

# On Interference of Signals and Generalization in Feedforward Neural Networks

Artur Rataj  
Institute of Theoretical and Applied Computer Science,  
Bałtycka 5, Gliwice, Poland

Technical Report IITiS-2002-08-1-1.02

## Abstract

Interference of signals in a feedforward neural network may improve generalization. In this paper it is discussed that the interference may also cause highly random generalization.

## 1 INTRODUCTION

The problem of generalization deterioration, observed in feedforward neural networks during the learning process, is in several articles explained by overfitting (Schaffer, 1991; Rosin and Fierens, 1995; Lawrence et al., 1997; Lawrence and Giles, 2000). In this paper it is discussed that the deterioration can also be caused by the *interference* of signals, propagated through a feedforward neural network, that is summing and common further processing of values at different inputs of a neuron.

The interference may improve generalization. For example, in the case of a multi-dimensional data set, processing of values from one input of a neural network can be influenced by values at another input of the neural network, what may model well the patterns in the training set. The error-minimizing learning process can prevent harmful interference if the interference would increase the neural network error of approximation of the training set. The values propagated from attributes of observations that are absent in the training set, however, can be interfered with no effect on the error. Therefore, the interference can affect the generalization ability of the network. The possible resulting deterioration of generalization can be very different from that caused by overfitting. While excessive fitting of the neural network function to the training set means only that some particular patterns of the set are memorized, the interference discussed may introduce *highly random* changes to the generalizing function of the neural network.

## 2 STRONG PROPAGATION REGIONS

In this section the so-called strong propagation regions in the input spaces of neurons will be discussed. The notion will be used further in this paper to describe the discussed interference of signals.

A neuron with linear weight functions and a hyperbolic tangent activation function has its output value equal to a given value  $r$  for its input values that, in the neuron

input space, create a hyperplane  $P_r$ , except the special case where all weights in the neuron are equal to 0. Specifically, there is a hyperplane  $P_0$  for the neuron output value equal to 0. Because the hyperbolic tangent activation functions have the greatest value of derivative at 0, the hyperplane  $P_0$  is the region in the neuron input space for which there is the strongest propagation of signals through the neuron. As the distance from this hyperplane increases, the derivative of the activation function decreases and in effect the propagation becomes weaker. Let us call the region with relatively strong level of propagation a strong propagation region. Let the region consist of points whose distances to  $P_0$  in the input space of the neuron do not exceed a certain value.

Let there be two fully connected subsequent layers  $L_i$  and  $L_{i+1}$  in a feedforward neural network. Let there be  $N_i$  and  $N_{i+1}$  neurons in the layers, respectively. Let us discuss the input spaces of the neurons in the layer  $L_{i+1}$ . Each of the neurons in the layer  $L_{i+1}$  has  $N_i + 1$  inputs,  $N_i$  of which are from the neurons in the preceding layer and a single input is from the bias element. Therefore, the transformation made in the layer  $L_{i+1}$  can be represented by parameterized  $N_{i+1}$   $N_i$ -dimensional input spaces of the neurons in  $L_{i+1}$ , where the parameters in the spaces are the values of functions of the respective neurons that correspond to the spaces.

An example of the input spaces of neurons in  $L_{i+1}$  is shown in Figure 1. The lines

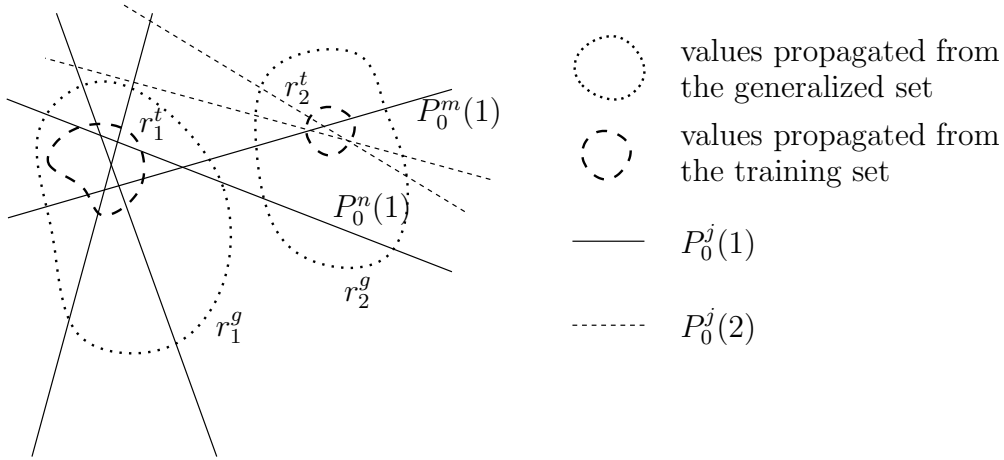


Figure 1: An example diagram of input spaces of neurons in a layer.

represent the hyperplanes  $P_0^j$  of all neurons  $j$ ,  $j = 0, 1, \dots, N_{i+1} - 1$ , in the layer  $L_{i+1}$ , that is they show the zeroes of the functions of the neurons, as it was described in Sec. 2. This is not a full representation of the input spaces of the neurons in the discussed layer, because the values of functions of the neurons are not given, yet this diagram shows the regions with the strong propagation of signals, being on and near the hyperplanes  $P_0^j$ . The values propagated to the neurons in the layer  $L_{i+1}$  are either the direct values of attributes of observations if  $L_{i+1}$  is the first hidden layer, or images of the attributes if  $L_{i+1}$  is any of the succeeding layers. Anyway, the region  $r_t$  of values propagated from the observations in the training set and the region  $r_g$  of values propagated from the observations in the generalized set can be shown in the input spaces of the neurons, as it is done in Figure 1. In the example diagram, the region  $r_t$  consists of two regions  $r_p^t$ ,  $p = 1, 2$ , and the region  $r_g$

consists of another two regions  $r_q^g$ ,  $q = 1, 2$ . The regions are schematically shown by solid regions in the diagrams, but they are sets of discrete points, where each point corresponds to one or more observations.

Let each observation has its input attributes, that is these that are propagated from the inputs of a neural network, and its output attributes, that is these that are compared to values at the outputs of the network. The hyperplanes  $P_0^j$  in the example diagram generally concentrate in or near the regions  $r_p^t$ . This may happen during the training process if there are relatively large differences between the values of output attributes of observations whose input attributes are propagated through such regions. Thus, relatively high values of derivatives of functions of the neurons in  $L_{i+1}$  may correspond to relatively large differences between the output attributes of observations in the training set. The hyperplanes  $P_r$ , by extending infinitely in the space, may allow for generalization to the points outside  $r_t$ , also to these that are relatively far from  $r_t$ .

### 3 INTERFERENCE OF SIGNALS

Let us discuss again the diagram of input spaces of neurons in Figure 1. Let there be several hyperplanes  $P_0^j(i)$ ,  $i = 1, 2$ , that were placed during the learning process near  $r_i^t$ , to minimize the component of  $\xi_l$  caused by the observations in the training set, whose attributes propagate through  $r_i^t$ . They are marked in the diagram by solid lines for  $i = 1$  and by dotted lines for  $i = 2$ .

Let the regions  $r_1^g$  and  $r_2^g$  be overlapping or be near to  $r_1^t$  or  $r_2^t$ , respectively. Let the observations whose input attributes are propagated through the regions  $r_1^g$  and  $r_2^g$  be generalized well because of the hyperplanes  $P_0^j(1)$  and  $P_0^j(2)$ , respectively. This is possible because  $P_0^j(1)$  extend from  $r_1^t$  and  $P_0^j(2)$  extend from  $r_2^t$ , thus ‘extrapolating’ the patterns in the region  $r_t$ .

Now, if a hyperplane  $P_0^j(i)$  would ‘intersect’  $r_{3-i}^t$ , like  $P_0^m(1)$  does, it could possibly increase  $\xi_l$ , and thus in the possible further training the intersecting hyperplane could, for example, be driven out of  $r_{3-i}^t$ . Yet if the hyperplane would intersect  $r_{3-i}^g$ , like  $P_0^n(1)$  does, it could intervene the generalization from  $r_{3-i}^t$  to  $r_{3-i}^g$  without any reaction in the training process. More, a region  $r_i^t$  could, during the training, be placed itself in  $r_{3-i}^g$ , thus causing all  $P_0^j(i)$ , associated with  $r_i^t$ , to intervene the generalization to  $r_{3-i}^g$ .

The interference of signals, causing a possibly high randomness of generalization, could be reduced if the strong propagation region of a neuron would not extend itself infinitely in space. This is like in the radial basis function neural networks (Broomhead and Lowe, 1988; Moody and Darken, 1989; Poggio and Girosi, 1989). On the other hand, such forms of finite strong propagation regions like in the radial basis function networks could worsen the ability of generalization of a neural network for sets where long strong propagation regions are needed for good generalization. A possible method of finding a good trade-off between infinite and finite strong propagation regions could be using adaptive activation functions. Such adaptive activation functions could, during training with a special learning algorithm, smoothly adapt their form, for example in the range between a radial basis function and a hyperbolic tangent.

## 4 TESTS

Because in the relatively simple generalization problems the discussed random generalization seemed to be rather rarely observed – usually the trained neural networks after some time began only to overfit the data, showing some randomness connected with limited flexibility – in this test a relatively complex training set will be used.

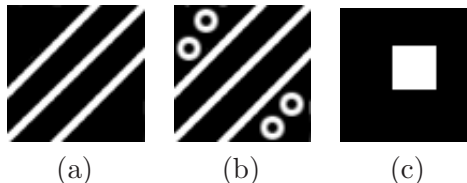


Figure 2: The data sets (a)  $\theta_l$ , (b)  $\theta_c$  and (c) the training subsets mask.

Let there be two three-dimensional sets  $\theta_l$  and  $\theta_c$ , as illustrated in Figures 2(a) and 2(b). The sets are  $64 \times 64$  images, whose pixel coordinates determine the neural network input vector values, a single value for each dimension, and the pixels brightnesses determine corresponding values in the neural network output vectors. The pixel at the lower left corner has coordinates  $(-0.5, -0.5)$  and the pixel at the upper right corner has coordinates  $(0.5, 0.5)$ . The brightness of the pixels represents the range from  $-0.5$  for black to  $0.5$  for white. Feedforward layered networks with two inputs, a single neuron in the output layer and two hidden layers of 16 neurons each are trained by subsets of either  $\theta_l$  or  $\theta_c$ . The neural networks have hyperbolic tangent activation functions. There is a weight decay at a rate of  $2 \cdot 10^{-7}$  to improve generalization (Krogh and Hertz, 1992). An online training is used with a learning step of 0.02. The training subsets are represented by the image in Figure 2(c). Black pixels in the image mean that the corresponding pixels in Figures 2(a) and 2(b) represent the training subsets of the respective sets.

Let there be four neural networks  $\mathcal{N}_i^l$ ,  $i = 0 \dots 3$ , trained with the subset of  $\theta_l$ , and four another neural networks  $\mathcal{N}_i^c$ ,  $i = 0 \dots 3$ , trained with the subset of  $\theta_c$ . Let these networks be tested at the 10000000th, 31622777th and 100000000th iteration. Let the generalizing functions of the networks be sampled and the weights of the neurons in the first input layer saved at each of the iterations. The results are illustrated in Figure 3. There is a table for each iteration, with sampled generalization functions in the upper row and diagrams representing input spaces of neurons in the first input layer in the lower row. The representation of the generalization functions is analogous to that of the sets  $\theta_l$  and  $\theta_c$ . Each of the input space diagrams shows with translucent lines the zeroes of the outputs of the first hidden layer neurons, that is the hyperplanes  $P_0^j$ , against the neurons common input values from the input layer. The lower left corner of the dotted rectangles drawn within the diagrams represents input values  $(-0.5, -0.5)$  and the upper right corner of the rectangles represents input values  $(0.5, 0.5)$ . Therefore, the input attributes of the observations in the sets  $\theta_l$  and  $\theta_c$  are propagated into the space marked in the diagrams by the dotted rectangles. The propagation is with no transformation of course, because the nodes in the input layer only pass signals to the first hidden layer.

Let us look at the diagrams of the input spaces of the neurons in the first hidden layer. Because of the direct relation between the space of the input attributes of the observations

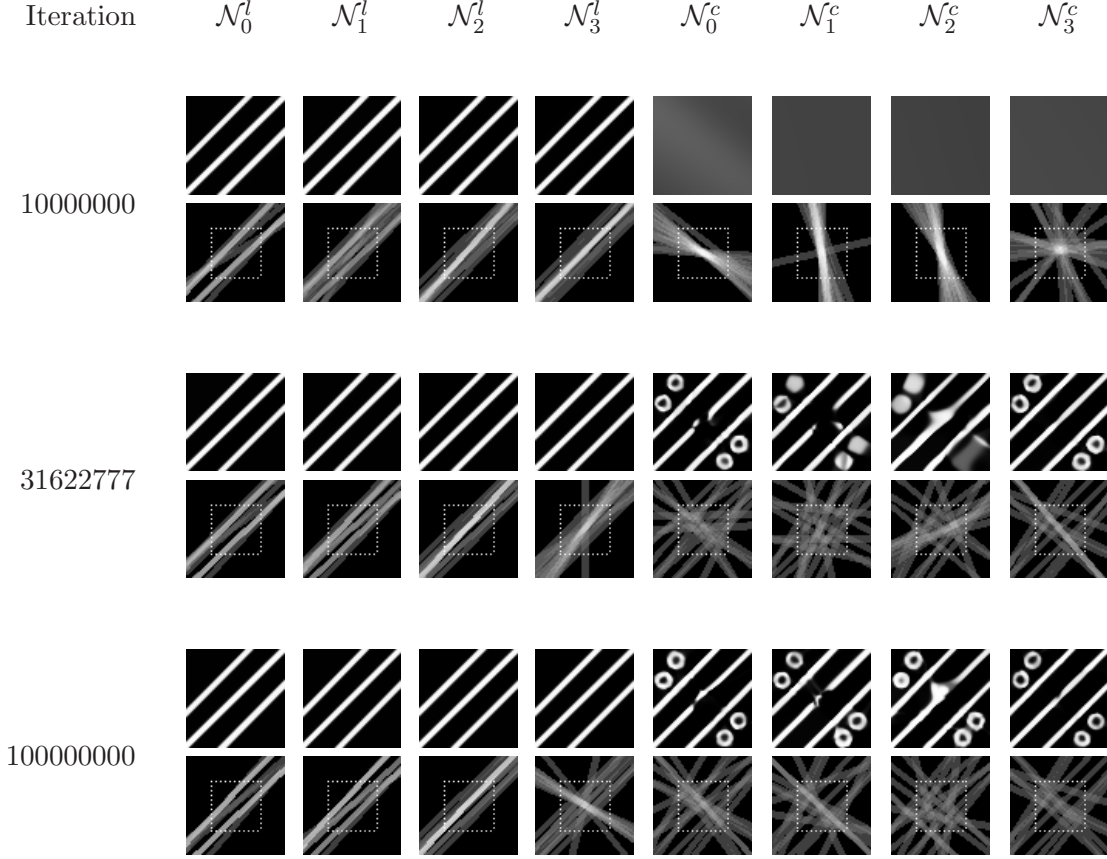


Figure 3: The generalizing functions and diagrams of the zeroes of the first hidden layer neurons.

and the input spaces of the first hidden layer neurons it can be said that in the cases of both  $\mathcal{N}_i^l$  and  $\mathcal{N}_i^c$  the hyperplanes  $P_0^j$  generally concentrate as it was discussed in Sec. 2. In the trained  $\mathcal{N}_i^c$ , generally some hyperplanes concentrate near the linear features  $f_l$  and some concentrate near the circular features  $f_c$ . In effect, the lines in the diagrams concentrated near  $f_c$  cross these concentrated near  $f_l$ . Additionally, the crossings occur partially in the region not covered by the training set. These are exactly the conditions prone to the random generalization, discussed in Sec. 3. In fact, unlike  $\mathcal{N}_i^l$ , where the hyperplanes finely ‘extrapolate’ the regions in the training file, in the functions of  $\mathcal{N}_i^c$  a highly random generalization can be seen.

## 5 CONCLUSIONS

It was theoretically discussed that the interference of signals in feedforward neural networks can cause highly random generalization. It was also discussed that the method of reduction of the described random generalization could be based on adaptive activation functions. An example training of some neural networks, with and without the highly random generalization, was studied.

## References

- D. S. Broomhead and D. Lowe. Multivariable functional interpolation and adaptive networks. *Complex Systems*, 2:321–355, 1988.
- Anders Krogh and John A. Hertz. A simple weight decay can improve generalization. In John E. Moody, Steve J. Hanson, and Richard P. Lippmann, editors, *Advances in Neural Information Processing Systems*, volume 4, pages 950–957. Morgan Kaufmann Publishers, Inc., 1992.
- Steve Lawrence and C. Lee Giles. Overfitting and neural networks: Conjugate gradient and backpropagation. In *Proceedings of the IEEE International Conference on Neural Networks*, pages 114–119. IEEE Press, 2000.
- Steve Lawrence, C. Lee Giles, and A. C. Tsoi. Lessons in neural network training: Overfitting may be harder than expected. In *Proceedings of the Fourteenth National Conference on Artificial Intelligence, AAAI-97*, pages 540–545. AAAI Press, Menlo Park, California, 1997.
- J. Moody and C. Darken. Fast learning in networks of locally tuned units. *Neural Computations*, 1(2):281–294, 1989.
- Tomaso Poggio and Federico Girosi. A theory of networks for approximation and learning. Technical Report AIM-1140, 1989.
- P. Rosin and F. Fierens. Improving neural network generalisation, 1995.
- C. Schaffer. Overfitting avoidance as bias. In *IJCAI-91 Workshop on Evaluating and Changing Representation in Machine Learning*, Sydney, 1991.