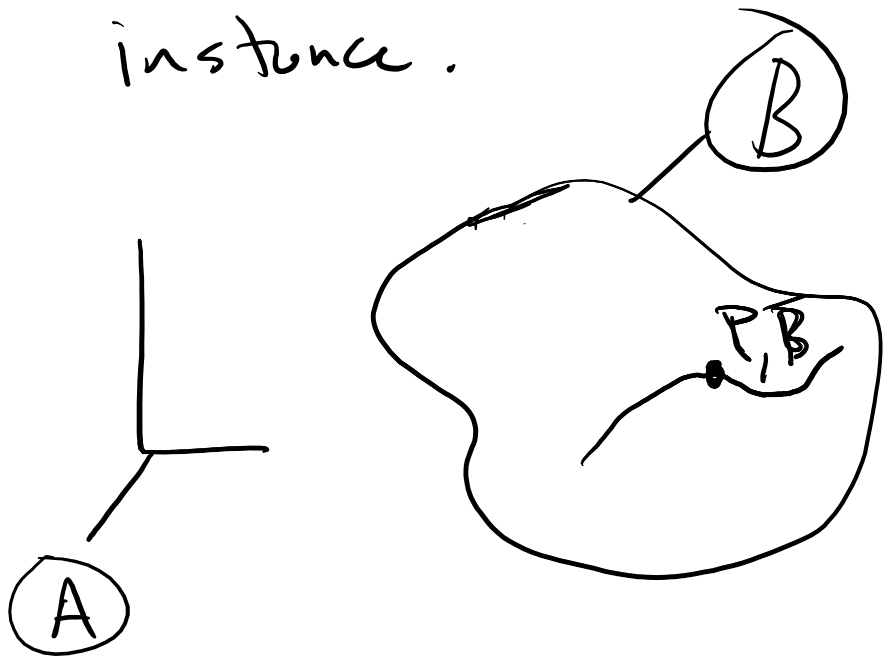
# MAE223-L6-01

Monday, October 16, 2017 9:58 AM

One point P moving in RF/RB (B) while (B) moves in another RF (A). Let $\bar{B}$ be a point ~~fixed~~ fixed in (B) that coincides with P at a particular instance.

$$\boxed{{}^A\bar{V}^P = {}^B\bar{V}^P + {}^A\bar{V}^{\bar{B}}}$$




$${}^B\bar{V}^{\bar{B}} = 0$$

Acceleration

$$\boxed{{}^A\bar{a}^P = {}^B\bar{a}^P + {}^A\bar{a}^{\bar{B}} + \underbrace{2\,{}^A\bar{\omega}^B \times {}^B\bar{V}^P}_{\text{Coriolis acceleration term}}}$$

relative acc

$\bar{\omega}\times(\bar{\omega}\times\bar{r})$
${}^A\alpha^{\bar{B}}$
${}^B\bar{a}^{\bar{B}}$

State variables of a set $S$ of $\nu$ particles $P_i$ $i = 1, 2, \ldots, \nu$ in a reference frame $A$ consists of two parts:

a) configuration of $S$ in $A$ (where they are in $A$)

b) motion of $S$ in $A$ (how they are moving in $A$)

Fundmental question: "How does the state change with time?

Configuratin - charactized by position vectors of each partich

Motion - charactized by velocity vectors of each partich

When motion is unconstrained we need $2\nu$ vectors or $6\nu$ measure numbers.

Constraints reduce the number of variables.

Configuration variables are called "generalized coordinates of $S$ in $A$".

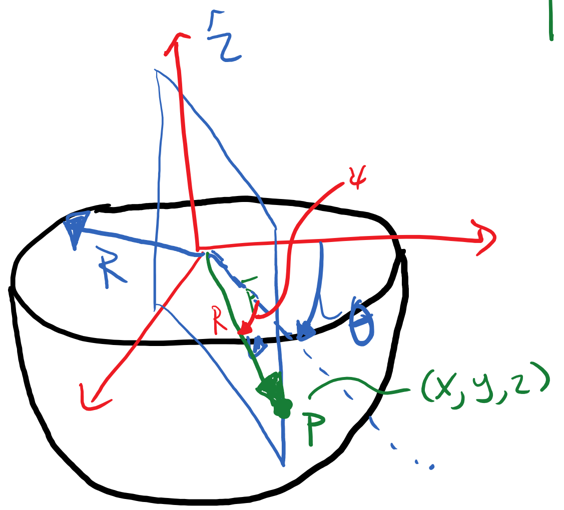
Motion variables are called "generalized speeds of $S$ in $A$"

Both the GC's and GS's are functions of time and both can be chosen in an infinite # of ways.

ex hemispherical bowl of radius R
with single particle moving on
surface.

$|\bar{p}| = R = x^2+y^2+z^2$ configuration constraint

$$\boxed{x^2+y^2+z^2-R=0}$$




GC's : x, y, z

alt. G.C's ?? $(\theta,\psi)$

$\psi < 0 \Rightarrow$ a config constraint

Configuration constraint equations that can be written as follows:

$$f(x_1, y_1, z_1, \dots, x_v, y_v, z_v, t) = 0$$

no velocity terms!

holonomic constraint eq

functions of positions and time

Holomic constraint types:

a) Rheonomic: when time is explicit in the constraint

b) Scleronomic: when time is implicit

Not only is $f(..)=0$ but $\frac{df}{dt}=0$

$$f = x^2+y^2+z^2-R=0$$

$$\frac{df}{dt} = \boxed{x\dot{x}+y\dot{y}+z\dot{z}=0}$$

this is "new" version of the config constraint that includes velocities.

if integrable this is a redundant way of stating $f=0$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial t}$$

to be integrable

$$\frac{\partial f^2}{\partial x \partial y} = \frac{\partial f^2}{\partial y \partial x} \Rightarrow \text{mixed partials must commute}$$

$$\left.\begin{array}{l} \frac{\partial f}{\partial x} = x \Rightarrow \frac{\partial f^2}{\partial y \partial x} = 1 \\ \frac{\partial f}{\partial y} = y \Rightarrow \frac{\partial f^2}{\partial x \partial y} = 1 \end{array}\right\} \text{equal!}$$

# MAE223-L6-04

Monday, October 16, 2017 11:29 AM

"Intelligent" choices of G.C.s and G.S.s are part of "art" of dynamics.

How do we choose these?

Every system has a minimum, $n$, of G.C.'s required to specify the configuration of the system uniquely.

All $n$ coordinates must be independent to be the minimal set, i.e. there are no holonomic constraints if $n$ is minimal.

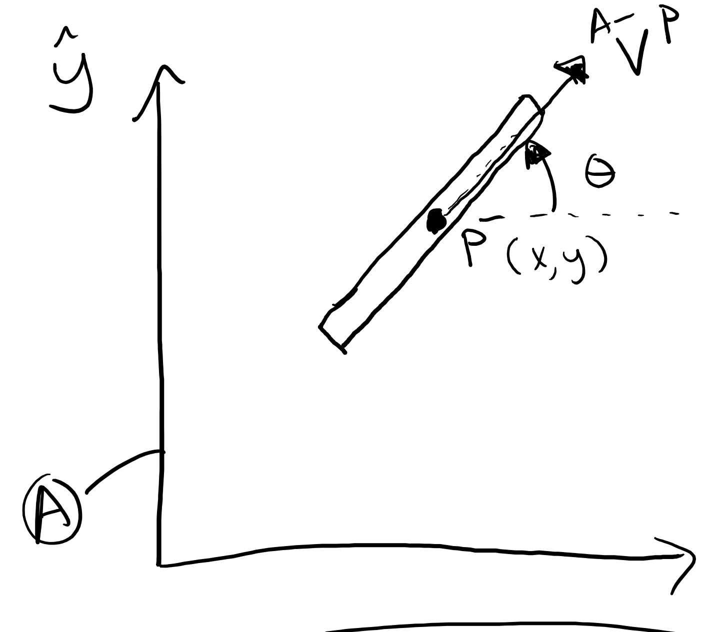
All other constraints are called <u>nonholonomic</u> <u>constraints</u>.

Nonholonomic constraints must involve velocities but are not able to be integrated to remove the velocity dependence.

NH constraints are essential velocity constraints.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

<u>ex</u> ice skate blade that can slide only along its length




3 G.C.'s $x, y, \theta$

$\dot{x}, \dot{y}$ are measure numbers of ${}^A\bar{V}^P$

constraint

$$\boxed{\tan\theta = \frac{\dot{y}}{\dot{x}}} \rightarrow \text{non holonomic!}$$

$$f(x,y,\theta,t) = \boxed{\dot{x}\tan\theta - \dot{y} = 0}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial \theta}\frac{d\theta}{dt} + \frac{\partial f}{\partial t}$$

is it integrable?

$$\frac{\partial f}{\partial x} = \tan\theta \qquad \frac{\partial f}{\partial y} = -1 \qquad \frac{\partial f}{\partial \theta} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \qquad \frac{\partial^2 f}{\partial x \partial \theta} \stackrel{?}{=} \frac{\partial^2 f}{\partial \theta \partial x}$$

$$0 = 0 \ \checkmark \qquad 0 \neq \sec^2\theta \ \times$$