

Causes and Explanations: A Structural-Model Approach

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June 1, 2013

Abstract

We propose new definitions of *actual causes* and *explanations*, using *structural equations* to model counterfactuals. We show that these definitions yield a plausible and elegant account of causation and explanation that handles well examples which have caused problems for other definitions and resolves major difficulties in the traditional account.

*Supported in part by NSF under grant IRI-96-25901

†Supported in part by grants from NSF, ONR, AFOSR, and MICRO.

1 Introduction

What does it mean that an event A *actually causes* event B ? This is a question that goes beyond mere philosophical speculation. As Good (1993) and Michie (1997) argue persuasively, in many legal settings, what needs to be established (for determining responsibility) is not a counterfactual kind of causation, but “cause in fact.” A typical example (Wright 1988) considers two fires advancing toward a house. If fire A burned the house before fire B , we (and many juries nationwide) would consider fire A “the actual cause” for the damage, even supposing the house would have definitely burned down by fire B , if it were not for A . Actual causation is also important in artificial intelligence applications. Whenever we undertake to *explain* a set of events that unfold in a specific scenario, the explanation produced must acknowledge the actual cause of those events. The automatic generation of adequate explanations, a task essential in planning, diagnosis and natural language processing, therefore requires a formal analysis of the concept of actual cause.

Giving a precise and useful definition of actual causality is notoriously difficult. The philosophical literature has been struggling with this notion since the days of Hume (1739); it continues to struggle with it to this day. (See (Sosa and Tooley 1993), (Hall 1998), and (Pearl 2000) for some recent discussions.) To borrow just one example from Hall (1998), suppose a bolt lightning hits a tree and starts a forest fire. It seems reasonable to say that the lightning bolt is a cause of the fire. (Indeed, the description “the lightning bolt . . . starts a forest fire” can be viewed as saying this.) But what about the oxygen in the air and the fact that the wood was dry? Presumably, if there has not been oxygen or the wood was wet there would not have been a fire. Carrying this perhaps to the point of absurdity, what about the Big Bang? This problem is relatively easy to deal with, but there are a host of other, far more subtle, difficulties that have been raised over the years.

Here we give a definition of actual causality based on the language of *structural equations*; we then give a definition of (*causal*) *explanation* using the definition of causality. An explanation adds information to an agent’s knowledge; very roughly, an explanation of φ is a minimal elaboration of events that suffice to cause φ even in the face of uncertainty about the actual situation at hand. The use of structural equations as a model for causal relationships is standard in the social sciences, and seems to go back to the work of Sewall Wright in the 1920s (see (Goldberger 1972) for a discussion); the particular framework that we use here is due to Pearl (1995), and is further developed in (Galles and Pearl 1997; Galles and Pearl 1998; Halpern 1998; Pearl 2000). While it is hard to argue that our definition (or any other definition, for that matter) is the “right definition”, we show that it deals well with the difficulties that have plagued other approaches in the past, especially those exemplified by the rather extensive compendium of Hall (1998).

Formally, the truth of every claim must be evaluated relative to a particular model of the world and, naturally, our definition will only allow us to claim that A causes B in a (particular context in a) particular structural model. It is possible to construct

two closely related structural models such that A causes B in one and some other event A' causes B in another. Among other things, the modeler must decide which variables (events) to reason about and which to leave in the background. The lightning and the forest fire example already shows the impact of such decisions: if we include “oxygen” in the model, then it becomes a cause; if we leave it in the background, it does not.

We view this as a feature of our model, not a bug. It moves the question of actual causality to the right arena—debating which of two (or more) models of the world is a better representation. This, indeed, is the type of debate that goes on in informal (and legal) arguments all the time.

To keep this paper to manageable length, we spend only minimal time describing other approaches and comparing ours to them. We refer the reader to (Hall 1998; Sosa and Tooley 1993; Pearl 2000) for details and criticism of the probabilistic and logical approaches to causality in the philosophy literature, and to (Chajewska and Halpern 1997; Gärdenfors 1988; Hempel 1965; Pearl 1988; Salmon 1989) for some background on explanation. There has been other recent work in the AI literature on causality; see, for example, (Heckerman and Shachter 1995). Typically, this work considers when a random variable X is the cause of a random variable Y ; by way of contrast, we focus on when an *event* such as $X = x$ causes an event such as $Y = y$. Nevertheless, many of the examples that are problematic for the causality relation between events considered in the philosophy literature apply with essentially no change to the definition causality relation between random variables.

The best judge of the adequacy of an approach are the intuitive appeal of the definitions and how well it deals with examples; we believe that this paper shows that our approach fares well on both counts. We remark that although the definition of causality in this paper is inspired by an earlier paper of the second author (Pearl 1998) which defined actual causality in terms of a construction called a *causal beam*, (see also (Pearl 2000, Chapter 10)), the technical details are quite different, and the resulting definition is more transparent.

The remainder of the paper is organized as follows. In the next section, we review structural models. In Section 3 we give our definition of actual causality. In Section 4, we show how our approach deals with some examples of causality that have been problematic for other accounts. In Section 5 we define explanation and show how it again gives the “intuitively reasonable” answer in a number of examples. We conclude in Section 6 with some discussion.

2 Causal Models: A Review

In this section we review the basic definitions of causal models, as defined in terms of structural equations, and the syntax and semantics of a language for reasoning about causality.

Causal Models: The description of causal models given here is taken from (Halpern 1998); the reader is referred to (Galles and Pearl 1997; Galles and Pearl 1998; Halpern 1998) for more details, motivation, and intuition.

The basic picture here is that we are interested in the values of random variables. If X is a random variable, a typical event has the form $X = x$. (In terms of possible worlds, this just represents the set of possible worlds where X takes on value x , although the model does not describe the set of possible worlds.) Some random variables may have a causal influence on others. This influence is modeled by a set of *structural equations*, where each equation represents a distinct mechanism (or law) in the world, one that may be modified (by external actions) without altering the others. In practice, it seems useful to split the random variables into two sets, the *exogenous* variables, whose values are determined by factors outside the model, and the *endogenous* variables. It is these endogenous variables whose values are described by the structural equations.

More formally, a *signature* \mathcal{S} is a tuple $(\mathcal{U}, \mathcal{V}, \mathcal{R})$, where \mathcal{U} is a finite set of exogenous variables, \mathcal{V} is a set of endogenous variables and \mathcal{R} associates with every variable $Y \in \mathcal{U} \cup \mathcal{V}$ a nonempty set $\mathcal{R}(Y)$ of possible values for Y (that is, the set of values over which Y ranges). A *causal model* (or *structural model*) over signature \mathcal{S} is a tuple $M = (\mathcal{S}, \mathcal{F})$ where \mathcal{F} associates with each variable $X \in \mathcal{V}$ a function denoted F_X such that $F_X : (\times_{U \in \mathcal{U}} \mathcal{R}(U)) \times (\times_{Y \in \mathcal{V} - \{X\}} \mathcal{R}(Y)) \rightarrow \mathcal{R}(X)$. F_X tells us the value of X given the values of all the other variables in $\mathcal{U} \cup \mathcal{V}$. We typically write $X = F_X(\mathcal{U}, \mathcal{V} - \{X\})$, to show that X is determined by the setting of the remaining variables. The functions F_X define a set of (*modifiable*) *structural equations*, relating the values of the variables. Because F_X is a function, there is a unique value of X once we have set all the other variables. Notice that we have such functions only for the endogenous variables. The exogenous variables are taken as given; it is their effect on the endogenous variables (and the effect of the endogenous variables on each other) that we are modeling with the structural equations.

An equation such as $X = F_X(\vec{u}, y)$ should be thought of as saying that in a context where the exogenous variables have values \vec{u} , if Y were set to y , then X would take on the value x . Thus, structural equations encode counterfactual information. They may be give a “closest world” interpretation: $X = F_X(\vec{u}, y)$ if $X = x$ in the “closest world” to \vec{u} where $Y = y$. Although setting Y to y may cause X to take on value x , it does not follow that setting X to x would have an influence on the value that Y takes. Thus, there may not be an equation $Y = F_Y(\vec{u}, x)$ corresponding to the equation for X . More precisely, there will be an equation for Y (as there is for every endogenous variable), but it may not be the result of rewriting the equation for X to put Y on the left-hand side of the equality. For example, if F_X is $Y + Z$, it does not follow that F_Y is $Z - X$. (That is, just because there is an equation saying that $X = Y + Z$, there is not necessarily a corresponding equation saying that $Y = Z - X$.) The equal sign in a structural equation does more than just describe an equality relationship between variables; it acts more like an assignment statement in programming languages. This should become clearer in our examples.

Example 2.1: Suppose that we want to reason about a forest fire that could be caused by either lightning or a match lit by an arsonist. Then the causal model would have the following endogenous variables (and perhaps others):

- F for fire ($F = 1$ if there is one, $F = 0$ otherwise)
- L for lightning ($L = 1$ if lightning occurred, $L = 0$ otherwise)
- ML for match lit ($ML = 1$ if the match was lit and 0 otherwise).

The set \mathcal{U} of exogenous variables includes things we need to assume so as to render all relationships deterministic (such as whether the wood is dry, there is enough oxygen in the air, etc.). Suppose that \vec{u} is a setting of the exogenous variables that makes a forest fire possible (i.e., the wood is sufficiently dry, there is oxygen in the air, and so on). Then, for example, $F_F(\vec{u}, L, ML)$ is such that $F = 1$ if either $L = 1$ or $ML = 1$. ■

Given a causal model $M = (\mathcal{S}, \mathcal{F})$, a (possibly empty) vector \vec{X} of variables in \mathcal{V} , and vectors \vec{x} and \vec{u} of values for the variables in \vec{X} and \mathcal{U} , respectively, we can define a new causal model denoted $M_{\vec{X} \leftarrow \vec{x}}$ over the signature $\mathcal{S}_{\vec{X}} = (\mathcal{U}, \mathcal{V} - \vec{X}, \mathcal{R}|_{\mathcal{V} - \vec{X}})$.¹ $M_{\vec{X} \leftarrow \vec{x}}$ is called a *submodel* of M by Pearl (2000). Intuitively, this is the causal model that results when the variables in \vec{X} are set to \vec{x} by external action, the cause of which is not modeled explicitly. Formally, $M_{\vec{X} \leftarrow \vec{x}} = (\mathcal{S}_{\vec{X}}, \mathcal{F}^{\vec{X} \leftarrow \vec{x}})$, where $F_Y^{\vec{X} \leftarrow \vec{x}}$ is obtained from F_Y by setting the values of the variables in \vec{X} to \vec{x} .

It may seem strange that we are trying to understand causality using causal models, which clearly already encode causal relationships. Our reasoning is not circular. Our aim is not to reduce causation to noncausal concepts, but to interpret questions about causes of specific events in fully specified scenarios in terms of generic causal knowledge such as what we obtain from the equations of physics. The causal models encode background knowledge about the tendency of certain event types to cause other event types (such as the fact that lightning can cause forest fires). We use the models to determine the causes and explanations of single (or token) events, such as whether it was arson that caused the fire of June 10, 2000, given what is known or assumed about that particular fire.

Notice that, in general, there may not be a unique vector of values that simultaneously satisfies the equations in $M_{\vec{X} \leftarrow \vec{x}}$; indeed, there may not be a solution at all. For simplicity in this paper, we restrict attention to what are called *recursive* (or *acyclic*) equations. This is the special case where there is some total ordering \prec of the variables in \mathcal{V} such that if $X \prec Y$, then F_X is independent of the value of Y ; i.e., $F_X(\dots, y, \dots) = F_X(\dots, y', \dots)$ for all $y, y' \in \mathcal{R}(Y)$. Intuitively, if a theory is recursive, there is no feedback. If $X \prec Y$, then the value of X may affect the value of Y , but the value of Y has no effect on the value of X . We do not lose much generality by restricting to recursive models (that

¹We are implicitly identifying the vector \vec{X} with the subset of \mathcal{V} consisting of the variables in \vec{X} . $\mathcal{R}|_{\mathcal{V} - \vec{X}}$ is the restriction of \mathcal{R} to the variables in $\mathcal{V} - \vec{X}$.

is, ones whose equations are recursive). As suggested in the latter half of Example 4.2, it is always possible to timestamp events to impose an ordering on variables and thus construct a recursive model corresponding to a story. In any case, in the appendix, we sketch the necessary modifications of our definitions to deal with nonrecursive models.

It should be clear that if M is a recursive causal model, then there is always a unique solution to the equations in $M_{\vec{X} \leftarrow \vec{x}}$, given a setting \vec{u} for the variables in \mathcal{U} (we call such a setting \vec{u} a *context*). We simply solve for the variables in the order given by \prec .

We can describe (some salient features of) a causal model M using a *causal network*. This is a graph with nodes corresponding to the random variables in \mathcal{V} and an edge from a node labeled X to one labeled Y if F_Y depends on the value of X . This graph is a *dag*—a directed, acyclic graph. The acyclicity follows from the assumption that the equations are recursive. Intuitively, variables can have a causal effect only on their descendants in the causal network; if Y is not a descendant of X , then a change in the value of X has no affect on the value of Y .

These causal networks, which are similar in spirit to the Bayesian networks used to represent and reason about dependences in probability distributions (Pearl 1988), will play a significant role in our definitions. They are quite similar in spirit to Lewis’s *neuron diagrams* (1986), but there are significant differences as well. Roughly speaking, neuron diagrams display explicitly the functional relationships (among variables in \mathcal{V}) for each specific context \vec{u} . The class of functions represented by neuron diagram is limited to those described by “stimulatory” and “inhibitory” binary inputs. Causal networks represent arbitrary functional relationships, although the exact nature of the functions is specified in the structural equations and is not encoded in the diagram. The structural equations carry all the information we need to do causal reasoning, including all the information about belief, causation, intervention, and counterfactual behavior.

As we shall see, there are many nontrivial decisions to be made when choosing the structural model. The exogenous variables to some extent encode the background situation, that which we wish to take for granted. Other implicit background assumptions are encoded in the structural equations themselves. Suppose that we are trying to decide whether a lightning bolt or a match was the cause of the forest fire, and we want to take for granted that there is sufficient oxygen in the air and the wood is dry. We could model the dryness of the wood by an exogenous variable D with values 0 (the wood is wet) and 1 (the wood is dry).² By making D exogenous, its value is assumed to be given and out of the control of the modeler. We could also take the amount of oxygen as an exogenous variable (for example, there could be a variable O with two values—0, for insufficient oxygen, and 1, for sufficient oxygen); alternatively, we could choose not to model oxygen explicitly at all. For example, suppose we have, as before, a random variable ML for match lit, and another variable WB for wood burning, with values 0 (it’s not) and 1 (it is). The structural equation F_{WB} would describe the dependence of WB on D and ML .

²Of course, in practice, we may want to allow D to have more values, indicating the degree of dryness of the wood, but that level of complexity is unnecessary for the points we are trying to make here.

By setting $F_{WB}(1, 1) = 1$, we are saying that the wood will burn if the match is lit and the wood is dry. Thus, the equation is implicitly modeling our assumption that there is sufficient oxygen for the wood to burn. If we were to explicitly model the amount of oxygen in the air (which certainly might be relevant if we were analyzing fires on Mount Everest), then F_{WB} would also take values of O as an argument.³

Besides encoding some of our implicit assumptions, the structural equations can be viewed as encodings the causal mechanisms at work. Changing the underlying causal mechanism can affect what counts as a cause. We shall see several examples of the importance of the choice of random variables and the choice of causal mechanism in Section 4.

Note that we are not claiming that it is always straightforward to decide what the “right” causal model is in a given situation, nor that it is always obvious which of two causal models is “better” in some sense. These may be difficult decisions and often lie at the heart of determining actual causality in the real world. Nevertheless, we believe that the tools we provide here should help in making principled decisions about those choices.

Syntax and Semantics: To make the definition of actual causality precise, it is helpful to have a logic with a formal syntax. Given a signature $\mathcal{S} = (\mathcal{U}, \mathcal{V}, \mathcal{R})$, a formula of the form $X = x$, for $X \in \mathcal{V}$ and $x \in \mathcal{R}(X)$, is called a *primitive event*. A *basic causal formula* (over \mathcal{S}) is one of the form $[Y_1 \leftarrow y_1, \dots, Y_k \leftarrow y_k]\varphi$ where

- φ is a Boolean combination of primitive events,
- Y_1, \dots, Y_k, X are variables in \mathcal{V} , with Y_1, \dots, Y_k are distinct,
- $x \in \mathcal{R}(X)$, and
- $y_i \in \mathcal{R}(Y_i)$.

Such a formula is typically abbreviated as $[\vec{Y} \leftarrow \vec{y}]\varphi$. The special case where $k = 0$ is abbreviated as φ . $[Y_1 \leftarrow y_1, \dots, Y_k \leftarrow y_k]\varphi$ says that φ holds in the counterfactual world that would arise if Y_i is set to y_i , $i = 1, \dots, k$. As we now show, the semantics guarantees that this counterfactual world is the one where the values of the variables are those in the unique solution to the equations obtained after setting the variables in \vec{Y} to the values \vec{y} .

A basic causal formula is true or false in a causal model, given a context \vec{u} . We write $(M, \vec{u}) \models \varphi$ if the causal formula φ is true in causal model M given \vec{u} .⁴ $(M, \vec{u}) \models [\vec{Y} \leftarrow$

³If there are other variables in the model, these would be arguments to F_{WB} as well; we have ignored other variables here just to make our point.

⁴We remark that in (Galles and Pearl 1997; Galles and Pearl 1998; Halpern 1998), the context \vec{u} does not appear on the left-hand side of \models ; rather, it is incorporated in the formula φ on the right-hand (so that a basic formula becomes $X(\vec{u}) = x$). Additionally, Pearl (2000) abbreviated $(M, \vec{u}) \models [\vec{Y} \leftarrow \vec{y}](X = x)$ as $X_y(u) = x$. The presentation here makes certain things more explicit, although they are technically equivalent.

$\vec{y}] (X = x)$ if in the (unique, since we are dealing with recursive models) solution to the equations in $M_{\vec{Y} \leftarrow \vec{y}}$ in context \vec{u} (that is, the unique vector of values for the exogenous variables that simultaneously satisfies all equations $F_Z^{\vec{Y} \leftarrow \vec{y}}$, $Z \in \mathcal{V} - \vec{Y}$, with the variables in \mathcal{U} set to \vec{u}) the variable X has value x . $(M, \vec{u}) \models [\vec{Y} \leftarrow \vec{y}] \varphi$ for an arbitrary Boolean combination φ of formulas of the form $\vec{X} = \vec{x}$ is defined similarly.

Note that the structural equations are deterministic. We can make sense out of probabilistic counterfactual statements, even conditional ones (the probability that X would be 3 if Y_1 were 2, given that Y is in fact 1) in this framework (see (Balke and Pearl 1994)), by putting a probability on the set of possible contexts. This will not be necessary for our discussion of causality, although it will play a more significant role in the discussion of explanation.

3 The definition of cause

With all this notation in hand, we can now give our definition of actual cause (“cause” for short). We want to make sense out of statements of the form “event A is an actual cause of event B (in context \vec{u})”.⁵ As we said earlier, the context is the background information that has usually been left implicit in many other treatments of causality; here we make it explicit. The picture here is that the context (and the structural equations) are given. Intuitively, they encode the background knowledge. All the relevant events are known. The only question is picking out which of them are the causes of φ or, alternatively, testing whether a given set of events can be considered the cause of φ .⁶

The types of events that we consider as actual causes are ones of the form $X_1 = x_1 \wedge \dots \wedge X_k = x_k$ —that is, conjunctions of primitive events; we typically abbreviate this as $\vec{X} = \vec{x}$. The events that can be caused are arbitrary Boolean combinations φ of primitive events.

In fact, in all our examples, the cause is just a primitive event of the form $X = x$. This is typically the case in all the examples from the philosophy literature as well. However, as we show in the appendix, we gain some important generality by allowing conjunctions of primitive events as causes. We might consider generalizing further to allow disjunctive. We do not believe that we lose much by disallowing disjunctive causes here. Since for causality we are assuming that the structural model and all the relevant facts are known, the only reasonable definition of “ A or B causes φ ” seems to be that “either A causes φ or B causes φ ”. There are no truly disjunctive causes once all the relevant facts are

⁵Note that we are using the word “event” here in the standard sense of “set of possible worlds” (as opposed to “transition between states of affairs”); essentially we are identifying events with propositions.

⁶We use both past tense and present tense in our examples (“was the cause” versus “is the cause”), with the usage depending on whether the scenario implied by the context \vec{u} is perceived to have taken place in the past or to persist through the present.

known.⁷ For similar reasons, there is no need to talk about probabilistic causality at the model level, since the model is assumed deterministic (although, as we shall see, probability will play a much more important role in explanation; see also the discussion of ranking functions after Example 16.

Definition 3.1: (Actual cause) $\vec{X} = \vec{x}$ is an *actual cause* of φ in (M, \vec{u}) if the following three conditions hold:

- AC1. $(M, \vec{u}) \models (\vec{X} = \vec{x}) \wedge \varphi$. (That is, both $\vec{X} = \vec{x}$ and φ are true in the actual world.)
- AC2. There exists a partition (\vec{Z}, \vec{W}) of \mathcal{V} with $\vec{X} \subseteq \vec{Z}$ and some setting (\vec{x}', \vec{w}') of the variables in (\vec{X}, \vec{W}) such that if $(M, \vec{u}) \models Z = z^*$ for $Z \in \vec{Z}$, then
 - (a) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}'] \neg \varphi$. In words, changing (\vec{X}, \vec{W}) from (\vec{x}, \vec{w}) to (\vec{x}', \vec{w}') changes φ from true to false
 - (b) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}', \vec{Z}' \leftarrow \vec{z}^*] \varphi$ for all subsets \vec{Z}' of \vec{Z} . In words, setting \vec{W} to \vec{w}' should have no effect on φ as long as \vec{X} is kept at its current value \vec{x} , even if all the variables in an arbitrary subset of \vec{Z} are set to their original values in the context \vec{u} .
- AC3. \vec{X} is minimal; no subset of \vec{X} satisfies conditions AC1 and AC2. Minimality ensures that only those elements of the conjunction $\vec{X} = \vec{x}$ that are essential for changing φ in AC2(a) are considered part of a cause; inessential elements are pruned. ■

Note that we allow $X = x$ to be a cause of itself. While we do not find such trivial causality terribly bothersome, it can be avoided by requiring that $\vec{X} = \vec{x} \wedge \neg \varphi$ be consistent for $\vec{X} = \vec{x}$ to be a cause of φ . The core of this definition lies in AC2. Informally, the variables in \vec{Z} should be thought of as describing the “active causal process” from \vec{X} to φ (also called “intrinsic process” by Lewis (1986)).⁸ These are the variables that mediate between \vec{X} and φ . Indeed, we can define an *active causal process* from $\vec{X} = \vec{x}$ to φ as a minimal set \vec{Z} that satisfies AC2. We would expect that the variables in an active causal process are all on a path from a variable in \vec{X} to a variable in φ . This is indeed the case. Moreover, it can easily be shown that the variables in an active causal process all change their values when (\vec{X}, \vec{W}) is set to (\vec{x}', \vec{w}') as in AC2. Any variable that does not change in this transition can be moved to \vec{W} , while retaining its value in \vec{w}' —the remaining variables in \vec{Z} will still satisfy AC2. (See the appendix for a formal proof.) AC2(a) says that there exists a setting \vec{x}' of \vec{X} that changes φ to $\neg \varphi$, as long as the variables not involved in the causal process (\vec{W}) take on value \vec{w}' . AC2(a) is reminiscent of the traditional

⁷Having said that, see the end of Example 3.2 for further discussion of this issue. Disjunctive *explanations* seem more interesting, although we cannot handle them well in our framework; see Section 5.

⁸Recently, Lewis (2000) has abandoned attempts to define “intrinsic process” formally. Pearl’s “causal beam” (Pearl 2000, p. 318) is a special kind of active causal process, in which AC2(b) is expected to hold (with $\vec{Z} = \vec{z}^*$) for all settings \vec{w}' of \vec{W} , not necessarily the one used in (a).

counterfactual criterion of Lewis (1986), according to which φ should be false if it were not for \vec{X} being \vec{x} . However, AC2(a) is more permissive than the traditional criterion; it allows the dependence of φ on \vec{X} to be tested under special circumstances in which the variables \vec{W} are held constant at some setting \vec{w}' . This modification of the traditional criterion was proposed by Pearl (1998, 2000) and was named *structural contingency*—an alteration of the model M that involves the breakdown of some mechanisms (possibly emerging from external action) but no change in the context \vec{u} . The need to invoke such contingencies will be made clear in Example 3.2, and is further supported by the examples of Hitchcock (1999).

AC2(b) is an attempt to counteract the “permissiveness” of AC2(a) with regard to structural contingencies. Essentially, it ensures that \vec{X} alone suffices to bring about the change from φ to $\neg\varphi$; setting \vec{W} to \vec{w}' merely eliminates spurious side effects that tend to mask the action of \vec{X} . It captures the fact that setting \vec{W} to \vec{w}' does not affect the causal process by requiring that changing \vec{W} from \vec{w} to \vec{w}' has no effect on the value of φ . Moreover, although the values in the variables \vec{Z} involved in the causal process may be perturbed by the change, the perturbation has no impact on the value of φ . The upshot of this requirement is that we are not at liberty to conduct the counterfactual test of AC2(a) under an arbitrary alteration of the model. The alteration considered must not have an effect of its own on the causal process. Clearly, if the contingencies considered are limited to “freezing” variables at their actual value (a restriction used by Hitchcock (1999)), so that $(M, \vec{u}) \models \vec{W} = \vec{w}$, then AC2(b) is satisfied automatically. However, as the examples below show, genuine causation may sometimes be revealed only through a broader class of counterfactual tests in which variables in \vec{W} are set to values that differ from their actual values. Interestingly, in all our examples in Section 4, changing \vec{W} from \vec{w} to \vec{w}' has no impact on the value of the variables in \vec{Z} . That is, $(M, \vec{u}) \models [\vec{W} \leftarrow \vec{w}'](Z = z^*)$ for all $Z \in \vec{Z}$. Thus, to check AC2(b) in these examples, it suffices to show that $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}']\varphi$. We provide an example in the appendix to show that there are cases where the variables in \vec{Z} can change value, so the full strength of AC2(b) is necessary.

AC3 is a minimality condition. Interestingly, in all the examples we have considered, AC3 forces the cause to be a single conjunct of the form $X = x$. We conjecture that this is in fact a consequence of our definition although we have not been able to prove it.

How reasonable are these requirements? In particular, is it appropriate to invoke structural changes in the definition of actual causation? The following example may help illustrate why we believe it is.

Example 3.2: Suppose that two arsonists drop lit matches in different parts of a dry forest, and both cause trees to start burning. Consider two scenarios. In the first, called “disjunctive,” either match by itself suffices to burn down the whole forest. That is, even if only one match were lit, the forest would burn down. In the second scenario, called “conjunctive,” both matches are necessary to burn down the forest; if only one match

were lit, the fire would die down. We can describe the essential structure of these two scenarios using a causal model with four variables:

- an exogenous variable U which determines, among other things, the motivation and state of mind of the arsonists. For simplicity, assume that $\mathcal{R}(U) = \{u_{00}, u_{10}, u_{01}, u_{11}\}$; if $U = u_{ij}$, then the first arsonist intends to start a fire iff $i = 1$ and the second arsonist intends to start a fire iff $j = 1$. In both scenarios $U = u_{11}$.
- endogenous variables ML_1 and ML_2 , each either 0 or 1, where $ML_i = 0$ if arsonist i doesn't drop the match and $ML_i = 1$ if he does, for $i = 1, 2$.
- an endogenous variable FB for forest burns down, with values 0 (it doesn't) and 1 (it does).

Both scenarios have the same causal network (see Figure 1); they differ only in the equation for FB . For the disjunctive scenario we have $F_{FB}(u, 1, 1) = F_{FB}(u, 0, 1) = F_{FB}(u, 1, 0) = 1$ and $F_{FB}(u, 0, 0) = 0$ (where $u \in \mathcal{R}(U)$); for the conjunctive scenario we have $F_{FB}(u, 1, 1) = 1$ and $F_{FB}(u, 0, 0) = F_{FB}(u, 1, 0) = F_{FB}(u, 0, 1) = 0$. In general, we expect that the causal model for reasoning about forest fires would involve many other variables; in particular, variables for other potential causes of forest fires such lightning and unattended campfires; here we focus on that part of the causal model that involves forest fires started by arsonists. Since for causality we assume that all the relevant facts are given, we can assume here that it is known that there were no unattended campfires and there was no lightning, which makes it safe to ignore that portion of the causal model. Denote by M_1 and M_2 the (portion of the) causal models associated with the disjunctive and conjunctive scenarios, respectively. The causal network for the relevant portion of M_1 and M_2 is described in Figure 1.

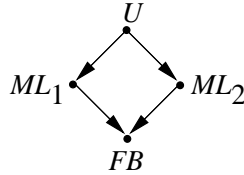


Figure 1: The causal network for M_1 and M_2 .

Despite the differences in the underlying models, each of $ML_1 = 1$ and $ML_2 = 1$ is a cause of $FB = 1$ (representing φ) in both scenarios. We present the argument for $ML_1 = 1$ here. To show that $ML_1 = 1$ is a cause in M_1 let $\vec{Z} = \{ML_1, FB\}$, so $\vec{W} = \{ML_2\}$. It is easy to see that the contingency $ML_2 = 0$ satisfies the two conditions in AC2. AC2(a) is satisfied because, in the absence of the second arsonist ($ML_2 = 0$), the first arsonist is necessary and sufficient for the fire to occur ($FB = 1$). AC2(b) is satisfied because, if the first match is lit ($ML_1 = 1$) the contingency $ML_2 = 0$ does not prevent the fire from burning the forest. Thus, $ML_1 = 1$ is a cause of $FB = 1$ in M_1 . (Note that we needed to

set ML_2 to 0, contrary to facts, in order to reveal the latent dependence of FB on ML_1 . Such a setting constitutes a structural change in the original model, since it involves the removal of some structural equations.)

To see that $ML_1 = 1$ is also a cause of $FB = 1$ in M_2 , again let $\vec{Z} = \{ML_1, FB\}$ and $\vec{W} = \{ML_2\}$. Since $(M_2, u_{11}) \models [ML_1 \leftarrow 0, ML_2 \leftarrow 1](FB = 0)$, AC2(a) is satisfied. Moreover, since the value of ML_2 required for AC2(a) is the same as its current value (i.e., $w' = w$), AC2(b) is satisfied trivially.

This example also illustrates the need for the minimality condition AC3. For example, if lighting a match qualifies as the cause of fire then lighting a match and sneezing would also pass the tests of AC1 and AC2 and awkwardly qualify as the cause of fire. Minimality serves here to strip “sneezing” and other irrelevant, over-specific details from the cause.

It might be argued that allowing disjunctive causes would be useful in this case to distinguish M_1 from M_2 as far as causality goes. A purely counterfactual definition of causality would make $ML_1 = 1 \vee ML_2 = 1$ a cause of $FB = 1$ in M_1 (since, if $ML_1 = 1 \vee ML_2 = 1$ were not true, then $FB = 1$ would not be true), but would make neither $ML_1 = 1$ nor $ML_2 = 1$ individually a cause (since, for example, if $ML_1 = 1$ were not true in M_1 , $FB = 1$ would still be true). Clearly, our definition does not enforce this intuition. As is well known (and as the examples in Section 4 show) purely counterfactual definitions of causality have other problems. We do not have a strong intuition as to the best way to deal with disjunction in the context of causality, and believe that disallowing it is reasonably consistent with intuitions.

Interestingly, as we shall see in Section 5, our definition of explanation *does* distinguish M_1 from M_2 ; each of $ML_1 = 1$ and $ML_2 = 1$ is an explanation of $FB = 1$ in M_1 under our definition of explanation, but neither is an explanation of $FB = 1$ in M_2 . In M_2 , the explanation of $FB = 1$ is $ML_1 = 1 \wedge ML_2 = 1$: both matches being lit are necessary to explain the forest burning down. ■

This is a good place to illustrate the need for structural contingencies in the analysis of actual causation. The reason we consider $ML_1 = 1$ to be a cause of $FB = 1$ in M_1 is that *if* ML_2 had been 0, rather than 1, FB would depend on ML_1 . In words, we imagine a situation in which the second match is not lit, and we then reason counterfactually that the forest would not have burned down if it were not for the first match. But how do we express this formally in a context u in which $ML_2 = 1$ is in fact true? To (hypothetically) suppress $ML_2 = 1$ in the context created by u_{11} , we must use a structural contingency and imagine that ML_2 is set to 0 by some external intervention (or “miracle”) that violates whatever causal laws (or equations) made $ML_2 = 1$ in u_{11} . For example, if u_{11} includes the motivations and conspiratorial plans of the two arsonists, then ML_2 may be 0 due to a mechanical failure (say, the match box fell into a creek) or to arsonist 2 having second thoughts. We know perfectly well that these changes did not occur, yet we are committed to contemplating such changes by the very act of representing our story in the form of a multi-stage causal model, with each stage representing an autonomous mechanism.

Recalling that a causal model actually stands, not for one, but for a whole set of models, one for each possible setting of the endogenous variables, contemplating interventional contingencies is an intrinsic part of every causal thought. In other words, the autonomy of the mechanisms in the model means that each mechanism advertises its possible breakdown, and these breakdowns signal contingencies against which causal expressions should be evaluated. It is reasonable, therefore, that we build such contingencies into the definition of actual causation.

Although $ML_1 = 1$ is a cause of $FB = 1$ in both the disjunctive and conjunctive scenarios, the models M_1 and M_2 differ in regard to explanation, as we shall see in Section 5. In the disjunctive scenario, the lighting of one of the matches constitutes a reasonable explanation of the forest burning down; not so in the conjunctive scenario. Intuitively, we feel that if both matches are needed for establishing a forest fire, then both $ML_1 = 1$ and $ML_2 = 1$ together would be required to fully explain the unfortunate fate of the forest; pointing to just one of these events would only beg another “How come?” question, and would not stop any serious investigating team from continuing its search for a more complete answer.

Finally, a remark concerning a *contrastive* extension to the definition of cause. When seeking a cause of φ , we are often not just interested the occurrence versus nonoccurrence of φ , but also the manner in which φ occurred, as opposed to some alternative way in which φ could have occurred (Hitchcock 1996). We say, for example, “ $X = x$ caused a fire in June as opposed to a fire in May.” If we assume that there is only enough wood in the forest for one forest fire, the two contrasted events, “fire in May” and “fire in June”, exclude but do not complement each other (e.g., neither rules out a fire in April.) Definition 3.1 can easily be extended to accommodate *contrastive causation*. We define “ x caused φ , as opposed to φ' ”, where φ and φ' are incompatible but not exhaustive, by simply replacing $\neg\varphi$ with φ' in condition AC2(a) of the definition.

Contrast can also be applied to the antecedent, as in “Susan’s running, rather than walking, to music class caused her fall.” to be distinguished from the false sentence “Susan’s running to music rather than history class caused her fall.” We can capture sentences of the form “ $X = x$, rather than $X = x'$ for some value $x' \neq x$, caused φ ” by taking this to mean that $X = x$ caused φ and AC2(b) holds for $X = x'$ and φ . That is, the only reason that $X = x'$ is not the cause of φ is that $X = x'$ is not in fact what happened in the actual world.⁹ $X = x'$ did not cause φ . (More generally, we can make sense of “ $X = x$ rather than $Y = y$ caused φ .”) Contrasting both the antecedent and the consequent components is straightforward, and allows us to interpret sentences of the form: “Susan’s running, rather than walking to music class caused her to spend the night in the hospital, as opposed to her boyfriend’s apartment.”

⁹As Christopher Hitchcock [private communication, 2000] has pointed out, one of the roles of such contrastive statements is to indicate that $\mathcal{R}(X)$ should include x' . The sentence does not make sense without this assumption.

4 Examples

In this section we show how our definition of actual causality handles some examples that have caused problems for other definitions.

Example 4.1: The first example is due to Bennett (and appears in (Sosa and Tooley 1993, pp. 222–223)). Suppose that there was a heavy rain in April and electrical storms in the following two months; and in June the lightning took hold. If it hadn’t been for the heavy rain in April, the forest would have caught fire in May. The question is whether the April rains caused *the* forest fire. According to a naive counterfactual analysis, they do, since if it hadn’t rained, there wouldn’t have been a forest fire in June. Bennett says “That is unacceptable. A good enough story of events and of causation might give us reason to accept some things that seem intuitively to be false, but no theory should persuade us that delaying a forest’s burning for a month (or indeed a minute) is causing a forest fire.”

In our framework, as we now show, it is indeed false to say that the April rains caused *the* fire, but they were a cause of there being a fire in June, as opposed to May. This seems to us intuitively right. To capture the situation, it suffices to use a simple model with three endogenous random variables:

- AS for “April showers”, with two values—0 standing for did *not* rain heavily in April and 1 standing for rained heavily in April;
- ES for “electric storms”, with four possible values: (0,0) (no electric storms in either May or June), (1,0) (electric storms in May but not June), (0,1) (storms in June but not May), and (1,1) (storms in both April and May);
- and F for “fire”, with three possible values: 0 (no fire at all), 1 (fire in May), or 2 (fire in June).

We do not describe the context explicitly, either here or in the other examples. Assume its value \vec{u} is such that it ensures that there is a shower in April, there are electric storms in both May and June, there is sufficient oxygen, there are no other potential causes of fire (like dropped matches), no other inhibitors of fire (alert campers setting up a bucket brigade), and so on. That is, we choose \vec{u} so as to allow us to focus on the issue at hand and to ensure that the right things happened (there was both fire and rain).

We will not bother writing out the details of the structural equations—they should be obvious, given the story (at least, for the context \vec{u}); this is also the case for all the other examples in this section. The causal network is simple: there are edges from AS to F and from ES to F . It is easy to check that each of the following hold.

- $AS = 1$ is a cause of the June fire ($F = 2$) (taking $\vec{W} = \{ES\}$ and $\vec{Z} = \{AS, F\}$) but not of fire ($F = 2 \vee F = 1$).

- $ES = (1, 1)$ is a cause of both $F = 2$ and $(F = 1 \vee F = 2)$. Having electric storms in both May and June caused there to be a fire.
- $AS = 1 \wedge ES = (1, 1)$ is not a cause of $F = 2$, because it violates the minimality requirement of AC3; each conjunct alone is a cause of $F = 2$. Similarly, $AS = 1 \wedge ES = (1, 1)$ is not a cause of $(F = 1 \vee F = 2)$.

The distinction between April showers being a cause of the fire (which they are not, according to our analysis) and April showers being a cause of a fire in June (which they are) is one that seems not to have been made in the discussion of this problem (cf. (Lewis 2000)); nevertheless, it seems to us an important distinction. ■

Although we did not describe the context explicitly in Example 4.1, it still played a crucial role. If we decide that the presence of oxygen is relevant then we must take this factor out of the context and introduce it as an explicit endogenous variables. Doing so can affect the causality picture (see Example 4.3); as we shall see in Section 5, it can have an even bigger impact on explanations. The next example already shows the importance of choosing an appropriate granularity in modeling the causal process and its structure.

Example 4.2: The following story from (Hall 1998) is an example of an *overdetermined* event.

Suzy and Billy both pick up rocks and throw them at a bottle. Suzy’s rock gets there first, shattering the bottle. Since both throws are perfectly accurate, Billy’s would have shattered the bottle if Suzy’s had not occurred, so the shattering is overdetermined.

Common sense suggests that Suzy’s throw is the cause of the shattering, but Billy’s is not. This holds in our framework too, but only if we model the story appropriately. Consider first a coarse causal model, with three endogenous variables:

- ST for “Suzy throws”, with values 0 (Suzy does not throw) and 1 (she does);
- BT for “Billy throws”, with values 0 (he doesn’t) and 1 (he does);
- BS for “bottle shatters”, with values 0 (it doesn’t shatter) and 1 (it does).

Again, we have a simple causal network, with edges from both ST and BT to BS . In this simple causal network, BT and BS play absolutely symmetric roles, with $BS = ST \vee BT$, and there is nothing to distinguish one from the other. Not surprisingly, both Billy’s throw and Suzy’s throw are classified as causes of the bottle shattering.

The trouble with this model is that it cannot distinguish the case where both rocks hitting the bottle simultaneously (in which case it would be reasonable to say that both

$ST = 1$ and $BT = 1$ are causes of $BS = 1$), and the case at hand, where Suzy’s rock hits first. The model has to be refined to express this distinction. One way to gain expressiveness is to allow BS to be three valued, with values 0 (the bottle doesn’t shatter), 1 (it shatters as a result of being hit by Suzy’s rock), and 2 (it shatters as a result of being hit by Billy’s rock). We leave it to the reader to check that $ST = 1$ is a cause of $BS = 1$, but $BT = 1$ is not (if Suzy hadn’t thrown but Billy had, then we would have $BS = 2$). Thus, to some extent this solves our problem. But it borders on cheating; the answer is almost programmed into the model by invoking the relation “as a result of”, which requires the identification of the actual cause.

A more useful choice is to add two new random variables to the model:

- BH for “Billy’s rock hits the (intact) bottle”, with values 0 (it doesn’t) and 1 (it does); and
- SH for “Suzy’s rock hits the bottle”, again with values 0 and 1.

With this addition, we can go back to BS being two-valued. In this model, we have the causal network shown in Figure 2, with the arrow $SH \rightarrow BH$ being inhibitory; $BH = BT \wedge \neg SH$ (that is, $BH = 1$ iff $BT = 1$ and $SH = 0$). Note that, to simplify the presentation, we have omitted the exogenous variables from the causal network in Figure 2; we do so in some of the subsequent figures as well. In addition, we have only given the arrows for the particular context of interest, where Suzy throws first. In a context where Billy throws first, the arrow would go from BH to SH rather than going from SH to BH , as it does in the figure.

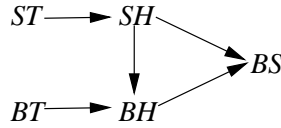


Figure 2: The rock-throwing example.

Now it is the case that $ST = 1$ is a cause of $BS = 1$. To satisfy AC2, we choose $\vec{W} = \{BT\}$ and $w' = 0$ and note that, because BT is *set* to 0, BS will track the setting of ST . Also note that $BT = 1$ is not a cause of $BS = 1$; there is no partition $\vec{Z} \cup \vec{W}$ that satisfies AC2. Attempting the symmetric choice $\vec{W} = \{BT\}$ and $w' = 0$ would violate AC2(b) (with $\vec{Z}' = \{BH\}$), because φ becomes false when we set $ST = 0$ and restore BH to its current value of 0.

This example illustrates the need for invoking subsets of \vec{Z} in AC2(b). (Additional reasons are provided in Example A.3 in the appendix.) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}'] \varphi$ holds if we take $\vec{Z} = \{BT, BH\}$ and $\vec{W} = \{ST, SH\}$, and thus without the requirement that AC2(b) hold for all subsets of \vec{Z} , $BT = 1$ would have qualified as a cause of $BS = 1$. Insisting that φ remains unchanged when both \vec{W} is set to \vec{w}' and \vec{Z}' is set to \vec{z}^* (for an

arbitrary subset \vec{Z}' of \vec{Z}) prevents us from choosing contingencies \vec{W} that interfere with the active causal paths from \vec{X} to φ .

This example also emphasizes an important moral. If we want to argue in a case of overdetermination that $X = x$ is the cause of φ rather than $Y = y$, then there must be a random variable (BH in this case) that takes on different values depending on whether $X = x$ or $Y = y$ is the actual cause. If the model does not contain such a variable, then it will not be possible to determine which one is in fact the cause. This is certainly consistent with intuition and the way we present evidence. If we want to argue (say, in a court of law) that it was X 's shot that killed C rather than Y 's, then we present evidence such as the bullet entered C from the left side (rather than the right side, which is how it would have entered had Y 's shot been the lethal one). The side from which the shot entered is the relevant random variable in this case. Note that the random variable may involve temporal evidence (if Y 's shot had been the lethal one, the death would have occurred a few seconds later), but it certainly does not have to be. This is indeed the rationale for Lewis's (1986) criterion of causation in terms of a counterfactual-dependence-chain. We shall see, however, that our definition goes beyond this criterion.

It may be argued, of course, that by introducing the intermediate variables SH and BH in Hall's story we have also programmed the desired answer into the problem; after all, it is the shattering of the bottle, not SH , which prevents BH . Pearl (2000, Section 10.3.5) analyzes a similar late-preemption problem in a dynamic structural equation models, where variables are time indexed, and shows that the selection of the first action as an actual cause of the effect follows from conditions (similar to) AC1–AC3 even without specifying the owner of the hitting ball. A simplified adaptation of this analysis is presented below.

Let t_1, t_2 , and t_3 stand, respectively, for the time that Suzy threw her rock, the time that Billy threw his rock, and the time that the bottle was found shattered. Let H_i and BS_i be variables indicating whether the bottle is hit (H_i) and shattered (BS_i) at time t_i (where $i = 1, 2, 3$ and $t_1 < t_2 < t_3$), with values 1 if hit (respectively, shattered), 0 if not. Roughly speaking, if we let T_i be a variable representing "someone throws the ball at time t_i and take BS_0 to be vacuously true (i.e., always 1), then we would expect the following time-invariant equations to hold for all times t_i (not just t_1, t_2 , and t_3):

$$\begin{aligned} H_i &= T_i \wedge \neg BS_i \\ BS_i &= BS_{i-1} \vee H_i. \end{aligned}$$

That is, the bottle is hit at time t_i if someone throws the ball at time t_i and the bottle wasn't already shattered at time t_i . Similarly, the bottle is shattered at time t_i either if it was already shattered at time t_{i-1} or it was hit at time t_i .

Since in this case we consider only times t_1, t_2 , and t_3 , we get the following structural equations, where we have left in the variable T_3 to bring out the essential invariance:

$$H_1 = ST$$

$$\begin{aligned}
BS_1 &= H_1 \\
H_2 &= BT \wedge \neg BS_1 \\
BS_2 &= BS_1 \vee H_2 \\
H_3 &= T_3 \wedge \neg BS_2 \\
BS_3 &= BS_2 \vee H_3.
\end{aligned}$$

The diagram associated with this model is shown in Figure 3. In addition to these generic equations, the story also specifies that the context is such that

$$ST = 1, BT = 1, T_3 = 0.$$

The following causal network describes the situation.

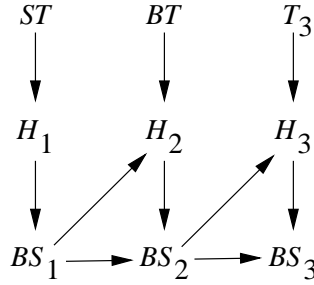


Figure 3: Time-invariant rock throwing.

It is not hard to show that $ST = 1$ is a cause of $BS_3 = 1$ (taking $\vec{W} = \{BT\}$ in AC2 and $w' = 0$). $BT = 1$ is not a cause of $BS_3 = 1$; it fails AC2(b) for every partition $\vec{Z} \cup \vec{W}$. To see this, note that to establish counterfactual dependence between BS_3 and BT , we must assign H_2 to \vec{Z} , BS_1 to \vec{W} , and impose the contingency $BS_1 = 0$. But this contingency violates condition AC2(b), since it results in $BS_3 = 0$ when we restore H_2 to 0 (its current value).

Two features are worth emphasizing in this example. First, Suzy’s throw is declared a cause of the outcome event $BS_3 = 1$ even though her throw did not hasten, delay, or change any property of that outcome. This can be made clearer by considering another outcome event, $J_4 =$ “Joe was unable to drink his favorite chocolate cocktail from that bottle on Tuesday night.” Being a consequence of BS_3 , J_4 will also be classified as having been caused by Suzy’s throw, not by Billy’s, although J_4 would have occurred at precisely the same time and in the same manner had Suzy not thrown the ball. This implies that hastening or delaying the outcome cannot be taken as the basic principle for deciding actual causation, a principle advocated by Paul (1998).

Second, Suzy’s throw is declared a cause of $BS_3 = 1$ even though there is no counterfactual dependence chain between the two (i.e., a chain $A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_k$ where each event is counterfactually dependent on its predecessor). The existence of such a chain was proposed by Lewis (1986) as a necessary criterion for causation in cases involving

preemption. In the actual context, BS_2 does not depend (counterfactually) on either BS_1 or on H_2 ; the bottle would be shattered at time t_2 even if it were unshattered at time t_1 (since Billy’s rock would have hit it), as well as if it were hit (miraculously) at time t_2 . The importance of this departure from Lewis’s account to one based on structural contingencies is further emphasized by Hitchcock (1999). ■

Example 4.3: Can *not* performing an action be (part of) a cause? Consider the following story, again taken from (Hall 1998):

Billy having stayed out in the cold too long throwing rocks, contracts a serious but nonfatal disease. He is hospitalized on Monday, but unfortunately the doctor forgets to administer the needed medication, so Billy is still sick on Tuesday.

Is the doctor’s omission to treat Billy on Monday a cause of Billy’s being sick on Tuesday? It seems that it should be, and indeed it is according to our analysis. Suppose that \vec{u} is the context where, among other things, Billy is sick on Monday and the situation is such that the doctor forgets to administer the medication Monday. (There is much more to the context \vec{u} , as we shall shortly see.) It seems reasonable that the model should have two random variables:

- MT for “Monday treatment”, with values 0 (the doctor does not treat Billy on Monday) and 1 (he does); and
- BMC for “Billy’s medical condition”, with values 0 (recovered) and 1 (still sick).

Sure enough, in the obvious causal model, $MT = 0$ is a cause of $BMC = 1$.

This may seem somewhat disconcerting at first. Suppose there are 100 doctors in the hospital. Although only one of them was assigned to Billy (and he forget to give medication), in principle, any of the other 99 doctors could have given Billy his medication. Is the fact that they didn’t give him the medication also part of the cause that he was still sick on Tuesday?

In the particular model that we have constructed, the other doctors’ failure to give Billy his medication is not a cause, since we have no random variables to model the other doctor’s actions, just as we had no random variable in Example 4.1 to model the presence of oxygen. Their lack of action is part of the context. We factor it out because (quite reasonably) we want to focus on the actions of Billy’s doctor. If we had included endogenous random variables corresponding to the other doctors, then they too would be causes of Billy’s being sick on Tuesday.

With this background, we continue with Hall’s modification of the original story.

Suppose that Monday’s doctor is reliable, and administers the medicine first thing in the morning, so that Billy is fully recovered by Tuesday afternoon. Tuesday’s doctor is also reliable, and would have treated Billy if Monday’s doctor had failed to ... And let us add a twist: one dose of medication is harmless, but two doses are lethal.

Is the fact that Tuesday doctor did *not* treat Billy the cause of him being alive (and recovered) on Tuesday afternoon?

The causal model for this story is straightforward. Since the story talks about the Tuesday treatment explicitly, we add a new random variable TT for Tuesday treatment, with values 0 and 1, just like MT (Monday treatment). We also need to augment the set of possible values of BMC to include the new possibilities. We now allow it to have 4 values—0 (recovered Tuesday morning, still alive and well Tuesday afternoon), 1 (sick Tuesday morning, recovered Tuesday afternoon), 2 (sick both Tuesday morning and afternoon), and 3 (recovered Tuesday morning, dead Tuesday afternoon). We can then describe Billy’s condition as a function of the four possible combinations of treatment/nontreatment on Monday and Tuesday.

In the causal network corresponding to this causal model, shown in Figure 4, there is an edge from MT to TT , since whether the Tuesday treatment occurs depends on whether the Monday treatment occurs, and there is an edge from both MT and TT to BMC , since Billy’s medical condition depends on both treatments.

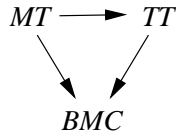


Figure 4: Billy’s medical condition.

In this causal model, it is true that $MT = 1$ is a cause of $BMC = 0$, as we would expect—because Billy is treated Monday, he is not treated on Tuesday morning, and thus recovers Tuesday afternoon.¹⁰ $MT = 1$ is also a cause of $TT = 0$, as we would expect, and $TT = 0$ is a cause of Billy’s being alive ($BMC = 0 \vee BMC = 1 \vee BMC = 2$). However, $MT = 1$ is *not* a cause of Billy’s being alive. It fails condition AC2(a): setting $MT = 0$ still leads to Billy’s being alive (with $W = \emptyset$). Note that it would not help to take $\vec{W} = \{TT\}$. For if $TT = 0$, then Billy is alive no matter what MT is, while if $TT = 1$, then Billy is dead when MT has its original value, so AC2(b) is violated (with $\vec{Z}' = \emptyset$).

This shows that causality is not transitive, according to our definitions. Although $MT = 1$ is a cause of $TT = 0$ and $TT = 0$ is a cause of $BMC = 0 \vee BMC = 1 \vee BMC = 2$,

¹⁰Lewis’s (1986) revised criterion of counterfactual-dependence-chain also fails in this example; BMC does not depend on either MT or TT in the context given.

$MT = 1$ is not a cause of $BMC = 0 \vee BMC = 1 \vee BMC = 2$. Nor is causality closed under *right weakening*: $MT = 1$ is a cause of $BMC = 0$, which logically implies $BMC = 0 \vee BMC = 1 \vee BMC = 2$, which is not caused by $MT = 1$.¹¹

Hall (1998) discusses the issue of transitivity of causality, and suggests that there is a tension between the desideratum that causality be transitive and the desideratum that we allow causality due to the failure of some event to occur. He goes on to suggest that there are actually two concepts of causation: one corresponding to counterfactual dependence and the other corresponding to “production”, whereby A causes B if A helped to produce B . Causation by production is transitive; causation by dependence is not.

Our definition certainly has some features of both counterfactual dependence and of production—AC2(a) captures some of the intuition of counterfactual dependence (if A hadn’t happened then B wouldn’t have happened if $\vec{W} = \vec{w}'$) and AC2(b) captures some of the features of production (A forced B to happen, even if $\vec{W} = \vec{w}'$). Nevertheless, we do not require two separate notions to deal with these concerns.

Moreover, whereas Hall attributes the failure of transitivity to a distinction between presence and absence of events, according to our definition, the requirement of transitivity causes problems whether or not we allow causality due to the failure of some event to occur. It is easy enough to construct a story whose causal model has precisely the same formal structure as that above, except that $TT = 0$ now means that the treatment was given and $TT = 1$ means it wasn’t. (Billy starts a course of treatment on Monday which, if discontinued once started, is fatal . . .) Again, we don’t get transitivity, but now it is because an event occurred (the treatment was given), not because it failed to occur.

Lewis (1986, 2000) insists that causality is transitive, partly to be able to deal with what is called in the literature *preemption* (Lewis 1986). An example of preemption (taken from (Hitchcock 1999)) is a scenario where an assassin-in-training, who is an excellent shot, fires and kills the victim. However, his supervisor (an equally skilled shot) is present on the mission in case the trainee loses his nerve. We would like to call the trainee the cause of the victim’s death, even though if the trainee hadn’t shot, the victim would have died anyway. As Hitchcock points out, our account handles preemption without needing to invoke transitivity: we simply take W to be such that the supervisor does not shoot. Of course, we could build transitivity into our definition of causality (by taking the transitive closure of our current definition). However, since our definition can deal with the difficulties encountered by Lewis’s definition without invoking transitivity and, as Lewis’s own examples show, assuming transitivity leads to counterintuitive conclusions regarding causality, it seems best not to build it in to the definition as a separate dictum.

■

¹¹Lewis (2000) implicitly assumes right weakening in his defense of transitivity. For example, he says “... it is because of Black’s move that Red’s victory is caused one way rather than another. That means, I submit, that in each of these cases, Black’s move did indeed cause Red’s victory. Transitivity succeeds.” But there is a critical (and to us, unjustifiable) leap in this reasoning. As we already saw in Example 4.1, the fact that April rains cause a fire in June does *not* mean that they cause the fire.

Example 4.4: This example considers the problem of what Hall calls *double prevention*. Again, the story is taken from Hall (1998):

Suzy and Billy have grown up, just in time to get involved in World War III. Suzy is piloting a bomber on a mission to blow up an enemy target, and Billy is piloting a fighter as her lone escort. Along comes an enemy fighter plane, piloted by Lucifer. Sharp-eyed Billy spots Lucifer, zooms in, pulls the trigger, and Lucifer’s plane goes down in flames. Suzy’s mission is undisturbed, and the bombing takes place as planned.

Does Billy deserve part of the cause for the success of the mission? After all, if he hadn’t pulled the trigger, Lucifer would have eluded him and shot down Suzy. Intuitively, it seems that the answer is yes, and the obvious causal model gives us this. Suppose we have the following random variables:

- *BPT* for “Billy pulls trigger”, with values 0 (he doesn’t) and 1 (he does)
- *LE* for “Lucifer eludes Billy”, with values 0 (he doesn’t) and 1 (he does);
- *LSS* for “Lucifer shoots Suzy”, with values 0 (he doesn’t) and 1 (he does);
- *SST* for “Suzy shoots target”, with values 0 (she doesn’t) and 1 (she does);
- *TD* for “target destroyed”, with values 0 (it isn’t) and 1 (it is).

The causal network corresponding to this model is just

$$BPT \longrightarrow LE \longrightarrow LSS \longrightarrow SST \longrightarrow TD.$$

In this model, $BPT = 1$ is a cause of $TD = 1$. This is all right, as far as it goes, but it seems to suggest that Suzy plays no role. This becomes particularly clear when we observe that $BPT = 1$ is also a cause of $SST = 1$: Billy’s pulling the trigger causes (or, perhaps better, results in, Suzy shooting the target.¹² The problem with this causal model is that it makes Suzy seem like an automaton. We would use the same causal model to describe the situation where Suzy’s plane is actually an unmanned (unwomaned?) plane pre-programmed to shoot at the target if it is not shot down. Under those circumstances, it seems perfectly reasonable to view $BPT = 1$ as the cause of both $SST = 1$ and $TD = 1$. We can bring Suzy more into the picture by having a random variable corresponding to Suzy’s plan or intention. Suppose that we add a random variable *SPS* for “Suzy plans to shoot the target”, with values 0 (she doesn’t) and 1 (she does). Assuming that Suzy shoots if she plans to, we then get the following causal network, where now *SST* depends on both *LSS* and *SPS*:

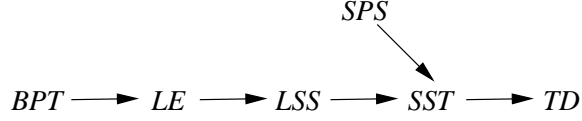


Figure 5: Blowing up the target.

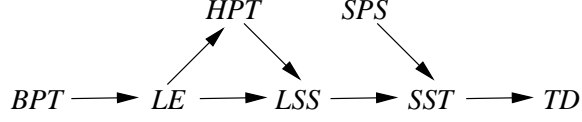


Figure 6: Blowing up the target (refined version).

In this case, it is easy to check that each of $BPT = 1$ and $SPS = 1$ is a cause of $TD = 1$.

Hall suggests that further complications arise if we add a second fighter plane escorting Suzy, piloted by Hillary. Billy still shoots down Lucifer, but if he hadn't, Hillary would have. The natural way of dealing with this is to add just one more variable HPT representing Hillary's pulling the trigger iff $LE = 1$ (see Figure 6), but then, using the naive counterfactual criterion, one might conclude that the target will be destroyed ($TD = 1$) regardless of Billy's action, and $BPT = 1$ would lose its "actual cause" status (of $TD = 1$). Fortunately, our definition goes beyond this naive criterion and classifies $BPT = 1$ as a cause of $TD = 1$, as expected.¹³ This can be seen by noting that the partition $\vec{Z} = \{BPT, LE, LSS, SST, TD\}$, $\vec{W} = \{HPT, SPS\}$ satisfies conditions AC1–AC3 (with w' such that $HPT = 0$ and $SPS = 1$). The intuition rests, again, on structural contingencies; although Billy's action seems superfluous under ideal conditions, it becomes essential under a contingency in which Hillary would fail her mission to shoot Lucifer. This contingency is represented by setting HPT to 0 (in testing AC2(a)), irrespective of LE . ■

Example 4.5: This example concerns what Hall calls the distinction between causation and determination. Again, we quote Hall (1998):

You are standing at a switch in the railroad tracks. Here comes the train: If you flip the switch, you'll send the train down the left-hand track; if you leave it where it is, the train will follow the right-hand track. Either way, the train will arrive at the same point, since the tracks reconverge up ahead. Your action is not among the causes of this arrival; it merely helps to determine

¹²It is also true that $SST = 1$ is a cause of $TD = 1$, as, for that matter, are $LE = 0$ and $LSS = 0$, but this does not affect the unreasonableness of $BPT = 1$ being a cause of $SST = 1$.

¹³Note that Lewis's revised criterion of counterfactual-dependence-chain (Lewis 1986) also fails in this example; LSS does not depend on either HPT or LE in the context given.

how the arrival is brought about (namely, *via* the left-hand track, or *via* the right-hand track).

Again, our causal model gets this right. Suppose we have three random variables:

- F for “flip”, with values 0 (you don’t flip the switch) and 1 (you do);
- T for “track”, with values 0 (the train goes on the left-hand track) and 1 (it goes on the right-hand track); and
- A for “arrival”, with values 0 (the train does not arrive at the point of reconvergence) and 1 (it does).

Now it is easy to see that flipping the switch ($F = 1$) does cause the train to go down the left-hand track ($T = 0$), but does not cause it to arrive ($A = 1$), thanks to AC2(a)—whether or not the switch is flipped, the train arrives.

However, our proposal goes one step beyond this simple picture. Suppose that we decide to model the tracks using *two* variables:

- LT for “left-track”, with values 1 (the train goes on the left-hand track) and 0 (it does not go on the left-hand track); and
- RT for “right-track”, with values 1 (the train goes on the right-hand track) and 0 (it does not go on the right-hand track).

The resulting causal diagram is shown in Figure 7; it is isomorphic to a class of problems Pearl (2000) calls “switching causation”.¹⁴ Lo and behold, this representation classifies $F = 1$ as a cause of A , which, at first sight, may seem counterintuitive: Can a change in representation turn a non-cause into a cause?

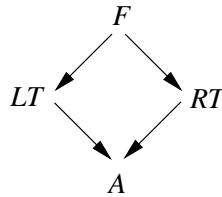


Figure 7: Flipping the switch.

It can and it should! The change to a two-variable model is not merely syntactic, but represents a profound change in the story. The two-variable model depicts the tracks as two independent mechanisms, thus allowing one track to be set (by action or mishap) to

¹⁴Pearl (2000) used a switch between two light sources, so as to avoid questions of RT and LT influencing each other. In our story, we might as well imagine that more than one train may arrive, so that RT and LT can simultaneously be true.

false (or true) without affecting the other. Specifically, this permits the disastrous mishap of flipping the switch while the left track is malfunctioning. Such abnormal eventualities are imaginable and expressible in the two-variable model, but not in the one-variable model. The potential for such eventualities is precisely what renders $F = 1$ a cause of the A in the model of Figure 7.¹⁵

Is flipping the switch a legitimate cause of the train’s arrival? Not in ideal situations, where all mechanisms work as specified. But this is not what causality (and causal modeling) are all about. Causal models earn their value in abnormal circumstances, created by structural contingencies, such as the possibility of a malfunctioning track. It is this possibility that should enter our mind whenever we decide to designate each track as a separate mechanism (i.e., equation) in the model and, keeping this contingency in mind, it should not be too odd to name the switch position a cause of the train arrival (or non-arrival). ■

Example 4.6: Consider an example originally due to McDermott (1995), and also considered by Lewis (2000) and Hitchcock (1999). A ball is caught by a fielder. A little further along its path there is a solid wall and, beyond that, a window. Does the fielder’s catch cause the window to remain unbroken? As Lewis (2000) says

We are ambivalent. We can think: Yes—the fielder and the wall between them prevented the window from being broken, but the wall had nothing to do with it, since the ball never reached the wall; so it must have been the fielder. Or instead we can think: No—the wall kept the window safe regardless of what the fielder did or didn’t do.

Lewis argues that our ambivalence in this case ought to be respected, and both solutions should be allowed. We can give this ambivalence formal expression in our framework. If we make both the wall and the fielder endogenous variables, then either one can be the cause. Note, however, by making them endogenous, we allow for the possibility both that the fielder did not catch the ball (which seems reasonable) and that the wall miraculously did not stop the ball if the fielder did not catch it. This seems perhaps less reasonable, but not totally surprising. By making the wall endogenous, we automatically proclaim the process of erecting (or maintaining) the wall subject to possible contingencies, since such contingencies are intrinsic to the semantics of endogenous variables. On the other hand, if we make the wall exogenous and part of the context, there is no nontrivial cause of the window not being shattered. It would remain safe no matter what the fielder did, in any structural contingency.

This example again stresses the importance of the choice of model, and thinking through what we want to vary and what we want to keep fixed. (Much the same point is made by Hitchcock (1999).) ■

¹⁵This can be seen by noting that condition AC2 is satisfied by the partition $\vec{Z} = \{F, LT, A\}$, $\vec{W} = \{RT\}$, and choosing w' as the setting $RT = 0$. The event $RT = 0$ conflicts with $F = 0$ under normal situations.

We conclude this section with two examples that show a potential problem for our definition, and suggest a solution.

Example 4.7: ¹⁶ Fred has his finger severed by a machine at the factory ($FS = 1$). Fortunately, Fred is covered by a health plan. He is rushed to the hospital, where his finger is sewn back on. A month later, the finger is fully functional ($FF = 1$). In this story, we would not want to say that $FS = 1$ is a cause of $FF = 1$ and, indeed, according to our definition, it is not, since $FF = 1$ whether or not $FS = 1$ (in all contingencies satisfying AC2(b)).

However, suppose we introduce a new element to the story, representing a nonactual structural contingency: Larry the Loanshark may be waiting outside the factory with the intention of cutting off Fred’s finger, as a warning to him to repay his loan quickly. Let LL represent whether or not Larry is waiting and let LC represent whether Larry cuts off the Fred’s finger. If Larry cuts off Fred’s finger, he will throw it away, so Fred will not be able to get it sewn back on. In the actual situation, $LL = LC = 0$; Larry is not waiting and Larry does not cut off Fred’s finger. So, intuitively, there seems to be no harm in adding this fanciful element to the story. Or is there? Suppose that, if Fred’s finger is cut off in the factory, then Larry will not be able to cut off the finger himself (since Fred will be rushed off to the hospital). Now $FS = 1$ becomes a cause of $FF = 1$. For in the structural contingency where $LL = 1$, if $FS = 0$ then $FF = 0$ (Larry will cut off Fred’s finger and throw it away, so it will not become functional again). Moreover, if $FS = 1$, then $LC = 0$ and $FF = 1$, just as in the actual situation. ■

This example seems somewhat disconcerting. Why should adding a fanciful scenario like Larry the Loanshark to the story affect (indeed, result in) the accident being a cause of the finger being functional one month later? While it is true that the accident would be judged a cause of Fred’s good fortune by anyone who knew of Larry’s vicious plan (many underworld figures owe their lives to “accidents” of this sort), the question remains how to distinguish genuine plans that just happened not to materialize from sheer fanciful scenarios that have no basis in reality. To some extent, the answer here is the same as the answer to essentially all the other concerns we have raised: it is a modeling issue. If we know of Larry’s plan, or it seems like a reasonable possibility, we should add it to the model (in which case the accident is a cause of the finger being functional); otherwise we shouldn’t.

But this answer makes the question of how reasonable a possibility Larry’s plan are into an all-or-nothing decision. One solution to this problem is to extend our notion of causal model somewhat, so as to be able to capture more directly the intuition that the Larry the Loanshark scenario is indeed rather fanciful. There a number of ways of doing this; we choose one based on Spohn’s notion of a *ranking function* (or *ordinal conditional function*) (Spohn 1988). A *ranking* κ on a space W is a function mapping subsets of

¹⁶We thank Eric Hiddleston for bringing this issue and this example to our attention.

W to $\mathcal{N}^* = \mathcal{N} \cup \{\infty\}$ such that $\kappa(W) = 0$, $\kappa(\emptyset) = \infty$, and $\kappa(A) = \min_{w \in A} (\kappa(\{w\}))$. Intuitively, an ordinal ranking assigns a degree of surprise to each subset of worlds in W , where 0 means unsurprising and higher numbers denote greater surprise. Let a *world* be a complete setting of the exogenous variables. Suppose that, for each context \vec{u} , we have a ranking $\kappa_{\vec{u}}$ on the set of worlds. The unique setting of the exogenous variables that holds in context \vec{u} is given rank 0 by $\kappa_{\vec{u}}$; other worlds are assigned ranks according to how “fanciful” they are, given context \vec{u} . Presumably, in Example 16, an appropriate ranking κ would give a world where Larry is waiting to cut off Fred’s finger (i.e., where $LL = 1$) a rather high κ ranking, to indicate that it is rather fanciful. We can then modify the definition of causality so that we can talk about $\vec{X} = \vec{x}$ being an actual cause of φ in (M, u) at rank k . The definition is a slight modification of condition AC2 in Definition 3.1 so the contingency (\vec{x}', \vec{w}') must hold in a world of rank at most k ; we omit the formal details here. We may then want to restrict actual causality so that the structural contingencies involved have at most a certain rank. This would be one way of ignoring very fanciful scenarios.

Example 4.8: ¹⁷ Consider Example 4.2, where both Suzy and Billy throw a rock at a bottle, but Suzy’s hits first. Now suppose that there is a noise which causes Suzy to delay her throw slightly, but still before Billy’s. Suppose that we model this situation using the approach described in Figure 3, adding two extra variables, N (where $N = 0$ if there is no noise and $N = 1$ if there is a noise) and $BS_{1.5}$ (where $BS_{1.5} = 1$ if the bottle is shattered at time $t_{1.5}$, where $t_1 < t_{1.5} < t_2$, and $BS_{1.5} = 0$ otherwise). In the actual situation, there is a noise and the bottle shatters at $t_{1.5}$, so $N = 1$ and $BS_{1.5} = 1$. Just as in Example 4.2, we can show that Suzy’s throw is a cause of the bottle shattering and Billy’s throw is not. Not surprisingly, $N = 1$ is a cause of $BS_{1.5} = 1$ (without the noise, the bottle would have shattered at time 1). Somewhat disconcertingly though, $N = 1$ is also a cause of the bottle shattering. That is, $N = 1$ is a cause of $BS_3 = 1$.

This seems unreasonable. Intuitively, the bottle would have shattered whether or not there had been a noise. However, this intuition is actually not correct in our causal model. Consider the contingency where $BS_1 = 0$. Under the contingency, the bottle does not shatter at time 1, even if Suzy’s throw hits it. However, if $N = 1$ and $BS_1 = 0$, then the bottle does shatter at time 1.5. Given this, it easily follows that, according to our definition, $N = 1$ is a cause of $BS_3 = 1$.

This problem can be dealt using ranking functions, much like Example 16. If it seems rather unlikely that the bottle would not shatter if Suzy’s rock hit it at t_0 , then we ascribe a high κ ranking to this contingency. If it ascribed rank k , then $N = 1$ would not be a cause of $BS_3 = 1$ at rank less than k . Note that if it does not seem so unreasonable that a rock hitting at time t_0 would not shatter the bottle (although a rock hitting at any other time would), then it does not seem so unreasonable to take the noise to be a cause of the bottle shattering. ■

¹⁷We thank Chris Hitchcock for bringing this example to our attention.

5 Explanation

Our definition of causality assumed that the causal model and all the relevant facts were given; the problem was to figure out which of the given facts were causes. In contrast, the role of explanation is to provide the information needed to establish causation. Roughly speaking, we view an explanation as a fact that is not known for certain but, if found to be true, would constitute a genuine cause of the explanandum, regardless of the agent’s initial uncertainty. Thus, what counts as an explanation depends on what you already know (or believe—we largely blur the distinction between knowledge and belief in this paper) and, naturally, the definition of an *explanation* should be relative to the agent’s epistemic state (as in Gärdenfors (1988)). It is also natural, from this viewpoint, that an explanation will include fragments of the causal model M , or reference to the physical laws which underly the connection between the cause and the effect. To borrow an example from (Gärdenfors 1988), if we want an explanation of why Mr. Johansson has been taken ill with lung cancer, the information that he worked in asbestos manufacturing for many years is not going to be a satisfactory explanation to someone who does not know anything about the effects of asbestos on people’s health. In this case, the causal model (or relevant parts of it) must be part of the explanation. On the other hand, for someone who knows the causal model but does not know that Mr. Johansson worked in asbestos manufacturing, the explanation would involve Mr. Johansson’s employment but would not mention the causal model.

5.1 The basic definition

The definition of explanation is motivated by the following intuitions. An individual in a given epistemic state K asks why it is the case that φ . What constitutes a good answer to his question? A good answer must (a) provide information that goes beyond K and (b) be such that the individual can see that it would, if true, be (or be very likely to be) a cause of φ . We may also want to require that (c) φ be true (or at least probable). Although our basic definition does not insist on (c), it is easy to add this requirement.

How do we capture the agent’s epistemic state in our framework? For ease of exposition, we first consider the case where the causal model is known and the context is uncertain. (The minor modifications required to deal with the general case are described in Section 5.3.) In that case, one way of describing an agent’s epistemic state by simply describing the set of contexts the agent considers possible.

Definition 5.1: (Explanation) Given a structural model M , $\vec{X} = \vec{x}$ is an *explanation* of φ relative to a set \mathcal{K} of contexts if the following conditions hold:

- EX1. $(M, \vec{u}) \models \varphi$ for each context $\vec{u} \in \mathcal{K}$. (That is, φ must hold in all contexts the agent considers possible—the agent considers what she is trying to explain as an established fact)

EX2. $\vec{X} = \vec{x}$ is a weak cause of φ in (M, \vec{u}) (that is, AC1 and AC2 hold, but not necessarily AC3) for each $\vec{u} \in \mathcal{K}$ such that $(M, \vec{u}) \models \vec{X} = \vec{x}$.

EX3. \vec{X} is minimal; no subset of \vec{X} satisfies EX2.

EX4. $(M, \vec{u}) \models \neg(\vec{X} = \vec{x})$ for some $\vec{u} \in \mathcal{K}$ and $(M, \vec{u}') \models \vec{X} = \vec{x}$ for some $\vec{u}' \in \mathcal{K}$. (This just says that the agent considers a context possible where the explanation is false, so the explanation is not known to start with, and considers a context possible where the explanation is true, so that it is not vacuous.) ■

Our requirement EX4 that the explanation is not known may seem incompatible with linguistic usage. Someone discovers some fact A and says “Aha! That explains why B happened.’ Clearly, A is not an explanation of why B happened relative to the epistemic state *after* A has been discovered, since at that point A is known. However, A can legitimately be considered an explanation of B relative to the epistemic state before A was discovered.

What does the definition of explanation tell us for the examples we considered earlier? Consider the arsonists in Example 3.2. If the causal model has only arsonists as the cause of the fire, there are two possible explanations in the disjunctive scenario: arsonist 1 did it or arsonist 2 did it (assuming \mathcal{K} consists of three contexts, where either 1, 2, or both set the fire). In the conjunctive scenario, no explanation is necessary, since the agent knows that both arsonists must have lit a match if arson is the only possible cause of the fire (assuming that the agent considers the two arsonists to be the only possible arsonists).

Perhaps more interesting is to consider a causal model with other possible causes, such as lightning and unattended campfires. Since the agent knows that there was a fire, in each of the contexts in \mathcal{K} , at least one of the potential causes must have actually occurred. If we assume that there is a context where only arsonist 1 lit the fire (and, say, there was lightning) and another where only arsonist 2 lit the fire then, in the conjunctive scenario, $ML_1 = 1 \wedge ML_2 = 1$ is an explanation of $FB = 1$, but neither $ML_1 = 1$ nor $ML_2 = 1$ by itself is an explanation (since neither by itself is a cause in all contexts in \mathcal{K} that satisfy the formula). On the other hand, in the disjunctive scenario, both $ML_1 = 1$ and $ML_2 = 1$ are explanations.

It is worth noting here that the minimality clause EX3 applies to all contexts. This means that our rough gloss of $\vec{X} = \vec{x}$ being an explanation of φ relative to a set \mathcal{K} of contexts if $\vec{X} = \vec{x}$ is a cause of φ in each context in \mathcal{K} where $\vec{X} = \vec{x}$ holds is not quite correct. For example, although $ML_1 = 1 \wedge ML_2 = 1$ is an explanation of fire (if \mathcal{K} includes contexts where there are other possible causes of fire), it is a cause of fire in none of the contexts in which it holds. The minimality condition AC3 would say that each of $ML_1 = 1$ and $ML_2 = 1$ is a cause, but their conjunction is not.

Now consider the story Example 4.1, with the forest fire caused by the electric storm. Suppose that the agent knows that there was an electric storm, but does not know when, and does not know whether there were April showers. Thus, \mathcal{K} consists of six contexts,

one corresponding to each of the values $(1,0)$, $(0,1)$, and $(1,1)$ of ES and the values 0 and 1 of AS . Then it is easy to see that $AS = 1$ is not an explanation of fire ($F = 1 \vee F = 2$), since, as we observed in Section 4, it is not a cause of fire (in any context in \mathcal{K}). Similarly, $AS = 0$ is not an explanation of fire. On the other hand, each of $ES = (1,1)$, $ES = (1,0)$, and $ES = (0,1)$ is an explanation of fire.

Now suppose that we are looking for an explanation of the June fire. Then the set \mathcal{K} of contexts can consist only of context compatible with there being a fire in June. Suppose that \mathcal{K} consists of three contexts, one where $AS = 1$ and $ES = (0,1)$, one where $AS = 1$ and $ES = (1,1)$, and one where $AS = 0$ and $ES = (0,1)$. In this case, each of $AS = 1$, $ES = (0,1)$, and $ES = (1,1)$ is an explanation of the June fire. (In the case of $AS = 1$, we need to consider the setting where $ES = (1,1)$.)

Finally, if the agent knows that there was an electric storm in May and June and heavy rain in April (so that \mathcal{K} consists of only one context), then there is no explanation of either fire or the fire in June. Formally, this is because it is impossible to satisfy EX4. Informally, this is because the agent already knows why there was a fire in June.

Note that, as for causes, we have disallowed disjunctive explanations. Here the motivation is less clear cut. It does make perfect sense to say that the reason that φ happened is either A or B (but I don't know which). There are some technical difficulties with disjunctive explanations, which suggest philosophical problems. For example, consider the conjunctive scenario of the arsonist example again. Suppose that the structural model is such that the only causes of fire are the arsonists, lightning, and unattended campfires and that \mathcal{K} consists of contexts where each of these possibilities is the actual cause of the fire. Once we allow disjunctive explanations, what is the explanation of fire? One candidate is "either there were two arsonists or there was lightning or there was an unattended campfire (which got out of hand)". But this does not satisfy EX4, since the disjunction is true in every context in \mathcal{K} . On the other hand, if we do not allow the disjunction of all possible causes, which disjunction should be allowed as an explanation? As a technical matter, how should the minimality condition EX3 be rewritten? We could not see any reasonable way to allow some disjunctions in this case without allowing the disjunction of all causes (which we have independent reasoning for rejecting as an explanation).

We believe that, in cases where disjunctive explanations seem appropriate, it is best to capture this directly in the causal model by having a variable that represents the disjunction. (Essentially the same point is made in (Chajewska and Halpern 1997).) For example, consider the disjunctive scenario of the arsonist example, where there are other potential causes of the fire. Do we want to allow "there was an arsonist" to be an explanation without specifically mentioning who the arsonist is? That depends on our goals. If this seems desirable, then it can be easily accomplished by replacing the variables ML_1 and ML_2 in the model by a variable ML which is Then $ML = 1$ becomes an explanation, without requiring disjunctive explanations.

Why not just add ML to the model rather than using to replace ML_1 and ML_2 ? We have implicitly assumed in our framework that all possible combinations of assignments

to the variables are possible (i.e., there is a structural contingency for any setting of the variables). If we add ML and view it as being logically equivalent to $ML_1 \vee ML_2$ (that is, $ML = 1$ *by definition* iff at least one of ML_1 and ML_2 is 1) then, for example, it is logically impossible for there to be a structural contingency where $ML_1 = 0$, $ML_2 = 0$, and $ML = 1$. Thus, in the presence of logical dependencies, it seems that we need to restrict the set of contingencies that can be considered to those that respect the dependencies. We have not yet considered the implications of such a change for our framework, so we do not pursue the matter here.¹⁸

One other point regarding the definition of explanation: When we ask for an explanation of φ , we usually expect that it not only explains φ , but that it is true in the actual world. Definition 5.1 makes no reference to the “actual world”, only to the agent’s epistemic state. There is no difficulty adding an actual world to the picture and requiring that the explanation be true in the world. We can simply talk about an explanation relative to a pair (\mathcal{K}, \vec{u}) , where \mathcal{K} is a set of contexts and $\vec{u} \in \mathcal{K}$. Intuitively, \vec{u} describes the context in the actual world. The requirement that $\vec{u} \in \mathcal{K}$ essentially says that \mathcal{K} represents the agent’s knowledge rather than the agent’s beliefs: the actual context is one of the ones the agent considers possible. The requirement that the explanation be true in the actual world then becomes $(M, \vec{u}) \models \vec{X} = \vec{x}$. Although we have not made this requirement part of the definition, adding it would not affect any of our other comments.

5.2 Partial explanations and explanatory power

Not all explanations are considered equally good. Some explanations are more likely than others. An obvious way to define the “goodness” of an explanation is by bringing probability into the picture. Suppose that the agent has a probability on the set \mathcal{K} of possible contexts. In this case, we can consider the probability of the set of contexts where the explanation $\vec{X} = \vec{x}$ is true. For example, if the agent has reason to believe that electric storms are quite common in both May and June, then the set of contexts where $ES = (1, 1)$ holds would have greater probability than the set where either $ES = (1, 0)$ or $ES = (0, 1)$ holds. Thus, $ES = (1, 1)$ would be considered a better explanation.

But the probability of an explanation is only part of the story; the other part concerns the degree to which an explanation fulfills its role (relative to φ) in the various contexts considered. This becomes clearer when we consider *partial* explanations. The following example, taken from (Gärdenfors 1988), is one where partial explanations play a role.

Example 5.2: Suppose I see that Victoria is tanned and I seek an explanation. Suppose that the causal model includes variables for “Victoria took a vacation in the Canary Islands”, “sunny in the Canary Islands”, and “went to a tanning salon”. The set \mathcal{K} includes contexts for all settings of these variables compatible with Victoria being tanned.

¹⁸As a related matter, note a model with logical dependencies means the model is no longer recursive. We discuss in the appendix how nonrecursive models can be dealt with.

Note that, in particular, there is a context where Victoria both went to the Canaries (and didn't get tanned there, since it wasn't sunny) and to a tanning salon. Gärdenfors points out that we normally accept “Victoria took a vacation in the Canary Islands” as a satisfactory explanation of Victoria being tanned and, indeed, according to his definition, it is an explanation. Victoria taking a vacation is not an explanation (relative to the context \mathcal{K}) in our framework, since there is a context $\bar{u}^* \in \mathcal{K}$ where Victoria went to the Canary Islands but it was not sunny, and in \bar{u}^* the actual cause of her tan is the tanning salon, not the vacation.

For us, the explanation would have to be “Victoria went to the Canary Islands *and* it was sunny.” In this case we can view “Victoria went to the Canary Islands” as a partial explanation (in a formal sense to be defined below). ■

In example 5.2 the partial explanation can be extended to a full explanation by adding a conjunct. But not every partial explanation can be extended to a full explanation. Roughly speaking, the full explanation may involve exogenous factors, which are not permitted in explanations. Assume, for example, that going to a tanning salon was not considered an endogenous variable in our model but, rather, an unnamed, inexplicable background contingency U_s that would make Victoria suntanned even in the absence of sun in Canary islands. Likewise, assume that the weather in Canary island was also part of the background context. In this case, we would still consider Victoria's vacation to provide a partial explanation of her sun tan, since the context where it fails to be a cause (no sun in the Canary island) is fairly unlikely, but we cannot add conjuncts to this event to totally exclude that context from the agent's realm of possibilities.

The situation actually is quite common, as the following example shows.

Example 5.3: Suppose that the sound on a television works but there is no picture. Suppose that the only cause of there being no picture that the agent is aware of is the picture tube being faulty. However, the agent is also aware that there are times when there is no picture even though the picture tube works perfectly well—intuitively, “for inexplicable reasons”. This is captured by the causal network described in Figure 8, where T describes whether or not the picture tube is working (1 if it is and 0 if it is not) and P describes whether or not there is a picture (1 if there is and 0 if there is not). The

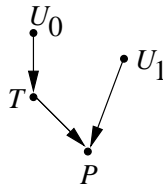


Figure 8: The television with no picture.

exogenous variable U_0 determines the status of the picture tube: $T = U_0$. The exogenous variable U_1 is meant to represent the mysterious “other possible causes”. If $U_1 = 0$, then

whether or not there is a picture depends solely on the status of the picture tube—that is, $P = T$. On the other hand, if $U_1 = 1$, then there is no picture ($P = 0$) no matter what the status of the picture tube. Thus, in contexts where $U_1 = 1$, $T = 0$ is *not* a cause of $P = 0$. Now suppose that \mathcal{K} includes a context \vec{u}_{00} where $U_0 = U_1 = 0$. Then it is easy to see that there is no explanation of $P = 0$. The only plausible explanation, that the picture tube is not working, is not a cause of $P = 0$ in the context \vec{u}_{00} . On the other hand, $T = 0$ is a cause of $P = 0$ in all other contexts in \mathcal{K} satisfying $T = 0$. If the probability of \vec{u}_{00} is small (capturing the intuition that it is unlikely that more than one thing goes wrong with a television at once), then we are entitled to view $T = 0$ as a quite good partial explanation of $P = 0$. ■

These examples motivate the following definition.

Definition 5.4: $\vec{X} = \vec{x}$ is an α -*partial explanation* of φ relative to (\mathcal{K}, Pr) (where Pr is a probability on \mathcal{K}) if there exists a subset \mathcal{K}' of \mathcal{K} such that $\vec{X} = \vec{x}$ is an explanation of φ relative to \mathcal{K}' and $\text{Pr}(\mathcal{K}' | \vec{X} = \vec{x}) \geq \alpha$.¹⁹ That is, roughly speaking, an α -partial definition of φ is one that explains φ on a set of contexts of probability at least α . $\vec{X} = \vec{x}$ is a *partial explanation* of φ if $\vec{X} = \vec{x}$ is an α -partial explanation of φ for some $\alpha > 0$.²⁰ The *explanatory power* of a partial explanation $\vec{X} = \vec{x}$ of φ is the sup over all α such that $\vec{X} = \vec{x}$ is an α -partial explanation. ■

In Example 5.2, if the agent believes that it is sunny in the Canary Islands with probability .9. (that is, the probability that it was sunny given that Victoria is suntanned and that she went to the Canaries is .9), then Victoria going to the Canaries is a good partial explanation of her being tanned (at the .9 level). The set \mathcal{K}' consists of those contexts where it is sunny in the Canaries. Similarly, in Example 5.3, if the agent believes that the probability of both the picture tube being faulty and the other mysterious causes being operative is .1, then $T = 0$ is a partial explanation of $P = 0$ at the .9 level (with \mathcal{K}' consisting of all the contexts where $U_1 = 1$).

A full explanation is clearly a 1-partial explanation, but we are quite often satisfied with partial explanations $\vec{X} = \vec{x}$ of lower explanatory power, especially if they have high probability (i.e., if $\text{Pr}(\vec{X} = \vec{x})$ is high). In general, there may be a tension between the explanatory power of an explanation and its probability. (Partial) explanations with higher explanatory power typically are more refined and, hence, less likely. There is no obvious way to resolve this tension. (See (Chajewska and Halpern 1997) for more discussion of this issue.)

¹⁹Here and elsewhere, a formula such as $\vec{X} = \vec{x}$ is being identified with the set of contexts where the formula is true. Recall, that since all contexts in \mathcal{K} are presumed compatible with φ there is no need to condition $\text{Pr}(\mathcal{K}' | \vec{X} = \vec{x})$ on φ ; this probability is already updated with the truth of the explanandum φ .

²⁰Our usage of *partial explanation* here is related to, but different from, that in (Chajewska and Halpern 1997).

5.3 The general definition

In general, an agent may be uncertain about the causal model, so an explanation will have to include information about it. (Gärdenfors (1988) and Hempel (1965) make similar observations, although they focus not on causal information, but on statistical and nomological information; we return to this point below.) It is relatively straightforward to extend our definition of explanation to accommodate this provision. Now an epistemic state \mathcal{K} consists not only of contexts, but of pairs (M, \vec{u}) consisting of a causal model M and a context \vec{u} . Call such a pair a *situation*. Intuitively, now an explanation should consist of some causal information (such as “prayers do not cause fires”) and the facts that are true. Thus, a (*general*) *explanation* has the form $(\psi, \vec{X} = \vec{x})$, where ψ is an arbitrary formula in our causal language and, as before, $\vec{X} = \vec{x}$ is a conjunction of primitive events. We think of the ψ component as consisting of some causal information (such as “prayers do not cause fires”, which corresponds to a conjunction of statements of the form $F \leftarrow i \Rightarrow [P = x](F = i)$, where P is a random variable describing whether or not prayer takes place). The first component in a general explanation is viewed as restricting the set of causal models. To make this precise, given a causal model M , we say ψ is *valid in* M , and write $M \models \psi$, if $(M, \vec{u}) \models \psi$ for all contexts \vec{u} consistent with M . With this background, it is easy to state the general definition.

Definition 5.5: $(\psi, \vec{X} = \vec{x})$ is an explanation of φ relative to a set \mathcal{K} of situations if the following conditions hold:

- EX1. $(M, \vec{u}) \models \varphi$ for each situation $(M, \vec{u}) \in \mathcal{K}$.
- EX2. For all $(M, \vec{u}) \in \mathcal{K}$ such that $(M, \vec{u}) \models \vec{X} = \vec{x}$ and $M \models \psi$, $\vec{X} = \vec{x}$ is a weak cause of φ in (M, \vec{u}) .
- EX3. $(\psi, \vec{X} = \vec{x})$ is minimal; there is no pair $(\psi', \vec{X}' = \vec{x}') \neq (\psi, \vec{X} = \vec{x})$ satisfying EX2 such that $\{M'' \in M(\mathcal{K}) : M'' \models \psi'\} \supseteq \{M'' \in M(\mathcal{K}) : M'' \models \psi\}$, where $M(\mathcal{K}) = \{M : (M, \vec{u}) \in \mathcal{K} \text{ for some } \vec{u}\}$, $\vec{X}' \subseteq \vec{X}$, and \vec{x}' is the restriction of \vec{x} to the variables in \vec{X}' .
- EX4. $(M, \vec{u}) \models \neg(\vec{X} = \vec{x})$ for some $(M, \vec{u}) \in \mathcal{K}$ and $(M', \vec{u}') \models \vec{X} = \vec{x}$ for some $(M', \vec{u}') \in \mathcal{K}$. ■

Note that in EX2, we now restrict to situations $(M, \vec{u}) \in \mathcal{K}$ that satisfy both parts of the explanation $(\psi, \vec{X} = \vec{x})$, in that $M \models \psi$ and $(M, \vec{u}) \models \vec{X} = \vec{x}$. Further note that, although both components of an explanation are formulas in our causal language, they play very different roles. The first component serves to restrict the set of causal models considered (to those with the appropriate structure); the second describes a cause of φ in the resulting set of situations.

Clearly Definition 5.1 is the special case of Definition 5.5 where there is no uncertainty about the causal structure (i.e., there is some M such that if $(M', \vec{u}) \in \mathcal{K}$, then $M = M'$). For in this case, it is clear that we can take ψ in the explanation to be *true*.

Definition 5.5 can also be extended to deal naturally with statistical information of the kind considered by Gärdenfors and Hempel. Let a *probabilistic causal model* be a tuple $M_{Pr} = (\mathcal{S}, \mathcal{F}, \text{Pr})$, where $M = (\mathcal{S}, \mathcal{F})$ is a causal model and Pr is a probability measure on the contexts defined by signature \mathcal{S} of M . Information like “with probability .9 $X = 3$ ” is a restriction on probabilistic models, and thus can be captured using a formula in an appropriate extension of our language that allows such probabilistic reasoning. With this extended language, the definition of explanation using probabilistic causal models remains unchanged.

As an orthogonal issue, there is also no difficulty considering a probability on the set \mathcal{K} of situations and defining partial explanation just as before.

Example 5.6: Using this general definition of causality, let us consider Scriven’s (1959) famous paresis example. Paresis develops only in patients who have been syphilitic for a long time, but only a small number of patients who are syphilitic in fact develop paresis. Furthermore, no other factor is known to be relevant in the development of paresis. This description is captured by a simple causal model M_P . There are two endogenous variables, S (for syphilis) and P (for paresis), and two exogenous variables, U_1 , the background factors that determine S , and U_2 , which intuitively represents “disposition to paresis”, i.e., the factors that determine, in conjunction with syphilis, whether or not paresis actually develops. An agent who knows this causal model and that a patient has paresis does not need an explanation of why: he knows without being told that the patient must have syphilis and that $U_2 = 1$. On the other hand, for an agent who does not know the causal model (i.e., considers a number of causal models of paresis possible), $(\{M_P\}, S = 1)$ is an explanation of paresis. ■

6 Discussion

We have presented a formal representation of causal knowledge and a principled way of determining actual causes and explanations from such knowledge. We have shown that the counterfactual approach to causation, in the tradition of Hume and Lewis, need not be abandoned; the language of counterfactuals, once supported with structural semantics, can yield a plausible and elegant account of actual causation that resolves major difficulties in the traditional account. While our account also has its problematic aspects, we are optimistic that they can be resolved along the lines we have sketched.

The essential principles of our account include

- the use of structural equation semantics of counterfactuals;

- the use of uniform counterfactual notation to encode and distinguish facts, actions, outcomes, processes and contingencies;
- using structural contingencies to uncover causal dependencies;
- careful screening of these contingencies to avoid tampering with the causal processes to be uncovered.
- treating a simple explanation as a proposition that conveys new knowledge and that, once believed, would constitute a cause (of the explanandum)

Our approach also stresses the importance of careful modeling. In particular, it shows that the choice of random variables can have a significant effect on the causality relation. This perhaps can be viewed as a significant deficiency in the approach. We prefer to think that it shows that, in the end, causality is a human construct. Thinking of a situation in terms of causality can be very useful and is something that humans often do. As a result, it is perhaps not surprising that we are good at choosing the appropriate random variables for doing so.

While our definitions still have some unsatisfying features, particularly (in our view) the difficulty of dealing with disjunctive explanations, and the need to appeal to something like ranking functions to deal with fanciful scenarios, particularly in Example 17, we hope that the examples have illustrated how well the definitions deal with many of the problematic cases found in the literature.

A Appendix: some technical issues

In this appendix, we consider some technical issues related to the definition of causality. The first is the argument that, we can assume without loss of generality that the variables in the set \vec{Z} in condition AC2 of the definition of causality can all be taken to be on a path from a path from a variable in \vec{X} to one a variable in φ . In fact, they can, without loss of generality, be assumed to change value when \vec{X} is set to \vec{x}' and \vec{W} is set to \vec{w}' . More formally, consider the following strengthening of AC2:

AC2'. There exists a partition (\vec{Z}, \vec{W}) of \mathcal{V} with $\vec{X} \subseteq \vec{Z}$ and setting (\vec{x}', \vec{w}') of the variables in (\vec{X}, \vec{W}) such that, if $(M, \vec{u}) \models Z = z^*$ for $Z \in \vec{Z}$, then

- (a) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}'](\neg\varphi \wedge Z \neq z^*)$ for all $Z \in \vec{Z}$.
- (b) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}', \vec{Z}' \leftarrow \vec{z}^*]\varphi$ for all subsets \vec{Z}' of \vec{Z} .

As we now show, we could have replaced AC2 by AC2'; it would not have affected the notion of causality. Say that $\vec{X} = \vec{x}$ is an *actual cause'* of φ if AC1, AC2', and AC3 hold.

Proposition A.1: $\vec{X} = \vec{x}$ is a cause of φ iff $\vec{X} = \vec{x}$ is a cause' of φ .

Proof: Define $\vec{X} = \vec{x}$ to be a *weak cause* (resp., *weak cause'*) of φ if AC1 and AC2 (resp., AC1 and AC2') hold, but not necessarily AC3. We show that $\vec{X} = \vec{x}$ is a weak cause of φ iff $\vec{X} = \vec{x}$ is weak cause' of φ . This clearly suffices to prove the result. The “if” direction is immediate, since AC2' clearly implies AC2.

For the “only if” direction, suppose that $\vec{X} = \vec{x}$ is a cause of φ . Let (\vec{Z}, \vec{W}) be the partition of \mathcal{V} and (\vec{x}', \vec{w}') the setting of the variables in (\vec{X}, \vec{W}) guaranteed to exist by AC2. Let $\vec{Z}' \subseteq \vec{Z}$ consist of the variables \vec{X} together with the variables $Z \in \vec{Z}$ such that $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}'](Z \neq z^*)$. Let $\vec{W}' = \mathcal{V} - \vec{Z}'$. Notice that \vec{W}' is a superset of \vec{W} . Finally, let \vec{w}'' be a setting of the variables in \vec{W} that agrees with \vec{w}' on the variables in \vec{W} and for $Z \in \vec{Z} \cap \vec{W}'$, sets Z to z^* (its original value). Since $Z = z^*$ in the unique solution to the equations in $M_{\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}'}$ and the equations in $M_{\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}'}$, it follows that (a) the equations in $M_{\vec{X} \leftarrow \vec{x}', \vec{W}' \leftarrow \vec{w}''}$ and $M_{\vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}'}$ have the same solutions and (b) the equations in $M_{\vec{X} \leftarrow \vec{x}, \vec{W}' \leftarrow \vec{w}''}$ and $M_{\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}'}$ have the same solutions. Thus, $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}', \vec{W}' \leftarrow \vec{w}''](\neg\varphi \wedge (Z \neq z^*))$ for all $Z \in \vec{Z}'$ and $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}'](\varphi \wedge (Z = z^*))$ for all $Z \in \vec{Z}'$. That is, AC2' holds for the pair (\vec{Z}', \vec{W}') , showing that $\vec{X} = \vec{x}$ is a weak cause' of φ . ■

Proposition A.1 shows that, without loss of generality, the variables in \vec{Z} can be taken to be “active” in the causal process, in that they change value when the variables in \vec{X} do. This means that each variable in \vec{Z} must be a descendant of some variable in \vec{X} in the causal graph. The next result shows that, without loss of generality, we can also assume that the variables in \vec{Z} are on a path from a variable in \vec{X} to a variable that appears in φ . Recall that we defined an active causal process to consist of a minimal set \vec{Z} that satisfies AC2.

Proposition A.2: *All the variables in an active causal process corresponding to a cause $\vec{X} = \vec{x}$ for φ in (M, \vec{u}) must be on a path from some variable in \vec{X} to a variable in φ in the causal network corresponding to M .*

Proof: Suppose that \vec{Z} is an active causal process, (\vec{Z}, \vec{W}) is the partition satisfying AC2 using the setting (\vec{x}', \vec{w}') . By Proposition A.2, all the variables in \vec{Z} must be descendants of a variable in \vec{X} . Suppose that some variable $Z \in \vec{Z}$ is not on a path from a variable in \vec{X} to a variable in φ . That means there is no path from Z to a variable in φ . It follows that there is no path from Z to a variable $Z' \in \vec{Z}$ that is on a path from a variable in \vec{X} to a variable in φ . That means that changing the value of Z cannot affect the value of φ nor of any a variable $Z' \in \vec{Z}$. Let $\vec{Z}' = \vec{Z} - \{Z\}$ and $\vec{W}' = \vec{W} \cup \{Z\}$. Extend \vec{w}' to \vec{w}'' by assigning Z to its original value z^* in context (M, \vec{u}) . It is now immediate from the preceding observations that (\vec{Z}', \vec{W}') is a partition satisfying AC2 using the setting (\vec{x}', \vec{w}'') . This contradicts the minimality of \vec{Z} . ■

The second issue we consider is clause AC2(b) in the definition. It is complicated by the need to check that no change in the value of the variables in \vec{Z} can affect the value of φ . In all the examples in Section 4, $Z = z^*$ for each variable $Z \in \vec{Z}$. Could we not just require this? The following example shows that we cannot.

Example A.3: Imagine that a vote takes place. For simplicity, two people vote. The measure is passed if at least one of them votes in favor. In fact, both of them vote in favor, and the measure passes. This version of the story is almost identical to Example 3.2. If we use V_1 and V_2 to denote how the voters vote ($V_i = 0$ if voter i votes against and $V_i = 1$ if she votes in favor) and P to denote whether the measure passes ($P = 1$ if it passes, $P = 0$ if it doesn't), then in the context where $V_1 = V_2 = 1$, it is easy to see that each of $V_1 = 1$ and $V_2 = 1$ is a cause of $P = 1$. However, suppose we now assume that there is a voting machine that tabulates the votes. Let M represent the total number of votes recorded by the machine. Clearly $M = V_1 + V_2$ and $P = 1$ iff $M \geq 1$. The following causal network represents this more refined version of the story. In this more

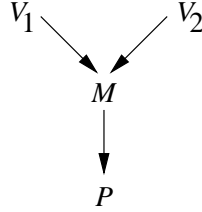


Figure 9: An example showing the need for AC2(b).

refined scenario, $V_1 = 1$ and $V_2 = 1$ are still both causes of $P = 1$. Consider $V_1 = 1$. Take $\vec{Z} = \{V_1, M, P\}$ and $\vec{W} = V_2$. Much like the simpler version of the story, if we choose the contingency $V_2 = 0$, then P is counterfactually dependent on V_1 , so AC2(a) holds. To check if this contingency satisfies AC2(b), we set V_1 to 1 (their original value) and check that setting V_2 to 0 does not change the value of P . This is indeed the case. Although M becomes 1, not 2 as it is when $V_1 = V_2 = 1$, nevertheless, $P = 1$ continues to hold, so AC2(b) is satisfied and $V_1 = 1$ is a cause of $P = 1$. However, if we had insisted in AC2(b) that $(M, u) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow w']Z = z^*$ for all variables $Z \in \vec{Z}$ (which in this case means that M would have to retain its original value of 2 when $V_1 = 1$ and $V_2 = 0$), then neither $V_1 = 1$ nor $V_2 = 1$ would be a cause of $P = 1$ (although with the obvious modification of the definition, $V_1 = 1 \wedge V_2 = 1$ would be a cause of $P = 1$.) Since, in general, one can always imagine that a change in one variable produces some feeble change in another, we cannot insist on the variables in \vec{Z} remaining constant; instead, we require merely that changes in \vec{Z} not affect φ . ■

Finally, we turn our attention to how the definition of causality (and hence explanation) can be modified to deal with nonrecursive models. In nonrecursive models, there may be more than one solution to an equation in a given context, or there may be none.

Thus, the truth of a formula such as $X = x$ must be taken relative not just to a context, but to a complete description (\vec{u}, \vec{v}) of the values of both the exogenous and the endogenous variables. That is, $(M, \vec{u}, \vec{v}) \models X = x$ if X has value x . Since the truth of $X = x$ depends on just \vec{v} , not \vec{u} , we sometimes write $(M, \vec{v}) \models X = x$. We then define $(M, \vec{u}, \vec{v}) \models [\vec{Y} \leftarrow \vec{y}](X = x)$ if in all solutions to the equations in $M_{\vec{Y} \leftarrow \vec{y}}$ in context \vec{u} , the variable X has value x . Since the truth of $[\vec{Y} \leftarrow \vec{y}](X = x)$ depends only on the context \vec{u} and not on \vec{v} , we typically write $(M, \vec{u}) \models [\vec{Y} \leftarrow \vec{y}](X = x)$.

The formula $\langle \vec{Y} \leftarrow \vec{y} \rangle (X = x)$ is the dual of $[\vec{Y} \leftarrow \vec{y}](X = x)$; that is, it is an abbreviation of $\neg[\vec{Y} \leftarrow \vec{y}](X \neq x)$. It is easy to check that $(M, \vec{u}, \vec{v}) \models \langle \vec{Y} \leftarrow \vec{y} \rangle (X = x)$ if in some solution to the equations in $M_{\vec{Y} \leftarrow \vec{y}}$ in context \vec{u} , the variable X has value x . For recursive models, it is immediate that $[\vec{Y} \leftarrow \vec{y}](X = x)$ is equivalent to $\langle \vec{Y} \leftarrow \vec{y} \rangle (X = x)$, since all equations have exactly one solution.

With these definitions in hand, it is easy to state our definition of causality for arbitrary models. Note it is now taken with respect to a tuple (M, \vec{u}, \vec{v}) , since we need the values of the exogenous variables to define the actual world.

Definition A.4: $\vec{X} = \vec{x}$ is an *actual cause* of φ in (M, \vec{u}, \vec{v}) if the following three conditions hold:

- AC1. $(M, \vec{v}) \models (\vec{X} = \vec{x}) \wedge \varphi$.
- AC2. There exists a partition (\vec{Z}, \vec{W}) of \mathcal{V} with $\vec{X} \subseteq \vec{Z}$ and some setting (\vec{x}', \vec{w}') of the variables in (\vec{X}, \vec{W}) such that if $(M, \vec{u}, \vec{v}) \models \vec{Z} = \vec{z}^*$, then
 - (a) $(M, \vec{u}) \models \langle \vec{X} \leftarrow \vec{x}', \vec{W} \leftarrow \vec{w}' \rangle \neg \varphi$.
 - (b) $(M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}, \vec{W} \leftarrow \vec{w}', \vec{Z}' \leftarrow \vec{z}^*] \varphi$ for all subsets \vec{Z}' of \vec{Z} . (Note that in part (a) we require that the value of φ change only in some solution to the equations, while in (b), we require that it stay true in *all* solutions.)
- AC3. \vec{X} is minimal; no subset of \vec{X} satisfies conditions AC1 and AC2. ■

There is a similar generalization of the definition of explanation.

While these seem like the most natural generalizations of our definition of causality and explanation to deal with nonrecursive models, we do not have examined examples to verify that these definitions give the expected result, partly because all the standard examples are most naturally modeled using recursive models.

Acknowledgments

We thank Christopher Hitchcock for many useful comments on earlier versions of the paper and pointing out Example 17, Zoltan Szabo for stimulating discussions, and Eric Hiddleston for pointing out Example 16.

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