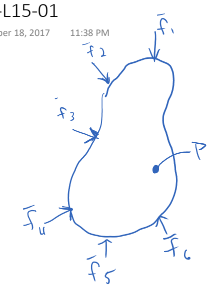
MAE223-L15-01

Saturday, November 18, 2017 11:38 PM

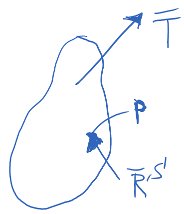


Set S of bound vectors

$S = \{\bar f_1, \bar f_2, \bar f_3, \bar f_4, \bar f_5, \bar f_6\}$

S has a resultant $\bar R^S$ and a moment about point P, $\bar M^{S/P}$

Set S can be replaced by a torque of a couple equal to $\bar M^{S/P}$ and a force at P that is equal to $\bar R^S$.



$\bar R^S$ acts at P but $\bar T^S$ is not bound because torques are the same value about every point

$$\tilde F_r = F_r + \sum_{s=p+1}^{n} A_{rs} F_s \qquad r=1,\dots,p$$

rth nonholonomic GAF (associated with independent speeds)

rth holonomic GAF (for each ind. speed)

sth holonomic GAF (associated with dep speeds)

sums through all dependent speeds

same

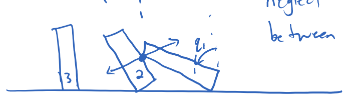
$$u_r = \sum_{s=1}^{p} A_{rs} u_s + B_r \qquad r=p+1,\dots,n$$

MAE223-L15-02

Saturday, November 18, 2017 11:50 PM

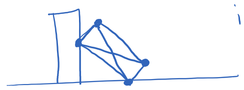
Non-contributing forces

a. contact forces on particles across smooth (frictionless) surfaces of rigid bodies



neglect contact forces between 1 and 2

b. any internal contact and body (distance) forces between any two points in a rigid body B in S



Ex: tensions and compressions in the strings that maintain the rigidity of this "quasi domino"

c. special case of a): rolling without slip
neglect all contact forces between rolling bodies

Generalized Active Forces for rigid bodies

Suppose S is comprised of one or more rigid bodies in addition to particles.

B is a rigid body in a nonholonomic system S with p DoF in RF A.
B has contact and/or distance forces that act on it which are equivalent to a couple of torque $\overline{T}$ together with resultant force $\overline{R}$ whose line of action passes through point Q of B.

$$\left(\tilde{F}_r\right)_B = {}^A\tilde{V}_r^{Q} \cdot \overline{R} + {}^A\tilde{\omega}_r^{B} \cdot \overline{T} \qquad r = 1, \ldots, p$$

Q can be mass center but doesn't have to be.

MAE223-L15-03

Saturday, November 18, 2017 11:56 PM

Generalized Inertia Forces

Holonomic and nonholonomic GIFs

Let $u_1, \dots, u_n$ GSs for S with p DoF in RFA.

If S contains ν particles:

$$\bar{F}_r^* \triangleq \sum_{i=1}^{\nu} {}^A\bar{V}_r^{P_i} \cdot \bar{R}_i^* \quad \text{where } \bar{R}_i^* = -m\bar{a}_i \qquad r = 1, \dots, n$$

inertial force

$\bar{a}_i$ → accel of P_i in A

${}^A\bar{V}_r^{P_i}$ → rth partial vel. of P_i in A

$\bar{F}_r^*$ ← rth holonomic GIF

Nonholonomic:

$$\tilde{F}_r^* = \sum_{i=1}^{\nu} {}^A\tilde{V}_r^{P_i} \cdot \bar{R}_i^* \qquad \tilde{F}_r^* = F_r^* + \sum_{s=p+1}^{n} F_s^* A_{sr}$$

For Rigid bodies:

$$(\tilde{F}_r^*)_B = \bar{\omega}_r \cdot \bar{T}^* + V_r^{B_0} \cdot \bar{R}^*$$

$\bar{\omega}_r$: rth nonholonomic angular velocity of RB B in A

$V_r^{B_0}$: rth nonholonomic partial velocity of B_0 (mass center of B)

$$\bar{T}^* = -\sum_{i=1}^{\beta} m_i \bar{r}_i \times \bar{a}_i$$

β: # particles comprising B

m_i: mass of ith particle

$\bar{r}_i$: position vector from B_0 to P_i

$\bar{a}_i$: acc. of P_i in A

$$\bar{R}^* = -M\bar{a}^{B_0} \qquad M = \sum_{i=1}^{\beta} m_i = \text{total mass}$$

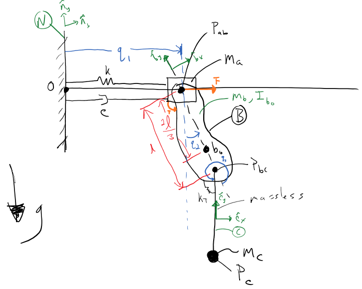
$\bar{a}^{B_0}$: acc. of B_0 in A

For distributed mass:

$$\bar{T}^* = -(\bar{\alpha}^B \cdot \bar{I}_{B_0} + \bar{\omega}^B \times \bar{I}_{B_0} \cdot \bar{\omega}^B)$$

$\bar{I}_{B_0}$: central moment of inertia of B

$\bar{\alpha}^B$: ang. acc. of B in A $\quad \bar{\omega}^B$ ang acc of B in A



G.Cs q_1, q_2, q_3 $n=3$ } holonomic

G.Ss u_1, u_2, u_3 $p=3$ # DoF

Particles: P_{ab}, P_c

RB: B (b_o, m_b, I_{b_o})

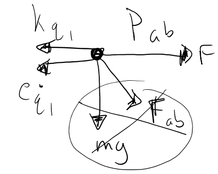
Loads (forces or torques):

weight of B, P_c

T, F specified forces, torques

translational spring and damper

rotational spring



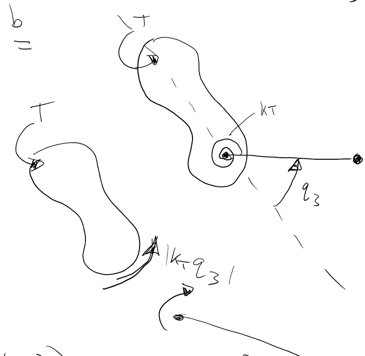
$$\bar{R}_{P_{ab}} = (F - kq_1 - c\dot{q}_1)\hat{n}_x$$

$$\bar{R}_{P_c} = -(m_c g)\hat{n}_y$$

$$\bar{R}_{b_o} = -m_b g \hat{n}_y$$

$$\bar{T}^b = T + k_T q_3$$

$$\bar{T}^b =$$



$${}^N F_r = \underbrace{{}^N\bar{V}_r^{P_{ab}} \cdot \bar{R}_{P_{ab}}}_{P_{ab}} + \underbrace{{}^N\bar{V}_r^{P_c} \cdot \bar{R}_{P_c}}_{P_c} + \underbrace{{}^N\bar{V}^{b_o} \cdot \bar{R}_{B_o} + {}^N\bar{\omega}_R^B \cdot \bar{T}^B}_{B} \qquad r = 1, \ldots, 3$$

$${}^N F_r^* = \underbrace{{}^N\bar{V}_r^{P_{ab}} \cdot -m_a {}^N\bar{a}^{P_{ab}}}_{P_{ab}} + \underbrace{{}^N\bar{V}_r^{P_c} \cdot -m_c {}^N a}_{P_c} + {}^N\bar{V}_r^{b_o} \cdot -m_b {}^N\bar{a}^{b_o} + {}^N\omega_r \cdot \left(-{}^N\bar{\alpha}^B \cdot \bar{I}_{B_o} + {}^N\bar{\omega}^B \times \bar{I}_{B_o} \cdot {}^N\bar{\omega}^B\right) \qquad r = 1, \ldots, 3$$