

Generating parity check equations for bounded-distance iterative erasure decoding

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Abstract—A generic (r, m) -erasure correcting set is a collection of vectors in \mathbb{F}_2^r which can be used to generate, for each binary linear code of codimension r , a collection of parity check equations that enables iterative decoding of all correctable erasure patterns of size at most m . That is to say, the only stopping sets of size at most m for the generated parity check equations are the erasure patterns for which there is more than one manner to fill in the erasures to obtain a codeword.

We give an explicit construction of generic (r, m) -erasure correcting sets of cardinality $\sum_{i=0}^{m-1} \binom{r-1}{i}$. Using a random-coding-like argument, we show that for fixed m , the minimum size of a generic (r, m) -erasure correcting set is linear in r .

Keywords: iterative decoding, binary erasure channel, stopping set

I. INTRODUCTION

This paper is motivated by the following well-known scheme for iterative decoding of a binary linear code C used on the binary erasure channel [1]. We are given a set \mathcal{H} of parity check equations for C . For a received word with E as set of erased positions, we inspect if one of the parity check equations from \mathcal{H} involves exactly one of the erasures. If so, we determine the value of the erasure involved in this equation and continue; if not, we stop the algorithm. In the latter case, the set E is called a *stopping set* for \mathcal{H} [1]. Different sets \mathcal{H} of parity check equations for C may result in different stopping sets. Note, however, that the support of a codeword is always a stopping set, as by definition each parity check vector has an even number of ones within the support of a codeword.

We are interested in the behavior of the iterative decoding algorithm for erasure patterns that are C -correctable, i.e., for which there is only one way to fill in the erasures to obtain a word from C . In fact, we wish to find parity check collections with which the iterative decoding algorithm decodes all C -correctable patterns of a sufficiently small cardinality. As related work, we mention that in [2], Weber and Abdel-Ghaffar construct collections of parity check equations for the Hamming code C_r of redundancy r with which the iterative algorithm decodes all C_r -correctable erasure patterns of size at most 3. In [3] and [4], Schwartz and Vardy study the minimum size of collections of parity check equations for a code C for which the iterative algorithm decodes all erasure patterns of size less than the minimum distance of C (note that all such erasure patterns are C -correctable).

In [5], we introduced and constructed so-called *generic (r, m) -erasure correcting sets*. These are subsets \mathcal{A} of \mathbb{F}_2^r such that for any code C of length n and codimension r , and any $r \times n$ parity check matrix H for this code, the collection of parity check equations

$$\{\mathbf{a}H \mid \mathbf{a} \in \mathcal{A}\}$$

allows the iterative decoding algorithm to correct all C -correctable erasure patterns of size at most m . Our aim is to construct generic (r, m) -erasure correcting sets of small size, and to investigate the minimum size $F(r, m)$ of such sets. At first sight, the definition of generic (r, m) -erasure correcting sets seems to be very restrictive. However, in [5] it was shown that if a set of linear combinations works for a parity check matrix H_r of the Hamming code of redundancy r , then it works for any parity matrix for any code of redundancy r – see Proposition 2.6 of the present paper for a more precise formulation and a proof.

The paper is organized as follows. In Section II, we introduce notations and definitions. In Section III, we present the explicit generic (r, m) -erasure correcting sets from [5]. These sets have size $\sum_{i=0}^{m-1} \binom{r-1}{i}$. In Section IV we show that $F(r, m) \geq r$. With a random-coding like argument we show that for each $m \geq 1$, there exist a constant c_m such that $F(r, m) \leq c_m r$ for each $r \geq m$. At present we do not have a *constructive* proof that $F(r, m)$ is linear in r .¹

II. PRELIMINARIES

In this section, we introduce some notations and definitions. Throughout this paper, we use boldface letters to denote row vectors. All vectors and matrices are binary. If there is no confusion about the length of vectors, we denote with $\mathbf{0}$ the vector consisting of only zeroes, and with \mathbf{e}_i the i -th unit vector, the vector that has a one in position i and zeroes elsewhere.

The size of a set A is denoted by $|A|$. If H is a $r \times n$ matrix and $E \subseteq \{1, 2, \dots, n\}$, then the *restriction $H(E)$ of H to E* denotes the $r \times |E|$ matrix consisting of those columns of H indexed by E . Similarly, if $\mathbf{x} \in \mathbb{F}_2^n$ and $E \subseteq \{1, 2, \dots, n\}$, then the restriction $\mathbf{x}(E)$ of \mathbf{x} to E is the vector of length $|E|$ consisting of the entries indexed by E .

¹In [6], we gave an explicit recursive construction, too involved to be included here, of generic $(r, 3)$ -erasure correcting sets of size $1+3(r-1)^{\log_2 3}$.

The *support* $\text{supp}(\mathbf{x})$ of a vector $\mathbf{x} \in \mathbb{F}_2^n$ is the set of its non-zero coordinates, that is,

$$\text{supp}(\mathbf{x}) = \{i \in \{1, 2, \dots, n\} \mid x_i \neq 0\},$$

and the weight $\text{wt}(\mathbf{x})$ of \mathbf{x} is the size $|\text{supp}(\mathbf{x})|$ of its support.

As usual, an $[n, k]$ code C is a k -dimensional subspace of \mathbb{F}_2^n ; the dual code of C , denoted by C^\perp , is the $[n, r]$ code with $r = n - k$ consisting of all vectors in \mathbb{F}_2^n that have inner product 0 with all words from C . The number r is referred to as the *codimension* or *redundancy* of the code. An $r \times n$ matrix is called a parity check matrix for C if its rows span C^\perp . When we speak about “code”, we will always mean binary linear code.

The following definitions are taken from [5].

Definition 2.1: Let $C \subseteq \mathbb{F}_2^n$ be a code. A set $E \subseteq \{1, 2, \dots, n\}$ is called *C-uncorrectable* if it contains the support of a non-zero codeword, and *C-correctable* otherwise.

The motivation for this definition is that a received word containing only correct symbols and erasures can be decoded unambiguously precisely when exactly one codeword agrees with this word in the non-erased positions; for linear codes this is the case precisely when the set of erasures does not contain the support of a non-zero codeword.

Definition 2.2: Let $C \subseteq \mathbb{F}_2^n$ be a code. A set $\mathcal{H} \subseteq C^\perp$ is called *m-erasure reducing* for C if for each erasure pattern $E \subseteq \{1, 2, \dots, n\}$ of size m that is C -correctable, there exists a parity check equation $\mathbf{h} \in \mathcal{H}$ with exactly one 1 in the positions indexed by E , that is, with $\text{wt}(\mathbf{h}(E)) = 1$. The set \mathcal{H} is called *m-erasure decoding* for C if it is m' -erasure reducing for C for all m' with $1 \leq m' \leq m$.

Definition 2.2 implies the following. If the iterative decoding algorithm is used with a set \mathcal{H} of parity check equations that is m -erasure reducing for C , then for each C -correctable erasure pattern of size m at least one erasure is resolved; if \mathcal{H} is m -erasure correcting for C , then the iterative decoding algorithm can correct all C -correctable erasure patterns of size at most m by removing one erasure at the time, without ever getting stuck. Note that this definition makes no requirements on the behaviour of the iterative decoding algorithm for erasure patterns that are *not* C -correctable.

The following example shows that an m -erasure reducing set for a code C need not be an m -erasure correcting set for C .

Example 2.3: Let C be the binary $[5, 1, 5]$ repetition code, and let \mathcal{H} consist of the four vectors $\mathbf{h}_1 = 10001$, $\mathbf{h}_2 = 01100$, $\mathbf{h}_3 = 01111$, and $\mathbf{h}_4 = 01010$. Note that \mathcal{H} spans the dual code C^\perp of C (which is just the even-weight code of length five). In the table below, we provide for each set of erasures of size four a parity check equation that has weight one inside

this erasure set.

non-erased position	parity check equation
1	\mathbf{h}_1
2	\mathbf{h}_2
3	\mathbf{h}_2
4	\mathbf{h}_4
5	\mathbf{h}_1

The set \mathcal{H} is therefore 4-erasure reducing for C . It is, however, not 4-erasure correcting for C , as $\{2, 3, 4\}$ is a stopping set that does not contain the support of a nonzero codeword. So for example the erasure set $\{1, 2, 3, 4\}$ is C -correctable, and can be reduced but not corrected by \mathcal{H} .

Finally, in [5] we introduced the notion of a “generic” m -erasure correcting and reducing set for codes of a fixed codimension. The idea is to describe which linear combinations to take given any parity check matrix for any such code.

Definition 2.4: Let $1 \leq m \leq r$. A set $\mathcal{A} \subseteq \mathbb{F}_2^r$ is called *generic (r, m)-erasure reducing* if for any $n \geq r$ and for any $r \times n$ matrix H of rank r , the collection $\{\mathbf{a}H \mid \mathbf{a} \in \mathcal{A}\}$ is m -erasure reducing for the code with parity check matrix H . The set $\mathcal{A} \subseteq \mathbb{F}_2^r$ is called *generic (r, m)-erasure correcting* if it is generic (r, m') -erasure reducing for all m' with $1 \leq m' \leq m$.

The following useful characterization of generic (r, m) -erasure reducing sets has been obtained in [5].

Proposition 2.5: A set $\mathcal{A} \subseteq \mathbb{F}_2^r$ is generic (r, m) -erasure reducing if and only if for any $r \times m$ matrix M of rank m there is a vector $\mathbf{a} \in \mathcal{A}$ such that $\text{wt}(\mathbf{a}M) = 1$.

The proof of this proposition can be outlined as follows. It can be shown that an erasure pattern E is C -correctable if and only if for any parity check matrix H for C , the restriction $H(E)$ has full rank. Hence, we need only consider $r \times m$ submatrices of full rank, and each $r \times m$ matrix of full rank can occur as such a submatrix. Using Proposition 2.5, it can be shown that for all codes of a fixed codimension, the Hamming code is the most difficult code to design generic erasure reducing sets for. The following proposition states this fact more precisely.

Proposition 2.6: Let $m \leq r$. Let H_r be a parity check matrix of the $[2^r - 1, 2^r - r - 1, 3]$ Hamming code C_r . A set $\mathcal{A} \subset \mathbb{F}_2^r$ is generic (r, m) -erasure reducing if and only if $\mathcal{H} = \{\mathbf{a}H_r \mid \mathbf{a} \in \mathcal{A}\}$ is m -erasure reducing for C_r .

Proof: Combination of Proposition 2.5 and the fact that any $r \times m$ matrix of rank m occurs, up to a column permutation, as a submatrix of H_r . ■

We are interested in generic (r, m) -erasure reducing sets of small size. This motivates the following definition.

Definition 2.7: For $1 \leq m \leq r$, we define $F(r, m)$ as the smallest size of any generic (r, m) -erasure reducing set.

Note that Proposition 2.5 implies that $\mathbb{F}_2^r \setminus \{\mathbf{0}\}$ is generic (r, m) -erasure reducing, so $F(r, m)$ is well-defined.

Example 2.3 shows that for particular codes C , the notions

" m -erasure reducing for C " and " m -erasure correcting for C " need not be the same; as we proceed to show, the notions "generic (r, m) -erasure reducing" and "generic (r, m) -erasure correcting", are, somewhat surprisingly, equivalent.

Proposition 2.8: Let $2 \leq m \leq r$. A generic (r, m) -erasure reducing set is a generic $(r, m-1)$ -erasure reducing set.

Proof: Let \mathcal{A} be a generic (r, m) -erasure-reducing set. Let M be a binary $r \times (m-1)$ matrix of rank $m-1$. We write

$$M = [M_0 \mid \mathbf{x}^\top],$$

where \mathbf{x}^\top denotes the rightmost column of M . Let \mathbf{y}^\top be a vector in \mathbb{F}_2^r that is not in the linear span of the columns of M , and let M' denote the $r \times m$ matrix defined as

$$M' = [M_0 \mid \mathbf{y}^\top \mid \mathbf{x}^\top + \mathbf{y}^\top].$$

As M' has rank m , there exists a vector $\mathbf{a} \in \mathcal{A}$ such that $\text{wt}(\mathbf{a}M') = 1$. We claim that $\text{wt}(\mathbf{a}M) = 1$. This is clear if $\text{wt}(\mathbf{a}M_0) = 1$, as then $\mathbf{a}\mathbf{x}^\top = \mathbf{a}\mathbf{y}^\top = 0$. If $\mathbf{a}M_0 = 0$, then $\mathbf{a}\mathbf{y}^\top = 0$ and $\mathbf{a}(\mathbf{x}^\top + \mathbf{y}^\top) = 1$, or vice versa. In either case, $\mathbf{a}\mathbf{x}^\top = \mathbf{a}\mathbf{y}^\top + \mathbf{a}(\mathbf{x}^\top + \mathbf{y}^\top) = 1$, from which we conclude that in this case also $\mathbf{a}M$ has weight 1. ■

Note that Proposition 2.8 implies that the parity check equations induced by a generic (r, m) -erasure reducing set can also be used to resolve an erasure from a correctable erasure set of size $m-1, m-2, \dots$ (as shown in Example 2.3, this need not hold for a specific m -erasure reducing set for a specific code). In other words, the following proposition holds.

Proposition 2.9: Any generic (r, m) -erasure reducing set is a generic (r, m) -erasure correcting set.

According to Proposition 2.9, the terms "generic (r, m) -erasure reducing" and "generic (r, m) -erasure correcting" can be used interchangeably. In the sequel, we use "correcting", and base our results on the characterization given in Proposition 2.5.

III. EXPLICIT GENERIC (r, m) -ERASURE CORRECTING SETS

In this section, we describe generic (r, m) -erasure correcting sets $\mathcal{A}_{r,m}$ for all r and m with $r \geq m \geq 2$ (see also [5]). We will show that the sets $\mathcal{A}_{r,3}$ are closely related to the sets found by Weber and Abdel-Ghaffar [2]. Modifications and generalizations of these sets can be found in [5] and [6].

Theorem 3.1: Let $2 \leq m \leq r$. The set $\mathcal{A}_{r,m}$ defined as $\mathcal{A}_{r,m} = \{\mathbf{a} = (a_1, a_2, \dots, a_r) \in \mathbb{F}_2^r \mid a_1 = 1 \text{ and } \text{wt}(\mathbf{a}) \leq m\}$ is a generic (r, m) -erasure correcting set of size

$$\sum_{i=0}^{m-1} \binom{r-1}{i}.$$

Proof: As $\mathcal{A}_{r,m}$ consists of all vectors that start with a one and have weight at most $m-1$ in the positions $2, 3, \dots, r$, the statement on the size of $\mathcal{A}_{r,m}$ is obvious.

In order to show that $\mathcal{A}_{r,m}$ is indeed generic (r, m) -erasure correcting, we will use Proposition 2.5. So let M be an $r \times m$ matrix of rank m . We have to show that there is a vector $\mathbf{a} \in \mathcal{A}_{r,m}$ such that $\text{wt}(\mathbf{a}M) = 1$. To this end, we proceed as follows. For $1 \leq i \leq r$, let \mathbf{m}_i denote the i -th row of M . Let $I \subseteq \{1, 2, \dots, r\}$ be such that $\{\mathbf{m}_i \mid i \in I\}$ forms a basis for \mathbb{F}_2^m . We distinguish two cases.

(i): $\mathbf{m}_1 \neq \mathbf{0}$.

We can and do choose I such that $1 \in I$. The set $\{\sum_{i \in I} x_i \mathbf{m}_i \mid (x_i)_{i \in I}, x_1 = 0\}$ is an $(m-1)$ -dimensional space and hence cannot contain all unit vectors. So there exists a vector $\mathbf{x} = (x_i)_{i \in I}$ with $x_1 = 1$ and $\text{wt}(\sum_{i \in I} x_i \mathbf{m}_i) = 1$. Now, let $\mathbf{a} \in \mathbb{F}_2^r$ be the vector that agrees with \mathbf{x} in the positions indexed by I and has zeroes elsewhere. Then $a_1 = x_1 = 1$ and $\text{wt}(\mathbf{a}) = \text{wt}(\mathbf{x}) \leq m$, hence $\mathbf{a} \in \mathcal{A}_{r,m}$ and $\mathbf{a}M = \sum_{i=1}^r a_i \mathbf{m}_i = \sum_{i \in I} x_i \mathbf{m}_i$, so $\text{wt}(\mathbf{a}M) = 1$.

(ii): $\mathbf{m}_1 = \mathbf{0}$.

In this case $1 \notin I$. As $\{\mathbf{m}_i \mid i \in I\}$ forms a basis, there are independent vectors $\mathbf{x}(j) = (x_i(j) \mid i \in I)$ such that $\mathbf{e}_j = \sum_{i \in I} x_i(j) \mathbf{m}_i$ for all j . As there is just one vector \mathbf{x} of weight m , and there are $m \geq 2$ unit vectors, there is an index j such that $\text{wt}(\mathbf{x}(j)) \leq m-1$. Now, let \mathbf{a} be the vector that agrees with $\mathbf{x}(j)$ in the positions indexed by I , has a "1" in the first position, and zeroes elsewhere. As $\text{wt}(\mathbf{x}(j)) \leq m-1$, the vector \mathbf{a} is in $\mathcal{A}_{r,m}$. Moreover, we have that $\mathbf{a}M = \sum_{i=1}^n a_i \mathbf{m}_i = a_1 \mathbf{m}_1 + \sum_{i \in I} a_i \mathbf{m}_i = \mathbf{0} + \mathbf{e}_j = \mathbf{e}_j$. ■

We now relate our result for $m = 3$ to that of Weber and Abdel-Ghaffar [2], which in our terminology states that

$$\mathcal{W}_r = \{\mathbf{e}_i \mid 1 \leq i \leq r\} \cup \{\mathbf{e}_1 + \mathbf{e}_i + \mathbf{e}_j \mid 2 \leq i < j \leq r\}$$

is generic $(r, 3)$ -erasure correcting. To this end, let S be the matrix with the all-one vector as leftmost column, and with \mathbf{e}_j^\top as j -th column for $2 \leq j \leq r$. Clearly, S is invertible, and for $2 \leq i \leq r$ and $2 \leq j < k \leq r$, we have that

$$\begin{aligned} \mathbf{e}_1 S &= \mathbf{e}_1, & (\mathbf{e}_1 + \mathbf{e}_i) S &= \mathbf{e}_i, \text{ and} \\ (\mathbf{e}_1 + \mathbf{e}_j + \mathbf{e}_k) S &= \mathbf{e}_1 + \mathbf{e}_i + \mathbf{e}_j \end{aligned}$$

As a consequence, we have that

$$\mathcal{W}_r = \{\mathbf{a}S \mid \mathbf{a} \in \mathcal{A}_{r,3}\}.$$

So \mathcal{W}_r and $\mathcal{A}_{r,3}$ are related via an element-wise multiplication with the invertible matrix S .

IV. UPPER AND LOWER BOUNDS ON $F(r, m)$

In this section, we show that $F(r, m)$ is of linear order in r . To be more precise, we will show that for each $m \geq 1$, there exists a constant c_m such that for each $r \geq m$, we have that $r \leq F(r, m) \leq c_m r$.

Concerning the lower bound, we have the following lemma.

Lemma 4.1: Any (r, m) -erasure decoding set spans \mathbb{F}_2^r . As a consequence, $F(r, m) \geq r$.

Proof: (cf. [5]) Suppose $\mathcal{A} \subset \mathbb{F}_2^r$ is such that $\text{span}(\mathcal{A}) \neq \mathbb{F}_2^r$. We will show that \mathcal{A} is not generic (r, m) -erasure decoding by constructing an $r \times m$ matrix M with rank m such that

for each $\mathbf{a} \in \mathcal{A}$, the vector $\mathbf{a}M$ does not have weight 1 (cf. Proposition 2.5).

Let \mathbf{v} be a non-zero vector that has inner product 0 with all words from \mathcal{A} . Let M be an invertible matrix such that the i -th row of M has odd weight if and only if $i \in \text{supp}(\mathbf{v})$, and let $\mathbf{a} \in \mathcal{A}$. As $(\mathbf{v}, \mathbf{a}) = 0$, the vector $\mathbf{a}M$ is the sum of an even number of (odd weight) rows of M indexed by integers from $\text{supp}(\mathbf{v})$, and some (even weight) rows of M indexed by integers outside $\text{supp}(\mathbf{v})$. As a consequence, $\mathbf{a}M$ has even weight. ■

The proof for the upper bound (cf. [6]) can be considered to be a random-coding argument: we will show that the collection of all subsets of \mathbb{F}_2^r of a sufficiently large size contains at least one generic (r, m) -erasure correcting set. The precise result is as follows.

Theorem 4.2: For all $m \geq 1$ and $r \geq m$, we have that

$$F(r, m) \leq \frac{m}{-\log_2(1 - m2^{-m})} \cdot r,$$

where \log_2 denotes the base-2 logarithm.

Proof: Let $1 \leq m \leq r$. We write $\mathcal{M}_{m,r}$ to denote the collection of all binary $r \times m$ matrices of rank m . Let N be some positive integer. Consider the following experiment. We randomly construct a binary $N \times r$ matrix A by setting each individual entry to zero or to one, each with probability $1/2$. We interpret this matrix as a sequence of N row vectors $\mathbf{a}_1, \dots, \mathbf{a}_N$, each of length r . For each matrix M in $\mathcal{M}_{m,r}$, we define the random variable X_M by

$$X_M = \begin{cases} 0, & \text{if there is an } i \text{ such that } \text{wt}(\mathbf{a}_i M) = 1; \\ 1, & \text{otherwise.} \end{cases}$$

So $X_M = 0$ if the matrix M is “good” with respect to the vectors $\mathbf{a}_1, \dots, \mathbf{a}_N$, and $X_M = 1$ if M is “bad”.

Furthermore, let the random variable X be defined as

$$X = \sum_{M \in \mathcal{M}_{m,r}} X_M.$$

The random variable X thus counts the number of bad matrices with respect to A ; if $X < 1$, then all matrices are “good” with respect to A , so that the collection $\mathcal{A} = \{\mathbf{a}_1, \dots, \mathbf{a}_N\} \subseteq \mathbb{F}_2^r$ satisfies the criterion in Proposition 2.5. Consequently, if $E[X] < 1$, then all matrices in $\mathcal{M}_{m,r}$ are good with respect to some matrix A , and so $F(r, m) \leq N$.

In order to compute $E[X]$, we start by fixing a matrix $M \in \mathcal{M}_{m,r}$ and compute the probability $\text{Prob}(X_M = 1)$ that X_M is equal to 1. As M has full rank, there are, for each $i = 1, 2, \dots, m$, exactly 2^{r-m} vectors \mathbf{a} such that $\mathbf{a}M = \mathbf{e}_i$. We conclude that there are $m2^{r-m}$ “good” vectors for M , and hence $2^m(1 - m2^{-m})$ “bad” vectors. Now the matrix M is bad if all the vectors $\mathbf{a}_1, \dots, \mathbf{a}_N$ are bad; we conclude that

$$\text{Prob}(X_M = 1) = (1 - m2^{-m})^N.$$

Since expectation is a linear operation, we have that

$$E[X] = \sum_{M \in \mathcal{M}_{m,r}} E[X_M] = |\mathcal{M}_{m,r}|(1 - m2^{-m})^N,$$

from which we conclude that $E[X] < 1$ if and only if

$$N > \frac{\log_2 |\mathcal{M}_{m,r}|}{-\log_2(1 - m2^{-m})}. \quad (1)$$

As a consequence of the foregoing, we have that $F(r, m) \leq N$ if N satisfies (1); as $|\mathcal{M}_{m,r}|$ is at most 2^{mr} , the cardinality of the set of all $r \times m$ matrices, it follows that $F(r, m) \leq N$ whenever

$$N \geq \frac{m}{-\log_2(1 - m2^{-m})} \cdot r. \quad \blacksquare$$

V. CONCLUSIONS

In this paper, we have introduced the notion of generic (r, m) -erasure correcting sets in \mathbb{F}_2^r ; such sets provide for each binary code C with redundancy r a collection of parity check equations for C that can be used to iteratively correct all C -correctable erasure patterns of size at most m . We provided an explicit construction of generic (r, m) -erasure correcting sets of size $\sum_{i=0}^{m-1} \binom{r-1}{i}$, generalizing the result for $m = 3$ from [2]. We also showed, by a random-coding-like argument, that for each fixed m , the minimal size of generic (r, m) -erasure correcting sets is *linear* in r .

The main remaining problem is to find *explicit* constructions for (r, m) -erasure correcting sets of size linear in r , especially in the first open case $m = 3$. In [6], we provide an explicit recursive construction of $(r, 3)$ -erasure correcting sets of cardinality $1 + 3(r-1)^{\log_2 3}$ – not linear in r , but much smaller than the $(r, 3)$ -erasure correcting sets from Section III for which the cardinality is quadratic in r .

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