pts** Can see only one page, link the remaining pages too