

Anonymous Oblivious Transfer

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(November 4th, 2000)

In this short note we want to introduce *anonymous oblivious transfer* a new cryptographic primitive which can be proven to be strictly more powerful than oblivious transfer. We show that all multi party protocols can robustly be realized by *anonymous oblivious transfer*. No assumption about possible collusions of cheaters or disruptors have to be made.

Furthermore we shortly discuss how to realize *anonymous oblivious transfer* with noisy broadcast channels. The protocol of *anonymous oblivious transfer* was inspired by a quantum protocol: the *anonymous quantum channel*.

I. INTRODUCTION

In [2,7,5] multi party protocols with oblivious transfer were presented which can tolerate a dishonest majority. These protocols work with perfect security if all players cooperate. But already one disruptor can abort the protocol without being detected. The contribution of [1] were protocols more robust against disruption. The idea was to replace two party subprotocols which failed by multi party protocols. Then either these protocols did work or a cheater could be identified.

Unfortunately replacing an oblivious transfer where the sender or the receiver refuses to cooperate by a multi party protocol weakens the security of the protocol. In [1] we can observe a trade off between the size of a tolerable collusion of active cheaters (including disruptors) and the size of a collusion of passive cheaters unable to obtain secret data.

In this paper we present the new cryptographic primitive *anonymous oblivious transfer* and prove that it is strictly more powerful than oblivious transfer. With this primitive we can realize multi party protocols which work with perfect security or a cheater can be identified unambiguously. As we cannot expect higher robustness and security than that we claim that anonymous oblivious transfer is the most powerful cryptographic primitive which can achieve unconditional security. We recently learned about independent work in this direction carried out by [6].

II. MULTI PARTY PROTOCOLS

In a multi party protocol a set P of players wants to correctly compute a function $f(a_1, \dots, a_n)$ which depends on secret inputs of n players. Some players might collude to cheat in the protocol as to obtain information about secret inputs of the other players or to modify the

result of the computation. Possible collusions of cheaters are modelled by *adversary structures*

Definition 1 *An adversary structure is a monotone set $\mathcal{A} \subseteq 2^P$, i. e., for subsets $S' \subseteq S$ of P the property $S \in \mathcal{A}$ implies $S' \in \mathcal{A}$.*

The main properties of a multi party protocol are:

1. A multi party protocol is said to be \mathcal{A} -secure if no single collusion from \mathcal{A} is able to obtain information about the secret inputs of other participants which cannot be derived from the result and the inputs of the colluding players.
2. A multi party protocol is \mathcal{A} -partially correct if no party can let the protocol terminate with a wrong result.
3. A multi party protocol is called \mathcal{A} -fair if no collusion from \mathcal{A} can reconstruct the result of the multi party computation earlier then all honest participants together. No collusion should be able to run off with the result.

We will be more strict here and demand robustness even against disruptors.

- 2' A multi party protocol is \mathcal{A} -correct whenever no single collusion from \mathcal{A} can abort the protocol, modify its result, or take actions such that some player gets to know a secret value.

A protocol is called \mathcal{A} -robust if it has all of the above properties. Note that we will allow only one collusion to cheat, but we think of every single player as being curious, i. e., even if he is not in the collusion actually cheating he will eavesdrop all information he can obtain without being detected cheating

With oblivious transfer all multi party protocols can be realized with perfect security if all players are cooperating [2,7,5]. But a collusion of players can abort the calculation, see next section.

III. IMPOSSIBILITY RESULTS

Lemma 2 *Let P be a set of players for which each pair of players is connected by a (private) oblivious transfer channel and each player has access to an authenticated broadcast channel. Then \mathcal{A} -robust multi party computations are possible for all functions if and only if no two sets of \mathcal{A} cover $P \setminus \{P_i\}$ for a player $P_i \in P$ or $|P| = 2$.*

Proof: Let A and B be two possible collusions covering $P \setminus \{P_i\}$, then oblivious transfer cannot be implemented \mathcal{A} -robustly between players of A and players of B . Between any two players Alice $\in A$ and Bob $\in B$ the oblivious transfer channel does not work, but it is not obvious for the player P_i who is refusing to cooperate. The player P_i must assist Alice and Bob. As no other player can assist we are in the three party situation with an

oblivious transfer channel only between Alice and P_i and Bob and P_i . For each bit being transferred from Alice to Bob the player P_i knows either as much as Alice about this bit or he knows as much as Bob. The players Alice and Bob cannot agree on a bit known to both without P_i knowing it, too. Hence oblivious transfer from Alice to Bob becomes impossible without P_i having to learn a secret of Alice or a secret of Bob. \square

IV. MULTI PARTY PROTOCOLS

In the multi party protocols of [5,1] a collusion of disruptors can abort the protocol if an assumption about possible collusions of disruptors is violated. We would like to have cryptographic primitives where every time a conflict arises a cheater can be identified. Two such primitives are *global bit commitment* and *undeniable oblivious transfer*. We will show in the following that these primitives, defined below, can realize the subprotocols needed in [5,1] relative to no assumptions about possible collusions.

Definition 3 A global bit commitment (GBC) binds a player to all other players to the same bit.

Definition 4 An undeniable oblivious transfer (UOT) protocol from a player Alice $\in P$ to a player Bob $\in P$ allows Alice to generate a GBC for a bit b in a way that Bob learns the bit b with probability $1/2$ and Alice cannot know if Bob learnt b .

Now we introduce the notions used for the multi party protocols.

Definition 5 A global bit commitment with Xor (GBCX) to a bit b is a GBC to bits $b_{1L}, b_{2L}, \dots, b_{mL}, b_{1R}, \dots, b_{mR}$ such that for each i $b_{iL} \oplus b_{iR} = b$.

Theorem 6 GBCX allow zero knowledge proofs of linear relations among several bits a player is committed to using GBCX. Especially (in)equality of bits or a bit string being contained in a linear code.

Furthermore GBCXs can be copied, as proofs may destroy a GBCX.

Proof: We will not state a full proof here as it can be found in [5]. But we will restate the copying procedure as it is an important subprotocol of all of the following protocols.

Suppose Alice is committed to Bob to a bit b and wants two instances of this commitment. Then Alice ceates $3m$ pairs of global bit commitments such that each pair Xors to b . Then all other player, by coin tossing, randomly partition these $3m$ pairs in three subsets of m pairs, thus obtaining three GBCX and ask Alice to prove the equality of the first new BCX with her GBCX for b . This

destroys the old GBCX and one of the new GBCX, but an honest Alice can thereby convince all players that the two remaining GBCX both stand for the value b . \square

The basic building block for multi party protocols of [5] are distributed bit commitments, where each player is committed to a share of a bit.

Definition 7 A distributed bit commitment (DBC) of a user Alice $\in P$ to a bit b consists of n GBCX one created by each player of P such that only Alice knows how to open all of them and the Xor of all values of the GBCX equals b .

An intermediate result DBC consists of n GBCX such that no subset of players unequal P can know how to open all of the GBCX.

Lemma 8 With a protocol for generating GBCX one can realize a DBC of a user.

Proof: Each player generates a GBCX and opens the commitment to Alice. In case of a conflict the player opens his GBCX publicly. Then Alice creates a GBCX such that the parity bit is the bit she wanted to create a DBC for. Only Alice knows how to open all commitments as she created one herself. \square

The intermediate result DBCs are automatically generated by the multi party protocols for these we need the key protocol of [5].

Definition 9 Given two players Alice and Bob where Alice is committed to bits b_0, b_1 and Bob is committed to a bit a . Then a committed oblivious transfer protocol (COT) is a protocol where Alice inputs her knowledge about her two commitments and Bob will input his knowledge about his commitment and the result will be that Bob is committed to b_a .

In a global committed oblivious transfer protocol (GCOT) all players are convinced of the validity of the commitments, i.e., that indeed Bob is committed to b_a after the protocol.

For the next result we use one-out-of-two UOT, which is the usual one-out-of-two OT, but the sender is committed to the two bits the receiver could choose from. The standard reduction from one-out-of-two OT to OT can be used to turn UOT into one-out-of-two UOT.

Lemma 10 With UOT and an authenticated broadcast channel one can realize GCOT.

Proof: We will essentially restate the GCOT protocol of [5] and see that with one-out-of-two UOT instead of one-out-of-two OT any conflict results in the identification of a cheater.

GCOT(a_0, a_1)(b)

1. All participants together choose one decodable $[m, k, d]$ linear code C with $k > (1/2 + 2\sigma)m$ and $d > \epsilon n$ for positive constants σ, ϵ , efficiently decoding t errors.

2. Alice randomly picks $c_0, c_1 \in \mathcal{C}$, commits to the bits c_0^i and c_1^i ($i \in \{1, \dots, m\}$) of the code words, and proves that the codewords fulfil the linear relations of \mathcal{C} .
3. Bob randomly picks $I_0, I_1 \subset \{1, \dots, M\}$, with $|I_0| = |I_1| = \sigma m$, $I_1 \cap I_0 = \emptyset$ and sets $b^i \leftarrow \bar{b}$ for $i \in I_0$ and $b^i \leftarrow b$ for $i \notin I_0$.
4. Alice runs $\text{OT}(c_0^i, c_1^i)(b^i)$ with Bob who gets w^i for $i \in \{1, \dots, m\}$. Bob tells $I = I_0 \cup I_1$ to Alice who opens c_0^i, c_1^i for each $i \in I$.
5. Bob checks that $w^i = c_0^i$ for $i \in I_0$ and $w^i = c_1^i$ for $i \in I_1$, sets $w^i \leftarrow c_b^i$, for $i \in I_0$ and corrects w using \mathcal{C} 's decoding algorithm, commits to w^i for $i \in \{1, \dots, m\}$, and proves that $w^1 \dots w^m \in \mathcal{C}$.
6. All players together randomly pick a subset $I_2 \subset \{1, \dots, m\}$ with $|I_2| = \sigma m$, $I_2 \cap I = \emptyset$ and Alice opens c_0^i and c_1^i for $i \in I_2$.
7. Bob proves that $w^i = c_b^i$ for $i \in I_2$.
8. Alice randomly picks and announces a privacy amplification function $h : \{0, 1\}^m \rightarrow \{0, 1\}$ such that $a_0 = h(c_0)$ and $a_1 = h(c_1)$ and proves $a_0 = h(c_0^1, \dots, c_0^m)$ and $a_1 = h(c_1^1, \dots, c_1^m)$.
9. Bob sets $a \leftarrow h(w)$, commits to a and proves $a = h(w^1 \dots w^m)$.

A conflict between Alice and Bob can only appear in connection with step 4. because all other steps can be checked by all other players and it becomes immediately clear who is cheating. In such a conflict Bob claims that Alice sent something inconsistent over the oblivious transfer or Alice accuses Bob to not have committed to what he received.

In case of a conflict Alice opens all bits of c_0, c_1 to which she is committed by the UOT also she opens her GBCX to these codewords, if she is not able to do it or unveils non code words or other inconsistent information she is detected cheating. The bits of c_0, c_1 do not give away any secret as these are random code words. If Alice's information is correctly unveiled and is consistent with all her past actions (proofs) then Bob was cheating if he did complain. If it was Alice complaining Bob has to prove zero knowledge that the bit string w he is committed to equals c_0 or equals c_1 if he is able to convince all other players Alice is detected cheating (conflicts appearing during the proofs can be resolved easily as it is obvious for every player who is cheating). \square

One other important property of multi party protocols is *fairness*. A multi party protocol is called *fair* if no collusion of players can reconstruct the result of the protocol earlier than all honest players. This problem is solved in the literature [3,7] and will not be discussed here.

Hence we have everything to follow the protocols of [5] robustly and in the following we need only to prove that a certain cryptographic primitive can realize GBC (or GBCX) and UOT and we know that it is capable of realizing all multi party protocols with perfect security and robustness.

V. ANONYMOUS OBLIVIOUS TRANSFER

We next define *anonymous oblivious transfer*.

Definition 11 An anonymous oblivious transfer (AOT) protocol allows a player Alice $\in P$ to send a bit string $b_1 \dots b_m$ to a player Bob $\in P$ such that Bob receives each bit of the bit string with probability $1/2$ or he receives \perp which indicates that he will not learn this bit. Alice cannot know which bits Bob received. Furthermore Bob does not know which player sent the bit string and Alice does not

For the following we will need two subprotocols which can easily be realized by AOT.

Lemma 12 With AOT and an authenticated broadcast channel one can realize anonymous message transfer and anonymous broadcast.

Proof: To send a message anonymously one has to encode the message with an error correcting code to cope with the erasures of the AOT.

For an anonymous broadcast Alice sends anonymously to a player P_i a message which this player shall broadcast. If he broadcasts something wrong Alice is in conflict with this player and picks another player P_j to start the procedure anew. Either the anonymous broadcast will be successful or Alice will leave the protocol as she is in conflict with all other players. \square

With these protocols we can realize GBCX.

Lemma 13 With AOT and an authenticated broadcast channel one can realize GBCX.

Proof: We let all players create GBCX according to the protocol of [5], but anonymously using AOT and anonymous broadcast. Then after some time no new conflicts occur for l anonymous GBCX of each player (l is a security parameter which is polynomial in n). If a player Alice was unable to create a GBCX we will split the set of players in a way that one set contains all honest players and the other set contains only cheaters. We explain this in more detail by the two cases which can occur:

1. If Alice was honest then, as a cheater cannot distinguish between the honest players after some time if the cheater keeps complaining about Alice this cheater will be in conflict with all honest players. Furthermore all honest players will know it. Now we can separate the set P of players in two subsets such that no two players having a conflict are in the same set. Then we can be sure that one of the two sets contains only cheaters and every honest player knows it.
2. If Alice was dishonest then we will also separate the set P . Alice will be in one group with all players complaining about the same players as Alice did

(these are all honest players if Alice were honest) all other players will be in the other group. As Alice is a cheater and hence in conflict with an honest player all players in her group must be cheaters, too.

□

Note that the protocol to create GBCX for all players needs only polynomial time in n , as only n^2 conflicts are possible.

After having realized GBC we need to implement UOT.

Lemma 14 *With AOT and an authenticated Broadcast channel one can realize UOT.*

Proof: Alice creates a GBCX and Bob publishes positions of two substrings of the strings Alice sent to him. One substring where he knows all the bits and one substring where he knows nothing. The substrings must have approximately the same length.

Alice publishes the bits of one of the substrings. Then Bob either learnt nothing new or he knows the bit Alice is committed to. We have realized UOT if we can show that no other player learns the bit Alice is committed to by the information published by Alice, but this is trivial as Alice sent different strings to different players. □

VI. REALIZING AOT

A. Quantum Protocols

Anonymous oblivious transfer was inspired by a quantum protocol [8]. But it cannot be realized by a quantum protocol unless no two possible collusions cover the set P of players.

The idea for the realization is to follow normal quantum multi party protocols if not two sets covering $P \setminus P_i$ are in conflict. In case of such a conflict the player P_i is not a disruptor or active cheater by assumption. This player can now forward quantum information between the two sets which are in conflict. Quantum cryptography allows to keep the player P_i from eavesdropping the quantum data excluding what happened in Lemma 2. As the player P_i can forward all quantum information in the same way and send quantum information himself this realizes an *anonymous quantum channel*. This yields:

Theorem 15 *Robust quantum multi party protocols for all functions are possible if and only if no two possible collusions cover the set P of players.*

These protocols become robust against a set of possible collusions after termination which may contain one and only one complement of a collusion tolerable during the execution of the protocol.

Especially a quantum channel can be more powerful than oblivious transfer (See Lemma 2). For details please refer to [9,8].

B. Noisy Broadcast

Noisy broadcast was, like OT, originally proposed to realize secure key exchange. In this section we will prove that the cryptographic power of noisy broadcast is equivalent to the power of AOT.

Definition 16 *A noisy broadcast channel is a broadcast channel which sends the messages with a bit error rate of ϵ such that the errors of different players are independent.*

With a noisy broadcast channel and an authenticated broadcast channel we can implement, using the techniques of [4], an *oblivious broadcast*:

Definition 17 *An oblivious broadcast channel is a protocol where a player inputs a bit string and every other player receives the output of an oblivious transfer of this string and the erasures are independent for the different players.*

Lemma 18 *An authenticated oblivious broadcast can realize a GBC.*

Proof: Alice sends, as a commitment, k bit strings of length m (k, m are security parameters which are polynomial in n) with parity b . Then the knowledge all other players have about b is negligible in m . Because the probability that a bit is received by at least one player is $1 - 1/2^n$ and the probability that all players together have knowledge about all m is $(1 - 1/2^n)^m$ which is negligible in m . If k strings are sent the probability remains negligible as k and m are polynomial in n .

If Alice wanted to change the bit she committed to she has to change k bits. The probability that any single player does not detect this change is negligible in k . □

Lemma 19 *An authenticated oblivious broadcast can realize UOT.*

Proof: Alice creates a GBC and Bob publishes positions of two substrings of the strings Alice sent over the oblivious broadcast. One substring where he knows all the bits and one substring where he knows nothing. The substrings must have approximately the same length.

Alice publishes the bits of one of the substrings. Then Bob either learnt nothing new or he knows the bit Alice is committed to. We have realized UOT if we can show that no other player learns the bit Alice is committed to by the information published by Alice. But as the substrings published are statistically independent of what the other players received this information just changes the probability of receiving a bit for each player. This change of probability can be coped with an suitable choice of the security parameters used in Lemma 18. □

VII. MAIN RESULT

Summarizing all of the above we can state:

Theorem 20 *The primitive of anonymous oblivious transfer is cryptographically strictly more powerful than oblivious transfer. Together with an authenticated broadcast channel it can realize all multi party protocols with a security and robustness which is independent from assumptions about possible collusions of cheaters or disruptors.*

Anonymous oblivious transfer can be realized by an authenticated noisy broadcast channel.

VIII. FUTURE WORK

An interesting question concerning the noisy broadcast channel is if we can cope with different players having different error probabilities. Maybe these probabilities are only known to lie in certain intervals for certain players. Then we would need an additional protocol comparable to advantage distillation to deal with the different error probabilities by public discussion.

There probably are many other primitives of equivalent cryptographic power. This has to be investigated to maybe find primitives which can be realized more easily or more efficiently (compare [6]).

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