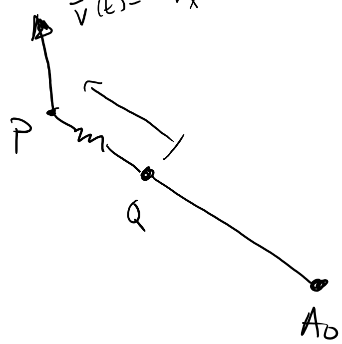
$\bar{v}(t) = v_x(t)\,\hat{n}_x + v_y(t)\,\hat{n}_y$



Taylor Series Expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = \underbrace{f(a) + \frac{f'(a)}{1!}(x-a)^1}_{\text{linear}}$$

Multivariate

$$f(x,y,z) = f(a,b,c) + (x-a)\frac{\partial f(a,b,c)}{\partial x} + (y-b)\frac{\partial f(a,b,c)}{\partial y} + (z-c)\frac{\partial f(a,b,c)}{\partial z}$$

Jacobian

$$J_f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$

if $\bar{f} \in \mathbb{R}^2$

3×2

$$J_{\bar{f}} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial x} \\ \frac{\partial f_1}{\partial y} & \frac{\partial f_2}{\partial y} \\ \frac{\partial f_1}{\partial z} & \frac{\partial f_2}{\partial z} \end{bmatrix}$$

$$\bar{F}_r + F_r^* = 0 = \bar{f}(\bar{q}, \bar{u}, \dot{\bar{u}}, t)$$

Vector form of Taylor Series

Vector form of Taylor Series

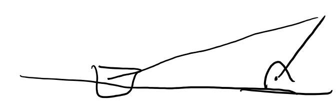
$$\bar f(x,y,z) = \bar f(a,b,c) + J_{\bar f}(a,b,c)\,(\bar v - \bar V_0)$$

$$\bar V = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \bar V_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

$$\bar f(\bar v) = \bar f(\bar V_0) + J_{\bar f}(\bar V_0)\,(\bar V - \bar V_0)$$

$$= \begin{bmatrix} f_1(x_0,y_0,z_0) \\ \vdots \\ f_n(x_0,y_0,z_0) \end{bmatrix} + \underset{n \times 3}{\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x} & & \frac{\partial f_n}{\partial z} \end{bmatrix}}\Bigg|_{\bar V_0 =} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

For the dynamics



$$\bar F_r + \bar F_r^* = 0 \qquad r = 1, \dots, p$$

$$J_{\bar f} \qquad \frac{\partial \bar f}{\partial \bar v}$$

$$\bar f(\bar q, \bar u, \dot{\bar u}) = 0$$

$$\bar V = \underset{(n+2p)\times 1}{\begin{bmatrix} \bar q \\ \bar u \\ \dot{\bar u} \end{bmatrix}} \qquad \bar V_0 = \begin{bmatrix} \bar q_0 \\ \bar u_0 \\ \dot{\bar u}_0 \end{bmatrix} = \begin{bmatrix} \bar q_0 \\ 0 \\ 0 \end{bmatrix}$$

$J_{\bar f}$ has size $p \times (n+2p)$