CSCI 2824 Written HW 6

Alex Garcia-Gonzalez

TOTAL POINTS

47.5 / 51

QUESTION 1

Problem 1 - Sets 12 pts

1.1 Set Equality with subsets 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** Correct definitions - forward direction
+ **1 pts** Proved LHS is a subset of RHS
+ **1 pts** Correct definitions - backward direction
+ **1 pts** Proved RHS is a subset of LHS
+ **1 pts** Some correct work, mostly incorrect
+ **0 pts** Incorrect / No solution

1.2 Set Equality with builder notation 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** Set builder notation
+ **1 pts** Using and mentioning intersection
+ **1 pts** Using and mentioning Cartesian product
+ **1 pts** Showing LHS = RHS
+ **1 pts** Correct work, but no conclusion
+ **1 pts** Some correct work, but mostly incorrect
+ **0 pts** Incorrect / No solution

1.3 Difference/Union of Sets 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** (i) Correct
+ **1 pts** (ii) Correct
+ **1 pts** (iii) Correct
+ **1 pts** (iv) Correct
+ **0 pts** Incorrect / No solution

QUESTION 2

Problem 2 - Cardinality 12 pts

2.1 Examples 4 / 4

✓ + **4 pts** **Correct Answer**

+ **1 pts** Choosing A, B as uncountable
+ **1 pts** i correct
+ **1 pts** ii correct
+ **1 pts** iii correct
+ **0 pts** Incorrect

2.2 Cantor diagonal argument 4 / 4

✓ + **4 pts** **Correct answer**

+ **0 pts** Incorrect answer
+ **3 pts** Partially correct answer

2.3 Show irrationals uncountable 4 / 4

✓ + **4 pts** **Correct answer**

+ **1 pts** For assuming irrationals as countable
+ **2 pts** For recognizing assumption implies R as countable
+ **1 pts** Concluding irrationals are indeed uncountable
+ **0 pts** Incorrect/ Not attempted

QUESTION 3

Recurrence Relations 9 pts

3.1 Closed form 1 1.5 / 3

+ **3 pts** Fully correct
+ **1 pts** Identified a pattern
+ **2 pts** Used recurrence
+ **0 pts** No answer provided/ incorrect answer
+ **1 pts** Only answer provided
+ **1.5 pts** Partially correct

✓ + **1.5 pts** **Missing steps that lead to the answer**

+ **1 pts** For attempting

3.2 Closed form 2 2 / 3

+ **3 pts** All correct

✓ + **2 pts** **Recurrence found**

+ **1 pts** Pattern found

+ **0 pts** Answer not provided/Incorrect
+ **1 pts** Only answer provided
+ **1.5 pts** Missing steps that lead to the answer
+ **1 pts** Partially correct answer

3.3 Proof of Solution 3 / 3

✓ + **3 pts** **Correct**
+ **0 pts** Incorrect answer/answer not provided/incorrect answer mapped to the question
+ **1 pts** Plugging in closed form
+ **2 pts** For simplification to a subscript n
+ **0.5 pts** For attempting
+ **1 pts** Proving by cases (not a preferred answer)
+ **1.5 pts** Partially correct
+ **2 pts** All steps correct, final answer incorrect

QUESTION 4

One-to-one and Onto 12 pts

4.1 Onto 1 3 / 3

✓ + **3 pts** **Correct**
+ **3 pts** Correct but explaination not given.
+ **1 pts** Wrong answer but given some reasonable explaination
+ **0 pts** Wrong
+ **0 pts** No solution

4.2 Onto 2 3 / 3

✓ + **3 pts** **Correct**
+ **3 pts** Correct but explanation not provided
+ **1 pts** Incorrect but some reasonable explanation given
+ **0 pts** Incorrect
+ **0 pts** No solution

4.3 Example function 1 3 / 3

✓ + **3 pts** **Correct**
+ **1 pts** domain range used correctly
+ **2 pts** Function
+ **0 pts** Wrong
+ **0 pts** No solution

4.4 Example function 2 3 / 3

✓ + **3 pts** **Correct**
+ **1 pts** not oneone but made some progress
+ **0 pts** Wrong
+ **0 pts** No solution

QUESTION 5

5 Style Points 5 / 6

✓ + **5 pts** **Not typed but everything else is fine**
+ **6 pts** Correct
+ **2 pts** Completeness
+ **3 pts** Neatness
+ **1 pts** typed

1. (a) Suppose P, Q, and R are non-empty sets. Prove that $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$ by showing that each side of this equation must be a **subset** of the other side, and concluding that the two sides must therefore be equal.

(b) Suppose that P, Q, and R are non-empty sets. Prove that $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$ by using **set builder notation** and set identities and definitions.

(c) Let U be the set of all integers. Let E be the set of all even integers, D the set of all odd integers, P the set of positive integers, and N the set of all negative integers. Find the following sets.

i. $E - P$

ii. $P \cup N$

iii. $D - E$

iv. $\bar{U}$

A. $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$

Suppose $(x, y) \in (P \times Q) \cap (P \times R)$

$(x, y) \in (P \times Q) \wedge (P \times R)$

x: (P)
y: (Q, R)

now lets assign X & Y elements

$(x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)$

now we see they have 'P' in common.... lets simplify!

$x \in P \wedge (y \in Q \wedge y \in R)$

$(x, y) \in P \times (Q \times R)$

$(P \times Q) \cap (P \times R) \subseteq P \times (Q \times R)$

$A \subseteq B$

suppose $(x, y) \in P \times (Q \cap R)$

$x \in P \wedge ((y \in Q) \wedge (y \in R))$

$(x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)$

$(x, y) \in P \times Q \wedge (x, y) \in P \times R$

$(x, y) \in (P \times Q) \cap (P \times R)$

$P \times (Q \cap R) \subseteq (P \times Q) \cap (P \times R)$.

$B \subseteq A$

$A = (P \times Q) \cap (P \times R)$
$B = P \times (Q \cap R)$

we want $A \subseteq B$, $B \subseteq A$ to prove . . .

now we have proved they are subsets of eachother Because the LHS = RHS.

B.

$P \times (Q \cap R) = \{(x, y) \mid x \in P \wedge y \in (Q \cap R)\}$

$\{(x, y) \mid x \in P \wedge ((y \in Q) \wedge (y \in R))\}$

$\{(x, y) \mid (x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)\}$

$P \times (Q \cap R) = (P \times Q) \cap (P \times R)$

1.1 Set Equality with subsets 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** Correct definitions - forward direction
+ **1 pts** Proved LHS is a subset of RHS
+ **1 pts** Correct definitions - backward direction
+ **1 pts** Proved RHS is a subset of LHS
+ **1 pts** Some correct work, mostly incorrect
+ **0 pts** Incorrect / No solution

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(b) Suppose that P, Q, and R are non-empty sets. Prove that $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$ by using **set builder notation** and set identities and definitions.

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iv. $\bar{U}$

A. $P \times (Q \wedge R) = (P \times Q) \cap (P \times R)$

Suppose $(x, y) \in (P \times Q) \cap (P \times R)$

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$x : (P)$
$y : (Q, R)$

now lets assign X&Y elements

$(x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)$

now we see they have 'P' in common.... lets simplify!

$x \in P \wedge (y \in Q \wedge y \in R)$

$(x, y) \in P \times (Q \times R)$

$(P \times Q) \cap (P \times R) \subseteq P \times (Q \times R)$

$A \subseteq B$

suppose $(x, y) \in P \times (Q \cap R)$

$x \in P \wedge ((y \in Q) \wedge (y \in R))$

$(x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)$

$(x, y) \in P \times Q \wedge (x, y) \in P \times R$

$(x, y) \in (P \times Q) \cap (P \times R)$

$P \times (Q \cap R) \subseteq (P \times Q) \cap (P \times R)$.

$B \subseteq A$

$A = (P \times Q) \cap (P \times R)$
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now we have proved they are subsets of eachother Because the LHS = RHS.

B.

$P \times (Q \wedge R) = \{(x, y) \mid x \in P \wedge y \in (Q \wedge R)\}$

$\{(x, y) \mid x \in P \wedge ((y \in Q) \wedge (y \in R))\}$

$\{(x, y) \mid (x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)\}$

$P \times (Q \wedge R) = (P \times Q) \cap (P \times R)$

1.2 Set Equality with builder notation 4 / 4

- ✓ + **4 pts** **Correct**
- + **1 pts** Set builder notation
- + **1 pts** Using and mentioning intersection
- + **1 pts** Using and mentioning Cartesian product
- + **1 pts** Showing LHS = RHS
- + **1 pts** Correct work, but no conclusion
- + **1 pts** Some correct work, but mostly incorrect
- + **0 pts** Incorrect / No solution

C.

U: set of all integers
E: all even integers
D: all odd integers
P: positive integers
N: all negative integers

i: $E - P$ = all negative integers.

ii: $P \cup N$ = "U" set of all positive and negative integers.

iii: $D - E = D$ all odd integers

iiii: $\bar{U}$ = " all numbers except integers"

2. (a) Give an example of two uncountable sets A and B with a nonempty intersection, such that $A - B$ is

   i. finite

   ii. countably infinite

   iii. uncountably infinite

   (b) Use the Cantor diagonalization argument to prove that the number of real numbers in the interval $[3, 4]$ is uncountable.

   (c) Use a proof by contradiction to show that the set of irrational numbers that lie in the interval $[3, 4]$ is uncountable. (You can use the fact that the set of rational numbers ($\mathbb{Q}$) is countable and the set of reals ($\mathbb{R}$) is uncountable). Show all work.

a.

i: Finite A: $[0,1]$
B: $(0,1]$
only one number remains.

ii: countably infinite
A: $\mathbb{R}$
B: $\mathbb{R} \setminus \mathbb{Q}$ - set of all irrational numbers
$A - B = \mathbb{R} - (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{Q}$ = set of all rational numbers
Countably infinite

iii: $A = [0,2]$
$B = [0,1]$
$A - B$ = irrational

b. Suppose $f: N \to [3,4]$ is any function
we make expansion table of $f(1), f(2), \dots f(n)$
S'pose $f(1) = 3 + \frac{n}{10}$, $f(2) = 2 + \frac{37}{99}$,
$f(3) = 3 + \frac{1}{7}$, ... $f(n)$

n	f(n)
1	3 . 3 1 4 1 5 9 2 653 ...
2	3 . 3 7 3 7 3 3 7 3 77 ...
3	3 . 1 4 2 8 5 7 1 428 ...
4	3 . 7 0 7 1 0 6 7 8 11 ...
5	3 . 3 7 5 0 0 0 0 0 0 ...

once we add 1 to decmil digits, it no longe can be in the table. differs
(1) in first digit
(2) in second digit
(n) differs in it's nth digit.

therefore there are a lot of numbers not in table & n for any n in the table

1.3 Difference/Union of Sets 4 / 4

- ✓ **+ 4 pts** **Correct**
- **+ 1 pts** (i) Correct
- **+ 1 pts** (ii) Correct
- **+ 1 pts** (iii) Correct
- **+ 1 pts** (iv) Correct
- **+ 0 pts** Incorrect / No solution

C.

U: set of all integers
E: all even integers
D: all odd integers
P: positive integers
N: all negative integers

i: E − P = all negative integers.

ii: P ∪ N = "U" set of all positive and negative integers.

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a.

i: Finite A: [0,1]
B: (0,1]
only one number remains.

ii: countably infinite
A: R
B: R\Q - set of all irrational numbers
A−B = R−(R\Q) = Q = set of all rational numbers
Countably infinite

iii: A = [0,2]
B = [0,1]
A−B = irrational

b. Suppose f: N → [3,4] is any function
we make expansion table of f(1), f(2), ... f(n)

S'pose $f(1) = 3 + \frac{n}{10}$, $f(2) = 2 + \frac{37}{99}$, $f(3) = 3 + \frac{1}{7}$, ... f(n)

n	f(n)
1	3 . 3 1 4 1 5 9 2 6 5 3 ...
2	3 . 3 7 3 7 3 3 7 3 7 7 ...
3	3 . 1 4 2 8 5 7 1 4 2 8 ...
4	3 . 7 0 7 1 0 6 7 8 1 1 ...
5	3 . 3 7 5 0 0 0 0 0 0 ...

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therefore there are a lot of numbers not in table & n for any n in the table

2.1 Examples 4 / 4

✓ + **4 pts** **Correct Answer**

+ **1 pts** Choosing A, B as uncountable
+ **1 pts** i correct
+ **1 pts** ii correct
+ **1 pts** iii correct
+ **0 pts** Incorrect

C.

U: set of all integers
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$A - B = \mathbb{R} - (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{Q}$ = set of all rational numbers
Countably infinite

iii: A = [0,2]
B = [0,1]
A - B = irrational

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3	3 . 1 4 2 8 5 7 1 4 2 8 ...
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(1) in first digit
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therefore there are a lot of numbers not in table & n for any n in the table

2.2 Cantor diagonal argument 4 / 4

- ✓ + **4 pts** **Correct answer**
- + **0 pts** Incorrect answer
- + **3 pts** Partially correct answer

C. c. A = irrationals [3,4]
B = rational in [3,4]
$A \cup B = [3,4]$

If we assume A is countable, B is countable, & union of 2 countable sets is countable

thus [3,4] is countable

However cantors diagonal argument → ←
This is a contradiction of last example.
The irrationals in [3,4] are uncountable

3. (a) Find a closed form for the recurrence relation: $a_n = 2a_{n-1} - 2, a_0 = -1$
(b) Find a closed form for the recurrence relation: $a_n = (n+2)a_{n-1}, a_0 = 3$
(c) Show that $a_n = 5(-1)^n - n + 2$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$.

a.) $a_0 = -1$

$a_1 = 2(a_0) - 2 = 2(-1) - 2 = 0$

$a_2 = 2(a_1) - 2 = 2(0) - 2 = -2$

$a_3 = 2(a_2) - 2 = 2(-2) - 2 = -6$

$\vdots$

$a_n = 2(a_{n-1}) - 2 = 2(a_0)^{-2} =$

Thus the closed form recurrene relationship is $a_n = 2 - 3 \quad 2^{(n)}$

b.) $a_0 = 3$

$a_1 = (1+2)a_0 = (3)(3) = 9$

$a_2 = (2+2)a_1 = (4)(9) = 36$

$a_3 = (3+2)a_2 = (5)(36) = 180$

$\vdots$

$a_n = (n+2)a_{n-1} =$

Thus the closed form recurrene relationshi is $a_n = -n + 9$

$2^{(n-1)} - 2$

2.3 Show irrationals uncountable 4 / 4

✓ + **4 pts** **Correct answer**

+ **1 pts** For assuming irrationals as countable
+ **2 pts** For recognizing assumption implies R as countable
+ **1 pts** Concluding irrationals are indeed uncountable
+ **0 pts** Incorrect/ Not attempted

C. c. A = irrationals [3,4]
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$\vdots$
$a_n = 2(a_{n-1}) - 2 = 2(a_0)^{-2} =$

Thus the closed form recurrene relationship is $a_n = 2 - 3 \; 2^{(n)}$

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Thus the closed form recurrene relationshi is $a_n = -n + 9 \; 2^{(n-1)} - 2$

3.1 Closed form 1 **1.5 / 3**

- \+ **3 pts** Fully correct
- \+ **1 pts** Identified a pattern
- \+ **2 pts** Used recurrence
- \+ **0 pts** No answer provided/ incorrect answer
- \+ **1 pts** Only answer provided
- \+ **1.5 pts** Partially correct
- ✓ + **1.5 pts** **Missing steps that lead to the answer**
- \+ **1 pts** For attempting

C. c. A = irrationals [3,4]
B = rational in [3,4]
$A \cup B = [3,4]$

If we assume A is countable, B is countable, & union of 2 countable sets is countable

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3. (a) Find a closed form for the recurrence relation: $a_n = 2a_{n-1} - 2, a_0 = -1$
   (b) Find a closed form for the recurrence relation: $a_n = (n+2)a_{n-1}, a_0 = 3$
   (c) Show that $a_n = 5(-1)^n - n + 2$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$.

a.) $a_0 = -1$

$a_1 = 2(a_0) - 2 = 2(-1) - 2 = 0$

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$a_3 = 2(a_2) - 2 = 2(-2) - 2 = -6$

$\vdots$

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Thus the closed form recurrene relationship is $a_n = 2 - 3 \; 2^{(n)}$

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$a_2 = (2+2)a_1 = (4)(9) = 36$

$a_3 = (3+2)a_2 = (5)(36) = 180$

$\vdots$

$a_n = (n+2)a_{n-1} =$

Thus the closed form recurrene relationshi is $a_n = -n + 9 \; 2^{(n-1)} - 2$

3.2 Closed form 2 **2 / 3**

- \+ **3 pts** All correct
- ✓ + **2 pts** **Recurrence found**
- \+ **1 pts** Pattern found
- \+ **0 pts** Answer not provided/Incorrect
- \+ **1 pts** Only answer provided
- \+ **1.5 pts** Missing steps that lead to the answer
- \+ **1 pts** Partially correct answer

c.)

$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ recurrence relation

$a_n = 5 \cdot (-1)^n - n + 2$ closed form solution

$\Rightarrow a_{n-1} + 2a_{n-2} + 2n - 9$

$= 5 \cdot (-1)^{n-1} - (n-1) + 2 + 2(5 \cdot (-1)^{n-2} - (n-2) + 2) + 2n - 9$

$= 5 \cdot (-1)^{n-1} - n - 1 + 2 + 10\,(-1)^{n-2} - \cancel{2(n}-2) + \cancel{4} + 2n - 9$

$\cancel{-2n} \quad \cancel{-4}$

-1

$= 5 \cdot (-1)^{n-1} + 10 \cdot (-1)^{n-2} - n + 2$

$= -5 \cdot (-1)^n + 10\,(-1)^n - n + 2$

$= 5 \cdot (-1)^n - n + 2$

$= a_n$

3.3 Proof of Solution 3 / 3

✓ **+ 3 pts** **Correct**

+ 0 pts Incorrect answer/answer not provided/incorrect answer mapped to the question

+ 1 pts Plugging in closed form

+ 2 pts For simplification to a subscript n

+ 0.5 pts For attempting

+ 1 pts Proving by cases (not a preferred answer)

+ 1.5 pts Partially correct

+ 2 pts All steps correct, final answer incorrect

4. (a) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m, n) = 2m - n$. Is this function onto?

(b) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m, n) = m^2 - n^2$. Is this function onto?

(c) Define the set C = the set of all residents of Colorado. Define in words a function $f : C \to \mathbb{Z}$. Is your function one-to-one? Is it onto? Be sure that the f you defined is indeed a **function**. Be creative and have fun!

(d) Again, define the set C = the set of all residents of Colorado. Define in words a function $f : C \to \mathbb{Z}$. However this time, make sure that your function is one-to-one. (Make sure to give a different example from part (c)).

a.) $f(m,n) = 2m - n$ $f: \mathbb{Z} \times \mathbb{Z} = \mathbb{Z}$

f is onto

Proof: $K \in \mathbb{Z}$

find two integers such that $f(m,n) = K$

$2m - n = K$

$n = 2m - K$

& If $n = 2m - K$

then $f(m,n) = f(m, 2m - K)$

$= K$ $\quad f(m,n) = 2m - n$

hence for every $K \in \mathbb{Z}$, we obtained $(m, 2m-K) \in \mathbb{Z} \times \mathbb{Z}$ such as

$f(m, 2m - K) = K$

thus f is onto

b.) $m^2 - n^2$ Proof: $y \in \mathbb{Z}$

let $f(m,n) = y$

If $y = 6$, then 6 couldn't be written as the difference of squares of two integers.

forexample:

$6 \neq 3^2 - 1^2$

$6 \neq 4^2 - 2^2$

$6 \neq 4^2 - 3^2$

$\vdots$

Hence f is not onto

4.1 Onto 1 3 / 3

✓ + **3 pts** Correct

+ **3 pts** Correct but explaination not given.

+ **1 pts** Wrong answer but given some reasonable explaination

+ **0 pts** Wrong

+ **0 pts** No solution

4. (a) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m, n) = 2m - n$. Is this function onto?

(b) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m, n) = m^2 - n^2$. Is this function onto?

(c) Define the set C = the set of all residents of Colorado. Define in words a function $f : C \to \mathbb{Z}$. Is your function one-to-one? Is it onto? Be sure that the f you defined is indeed a **function**. Be creative and have fun!

(d) Again, define the set C = the set of all residents of Colorado. Define in words a function $f : C \to \mathbb{Z}$. However this time, make sure that your function is one-to-one. (Make sure to give a different example from part (c)).

a.) $f(m,n) = 2m-n$ $f: \mathbb{Z} \times \mathbb{Z} = \mathbb{Z}$

f is onto

Proof: $k \in \mathbb{Z}$

Find two integers such that $f(m,n) = k$

$2m - n = k$
$n = 2m - k$

& If $n = 2m-k$

then $f(m,n) = f(m, 2m-k)$
$= k$ $f(m,n) = 2m-n$

hence for every $k \in \mathbb{Z}$, we obtained $(m, 2m-k) \in \mathbb{Z} \times \mathbb{Z}$

such as

$f(m, 2m-k) = k$

thus f is onto

b.) $m^2 - n^2$ Proof: $y \in \mathbb{Z}$

let $f(m,n) = y$

If $y = 6$, then 6 couldn't be written as the difference of squares of two integers.

for example:

$6 \neq 3^2 - 1^2$
$6 \neq 4^2 - 2^2$
$6 \neq 4^2 - 3^2$
$\vdots$

Hence f is not onto

4.2 Onto 2 3 / 3

- ✓ **+ 3 pts** Correct
- **+ 3 pts** Correct but explanation not provided
- **+ 1 pts** Incorrect but some reasonable explanation given
- **+ 0 pts** Incorrect
- **+ 0 pts** No solution

c.) C = set of all residents in Colorado.

$C = \{1, 2 \ldots\ldots\}$

$f: C \to \mathbb{Z}$ by $f(x) = x$ $\forall x \in C$

associate each resident with a number $\mathbb{Z}$.

This becomes an identity map which is one-to-one as each resident is associated with a unique #.

f is not onto due to set C being finite & set $\mathbb{Z}$ is infinite, so there maybe $\in$ set $\mathbb{Z}$ which has no association with any resident.

d. Define

$f: C \to \mathbb{Z}$ by

$f(x) = x^2$ $\forall$ $x \in C$

If we associate each People is a number which is square of the ith number of person that is: 5th person will be associated is 25

Hence one to one as every person is associated is a unique #.

It is not onto as $7 \in \mathbb{Z}$ has no PreImage.

4.3 Example function 1 **3 / 3**

- ✓ **+ 3 pts** **Correct**
- **+ 1 pts** domain range used correctly
- **+ 2 pts** Function
- **+ 0 pts** Wrong
- **+ 0 pts** No solution

c.) C = set of all residents in Colorado.

$C = \{1, 2 \ldots \}$

$f: C \to \mathbb{Z}$ by $f(x) = x$ $\forall x \in C$

associate each resident with a number $\mathbb{Z}$.

This becomes an identity map which is one-to-one as each resident is associated with a unique #.

f is not onto due to set C being finite & set $\mathbb{Z}$ is infinite, so there maybe $\in$ set $\mathbb{Z}$ which has no association with any resident.

d. Define

$f: C \to \mathbb{Z}$ by

$f(x) = x^2 \; \forall \; x \in C$

If we associate each People is a number which is square of the ith number of person that is: 5th person will be associated is 25

Hence one to one as every person is associated is a unique #.

It is not onto as $7 \in \mathbb{Z}$ has no PreImage.

4.4 Example function 2 3 / 3

- ✓ + **3 pts** **Correct**
- + **1 pts** not oneone but made some progress
- + **0 pts** Wrong
- + **0 pts** No solution

5 Style Points 5 / 6

✓ + **5 pts** **Not typed but everything else is fine**

+ **6 pts** Correct
+ **2 pts** Completeness
+ **3 pts** Neatness
+ **1 pts** typed