

The Computational Complexity of $3k$ -CLIQUE

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Abstract: In this note, we show that the fastest deterministic and exact algorithm that solves the $3k$ -CLIQUE problem must run in $\Omega(n^{2k})$ time in the worst-case scenario on a classical computer, where n is the number of vertices in the graph.

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The $3k$ -CLIQUE problem is to determine whether a given undirected graph G contains a clique of size $3k$, where k is a positive integer that is not part of the input of the problem [3]. In this note, we show that the fastest deterministic and exact algorithm that solves $3k$ -CLIQUE must run in $\Omega(n^{2k})$ time in the worst-case scenario on a classical computer, where n is the number of vertices in the graph:

Let G be an undirected graph with n vertices. For every k -clique C in G , create a corresponding vertex $v(C)$ in an auxiliary graph G' . And for every two vertices $v(C_1)$ and $v(C_2)$ in G' , create an edge connecting them in G' if and only if $C_1 \cup C_2$ forms a $2k$ -clique in G . Note that the 3-CLIQUE problem on G' is equivalent to the $3k$ -CLIQUE problem on G [3].

Since graph G' has a maximum of $\Theta(n^k)$ vertices and $\Theta(n^{2k})$ edges, it must take $\Omega(n^{2k})$ time for any algorithm to solve 3-CLIQUE on G' in the worst-case scenario. Thus, since the 3-CLIQUE problem on G' is equivalent to the $3k$ -CLIQUE problem on G , it must also take $\Omega(n^{2k})$ time for any algorithm to solve the $3k$ -CLIQUE problem on G in the worst-case scenario. And this implies that $P \neq NP$ [1]. \square

This lower bound is confirmed by the fact that the fastest known deterministic and exact algorithm that solves $3k$ -CLIQUE was first published in 1985 and has a running-time of $\Theta(n^{\omega k})$, where $\omega \geq 2$ [1, 2, 3].

References

- [1] F. Eisenbrand and F. Grandoni, "On the complexity of fixed parameter clique and dominating set", *Theoretical Computer Science* 326(1-3): 57-67, 2004.
- [2] J. Nešetřil and S. Poljak, "On the complexity of the subgraph problem", *Comment. Math. Univ. Carolin.* 26 (1985) 415-419.
- [3] G. J. Woeginger, "Open problems around exact algorithms", *Discrete Applied Mathematics* 156(3): 397-405 (2008).