CSCI 2824 Written HW10

Alex Garcia-Gonzalez

TOTAL POINTS

43.5 / 51

QUESTION 1

Easter Candy 8 pts

1.1 Part (a) 4 / 4

✓ **+ 4 pts All correct**

+ **1 pts** n = 11
+ **1 pts** r = 30
+ **2 pts** for writing 40 C 10 or 40 C 30
+ **0 pts** Incorrect/Not answered
+ **1 pts** For attempting
+ **2 pts** Summation of C from n = 0 to n = 30

1.2 Part (b) 4 / 4

✓ **+ 4 pts All correct**

+ **1 pts** n = 11
+ **2 pts** r = 19
+ **1 pts** 29 C 19 or 29 C 10
+ **1 pts** For attempting
+ **0 pts** Incorrect/not answered
+ **2 pts** Partly correct

QUESTION 2

Passwords 11 pts

2.1 Part (a) 3 / 3

✓ **+ 3 pts Everything Correct**

+ **1 pts** Correct base i.e., 72
+ **1 pts** 72^7, 72^8, 72^9
+ **1 pts** Adding above mentioned terms
+ **0 pts** No Solution or Incorrect

2.2 Part (b) 3 / 3

✓ **+ 3 pts Everything Correct**

+ **1 pts** P(n,2) or n(n-1)
+ **1 pts** 100
+ **1 pts** 52^(n-2)
+ **0 pts** No solution or Incorrect

2.3 Part (c) 2.5 / 3

+ **3 pts** Everything Correct i.e., 72^9 - (46^9 + 62^9 - 36^9)

✓ **+ 1 pts 72^9**

✓ **+ 1 pts -(46^9+62^9)**

✓ **+ 1 pts 36^9**

+ **0 pts** No solution or Incorrect

- 0.5 Point adjustment

1 extra term

2.4 Part (d) 2 / 2

✓ **+ 2 pts All Correct i.e., 9log(72)**

+ **1 pts** 9
+ **1 pts** log(72)
+ **0 pts** No solution or incorrect

QUESTION 3

Yahtzee 14 pts

3.1 Part (a) 4 / 4

✓ **+ 4 pts Correct**

+ **2 pts** Correct numerator
+ **2 pts** Correct denominator
+ **0 pts** No solution

3.2 Part (b) 4 / 4

✓ **+ 4 pts Correct**

+ **2 pts** Correct numerator
+ **2 pts** Correct denominator
+ **0 pts** Incorrect / No solution

3.3 Part (c) 4 / 4

✓ **+ 4 pts Correct**

+ **2 pts** Correct numerator
+ **2 pts** Correct denominator
+ **0 pts** Incorrect / No solution

3.4 Part (d) 2 / 2

✓ + **2 pts** **Correct**
+ **1 pts** Correct answer, but no math shown
+ **1 pts** Some correct work, but no conclusion
+ **0 pts** Incorrect / No solution

QUESTION 4

Binomial Theorem 12 pts

4.1 Part (a) 3 / 3

✓ + **3 pts** **Correct Answer**
+ **1 pts** For n=500
+ **1 pts** For k=301
+ **1 pts** For correct answer
+ **0 pts** Incorrect

4.2 Part (b) 3 / 3

✓ + **3 pts** **Correct Answer**
+ **1 pts** For identifying (2x) term
+ **1 pts** For identifying (-1)
+ **1 pts** Right answer
+ **0 pts** Incorrect

4.3 Part (c) 0 / 3

+ **3 pts** Correct
+ **1 pts** Correct Answer
+ **2 pts** Valid Explanation

✓ + **0 pts** **Incorrect Answer**

4.4 Part (d) 0 / 3

+ **3 pts** Correct
+ **2 pts** Comination of terms
+ **1 pts** Permutation of terms

✓ + **0 pts** **Incorrect Answer**

QUESTION 5

5 Style Points 5 / 6

+ **6 pts** All correct

✓ + **5 pts** **Not typed**
+ **3 pts** Completeness
+ **2 pts** Neatness
+ **1 pts** Latex ed
+ **0 pts** No submission

1. Easter is approaching, and the Easter Bunny has given out a massive amount of candy already. It's unreal. However, a little known fact about the Easter bunny is how much he loves candy himself. So, he has been eating a ton of candy while also giving candy out to youngsters, and in all the excitement he lost track of how much candy he gave to youngsters and how much he ate himself.

   The Easter Bunny knows that 10 youngsters approached him looking for candy. The Easter Bunny also knows that he had 30 indistinguishable pieces of candy to start with, so he gave out at most 30 pieces of candy to these 10 youngsters. This can be represented as:

$$\sum_{k=1}^{10} x_k \leq 30$$

   where x_k represents the number of pieces of candy that the k^{th} youngster receives.

   (a) How many ways could the Easter Bunny have distributed at most 30 pieces of candy among the 10 youngsters?

   (b) Suddenly, the Easter Bunny remembers that he ate at least 11 pieces of candy. Now, how many ways are there for the Easter Bunny to have distributed the candy?

---

a.

$\sum_{k=1}^{11} x_K = 30$

will give us # of solutions to linear equation.

x_{11}: how much easter bunny ate

Combination w/o Repitition

$n = 11$ bins

$r = 30$ stars

$$C^{r}_{r+n-1} = C(30+11-1, 30)$$
$$= C(40, 30)$$
$$= \boxed{847{,}660{,}528}$$

b. $30 - 11 = 19$ left to distribute. -11 bins

Combination w/o Repitition

$r = 19$

$n = 11$

$$C(19+11-1, 30) = \boxed{20{,}030{,}010}$$

$$C(29, 19)$$

1.1 Part (a) **4 / 4**

✓ + **4 pts** **All correct**

+ **1 pts** n = 11

+ **1 pts** r = 30

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$r = 19$

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1.2 Part (b) **4 / 4**

- ✓ **+ 4 pts** **All correct**
- **+ 1 pts** n = 11
- **+ 2 pts** r = 19
- **+ 1 pts** 29 C 19 or 29 C 10
- **+ 1 pts** For attempting
- **+ 0 pts** Incorrect/not answered
- **+ 2 pts** Partly correct

2. You are making a new account on your favorite social media website. This great new website offers the most fun way to steal all of your personal information! Long story short, you need a new password. Suppose passwords can include upper-case letters, lower-case letters, numbers and the symbols you can get by holding `Shift` + a number. Characters may be repeated. Please do **not** simplify your answers.

   (a) Suppose you want to make a 7, 8 or 9-character long password. How many such passwords are there?

   (b) Now suppose you want to generate a password of length n that includes exactly one symbol and one number. Find an expression in terms of n for the number of such passwords.

   (c) Suppose you want to make a 9-character password, and passwords must contain at least one upper-case letter and at least one number. How many such passwords are there?

   (d) A password's entropy can be calculated as the $\log_2$ of the number of characters in the character set used, multiplied by the number of characters in the password itself. How many bits of entropy does a 9-character password have, if it may be chosen from the set of upper-case letters, lower-case letters, numbers and symbols (`Shift` + number)?

a.

first we calculate the number of possible Characters:

26 uppercase
26 lowercase
10 numbers (0-9)
+ 10 symbols
72 chacters Possible for Password.

Passwords for 7 character long password:

$$72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 = 72^7$$

Passwords for 8 character long password:

$$72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 = 72^8$$

Passwords for 9 character long password:

$$72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 = 72^9$$

Passwords for 7, 8, 9 character long passwords $= 72^7 + 72^8 + 72^9$

b.

#'s of Passwords • "n length characters"
such that • 2 characters for Symbol & number

• Since the number and symbol are distinguishable we use Permutations to pick out # of ways to pick 2 of the n characters.

$$P(n,2)$$

• # of ways to choose number & symbol (-10 symbols, -10 numbers)

$$= 10^2 = 10 \cdot 10$$

• for the remaining n-2 charcter places. for 1 number & 1 symbol we will have

72 characters
-10 numbers
-10 symbols
52 possibilities for numbers and symbols.

Hence we have 52^{n-2} ways

- repeat charcters
- order matters
- Permutation with repitition.

$$= P(n,2) \cdot 10^2 \cdot 52^{n-2}$$

total possible passwords

$$= \frac{n!}{(n-2)!} \cdot 100 \cdot 52^{n-2} = 100\, n(n-1) \cdot 52^{n-2}$$

2.1 Part (a) 3 / 3

- ✓ **+ 3 pts** **Everything Correct**
- **+ 1 pts** Correct base i.e., 72
- **+ 1 pts** 72^7, 72^8, 72^9
- **+ 1 pts** Adding above mentioned terms
- **+ 0 pts** No Solution or Incorrect

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Passwords for 9 character long password:

$$72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 \cdot 72 = 72^9$$

Passwords for 7, 8, 9 character long passwords $= 72^7 + 72^8 + 72^9$

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$$= P(n,2) \cdot 10^2 \cdot 52^{n-2}$$

total possible passwords

$$= \frac{n!}{(n-2)!} \cdot 100 \cdot 52^{n-2} = 100\, n(n-1) \cdot 52^{n-2}$$

2.2 Part (b) 3 / 3

✓ + **3 pts** **Everything Correct**

+ **1 pts** P(n,2) or n(n-1)
+ **1 pts** 100
+ **1 pts** 52^(n-2)
+ **0 pts** No solution or Incorrect

C. U = upper case (atleast 1)
N = number (atleast 1)
T = 9 character Password = 72^9

$\overline{|U \cap N|} = |\bar{U} \cup \bar{N}| = |\bar{U}| + |\bar{N}| - |\bar{U} \cap \bar{N}|$

$|\bar{U}|$ = # of Passwords w/o upperase let.
$= (72-26)^9 = 46^9$

$|\bar{N}|$ = # of Passwords w/o any numbers
$= (72-10)^9 = 62^9$

$|\bar{U} \cap \bar{N}|$ = # of Pass. w/o - uppercase letters - any number
$= (72-10-26)^9$
$= 36^9$

$T = |U \cap N| + \overline{|U \cap N|}$

# of 9 chacters Passwords w/o upper case or numbers or Both

What we want

# of 9 chacters Passwords w/o upper case or numbers or Both

what we have

what we need to solve for

# of 9 chacters Passwords w/o upper case or numbers or Both $= |T| - |\bar{U}| + |\bar{N}| - (\bar{U} \cap \bar{N}) = 72^9 - (46^9 - (46^9 + 62^9 - 36^9)$

d. 9 characters
72 possible chacters entropy $= 9 \cdot \log_2 72 = 55.5$ bits

---

3. Yahtzee is a game in which a player rolls five six-sided dice simultaneously. The actual rules are a bit more involved, but for the sake of simplicity, let us only consider the case of rolling all five dice at once.

(a) What is the probability of obtaining all unique outcomes on each of the five dice rolled? For example, $\{1, 2, 4, 5, 6\}$ is all unique outcomes, but $\{1, 2, 4, 5, 2\}$ repeats a 2, so those outcomes are not all unique.

(b) A "small straight" consists of four numbers all in a sequence, plus one other die that can be anything. For example, one such outcome could be $\{2, 3, 4, 5, 3\}$, and another could be $\{2, 3, 4, 5, 6\}$ (we are ignoring the distinction between a small and a large straight). What is the probability of rolling a small straight when you roll all five dice?
Caution: beware of counting specific outcomes multiple times!

(c) You roll all five dice but then your phone alerts you that you've caught a new Pokémon with your Fortnite. Nice! You become so engrossed in the video games that you don't see what the outcome of the dice rolls is. Your kind friend tells you that the dice are all unique. Given this information, what then is the probability of that you have rolled a small straight?

(d) Are the events [*roll a small straight*] and [*roll all unique outcomes*] independent? Fully justify your answer **with math**.

a.
Let A= the event of getting all outcomes unique:
-1st dice can shows any of the side,
I.e. in 6 different ways.

2nd die can show a face in 5 different ways
i.e. except that the 1st die shows

3rd die can show a face in 4 different ways
i.e. except that the 1st and 2nd die shows

Similarly the 4th and 5th dice shows the faces in 3 and 2 different ways respectively.

Thus |A| = 6*5*4*3*3 = 6!

Thus the probability of obtaining all unique outcomes on each of the 5 dice rolled is:

P(A)= (|A|) / (|Ω|) = (6!)/(6^5) = (6*5*4*3*3*2)/(6*6*6*6*6)
P(A)=(5)/(54)

P(6,5)

2.3 Part (c) 2.5 / 3

+ **3 pts** Everything Correct i.e., 72^9 - (46^9 + 62^9 - 36^9)

✓ + **1 pts** 72^9

✓ + **1 pts** -(46^9+62^9)

✓ + **1 pts** 36^9

+ **0 pts** No solution or Incorrect

- 0.5 Point adjustment

1 extra term

C. U = upper case (atleast 1)
N = number (atleast 1)
T = 9 character Password = 72^9

$|\overline{U \wedge N}| = |\bar{U} \vee \bar{N}| = |\bar{U}| + |\bar{N}| - |\bar{U} \wedge \bar{N}|$

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$|\bar{U} \wedge \bar{N}|$ = # of Pass. w/o - uppercase letters - any number
$= (72-10-26)^9 = 36^9$

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# of 9 chacters Passwords w/o upper case or numbers or Both

What we want

What we have

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3. Yahtzee is a game in which a player rolls five six-sided dice simultaneously. The actual rules are a bit more involved, but for the sake of simplicity, let us only consider the case of rolling all five dice at once.

(a) What is the probability of obtaining all unique outcomes on each of the five dice rolled? For example, $\{1, 2, 4, 5, 6\}$ is all unique outcomes, but $\{1, 2, 4, 5, 2\}$ repeats a 2, so those outcomes are not all unique.

(b) A "small straight" consists of four numbers all in a sequence, plus one other die that can be anything. For example, one such outcome could be $\{2, 3, 4, 5, 3\}$, and another could be $\{2, 3, 4, 5, 6\}$ (we are ignoring the distinction between a small and a large straight). What is the probability of rolling a small straight when you roll all five dice?
Caution: beware of counting specific outcomes multiple times!

(c) You roll all five dice but then your phone alerts you that you've caught a new Pokémon with your Fortnite. Nice! You become so engrossed in the video games that you don't see what the outcome of the dice rolls is. Your kind friend tells you that the dice are all unique. Given this information, what then is the probability of that you have rolled a small straight?

(d) Are the events [*roll a small straight*] and [*roll all unique outcomes*] independent? Fully justify your answer **with math**.

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Thus |A| = 6*5*4*3*3 = 6!

Thus the probability of obtaining all unique outcomes on each of the 5 dice rolled is:

P(A)= (|A|) / (|Ω|) = (6!)/(6^5) = (6*5*4*3*3*2)/(6*6*6*6*6)
P(A)=(5)/(54)

P(6,5)

2.4 Part (d) 2 / 2

- ✓ + **2 pts** **All Correct i.e., 9log(72)**
- + **1 pts** 9
- + **1 pts** log(72)
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C. U = upper case (atleast 1)
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$T = |U \wedge N| + |\overline{U \wedge N}|$

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(b) A "small straight" consists of four numbers all in a sequence, plus one other die that can be anything. For example, one such outcome could be $\{2, 3, 4, 5, 3\}$, and another could be $\{2, 3, 4, 5, 6\}$ (we are ignoring the distinction between a small and a large straight). What is the probability of rolling a small straight when you roll all five dice?
Caution: beware of counting specific outcomes multiple times!

(c) You roll all five dice but then your phone alerts you that you've caught a new Pokémon with your Fortnite. Nice! You become so engrossed in the video games that you don't see what the outcome of the dice rolls is. Your kind friend tells you that the dice are all unique. Given this information, what then is the probability of that you have rolled a small straight?

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Thus |A| = 6*5*4*3*3 = 6!

Thus the probability of obtaining all unique outcomes on each of the 5 dice rolled is:

P(A)= (|A|) / (|Ω|) = (6!)/(6^5) = (6*5*4*3*3*2)/(6*6*6*6*6)
P(A)=(5)/(54)

P(6,5)

3.1 Part (a) 4 / 4

✓ + **4 pts** **Correct**

+ **2 pts** Correct numerator

+ **2 pts** Correct denominator

+ **0 pts** No solution

b.

Probability of rolling small straight, when you roll all five dice.

$-(6^5)-$

Small straight w/o Large straight.

1234 w/o a 5

5th die is either (1-4 repeat / or 6 not repeat)

4 ways to choose repeat

$C(5,2)$ ways to choose 2 dice are repeated value.

3 remaing dice $=3!$

$4 \cdot C(5,2) \cdot 3! = 2 \cdot 5!$

once 5th die is a 6 $= +1$

$3 \cdot 5!$

2345 w/o a 1 or 6

only $2 \cdot 5$

3456 w/o a 2

$3 . 5!$

Small Straights All together $\rightarrow$ $\boxed{8 \cdot 5!}$

large Straights All together $\rightarrow$ $\boxed{2 \cdot 5!}$

Large straights

12345

23456

$\boxed{2 \cdot 5!} + \boxed{8 \cdot 5!} = 10 \cdot 5!$

ways to roll small straights including large straights.

Probabity of small straight $P(\text{Small Straight}) = \frac{10 \cdot 5!}{6^5} = 0.1543$

3.2 Part (b) **4 / 4**

- ✓ **+ 4 pts** **Correct**
- **+ 2 pts** Correct numerator
- **+ 2 pts** Correct denominator
- **+ 0 pts** Incorrect / No solution

C. P(small | unique) $P(S|U) = \frac{P \cap U}{P(U)}$

$$P(S \cap U) = \frac{\text{unique + small straight}}{\text{all outcomes}}$$

$$P(U) = \frac{5}{54}$$

$$P(S|U) = \frac{\frac{4 \cdot 5!}{6^5}}{\frac{5}{54}} = \frac{4 \cdot 5! \cdot 54}{5 \cdot 6^5} = \frac{4 \cdot 4 \cdot 3 \cdot 2 \cdot 9}{6^4} = \frac{2}{3}$$

d.

A= The events of rolling a small straight
B= The events of rolling all unique outcomes

Now:
|A|=2
|B|=6!
|A∩B|=2

From Part A and C we get
P(A)=1/3888
P(B)=5/54

Now:
P(A∩B)= |A∩B|/ (|Ω|) = 2/6^5=1/3888

We know that two events A and B one independent
If P(A∩B)= P(A)*P(B)

But P(A)*P(B)=1/3888*5/54 ≠ P(A∩B)

Thus the events A and B are not independent

3.3 Part (c) **4 / 4**

- ✓ **+ 4 pts** **Correct**
- **+ 2 pts** Correct numerator
- **+ 2 pts** Correct denominator
- **+ 0 pts** Incorrect / No solution

C. $P(\text{small} / \text{unique})\ P(S/U) = \frac{P \wedge U}{P(U)}$

$$P(S \cap U) = \frac{\text{unique + small straight}}{\text{all outcomes}}$$

$$P(U) = \frac{5}{54}$$

$$P(S|U) = \frac{\frac{4 \cdot 5!}{6^5}}{\frac{5}{54}} = \frac{4 \cdot 5! \cdot 54}{5 \cdot 6^5} = \frac{4 \cdot 4 \cdot 3 \cdot 2 \cdot 9}{6^4} = \frac{2}{3}$$

d.

A= The events of rolling a small straight
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Now:
|A|=2
|B|=6!
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From Part A and C we get
P(A)=1/3888
P(B)=5/54

Now:
P(A∩B)= |A∩B|/ (|Ω|) = 2/6^5=1/3888

We know that two events A and B one independent
If P(A∩B)= P(A)*P(B)

But P(A)*P(B)=1/3888*5/54 ≠ P(A∩B)

Thus the events A and B are not independent

3.4 Part (d) 2 / 2

✓ **+ 2 pts** **Correct**

+ 1 pts Correct answer, but no math shown

+ 1 pts Some correct work, but no conclusion

+ 0 pts Incorrect / No solution

4. Solve the following problems and be sure to show all your work.

(a) What is the coefficient on $x^{199}y^{301}$ in the expansion of $(x+y)^{500}$? Do not simplify your answer.

(b) What is the coefficient on $x^{199}y^{301}$ in the expansion of $(2x-y)^{500}$? Do not simplify your answer.

(c) In the expansion of $(x+y)^{500}$, is there a term whose only x and y components are $x^{198}y^{301}$? Why or why not?

(d) How many possible distinct rearrangements of the letters STRUCTURE are there? Do not simplify your answer.
Hint: Any two of the same letter are indistinguishable from one another. For example, the two Ts are indistinguishable.

a.
We have to find coefficient of x^199 y^301 in expansion of (x+4)^500
We know that general term of $(a + b)^n$ is

$T_{r+1} = {}^nC_r\,(a)^{n-r}\,.\,(b)^r$ ...(1)

-For (x+y)^500, the general term equation becomes:
=t(r+1)=500cr*x^(500-r)*4^(r)

-we want coefficient of x^199*4^301
Let r=301
So
=500-r
=500-301
=199
Follow up Equation becomes:
t(301+1)=500c_301 x^(199)y^(301)

Hence it is the 302th term which contains x^199 y^301
The coefficient of x^(199)y^(301)=500x_(301)

b.
We have to find coefficient of x^199 y^301 in expansion of (2x-4)^500

We know that general term of $(a + b)^n$ is

$T_{r+1} = {}^nC_r\,(a)^{n-r}\,.\,(b)^r$ ...(1)

-Hence to find expansion of (2x-4)^500,

-let (2x-4)^500, =(2x+(-4))^500,
So
a=2x
b=-y
=T(r+1)=500cr(2x)^(500-r)*(-4)^r
-therefore follow up general form equation:
-let r=301
T(r+1)=500cr(2x)^(500-r)*(-4)^r
Thus
=500-r
=500-301
=199

-therefore follow up general form equation:
T(301+1)=500c_(301)(2x)^199 * (-4)^301
T(301+1)=500c_(301)(2)^199*(x)^199 * (-1)^301(4)^301
T(301+1)=2^(199)*(-1) * 500c_(301)x^(199)*4^*(301)

Hence the coefficient of x^199 y^301 is:
=(-1)*2^(199)*500c_(301)

4.1 Part (a) 3 / 3

- ✓ **+ 3 pts** **Correct Answer**
- **+ 1 pts** For n=500
- **+ 1 pts** For k=301
- **+ 1 pts** For correct answer
- **+ 0 pts** Incorrect

4. Solve the following problems and be sure to show all your work.

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(b) What is the coefficient on $x^{199}y^{301}$ in the expansion of $(2x-y)^{500}$? Do not simplify your answer.

(c) In the expansion of $(x+y)^{500}$, is there a term whose only x and y components are $x^{198}y^{301}$? Why or why not?

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We have to find coefficient of x^199 y^301 in expansion of (2x-4)^500

We know that general term of $(a + b)^n$ is

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-therefore follow up general form equation:
-let r=301
T(r+1)=500cr(2x)^(500-r)*(-4)^r
Thus
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-therefore follow up general form equation:
T(301+1)=500c_(301)(2x)^199 * (-4)^301
T(301+1)=500c_(301)(2)^199*(x)^199 * (-1)^301(4)^301
T(301+1)=2^(199)*(-1) * 500c_(301)x^(199)*4^*(301)

Hence the coefficient of x^199 y^301 is:
=(-1)*2^(199)*500c_(301)

4.2 Part (b) 3 / 3

✓ + **3 pts** **Correct Answer**

+ **1 pts** For identifying (2x) term

+ **1 pts** For identifying (-1)

+ **1 pts** Right answer

+ **0 pts** Incorrect

c.
Absolutely Not because the expansion of (c+y)^500 in each term the sum sum of exponents of x and y will be:
For example 500 x^199 y^301 sum of exponents 199+301=500
But in x^198 y^301 sum of exponents is 499

d.
We know the number of permutations are

Permutation Formula

The number of permutations of “n” objects, “r” of which are alike, “s” of which are alike, ‘t” of which are alike, and so on, is given by the expression

$$\boxed{\frac{n!}{r! \times s! \times t! \ldots}}$$

Now in word STRUCTURE
T comes twice
R comes twice
U come twice

So total no. of distinct rearrangements
=9!/(2!)(2!)(2!)

4.3 Part (c) **0 / 3**

- \+ **3 pts** Correct
- \+ **1 pts** Correct Answer
- \+ **2 pts** Valid Explanation
- ✓ + **0 pts** **Incorrect Answer**

c.
Absolutely Not because the expansion of (c+y)^500 in each term the sum sum of exponents of x and y will be:
For example 500 x^199 y^301 sum of exponents 199+301=500
But in x^198 y^301 sum of exponents is 499

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Permutation Formula

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Now in word STRUCTURE
T comes twice
R comes twice
U come twice

So total no. of distinct rearrangements
=9!/(2!)(2!)(2!)

4.4 Part (d) 0 / 3

- \+ **3 pts** Correct
- \+ **2 pts** Comination of terms
- \+ **1 pts** Permutation of terms
- ✓ + **0 pts** **Incorrect Answer**

5 Style Points 5 / 6

+ **6 pts** All correct
✓ + **5 pts** **Not typed**
+ **3 pts** Completeness
+ **2 pts** Neatness
+ **1 pts** Latex ed
+ **0 pts** No submission