Assignment-4

ANGAD MANJUNATHA

1001718335

Task-1

Part-A

P(color is not green | vechicle is Truck)

$\therefore \dfrac{P(\text{color is red} \mid \text{vechicle is truck}) + P(\text{color is blue} \mid \text{vehicle is truck})}{P(\text{vehicle is truck})}$

P(vechicle is truck) = 0.04

$\therefore \dfrac{0.0060 + 0.0216}{0.04}$

$= \dfrac{0.0276}{0.04}$

$= 0.69$

Part - b

$P(red) = 0.0450 + 0.0495 + 0.0060 + 0.0495$
$= 0.15$
$P(blue) = 0.1620 + 0.1782 + 0.0216 + 0.1782$
$= 0.54$
$P(green) = 0.0930 + 0.1023 + 0.0124 + 0.1023$
$= 0.31$

$P(car) = 0.0495 + 0.1023 + 0.1782$
$= 0.33$
$P(Truck) = 0.0060 + 0.0124 + 0.0216$
$= 0.04$
$P(Van) = 0.0495 + 0.1023 + 0.1782$
$= 0.33$
$P(SUV) = 0.0495 + 0.1023 + 0.1782$
$= 0.33$

Red color

$P(red/van) = \frac{0.0495}{0.33} = 0.15$

$P(red/car) = \frac{0.0450}{0.3} = 0.15$

$P(red/truck) = \frac{0.0060}{0.04} = 0.15$

$P(red/SUV) = \frac{0.0495}{0.33} = 0.15$

$P(red) = P(red/car) = P(red/truck) = P(red/van) = P(red/suv)$

Green colour.

$P(green/car) = \frac{0.0930}{0.3} = 0.31$

$P(green/van) = \frac{0.1023}{0.33} = 0.31$

$P(green/Truck) = \frac{0.0124}{0.04} = 0.31$

$P(green/suv) = \frac{0.1023}{0.33} = 0.31$

$P(green) = P(green/car) = P(green/truck) = P(green/van) = P(green/suv)$

Blue colour

$P(blue/car) = \frac{0.1620}{0.3} = 0.54$

$P(blue/van) = \frac{0.1782}{0.33} = 0.54$

$P(blue/truck) = \frac{0.0216}{0.04} = 0.54$

$P(blue/suv) = \frac{0.1782}{0.33} = 0.54$

$P(blue) = P(blue/car) = P(blue/van) = P(blue/truck) = P(blue/suv)$

The color and the vehicle ~~to~~ of that color have the same probability value. Therefore the color and vehicle are totally independent.

Task - 2

Total no of variables = 12
The possible values of A = 8
The possible values of B = 5^{10}
The possible values of C = 6

Part A

Join Probability Distribution needs to store $8 \times 5^{10} \times 6$ numbers

468,75,0,000 in theory

468,749,999 in practice.

Part - b.

In practice

$P(A, B_1 \ldots B_{10}) = P(B_1|A)\, P(B_2|A) \ldots P(B_{10}|A)$

given conditionally independent

$P(B|A)$ takes $8 \times (5-1) = 32$ values

$P(A)$ takes $= 8 - 1 = 7$ values

$P(C)$ takes $6 - 1 = 5$ values

Total values $= 32 \times 10 + 7 + 5$

$= 320 + 12$

$= 332$

In theory

Total values $= 8 \times 5 \times 10 + 8 + 6$

$= 400 + 14$

$= 414$

Task - 3

Let baseball_game_on_TV : BGTV
George_watches_TV : GWTV
out_of_cat_food : OCF
George_feeds_cat : GFC

$$\begin{tikzcd}
 & \text{BGTV} \arrow[d] \\
 & \text{GWTV} \arrow[d] \\
\text{OCF} \arrow[r] & \text{GFC}
\end{tikzcd}$$

Task - 4

Refer attached file.

Task - 5

$P(\neg(\text{baseball\_game\_on\_TV}) \mid \neg(\text{George\_feeds\_cat}))$

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

$P(\neg(\text{baseball\_game\_on\_TV}) \mid \neg(\text{george\_feeds\_cat}))$

$$P(\neg bgtv, \neg g/c) = P(\neg(bgtv), \neg g/c, gwtv, o/c) + P(\neg(bgtv), \neg g/c, gwtv, \neg o/c) + P(\neg(bgTV), \neg g/c, \neg gwtv, o/c) + P(\neg(bgTV), \neg g/c, \neg gwtv, \neg o/c).$$

$$= P(\neg bgtv) \times P(o/c) \times P(gwtv \mid bgtv) \times P(\neg g/c \mid gwtv, o/c)$$
$$+ P(\neg bgtv) \times P(\neg o/c) \times P(gwtv \mid bgtv) \times P(\neg g/c \mid gwtv, \neg o/c)$$
$$+ P(\neg bgtv) \times P(o/c) \times P(\neg gwtv \mid bgtv) \times P(\neg g/c \mid \neg gwtv, o/c)$$
$$+ P(\neg bgtv) \times P(\neg o/c) \times P(\neg gwtv \mid bgtv) \times P(\neg g/c \mid \neg gwtv, \neg o/c)$$

$$= (0.6959 \times 0.1699 \times 0.9279 \times 0.9583) + (0.6959 \times 0.8301 \times 0.9279 \times 0.2956) + (0.6959 \times 0.1699 \times 0.0721 \times 0.6842) + (0.6959 \times 0.8301 \times 0.0721 \times 0.0412)$$

$$= \underline{0.12673}$$

$$P(g/c) = \frac{276}{365} = 0.7562$$

$$P(\neg g/c) = 1 - 0.7562 = 0.2438$$

$$\therefore P(\neg bgtv \mid \neg g/c) = \frac{P(\neg bgTV, \neg g/c)}{P(\neg g/c)}$$

$$= \frac{0.12673}{0.2438} = \underline{\underline{0.51962}}$$

Task 6.

a. Parents : I
children : R, S
other parents of children : M, O

Markov Blanket of N : I, R, S, M, O

b. $P(I, D)$ can be written as

$P(I/D) \cdot P(D)$ (Product rule)
$0.5 \times 0.5 = 0.25$

c. $P(M, \neg C / H)$

$$= \frac{P(M, \neg C, H)}{P(H)}$$

Markov Blanket of H is C, M

so

$P(H) = P(M, C, H) + P(M, \neg C, H) + P(\neg M, C, H)$
$+ P(\neg M, \neg C, H)$

Applying chain rule
$= P(M/H) + P(H/C) \cdot P(C) + P(M/H) P(H/\neg C) P(\neg C)$
$+ P(\neg M / H) P(H/c) P(C) + P(\neg M/H) P(H/\neg C)$
$P(\neg C)$

$$= 0.1 \times 0.6 \times 0.6 + 0.1 \times 0.1 \times 0.4 + 0.9 \times 0.6 \times 0.6 + 0.9 \times 0.1 \times 0.4$$

$$= 0.6 \times 0.6 + 0.1 \times 0.4$$

$$= \underline{\underline{0.4}}$$

$$P(M, \neg C, H) = P(M/H) \cdot P(H/\neg C) \cdot P(\neg C)$$

$$= 0.1 \times 0.1 \times 0.4$$

$$= 0.004$$

So,

$$P(M, \neg C / H) = \frac{0.004}{0.4} = \underline{\underline{0.01}}$$