CSCI 2824 Written HW 12

Alex Garcia-Gonzalez

TOTAL POINTS

49 / 51

QUESTION 1

6 pts

1.1 part (a) 3 / 3

- ✓ + **3 pts Correct**
- + **1 pts** Correct unrolling step
- + **1 pts** Correct GP formula
- + **1 pts** Correct conclusion
- + **0 pts** Incorrect / No solution

1.2 part (b) 3 / 3

- ✓ + **3 pts Correct**
- + **1 pts** Correct homogeneous solution
- + **1 pts** Correct particular solution
- + **1 pts** Correct conclusion
- + **0 pts** Incorrect / No solution

QUESTION 2

10 pts

2.1 part (a) 1 / 2

- + **2 pts** Correct
- ✓ + **1 pts r^2 - r - 2 = 0**
- + **1 pts** a = r^n
- + **0 pts** No answer/Incorrect

2.2 part (b) 2 / 2

- ✓ + **2 pts Correct**
- + **1 pts** A(-1)^n
- + **1 pts** B(2)^n
- + **0 pts** Incorrect/No answer
- + **1 pts** Partly correct

2.3 part (c) 2 / 2

- ✓ + **2 pts Correct**
- + **1 pts** Writing the correct quadratic term -n^2
- + **1 pts** Writing the correct linear term with constant (-5n - 8)
- + **1 pts** No coefficients written but a general quadratic
- + **0 pts** Incorrect/No answer

2.4 part (d) 2 / 2

- ✓ + **2 pts Correct**
- + **1 pts** Writing the correct homogenous solution
- + **1 pts** Writing the correct particular solution
- + **0 pts** Not shown/Incorrect

2.5 part (e) 2 / 2

- ✓ + **2 pts Correct**
- + **1 pts** For solving
- + **1 pts** Correct answer
- + **0 pts** Not answered?Incorrect

QUESTION 3

3 8 / 8

- ✓ + **8 pts Everything Correct (1*2^n-8n*2^n+9*3^n)**
- + **2 pts** r=2,2
- + **1 pts** Homogenous solution: A*(2^n)+b*n*(2^n)
- + **1 pts** Particular solution: C*(3^n)
- + **1 pts** Homogenous + Particular
- + **1 pts** C=9
- + **1 pts** A=1
- + **1 pts** B=-8
- + **0 pts** Incorrect solution or No solution

QUESTION 4

12 pts

4.1 part (a) 3 / 3

- ✓ + **3 pts Correct**
- + **1 pts** For 2an-1 term

+ **1 pts** for 2an-2 term
+ **1 pts** Explanation
+ **0 pts** Incorrect

4.2 part (b) 3 / 3

✓ + **3 pts Correct**
+ **1 pts** a0 orrect
+ **1 pts** a1 correct
+ **1 pts** a2 correct
+ **0 pts** Incorrect

4.3 part (c) 3 / 3

✓ + **3 pts Correct**
+ **1 pts** Equation is correct
+ **1 pts** A correct
+ **1 pts** B correct
+ **0 pts** Incorrect

4.4 part (d) 3 / 3

✓ + **3 pts Correct**
+ **1 pts** Calculation mistake
+ **0 pts** Not attempt/incorrect

QUESTION 5

9 pts

5.1 part (a) 3 / 3

✓ + **3 pts Everything Correct**
+ **1 pts** Reflexive
+ **1 pts** Symmetric
+ **1 pts** Not transitive
+ **0 pts** No solution or Wrong solution

5.2 part (b) 3 / 3

✓ + **3 pts Everything Correct**
+ **1 pts** No reflexive
+ **1 pts** No symmetric
+ **1 pts** Transitive
+ **0 pts** No solution or wrong solution

5.3 part (c) 3 / 3

✓ + **3 pts EverythingCorrect**
+ **1 pts** Reflexive
+ **1 pts** Symmetric
+ **1 pts** Transitive
+ **0 pts** Wrong solution or No solution

QUESTION 6

Style Points 6 pts

6.1 completeness 3 / 3

✓ + **3 pts Correct**
+ **0 pts** Click here to replace this description.

6.2 neatness 2 / 2

✓ + **2 pts Correct**
+ **0 pts** Incorrect

6.3 Latexed 0 / 1

+ **1 pts** Correct
✓ + **0 pts Not typed**

1. (a) Use **recursive-plugging-back-in** (also known as **iteration** or **unrolling**) to find a closed form solution to the the recurrence relation $a_n = -2a_{n-1} + 6$, with the initial condition $a_0 = 3$.

   (b) Consider again the recurrence relation $a_n = -2a_{n-1} + 6$, with the initial condition $a_0 = 3$. Verify the closed form solution you found in part (a) by combining the solution to the associated homogeneous recurrence and a particular solution to the full nonhomogeneous recurrence.

---

a. given $an = -2a_{n-1} + 6$ $\qquad a0 = 3$

$$an = -2a_{n-1} + 6$$
$$an = -2(-2a_{n-1} + 6) + 6$$
$$= (-2)^2 a_{n-2} + 6(1-2)$$
$$-2((-2)^2 a_{n-2} + 6(1-2)) + 6$$
$$= (-2)^3 a_{n-3} + 6(1-2+4)$$
$$= (-2)^n a_0 + 6 \sum_{k=0}^{n-1} (-2)^k$$
$$= 3(-2)^n + 6\frac{(-2)^n - 1}{(-2) - 1}$$
$$3(-2)^n - 2((-2)^n - 1$$
$$= (-2)^n + 2$$

b. $an = an^h + an^p = A(-2)^n + 2$

$an = -2an-1$

an^p

- non homogenous is 6 which looks like B

$-2(B) + 6 = -2an-1+6$

$3B = 6 \Rightarrow \underline{A_n^p = 2}$

an^h

$an = r^n = -2r^{n-1}$

$= r = -2$

$\underline{an^h = A(-2)^n}$

inital condition $a_0 = 3 = A(-2)^0 + 2 = A + 2 \qquad A = 1$

Solution: $an = (-2)^n + 2$

1.1 part (a) **3 / 3**

- ✓ **+ 3 pts** **Correct**
- **+ 1 pts** Correct unrolling step
- **+ 1 pts** Correct GP formula
- **+ 1 pts** Correct conclusion
- **+ 0 pts** Incorrect / No solution

1. (a) Use **recursive-plugging-back-in** (also known as **iteration** or **unrolling**) to find a closed form solution to the the recurrence relation $a_n = -2a_{n-1} + 6$, with the initial condition $a_0 = 3$.

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a.

given $a_n = -2a_{n-1} + 6$ $\qquad a_0 = 3$

$a_n = -2a_{n-1} + 6$

$a_n = -2(-2a_{n-1} + 6) + 6$

$= (-2)^2 a_{n-2} + 6(1-2)$

$-2((-2)^2 a_{n-2} + 6(1-2)) + 6$

$= (-2)^3 a_{n-3} + 6(1-2+4)$

$= (-2)^n a_0 + 6 \sum_{k=0}^{n-1} (-2)^k$

$= 3(-2)^n + 6 \dfrac{(-2)^n - 1}{(-2) - 1}$

$3(-2)^n - 2((-2)^n - 1$

$= (-2)^n + 2$

b. $a_n = a_n^h + a_n^p = A(-2)^n + 2$

$a_n = -2a_{n-1}$

a_n^p

- non homogenous is 6 which looks like B

$-2(B) + 6 = -2a_{n-1} + 6$

$3B = 6 \Rightarrow \underline{A_n^p = 2}$

a_n^h

$a_n = r^n = -2r^{n-1}$

$= r = -2$

$\underline{a_n^h = A(-2)^n}$

initial condition $a_0 = 3 = A(-2)^0 + 2 = A + 2 \qquad A = 1$

Solution: $a_n = (-2)^n + 2$

1.2 part (b) 3 / 3

- ✓ **+ 3 pts** **Correct**
- **+ 1 pts** Correct homogeneous solution
- **+ 1 pts** Correct particular solution
- **+ 1 pts** Correct conclusion
- **+ 0 pts** Incorrect / No solution

2. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

   Find a closed form solution for the given recurrence relation. In your solution, put a box around each of the following, and clearly label them:

   (a) the characteristic polynomial

   (b) the solution to the associated homogeneous recurrence relation ($a_n^{(h)}$)

   (c) the full particular solution **guess** that you are plugging into the full nonhomogeneous recurrence relation ($a_n^{(p)}$)

   (d) the full general solution (with unknown coefficients still) (a_n)

   (e) the full solution to the initial value problem (having now solved for any unknown coefficients)

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a.

Closed form solution for an=-2an-1+5 initial condition: a0=3

Given the Equation: an=an-a+an-2+2n^2
Initial Conditions: a0=0, a1=7

Solving for homogenous recurrence relation
= an=an-a+an-2+2n^2=0
= an-an01-2an02=2n^2

b.

Characteristic polynomial is given by [**r^2-3-2=0**]

Not let's solve for characters polynomial

r^2-r-2=0
r^2-2r+r-2=0
(r-2)(r+1)=0
r=2 r=-1

Thus the solution associated to homogenous solution is:

(an^(h))= (c1)2^n+(c2)(-1)^n

c.

Solving for particular solution guess plugged into full non homogenous recurrence relation
(an^(p))=an^2+bn+c
And plug it in the solution
(an^(p))=(an-1^(p))=2(an-2^(p))=2n^2

2(a(n-1)^2+b(n-2)+c) -(a(n-1)^2+b(n-1)+c) - (an^2+bn+c) =2n^2
2(an^2+4an+bn-2b+c)- (an^2+a-2an+bn-b+c) - (an^2+bn+c) =2n^2
-2an^2+2a+8a-2b)n + (-a+b-8a+4b-2c)=2n^2
-2an+n(10a-2b)+(-9a+5b-2c)=2n^2
-2a=2= [a=-1]

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2b=10a=10(-1)
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=9a+5b-2c=0
=-9a+5b
=-9(-a)+5(-5)
[c=-8]

Thus an^p=-n^2-5n-8

d.

The full general solution of
an=an^h+ an^p
[an= (c1)2^n +c2(-1)^n-n^2-5n-8]

e.

Applying initial condition

Given a0=0

0=(c1)2^0 +c2(-1)^0-0^2-5(0)-8
0= (c1)+(c2)-8=1
6=c1+c2

Given a1=7
7=(c1)w^2+(c2)(-1)^1-1^2-5(1)-8
7=2(c1)-(c2)-14
21=2(c1)-c(2)

c1= 29/2
c2= -5/3

Full solution: [(29/3)(2)^n - 5/3(-1)^n-n^2-5n-8]

2.1 part (a) 1 / 2

+ **2 pts** Correct

✓ + **1 pts** **r^2 - r - 2 = 0**

+ **1 pts** a = r^n

+ **0 pts** No answer/Incorrect

2. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

   Find a closed form solution for the given recurrence relation. In your solution, put a box around each of the following, and clearly label them:

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Characteristic polynomial is given by [**r^2-3-2=0**]

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r^2-r-2=0
r^2-2r+r-2=0
(r-2)(r+1)=0
r=2 r=-1

Thus the solution associated to homogenous solution is:

(an^(h))= (c1)2^n+(c2)(-1)^n

c.

Solving for particular solution guess plugged into full non homogenous recurrence relation
(an^(p))=an^2+bn+c
And plug it in the solution
(an^(p))=(an-1^(p))=2(an-2^(p))=2n^2

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Given a0=0

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2.2 part (b) 2 / 2

- ✓ **+ 2 pts** **Correct**
- **+ 1 pts** A(-1)^n
- **+ 1 pts** B(2)^n
- **+ 0 pts** Incorrect/No answer
- **+ 1 pts** Partly correct

2. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

Find a closed form solution for the given recurrence relation. In your solution, put a box around each of the following, and clearly label them:

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The full general solution of
an=an^h+ an^p
[an= (c1)2^n +c2(-1)^n-n^2-5n-8]

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Applying initial condition

Given a0=0

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Full solution: [(29/3)(2)^n - 5/3(-1)^n-n^2-5n-8]

2.3 part (c) **2 / 2**

- ✓ **+ 2 pts** **Correct**
- **+ 1 pts** Writing the correct quadratic term -n^2
- **+ 1 pts** Writing the correct linear term with constant (-5n - 8)
- **+ 1 pts** No coefficients written but a general quadratic
- **+ 0 pts** Incorrect/No answer

2. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

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2.4 part (d) **2 / 2**

- ✓ **+ 2 pts** **Correct**
- **+ 1 pts** Writing the correct homogenous solution
- **+ 1 pts** Writing the correct particular solution
- **+ 0 pts** Not shown/Incorrect

2. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

Find a closed form solution for the given recurrence relation. In your solution, put a box around each of the following, and clearly label them:

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Characteristic polynomial is given by [**r^2-3-2=0**]

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-2an+n(10a-2b)+(-9a+5b-2c)=2n^2
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The full general solution of
an=an^h+ an^p
[an= (c1)2^n +c2(-1)^n-n^2-5n-8]

e.
Applying initial condition

Given a0=0

0=(c1)2^0 +c2(-1)^0-0^2-5(0)-8
0= (c1)+(c2)-8=1
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21=2(c1)-c(2)

c1= 29/2
c2= -5/3

Full solution: [(29/3)(2)^n - 5/3(-1)^n-n^2-5n-8]

2.5 part (e) **2 / 2**

✓ + **2 pts** **Correct**

+ **1 pts** For solving

+ **1 pts** Correct answer

+ **0 pts** Not answered?Incorrect

3. Find the closed form solution to the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 3^n$, $a_0 = 10$, $a_1 = 13$.

Given: an=4an-1-4an-2+3^n,
Initial Condition: a0=10, a1=13

Homogenous part has characetersitc equation
x^2+4x+4=0
Roots: x=2
Solution form:
hn=(d1+d2 n)2^n

Solution for we need:
pn=3^n*c
pn=4x+3^n-1 -4c*3^0+3^2

Let n=2
c*3^2=4x*3^1-4c-3^0+3^2
9c=12c-4c=9
c=9

pn=9*3^n

Proceding:
an=hn+pn
an= (d1+d2 n)2^n+9*3^n

Find constants with initial conditions
a0=a0= (d1+d2 (0))2^0+9*3^0
10=d1+9
d1=1

a1=d1+d2(1)2^1+9(3)^1
a1=(d1+d2)2^1+27
13=(2d1+2d2)+27

Plug d1=1 to solve for d2

13=(2d1+2d2)+27
13=(2(1)+2d2)+27
13=29+2d2
13+29=2d2
-16=2d2
d2=-16/2
d2=-8

Thus an=(1+-8n)2^n+9-3^n
an=(1-8n)2^n+9*3^n

$an = 2^n - 8n \cdot 2^n + 9 \cdot 3^n$

3 8 / 8

- ✓ **+ 8 pts** **Everything Correct (1*2^n-8n*2^n+9*3^n)**
- **+ 2 pts** r=2,2
- **+ 1 pts** Homogenous solution: A*(2^n)+b*n*(2^n)
- **+ 1 pts** Particular solution: C*(3^n)
- **+ 1 pts** Homogenous + Particular
- **+ 1 pts** C=9
- **+ 1 pts** A=1
- **+ 1 pts** B=-8
- **+ 0 pts** Incorrect solution or No solution

4. Popeye and Olive Oyl frequently send each other text messages that are just contiguous strings of the three emojis , , and . For instance, one particular length-5 emoji string might be .

(a) Find a recurrence relation for the number of possible length-n emoji strings that do not contain two consecutive winkey emojis, .

(b) What are the initial conditions for the recurrence relation?

(c) Find a closed-form solution to the recurrence relation you found in part (a) by solving for the roots of the characteristic polynomial and then using initial conditions to determine the constants.

(d) Use your closed form expression to determine the number of length-7 emoji strings that do not contain 2 consecutive winkey emojis. [Note: You may use Python to help you answer this question.]

---

a.

an = the number of ways that emoji can line up such that there are no two winky emoji next to each other.

If the last person is not R, then they must be J or S, so there are 2 ways to do this.

you add this for both sides in the case it is in the front or last. There will be a total of an=2an-1 + 2an-2

b.

Initial Conditions:

1 way to arrange 0 emoji, so a0=1
3 way to arrange 1 emoji, so a1=3

There are a total of 3^2 = 9 lines of length 2, minus the double emoji line, thus a2=8

c.

We plug in the solution guess an = r^n

an =r^n = 2r^(n−1) +2r^(n−2). r^2-2r-2=0

To find the roots we move everything to the left and set to zero. Then
We can use the quadratic formula to find the roots:
This will give us after plugging in:

r^2-2r-2=0
r=2+sqrt(12)/(2)=1+sqrt(3)

an= A(1+sqrt(3))^n + B(1-sqrt(3))^n Now we use the initial conditions to determine A and B

$$A = \frac{2+\sqrt{3}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9+2\sqrt{3}}{6}$$

$$B = 1 - \frac{2+\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{3}-2-\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}-2}{2\sqrt{3}}$$

$$a_n = \frac{2+\sqrt{3}}{2\sqrt{3}}(1+\sqrt{3})^n + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^n$$

d. $$a_7 = \frac{2+\sqrt{3}}{2\sqrt{3}}(1+\sqrt{3})^7 + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^7$$

$$= 1224$$

4.1 part (a) 3 / 3

✓ + **3 pts** **Correct**

+ **1 pts** For 2an-1 term
+ **1 pts** for 2an-2 term
+ **1 pts** Explanation
+ **0 pts** Incorrect

4. Popeye and Olive Oyl frequently send each other text messages that are just contiguous strings of the three emojis 😃, 😐, and 😑. For instance, one particular length-5 emoji string might be 😃😐😃😐😑.

(a) Find a recurrence relation for the number of possible length-n emoji strings that do not contain two consecutive winkey emojis, 😐😐.

(b) What are the initial conditions for the recurrence relation?

(c) Find a closed-form solution to the recurrence relation you found in part (a) by solving for the roots of the characteristic polynomial and then using initial conditions to determine the constants.

(d) Use your closed form expression to determine the number of length-7 emoji strings that do not contain 2 consecutive winkey emojis. [Note: You may use Python to help you answer this question.]

---

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If the last person is not R, then they must be J or S, so there are 2 ways to do this.

you add this for both sides in the case it is in the front or last. There will be a total of an=2an-1 + 2an-2

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Initial Conditions:

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d.

$$a_7 = \frac{2+\sqrt{3}}{2\sqrt{3}}(1+\sqrt{3})^7 + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^7$$

$$= 1224$$

4.2 part (b) 3 / 3

- ✓ + **3 pts** **Correct**
- + **1 pts** a0 orrect
- + **1 pts** a1 correct
- + **1 pts** a2 correct
- + **0 pts** Incorrect

4. Popeye and Olive Oyl frequently send each other text messages that are just contiguous strings of the three emojis 😬, 😐, and 😑. For instance, one particular length-5 emoji string might be 😬😐😬😐😑.

(a) Find a recurrence relation for the number of possible length-n emoji strings that do not contain two consecutive winkey emojis, 😐😐.

(b) What are the initial conditions for the recurrence relation?

(c) Find a closed-form solution to the recurrence relation you found in part (a) by solving for the roots of the characteristic polynomial and then using initial conditions to determine the constants.

(d) Use your closed form expression to determine the number of length-7 emoji strings that do not contain 2 consecutive winkey emojis. [Note: You may use Python to help you answer this question.]

---

a.

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If the last person is not R, then they must be J or S, so there are 2 ways to do this.

you add this for both sides in the case it is in the front or last. There will be a total of an=2an-1 + 2an-2

b.

Initial Conditions:

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There are a total of 3^2 = 9 lines of length 2, minus the double emoji line, thus a2=8

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$$a_7 = \frac{2+\sqrt{3}}{2\sqrt{3}}(1+\sqrt{3})^7 + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^7$$

$$= 1224$$

4.3 part (c) 3 / 3

- ✓ **+ 3 pts** **Correct**
- **+ 1 pts** Equation is correct
- **+ 1 pts** A correct
- **+ 1 pts** B correct
- **+ 0 pts** Incorrect

4. Popeye and Olive Oyl frequently send each other text messages that are just contiguous strings of the three emojis , , and . For instance, one particular length-5 emoji string might be .

(a) Find a recurrence relation for the number of possible length-n emoji strings that do not contain two consecutive winkey emojis, .

(b) What are the initial conditions for the recurrence relation?

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(d) Use your closed form expression to determine the number of length-7 emoji strings that do not contain 2 consecutive winkey emojis. [Note: You may use Python to help you answer this question.]

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$$= 1224$$

4.4 part (d) 3 / 3

- ✓ + **3 pts** **Correct**
- + **1 pts** Calculation mistake
- + **0 pts** Not attempt/incorrect

5. Determine whether each of the following relations $R \subseteq A \times A$, where A is the set of all CU students, is reflexive, symmetric, transitive, and/or an equivalence relation. Briefly justify each conclusion.

(a) $(a, b) \in R$ if and only if a shares at least one class with b.

(b) $(a, b) \in R$ if and only if a has a higher GPA than b.

(c) $(a, b) \in R$ if and only if a is roommates with b.

---

a) It is symmetric and reflexive.

If a and b have a common case, then so does b and a.
a will have a common class with a.
not transitive because relation cannot guarantee "a" and "c" share any classes.

b) It is transitive.

Transitive because $(a,b) \in R$ and $(b,c) \in R$
Meaning if a has a higher GPA than b and b has a higher GPA than c, then a has a higher GPA than c.
it must be true that $(a, c) \in R$.

c.) It is symmetric, transitive and reflexive.

(Reflexive) because roommate a lives in the same place as herself.

If a and b are born on same day, it means b and a are born on same day as well. (symmetric)

If a and b are roomies and b and c are roomies too, then a and c are roomies as well. (Transitive)

5.1 part (a) 3 / 3

- ✓ + **3 pts** **Everything Correct**
- + **1 pts** Reflexive
- + **1 pts** Symmetric
- + **1 pts** Not transitive
- + **0 pts** No solution or Wrong solution

5. Determine whether each of the following relations $R \subseteq A \times A$, where A is the set of all CU students, is reflexive, symmetric, transitive, and/or an equivalence relation. Briefly justify each conclusion.

   (a) $(a, b) \in R$ if and only if a shares at least one class with b.

   (b) $(a, b) \in R$ if and only if a has a higher GPA than b.

   (c) $(a, b) \in R$ if and only if a is roommates with b.

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b) It is transitive.

Transitive because (a,b)∈R and(b,c)∈ R
Meaning if a has a higher GPA than b and b has a higher GPA than c, then a has a higher GPA than c.
it must be true that (a, c) ∈ R.

c.) It is symmetric, transitive and reflexive.

(Reflexive) because roommate a lives in the same place as herself.

If a and b are born on same day, it means b and a are born on same day as well. (symmetric)

If a and b are roomies and b and c are roomies too, then a and c are roomies as well. (Transitive)

5.2 part (b) 3 / 3

- ✓ + **3 pts** **Everything Correct**
- + **1 pts** No reflexive
- + **1 pts** No symmetric
- + **1 pts** Transitive
- + **0 pts** No solution or wrong solution

5. Determine whether each of the following relations $R \subseteq A \times A$, where A is the set of all CU students, is reflexive, symmetric, transitive, and/or an equivalence relation. Briefly justify each conclusion.

   (a) $(a, b) \in R$ if and only if a shares at least one class with b.

   (b) $(a, b) \in R$ if and only if a has a higher GPA than b.

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---

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If a and b are born on same day, it means b and a are born on same day as well. (symmetric)

If a and b are roomies and b and c are roomies too, then a and c are roomies as well. (Transitive)

5.3 part (c) 3 / 3

- ✓ + **3 pts** **EverythingCorrect**
- \+ **1 pts** Reflexive
- \+ **1 pts** Symmetric
- \+ **1 pts** Transitive
- \+ **0 pts** Wrong solution or No solution

CSCI 2824
Spring 2019

Write **clearly**:

Name:	Alex Garcia Gonzalez
Student ID:	104288519
Section number:	001 COX
Assignment:	HW12

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn one extra credit point per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

6.1 completeness 3 / 3

✓ **+ 3 pts** **Correct**

+ 0 pts Click here to replace this description.

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6.2 neatness 2 / 2

✓ + 2 pts Correct

+ 0 pts Incorrect

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Spring 2019

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6.3 Latexed **0 / 1**

+ **1 pts** Correct

✓ **+ 0 pts Not typed**