MAE223-L4-01

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Euler's Equations

$$\Sigma \bar{M}_c = \frac{{}^N d\bar{H}_c}{dt}$$

$\bar{H}_c$: central angular momentum

if the body is symmetric then the 3 principal moments of inertia $\Rightarrow I_1, I_2, I_3$

$\bar{H}_c$ expressed in the RF B which is fixed to RB (B)

$$\bar{H}_c = I_1 \omega_1 \hat{b}_1 + I_2 \omega_2 \hat{b}_2 + I_3 \omega_3 \hat{b}_3$$

$${}^N\bar{\omega}^B = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3$$

inertial frame

$\hat{b}_1, \hat{b}_2, \hat{b}_3 \Rightarrow$ principal axes

$$\frac{{}^N d\bar{H}_c}{dt} = \frac{{}^B d\bar{H}_c}{dt} + {}^N\bar{\omega}^B \times \bar{H}_c$$

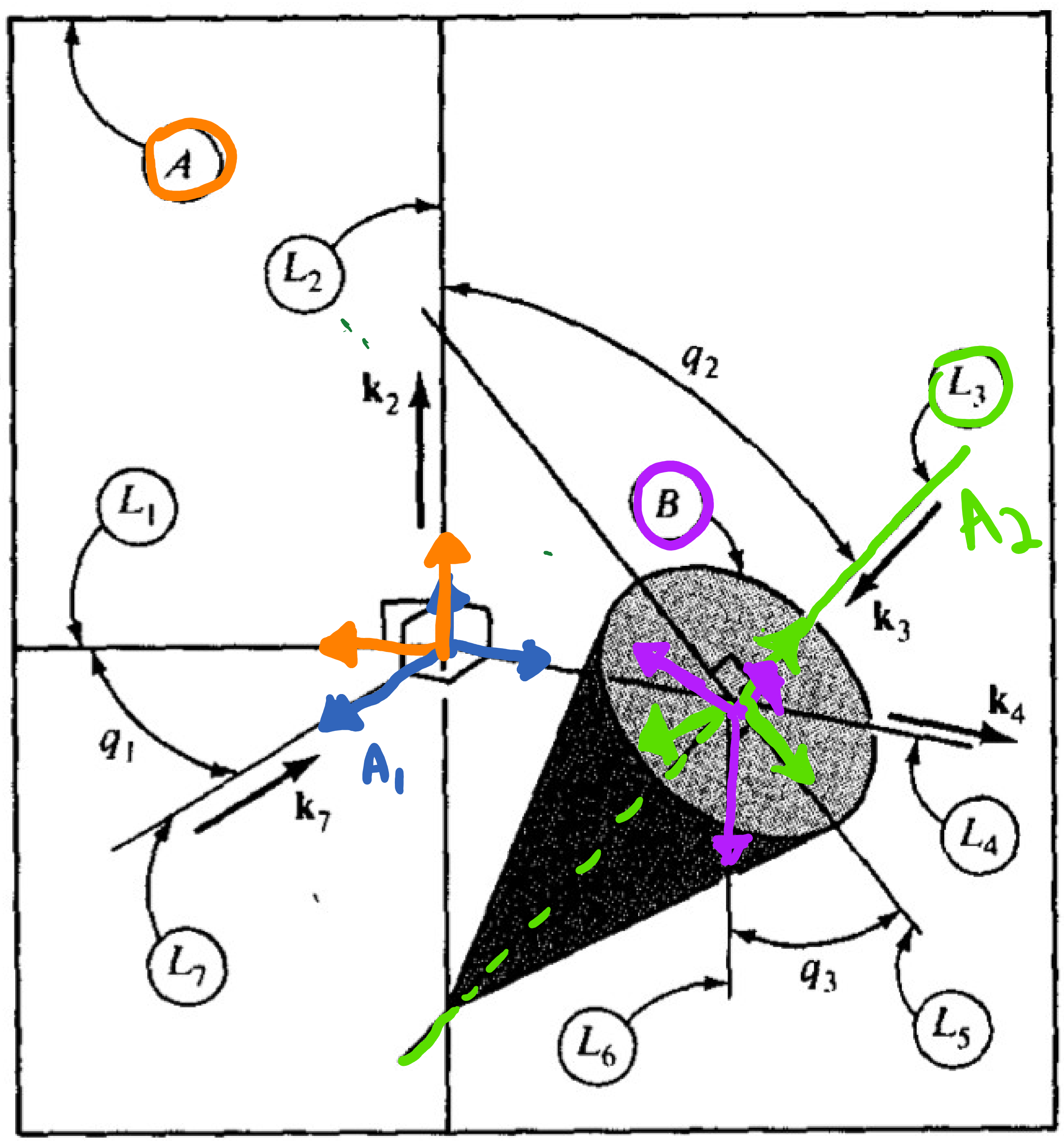

Figure 2.4.1

$\hat{k}_2$: unit vector aligned with L_2

$\hat{k}_7$: unit vector aligned with L_7

$\hat{k}_3$: unit vector aligned with L_3

Auxiliary reference frames

reference frames between two other frames whose orientation and angular velocities we want. Through a sequence of simple rotations we build or create the angular velocity of the outside ~~frames~~ frames using the addition thereom.

$${}^{A}\bar{\omega}^{B} = -{}^{B}\bar{\omega}^{A}$$

$$\boxed{{}^{A}\bar{\omega}^{B} = {}^{A}\bar{\omega}^{A_1} + {}^{A_1}\bar{\omega}^{A_2} + \dots + {}^{A_{n-1}}\bar{\omega}^{A_n} + {}^{A_n}\bar{\omega}^{B}}$$

n auxiliary RFs

$${}^A\bar\omega^B = {}^A\bar\omega^{A_1} + {}^{A_1}\bar\omega^{A_2} + {}^{A_2}\bar\omega^B$$

$${}^A\bar\omega^B = \dot q_1 \hat k_2 + \dot q_2 \hat k_7 + \dot q_3 \hat k_3$$

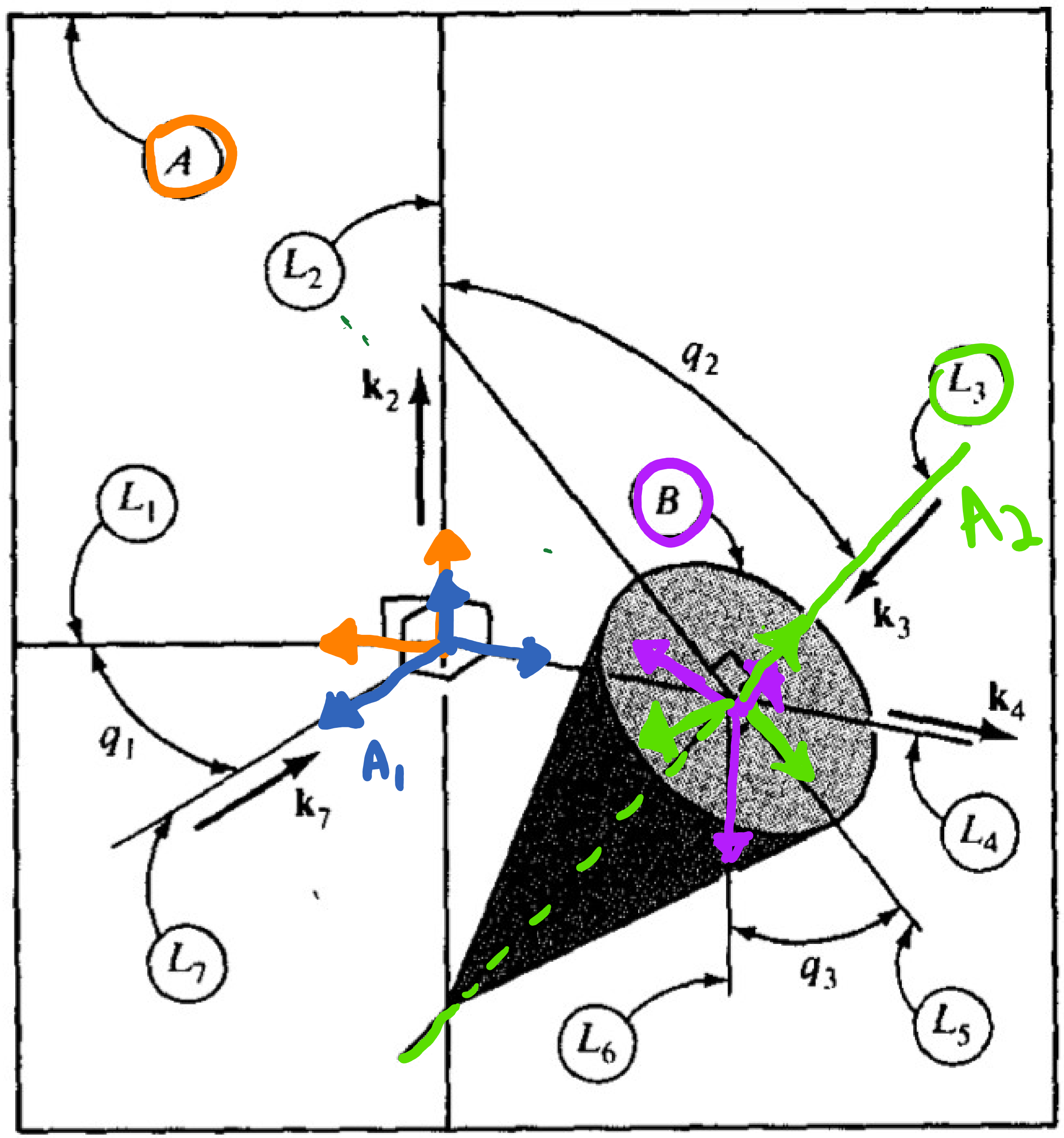

Figure 2.4.1

${}^A\bar\alpha^B$: angular acceleration vector

$${}^A\bar\alpha^B \triangleq \frac{{}^A d\,{}^A\bar\omega^B}{dt}$$

$$\boxed{\frac{{}^A d\,{}^A\bar\omega^B}{dt} = \frac{{}^B d\,{}^A\bar\omega^B}{dt}} + \underbrace{{}^A\bar\omega^B \times {}^A\bar\omega^B}_{0}$$

$$\| \; {}^A\bar\alpha^B$$

$${}^A\bar\alpha^B = \frac{{}^A d}{dt}\left(\dot q_1 \hat k_2 + \dot q_2 \hat k_7 + \dot q_3 \hat k_3\right)$$

$$\ddot q_1 \hat k_2 + \dot q_1 \frac{{}^A d \hat k_2}{dt} + \ddot q_2 \hat k_7 + \dot q_2 \frac{{}^A d \hat k_7}{dt} + \ddot q_3 \hat k_3 + \dot q_3 \frac{{}^A d \hat k_3}{dt}$$

$$= \ddot{q}_1 \hat{k}_2 + \dot{q}_1 \frac{d\hat{k}_2}{dt} + \ddot{q}_2 \hat{k}_2 + \dot{q}_2 \frac{d\hat{k}_2}{dt} + \ddot{q}_3 \hat{k}_3 + \dot{q}_3 \frac{d\hat{k}_3}{dt}$$

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$$\frac{{}^{A}d\hat{k}_2}{dt} = 0 \qquad \hat{k}_2 \text{ is fixed in the A frame}$$

$$\frac{{}^{A}d\hat{k}_3}{dt} = {}^{A}\bar{\omega}^{B} \times \hat{k}_3 \qquad \hat{k}_3 \text{ is fixed in the B frame}$$

$$= \dot{q}_1 \hat{k}_2 \times \hat{k}_3 + \dot{q}_2 \hat{k}_7 \times \hat{k}_3$$

$$\frac{{}^{A}d\hat{k}_7}{dt} = \dot{q}_1 \hat{k}_2 \times \hat{k}_7 \qquad \hat{k}_7 \text{ is fixed in } A_1 \text{ and } {}^{A}\bar{\omega}^{A_1} = \dot{q}_1 \hat{k}_2$$

note that ${}^{A}\bar{\alpha}^{A_1} = \ddot{q}_1 \hat{k}_2$, ${}^{A_1}\bar{\alpha}^{A_2} = \ddot{q}_1 \hat{k}_7$, ${}^{A_2}\bar{\alpha}^{B} = \ddot{q}_3 \hat{k}_3$ simple rotations!

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$$ {}^{A}\bar{\alpha}^{B} = \ddot{q}_1 \hat{k}_2 + \ddot{q}_2 \hat{k}_7 + \dot{q}_2 \dot{q}_1 \hat{k}_2 \times \hat{k}_7 + \ddot{q}_3 \hat{k}_3 $$

$$ + \dot{q}_3 \left(\dot{q}_1 \hat{k}_2 \times \hat{k}_3 + \dot{q}_2 \hat{k}_7 \times \hat{k}_3 \right) $$

$$ {}^{A}\bar{\alpha}^{B} \neq {}^{A}\bar{\alpha}^{A_1} + {}^{A_1}\bar{\alpha}^{A_2} + {}^{A_2}\bar{\alpha}^{B} $$

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Linear Velocity and Acceleration

Vel and Acc of Points

Let $\bar{p}$ = position vector from any point O fixed in RF A to a point P moving in A.

$$\boxed{{}^A\bar{V}^P \triangleq \frac{{}^A d\bar{p}}{dt}} \qquad \boxed{{}^A\bar{a}^P \triangleq \frac{{}^A d\left({}^A\bar{V}^P\right)}{dt}}$$

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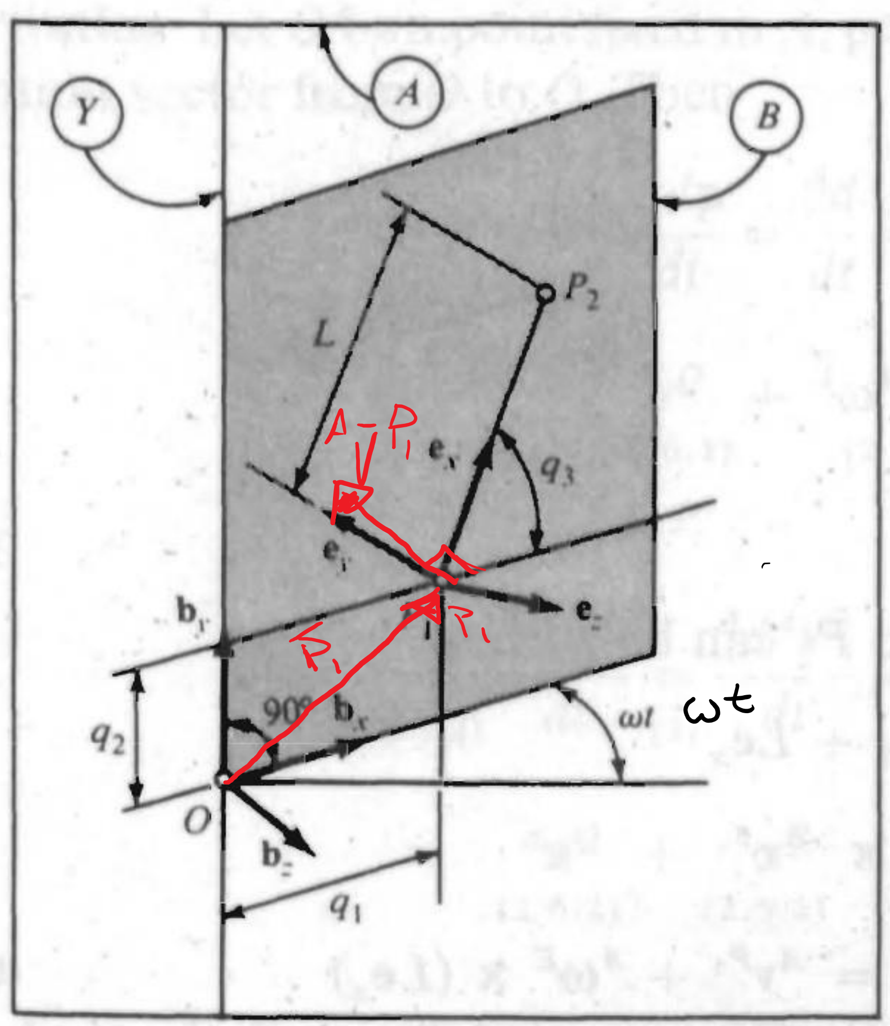

Figure 2.6.1

$\bar{P}_1$ = position vector from O to P_1

$\boxed{\bar{r}^{P_1/O} \text{ alt notation}}$

$$\bar{P}_1 = q_1\hat{b}_x + q_2\hat{b}_y$$

$$^A\bar{\omega}^B = \omega\hat{b}_y$$

$$\bar{P}_2 = \bar{P}_1 + L\hat{e}_x$$

$$\hat{b}_x = \hat{e}_x c_3 - \hat{e}_y s_3$$

$$\hat{b}_y = \hat{e}_x s_3 + \hat{e}_y c_3$$

$$\hat{b}_z = \hat{e}_z$$

$$\bar{P}_1 = q_1(\hat{e}_x c_3 - \hat{e}_y s_3) + q_2(\hat{e}_x s_3 + \hat{e}_y c_3)$$

$$^A\bar{V}^{P_1} = \frac{^A d\bar{P}_1}{dt} = \frac{^B d\bar{P}_1}{dt} + {}^A\bar{\omega}^B \times \bar{P}_1$$

$$^A\bar{V}^{P_1} = \dot{q}_1\hat{b}_x + \cancel{q_1\dot{\hat{b}}_x} + \dot{q}_2\hat{b}_y + \cancel{q_2\dot{\hat{b}}_y} \to 0 + \overbrace{\left(\omega\hat{b}_y \times (q_1\hat{b}_x + q_2\hat{b}_y)\right)}^{-\omega q_1 \hat{b}_z}$$