$$\bar{u} = Y\dot{q} + \bar{Z} \qquad \bar{u}_r = \sum_{s=1}^{n} Y_{rs}\dot{q}_s + Z_r$$

Kinematical differential equations (modeler chooses)

$$\begin{bmatrix}\bar{V}_1\\ \bar{V}_2\\ \bar{V}_3\end{bmatrix} = \overset{A}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} + \overset{B}{\begin{bmatrix} \quad \\ \quad \end{bmatrix}}$$

Partial Velocities

$$\bar{V} = \sum_{r=1}^{n} \bar{V}_r u_r + \bar{V}_t$$

r^{th} holomic partial velocity

for n G.S.

if motion constraint $p = n - m$

independ G.S.

Nonholonomic

$$\bar{V} = \sum_{r=1}^{p} \tilde{V}_r u_r + \tilde{V}_t$$

r^{th} nonholonomic partial velocity

$$\bar{\omega} = \sum_{r=1}^{p} \tilde{\omega}_r u_r + \tilde{\omega}_t$$

r^{th} nonholonomic angular vel

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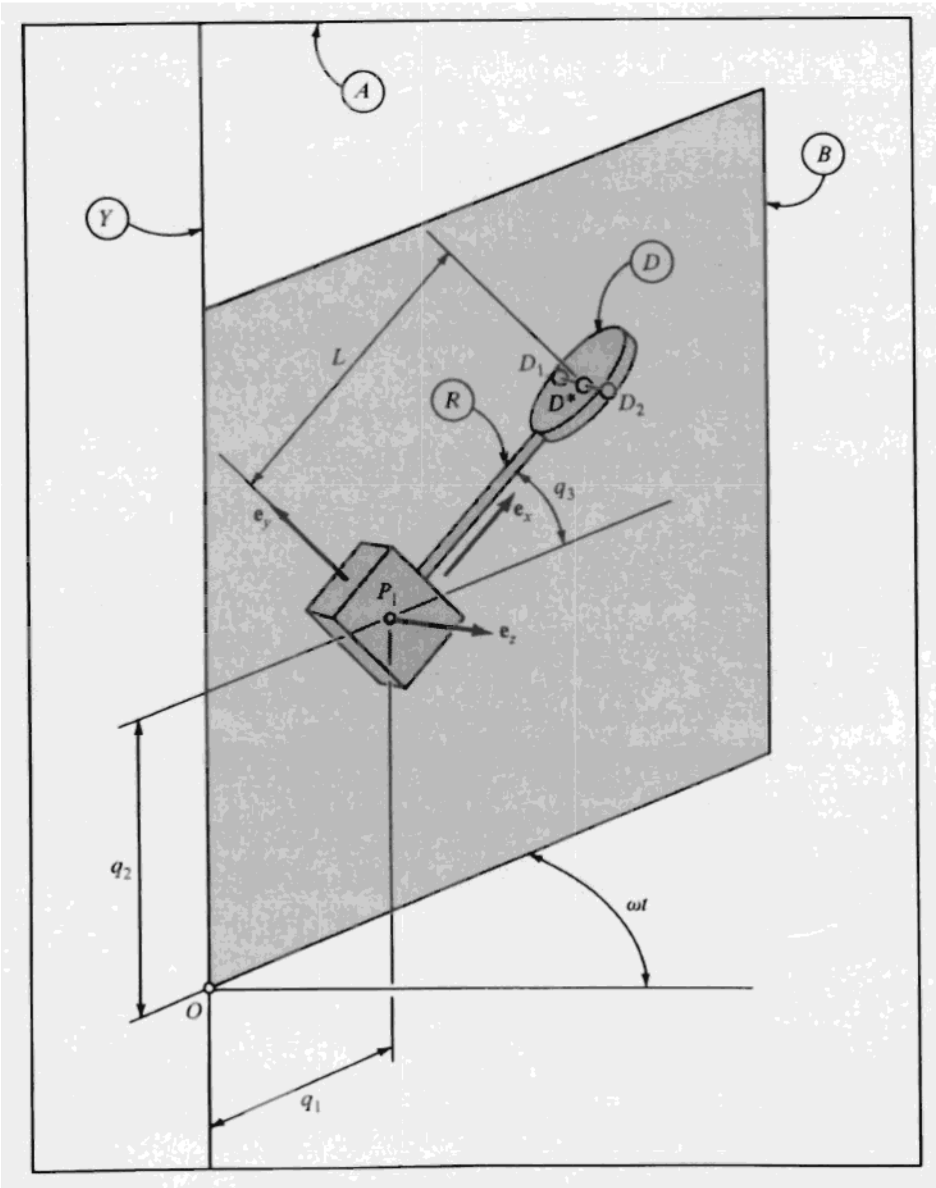

Figure 2.13.1

3 G.C.'s $\quad q_1, q_2, q_3$

$${}^A\bar{V}^{P_1} = u_1\hat{e}_x + u_2\hat{e}_y - \omega q_1\hat{e}_z$$

$u_1 = \dot{q}_1$

$u_2 = \dot{q}_2$

$${}^A\bar{V}^{D^*} = {}^A\bar{V}^{P_1} + {}^A\bar{\omega}^E \times L\hat{e}_x$$

$${}^A\bar{\omega}^E = {}^A\bar{\omega}^B + {}^B\bar{\omega}^E = \omega s_3\hat{e}_x + \omega c_3\hat{e}_y + u_3\hat{e}_z$$

$u_3 = \dot{q}_3$

$${}^A\bar{V}^{D^*} = u_1\hat{e}_x + (u_2 + Lu_3)\hat{e}_y - \omega(q_1 + Lc_3)\hat{e}_z$$

No lateral slip constraint

$${}^B\bar{V}^{D^*}\cdot\hat{e}_y = 0 \Rightarrow u_2 + Lu_3 = 0$$

$$u_3 = \frac{-u_2}{L}$$

ind. GS.

dep. GS.

We get to choose which are ind. and dep.

Partial Vels

${}^A\bar{V}_1^{D^*} = \hat{e}_x \quad {}^A\bar{V}_2^{D^*} = \hat{e}_y \quad {}^A\bar{V}_3^{D^*} = L\hat{e}_y \quad {}^A\bar{V}_t^{D^*} = -\omega(q_1 + Lc_3\hat{e}_z)$

${}^A\tilde{V}_1^{D^*} = \hat{e}_x \quad {}^A\tilde{V}_2^{D^*} = 0 \quad {}^A\tilde{V}_t^{D^*} = -\omega(q_1 + Lc_3\hat{e}_z)$

${}^A\bar{\omega}_1^E = 0 \quad {}^A\bar{\omega}_2^E = 0 \quad {}^A\bar{\omega}_3^E = \hat{e}_z \quad {}^A\bar{\omega}_t^{D^*} = \omega s_3\hat{e}_x + \omega c_3\hat{e}_y$

${}^E\tilde{\omega}_1^A = 0$

${}^E\tilde{\omega}_2^A = -\frac{\hat{e}_z}{L}$

${}^E\bar{\omega}_t^A = \omega s_3\hat{e}_x + \omega c_3\hat{e}_y$

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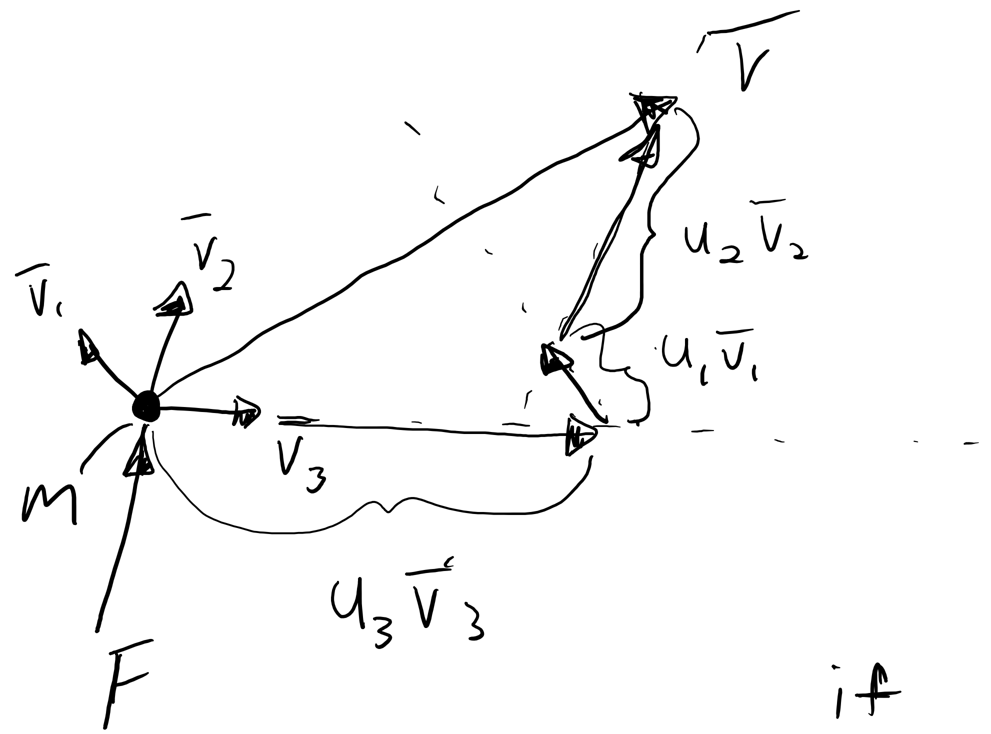

$\bar{a} = \frac{{}^N d \bar{V}}{dt}$

$\frac{\bar{F}}{m} = \bar{a}$

$\frac{\bar{F}}{m} \cdot \underbrace{\bar{V}_1}_{\text{direction associated with } u_1} \Rightarrow$ the component of force that causes motion in the u_1 G.S.

if spatial velocity is $\perp$ to $\bar{F}$, then $\bar{F}$ ~~doesn't~~ doesn't cause any motion in the r^{th} direction

so if $V_r \perp \bar{F}$

Mass Distribution

particles & RBs

m $\qquad$ $m = \int_{\text{volume}} \rho(\bar{r})\, d\forall$

zeroth moment of mass

Mass center of a rigid body (where the mass is on average)

Given a set of particles of mass $m_1, \ldots, m_\nu$ located at positions $\bar{r}_1, \ldots, \bar{r}_\nu$, there is a point S^* so that $\sum m_i \bar{r}_i = 0$ where $\bar{r}_i$ is the vector from S^* to particle i.

S^* is the <u>mass center</u>.

The "first moment of mass" is zero relative to the mass center.

Or if we choose arbitrary point O, from which $\bar{r}_i$ are measured then position vector to S^* from O; $\bar{p}^*$ is given by

$$\bar{p}^* = \frac{\sum_{i=1}^{\nu} m_i \bar{r}_i}{\underbrace{\sum_{i=1}^{\nu} m_i}_{m}} = \frac{\text{first moment}}{\text{zeroth moment}}$$

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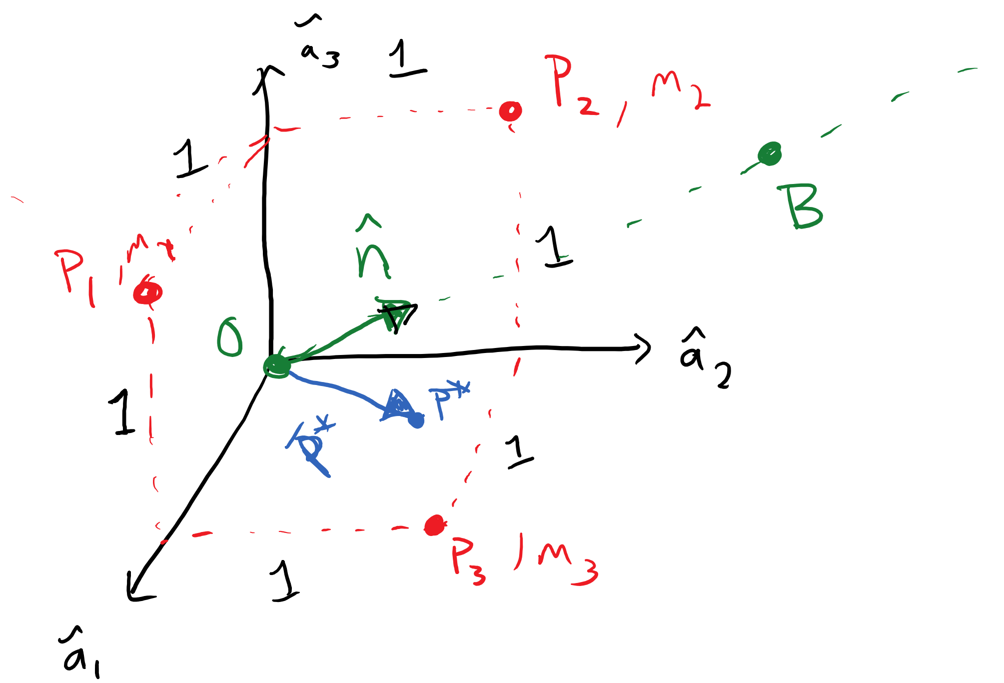
Ex 3 particles



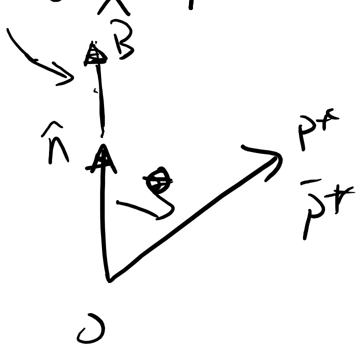
Let $\bar p^*$ = position vector from O to S^*

say $m_1 = m$, $m_2 = 2m$, $m_3 = \mu$

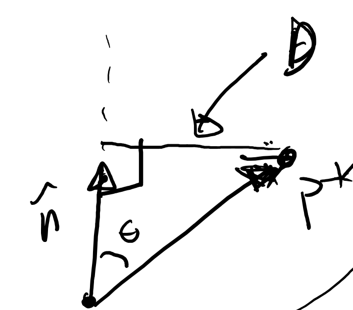
$$\bar p^* = \frac{\sum_{i=1}^{3} \bar p_i m_i}{\sum_{i=1}^{3} m_i} = \frac{m(\hat a_2 + \hat a_3) + 2m(\hat a_1 + \hat a_3) + \mu(\hat a_1 + \hat a_2)}{m + 2m + \mu}$$

Distance from p^* to $\overline{OB}$?

$$\hat n^{B/O} \times \bar r^{p^*/O}$$



$$\hat n \times \bar p^* = |p^*| \sin\theta \Rightarrow$$



$$\sin\theta = \frac{|\hat n \times \bar p^*|}{|p^*|}$$

$$\frac{D}{|\bar p^*|} = \sin\theta$$

Can we vary m_3 some amount to move the mass center to get mass center as close as possible to line $\overline{OB}$?

Now assume $\hat n = \dfrac{(\hat a_1 + \hat a_2 + \hat a_3)}{\sqrt{3}}$

$$D = \left| \bar p^* - (\bar p^* \cdot \hat n)\hat n \right| = |\hat n \times \bar p^*|$$

$$\frac{dD^2}{d\mu} = 0$$

$$D^2 = \frac{2(\mu^2 - 3\mu m + 3m^2)}{3(m+\mu)^2}$$

$$\frac{dD}{d\mu} = 0 \Rightarrow$$

$$\frac{dD}{d\mu} = -\frac{m\sqrt{6}\,(9m - 5\mu)}{18(m^2 + 2m\mu + \mu^2)\sqrt{3m^2 - 3m\mu + \mu^2}}$$

$$\frac{dD}{d\mu} = 0 \Rightarrow \boxed{\mu = \frac{5m}{3}}$$

$3(m+u)^2$ $\quad du \quad \frac{d}{du} = 0 \Rightarrow \boxed{\mu = \frac{5m}{3}}$

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Inertia Matrix 3x3

$$\begin{array}{c|ccc} & \hat{x} & \hat{y} & \hat{z} \\ \hline \hat{x} & I_{xx} & I_{xy} & I_{xz} \\ \hat{y} & I_{yx} & I_{yy} & I_{yz} \\ \hat{z} & I_{zx} & I_{zy} & I_{zz} \end{array}$$

products of inertia

moments of inertia

Specific RF

$$I_{xy} = I_{yx}$$