HW #8: 5.5

4) $(-3-\lambda)(-5-\lambda)+2=0$

$15+8\lambda+\lambda^2+2=0$

$\lambda^2+8\lambda+17=0$

$$\lambda=\frac{-8\pm\sqrt{64-68}}{2}=\frac{-8\pm 2i}{2}=-4\pm i$$

$-4+i$:

$$\begin{bmatrix}-3+4-i & -1 & 0\\ 2 & -5+4-i & 0\end{bmatrix}=\begin{bmatrix}1-i & -1 & 0\\ 2 & -1-i & 0\end{bmatrix}$$

$x_1-ix_1-x_2=0 \quad x_2=-x_1+ix_1$

$2x_1 \quad -x_2-ix_2=0$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}x_1\\-x_1+ix_1\end{bmatrix}=\begin{bmatrix}1\\-1+i\end{bmatrix}$$

eigenspace basis $=\left\{\begin{bmatrix}1\\-1+i\end{bmatrix},\begin{bmatrix}1\\-1-i\end{bmatrix}\right\}$

10) $A=\begin{bmatrix}3 & 3\\-3 & 3\end{bmatrix}$

$b=-3 \quad a=3$

$\lambda=3\pm(-3)$

$r=\sqrt{18}=3\sqrt{2}$

$$\begin{bmatrix}3\sqrt{2} & 0\\ 0 & 3\sqrt{2}\end{bmatrix}\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix}=\begin{bmatrix}3 & 3\\-3 & 3\end{bmatrix}$$

$$\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix}=\frac{1}{18}\begin{bmatrix}3\sqrt{2} & 0\\ 0 & 3\sqrt{2}\end{bmatrix}\begin{bmatrix}3 & 3\\-3 & 3\end{bmatrix}$$

$$=\frac{1}{18}\begin{bmatrix}9\sqrt{2} & 9\sqrt{2}\\-9\sqrt{2} & 9\sqrt{2}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$$

$\cos\phi=\frac{\sqrt{2}}{2}$ $\quad \phi=\frac{7\pi}{4}=-\frac{\pi}{4}$

$\sin\phi=-\frac{\sqrt{2}}{2}$

16) $A=\begin{bmatrix}-3 & -1\\ 2 & -5\end{bmatrix}$

$(-3-\lambda)(-5-\lambda)+2=0$

$15+8\lambda+\lambda^2+2=0$

$$\lambda=\frac{-8\pm\sqrt{64-68}}{2}=-4\pm i$$

$\lambda=-4+i$ $\quad$ $\lambda=a\pm b$, $a=-4$, $b=1$ $\quad C=\begin{bmatrix}-4 & -1\\ 1 & -4\end{bmatrix}$

$$\begin{bmatrix}1-i & -1 & 0\\ 2 & -1-i & 0\end{bmatrix}$$

$x_1-ix_1-x_2=0 \quad x_2=x_1-ix_1 \quad$ let $x_1=1$;

$2x_1-x_2-ix_2=0 \quad x_2=1-i$

$V=\begin{bmatrix}1\\1-i\end{bmatrix}$

$P=\begin{bmatrix}\operatorname{Re} v & \operatorname{Im} v\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix}$

24) False since complex come in pairs of $a\pm bi$

26) False since the eigenvectors must have a Re & Im component

27) all must follow the rule of $A^T=A$ and that conjugates are paired and solve one from one from the other

28) $Ax=\lambda x$

$x^TAx=x^T\lambda x$

$x^TAx=\lambda x^Tx$

by #27: this should equal a real vector q, which means the real part of x is an eigenvector of A

HW #8: 6.1

6) $\left(\frac{x \cdot w}{x \cdot x}\right) x = \left(\frac{18+2-15}{36+4+9}\right) x = \frac{5}{49} x = \begin{bmatrix} 30/49 \\ -10/49 \\ 15/49 \end{bmatrix}$

10) $v = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$ $\|v\| = \sqrt{9+36+9}$
$= 3\sqrt{6}$

$u = \begin{bmatrix} \sqrt{6}/6 \\ \sqrt{6}/3 \\ -\sqrt{6}/6 \end{bmatrix}$

14) $\sqrt{16+16+4} = \sqrt{36} = 6$

18) $-3 - 56 + 60 = 1$
not orthogonal

20) True since dot product is commutative for any vector $u, v \in \mathbb{R}^n$

26) True since scalar multiplication is commutative for any vector $u, v \in \mathbb{R}^n$ and any scalar $c \in \mathbb{R}$

39) let $x \in W$
$\therefore$ for every $y \in W^\perp$
$y \cdot x = 0$
y is orthogonal to x
if $x \in W$ & $x \in W^\perp$
$y = x$
$\therefore x \cdot x = 0$

Hw #8: 6.2

2) $\begin{bmatrix}1\\-2\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\\2\end{bmatrix} = 0-2+2=0$

$\begin{bmatrix}1\\-2\\1\end{bmatrix}\cdot\begin{bmatrix}-5\\-2\\1\end{bmatrix} = -5+4+1=0$

$\begin{bmatrix}0\\1\\2\end{bmatrix}\cdot\begin{bmatrix}-5\\-2\\1\end{bmatrix} = 0-2+2=0$

$\therefore$ this set is orthogonal

10) $\left.\begin{aligned} u_1\cdot u_2 &= 0\\ u_1\cdot u_3 &= 0\\ u_2\cdot u_3 &= 0\end{aligned}\right\}$ $\therefore$ orthogonal set

let $U = [u_1\ u_2\ u_3]$

$$U = \begin{bmatrix}3 & 2 & 1\\ -3 & 2 & 1\\ 0 & -1 & 4\end{bmatrix} = \begin{bmatrix}3 & 2 & 1\\ 0 & 4 & 2\\ 0 & -1 & 4\end{bmatrix} = \begin{bmatrix}3 & 2 & 1\\ 0 & -1 & 4\\ 0 & 0 & 18\end{bmatrix}$$

pivot in every col

$\therefore$ cols of $U = [u_1\ u_2\ u_3]$ are a basis for $\mathbb{R}^3$

$\therefore \{u_1\ u_2\ u_3\}$ is an orthogonal basis

$$x = \begin{bmatrix}5\\-3\\1\end{bmatrix} = \frac{4}{3}u_1 - \frac{3}{2}u_2 + \frac{1}{3}u_3$$

$$c_1 = \frac{x\cdot u_1}{u_1\cdot u_1} = \frac{15+9}{18} = \frac{24}{18} = \frac{4}{3}$$

$$c_2 = \frac{-8-16-3}{1+16+1} = \frac{-27}{18} = \frac{-3}{2}$$

$$c_3 = \frac{5-3+4}{1+1+16} = \frac{6}{18} = \frac{1}{3}$$

14)

$y = \hat{y} + z$

$$\hat{y} = \frac{y\cdot u}{u\cdot u}u$$

$$= \frac{12+6}{36+1}u = \frac{18}{37}u$$

$$= \frac{18}{37}\begin{bmatrix}6\\1\end{bmatrix} = \begin{bmatrix}108/37\\1/37\end{bmatrix}$$

$$z = \begin{bmatrix}(74-108)/37\\221/37\end{bmatrix} = \begin{bmatrix}-34/37\\221/37\end{bmatrix}$$

20)

$-\frac{2}{9}+\frac{2}{9} = 0$ $\therefore$ orthogonal

$\frac{4}{9}+\frac{1}{9}+\frac{4}{9} = \frac{9}{9} = 1$ $\therefore$ not orthonormal

$\frac{1}{9}+\frac{4}{9}+0 = \frac{5}{9} \neq 1$

$$\left\{\begin{bmatrix}-2/3\\1/3\\2/3\end{bmatrix}, \begin{bmatrix}\sqrt{5}/9\\2\sqrt{5}/9\\0\end{bmatrix}\right\}$$

30) $\text{proj}_v\, y = \frac{y\cdot v}{v\cdot v}v$ True

since the vector will be row equivalent to

$$\text{proj}_v\, y = \frac{y\cdot cv}{cv\cdot cv}v$$

32) True since $QQ^T = I$ $\therefore Q^T = Q^{-1}$

35) let $S = \{u_1 \dots u_n\}$ S is an orthonormal set

$U = [u_1 \dots u_n]$

$\therefore U$ is an orthogonal matrix since its columns are orthonormal

$\therefore UU^T = I$; $U^{-1} = U^T$

$\therefore U$ is invertible

36) since $UU^T = I$

let $V = U^T$

$\therefore VV^T = I$

$\therefore$ in this case V is orthogonal

$\therefore$ cols of V are orthogonal

$V = U^T$

$\therefore$ rows of U must also be orthogonal by defn

Hw 8: 6.3

6)

$0-1+1=0 \quad y=\begin{bmatrix}4\\4\\1\end{bmatrix}$

$\therefore \{u_1, u_2\}$ is orthogonal

$\hat{y}_1 = \text{Proj}_W y = \frac{-16-4+1}{18} u_1 = \frac{-19}{18} u_1 = \begin{bmatrix}19/9\\19/18\\-19/18\end{bmatrix}$

$\hat{y}_2 = \text{Proj}_W y = \frac{4+1}{2} u_2 = \frac{5}{2} u_2 = \begin{bmatrix}0\\5/2\\5/2\end{bmatrix}$

$\hat{y} = \hat{y}_1 + \hat{y}_2 = \begin{bmatrix}19/9\\64/18\\24/18\end{bmatrix}$

8) $y = \hat{y} + z$

$y = \begin{bmatrix}-1\\4\\3\end{bmatrix}$

$v_1 \cdot v_2 = 0$

$\hat{y} = \frac{-1+4+3}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{1+12-6}{1+9+4}\begin{bmatrix}-1\\3\\-2\end{bmatrix} = \begin{bmatrix}2\\2\\2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}-1\\3\\2\end{bmatrix} = \begin{bmatrix}3/2\\7/2\\3\end{bmatrix}$

$z = y - \hat{y} = \begin{bmatrix}-1-3/2\\4-7/2\\3-3\end{bmatrix} = \begin{bmatrix}-5/2\\1/2\\0\end{bmatrix}$

$\therefore y = \begin{bmatrix}3/2\\7/2\\3\end{bmatrix} + \begin{bmatrix}-5/2\\1/2\\0\end{bmatrix}$

14)

$\text{Proj}_W z = 7v_1 + 0v_2 = 7v_1$

24) if $y \in W$

then $\text{Proj}_W y = y$

$\therefore$ orthog proj must

be y since $y = \text{Proj}_W y$

True

28) True since

$y = \hat{y} + z$

where $z \in W^\perp$

& $\hat{y} = \text{Proj}_W y$

$\therefore z_1 = \hat{y} = \text{Proj}_W y$

32)

a) This must

be an orthogonal set

since $W^\perp$ has basis which

are orthogonal to every basis

in W

b) This spans $\mathbb{R}^n$ since

$W + W^\perp$ is the basis for

$\mathbb{R}^n$

c) Since set in part a spans

$\mathbb{R}^n$ & they are basis vectors

let $\dim\{w_1, \dots w_p\} = a$

& $\dim\{v_1 \dots v_q\} = b$

$a + b = n$ since $\{w_1, \dots w_p, v_1 \dots v_q\}$

spans $\mathbb{R}^n$