

Resilient $PC(l)$ of Order k Boolean Functions from AG-Codes

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Abstract

Propagation criterion of degree l and order k ($PC(l)$ of order k) and resiliency of vectorial Boolean functions are important for cryptographic purpose (see [1, 2, 3, 6, 7,8,10,11,16]. Kurosawa , Stoh [8] and Carlet [1] gave a construction of Boolean functions satisfying $PC(l)$ of order k from binary linear or nonlinear codes. In this paper, algebraic-geometric codes over $GF(2^m)$ are used to modify Carlet and

Kurosawa-Satoh's construction for giving vectorial resilient Boolean functions satisfying $PC(l)$ of order k . The new construction is compared with previously known results.

Index Terms—Cryptography, Boolean functions, algebraic-geometric codes

I. Introduction and Preliminaries

In cryptography vectorial Boolean functions are used in many applications (see [2]). Propagation criterion of degree l and order k is one of the most general properties of Boolean functions which has to be satisfied for their use in block ciphers. It was introduced in Preneel et al [11], which extends the property strictly avalanche criterion SAC in [16]. For a Boolean function $f(x) = (x_1, \dots, x_n)$ of n variables, set $\frac{Df}{D\alpha} = f(x) + f(x + \alpha)$, f satisfies $PC(l)$ if $\frac{Df}{D\alpha}$ is a balanced Boolean function for any α with $1 \leq wt(\alpha) \leq l$. When any function obtained from f by keeping any k variables fixed satisfies $PC(l)$, we say f have the property $PC(l)$ of order k . For a vectorial Boolean function $\mathbf{f} = (f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n))$ it is called $(n, m) - PC(l)$ of order k if any nonzero linear combination of f_1, \dots, f_m satisfies $PC(l)$ of order k . We say \mathbf{f} satisfies $SAC(k)$ if it has $PC(1)$ of order k property. A vectorial Boolean function $\mathbf{f} = (f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n))$ is called k -resilient, if any nonzero linear combination $\sum_i a_i f_i$ is a k -resilient. Resiliency of vectorial Boolean functions are relevant to quantum key distribution and pseudo-random sequence generators for stream ciphers (see [1], [2], [3], [4] and [17]).

We recall the Maiorana-MacFarland construction of vectorial Boolean functions. Let $\phi_i : GF(2)^s \longrightarrow GF(2)^s$ be vectorial Boolean functions for $i = 1, \dots, m$, the class of Maiorana-MacFarland $(r+s, m)$ Boolean functions is the set of the functions $F(x, y)$ of the form $F(x, y) = (x \cdot \phi_1(y) + h_1(y), \dots, x \cdot \phi_m(y) + h_m(y)) : GF(2)^{r+s} \longrightarrow GF(2)^m, (x, y) \in GF(2)^r \times GF(2)^s$. It is well known that $F(x, y)$ is at least t -resilient if $a_1 \phi_1(y) + \dots + a_m \phi_m(y)$, for any $(a_1, \dots, a_m) \in GF(2)^m$, has its Hamming weight at least $t + 1$ for all $y \in GF(2)^s$ (see [1], [2] and [3]).

It was known that $PC(n)$ Boolean functions of n variables are just the

perfect nonlinear functions introduced by W.Meier and O.Staffebach [10]. They exist only when n is even. Bent functions are example of this kind of functions (see [10] and [16]). People only have few constructions of $PC(l)$ of order k Boolean functions. In [1] and [8] $PC(l)$ of order k (vectorial) Boolean functions were constructed from binary linear or nonlinear codes. For satisfying the conditions of the construction the minimum distances of the binary codes and its dual have to be lower bounded. Some lower bounds on the minimum length (which is the half of the number of the number of the variables in Kurosawa-Satoh construction) of these binary linear codes were studied in [9].

From [1] and [8] we know the following results.

Kurosawa-Satoh Theorem ([8]). *Let C_1 be a linear binary code of length s and minimum distance d_1 and dual distance d'_1 , C_2 be a linear binary code of length t with minimum distance d_2 and dual distance d'_2 . Set $l = \min\{d'_1, d'_2\} - 1$ and $k = \min\{d_1, d_2\} - 1$. Then the Boolean functions of $s + t$ inputs satisfying $PC(l)$ of order k can be explicitly given.*

Corollary 1 ([8] and [9]). *Let C be a linear binary code with minimum distance at least $k+1$ and dual distance at least $l+1$. Then Boolean functions of $2n$ inputs satisfying $PC(l)$ of order k can be explicitly given.*

Carlet Theorem ([1]). *For a Boolean function $f(x, y) = x\phi(y) + g(y)$ from $GF(2)^{r+s}$ to $GF(2)$, f satisfies $PC(l)$ of order k if the following two conditions are satisfied.*

- 1) *the sum of at least 1 and at most l coordinates of ϕ is k -resilient;*
- 2) *if $b \in GF(2)^s$ is nonzero and has its weight smaller than or equal to l , at least $k + 1$ coordinates of the words $\phi(y + b)$ and $\phi(y)$ differ.*

In this paper the functions ϕ_i 's in the Mairana-MacFarland construction are of the form $A_i y + v_i$, where A_i is a fixed $r \times s$ matrix over $GF(2)$ and v_i is a fixed vector in $GF(2)^r$, for $i = 1, \dots, m$.

Let us now recall some basic facts about AG-codes (algebraic-geometric codes, see [12],[13] and [14]). Let X be an absolutely irreducible, projective and smooth curve defined over $GF(q)$ with genus g , $D = \{P_1, \dots, P_n\}$ be a set

of $GF(q)$ -rational points of X and G be a $GF(q)$ -rational divisor satisfying $\text{supp}(G) \cap D = \emptyset$, $2g - 2 < \deg(G) < n$. Let $L(G) = \{f : (f) + G \geq 0\}$ is the linear space (over $GF(q)$) of all rational functions with its divisor not smaller than $-G$ and $\Omega(B) = \{\omega : (\omega) \geq B\}$ be the linear space of all differentials with their divisors not smaller than B . Then the functional AG-code $C_L(D, G) \in GF(q)^n$ and residual AG-code $C_\Omega(D, G) \in GF(q)^n$ are defined. $C_L(D, G)$ is a $[n, k = \deg(G) - g + 1, d \geq n - \deg(G)]$ code over $GF(q)$ and $C_\Omega(D, G)$ is a $[n, k = n - \deg(G) + g - 1, d \geq \deg(G) - 2g + 2]$ code over $GF(q)$. We know that the functional code is just the evaluations of functions in $L(G)$ at the set D and the residual code is just the residues of differentials in $\Omega(G - D)$ at the set D .

We also know that $C_L(D, G)$ and $C_\Omega(D, G)$ are dual codes. It is known that for a differential η that has poles at P_1, \dots, P_n with residue 1 (there always exists such a η , see [12]) we have $C_\Omega(D, G) = C_L(D, D - G + (\eta))$, the function f corresponds to the differential $f\eta$. This means that functional codes and residue code are essentially the same. It is clear that if there exist a differential η such that $G = D - G + (\eta)$, then $C_L(P, G) = C_\Omega(P, G) = C_L(P, P - G + (\eta))$ is a self-dual code over $GF(q)$. For many examples of AG codes, including these self-dual AG-codes, we refer to [12], [13] and [14].

From the theory of algebraic curves over finite fields, there exist algebraic curves $\{X_t\}$ defined over $GF(q^2)$ with the property $\lim_{t \rightarrow \infty} \frac{N(X_t)}{g(X_t)} = q - 1$ (Drinfeld-Vladut bound)(see [5] and [13]), where $N(X_t)$ is the number of $GF(q^2)$ rational points on the curve X_t and $g(X_t)$ is the genus of the curve X_t . Actually for this family of curve $N(X_t) \geq (q-1)q^t + 1$, $g(X_t) = q^t - 2q^{\frac{t}{2}} + 1$ for t even and $g(X_t) = q^t - q^{\frac{t+1}{2}} - q^{\frac{t-1}{2}} + 1$ for t odd (see [5]).

For a AG-code over $GF(2^m)$ its expansion to some base B of $GF(2^m)$ over $GF(2)$ will be used in our construction. Let $\{e_1, \dots, e_m\}$ be a base of $GF(2^m)$ as a linear space over $GF(2)$. For a $[n, k, d]$ linear code $C \subseteq GF(2^m)^n$, the expansion with respect to the base B is the binary code $B(C) \subseteq GF(2)^{mn}$ consisting of all codewords $B(x) = (B(x_1), \dots, B(x_n))$, $x = (x_1, \dots, x_n) \in C$. Here $B(x_i)$ is a length m binary vector (x_i^1, \dots, x_i^m) , where $x_i = \sum_{j=1}^m x_i^j e_j \in GF(2^m)$. It is easy to verify that the binary linear code $B(C)$ is $[mn, mk, \geq d]$ code. It is well known that there exists a self-dual base B for any finite field

$GF(2^m)$ of characteristic 2. The following result is useful in our construction.

Proposition 1 ([6]). *Let B be a self-dual base of $GF(2^m)$ over $GF(2)$ and C be a linear code over $GF(2^m)$. Then the dual code $B(C)^\perp$ is just $B(C^\perp)$.*

A divisor G on the curve X is called effective if the coefficients of all points in the support G are non-negative. We say $G_1 \geq G_2$ if $G_1 - G_2$ is an effective divisor. This gives a partial order relation on the set of all divisors. Let U_1, \dots, U_m be divisors on the curve X , set $\max\{U_1, \dots, U_m\}$ the smallest divisor U such that $U - U_i$ is effective for all $i = 1, \dots, m$ and $\min\{U_1, \dots, U_m\}$ the biggest divisor U' such that $U_i - U'$ is effective for all $i = 1, \dots, m$. For m divisors U_1, \dots, U_m and it is clear the intersection $\bigcap_i L(U_i) = L(\min\{U_1, \dots, U_m\})$, $\bigcap_i \Omega(U_i) = \Omega(\max\{U_1, \dots, U_m\})$, the linear span of $L(U_1), \dots, L(U_m)$ is just $L(\max\{U_1, \dots, U_m\})$.

II. Main Result

The following Theorem 1 and Corollary 2 are the main results of this paper.

Theorem 1. *Let X (resp. X') be a projective, absolutely irreducible smooth curve of genus g (resp. g') defined over $GF(2^w)$ (resp. $GF(2^{w'})$), P (resp. P') be a set of n $GF(2^w)$ (resp. n' , $GF(2^{w'})$) rational points on X (resp. X'), U_1, \dots, U_m (resp. U'_1, \dots, U'_m) be $GF(2^w)$ (resp. $GF(2^{w'})$)-rational effective divisors on X (resp. X') with degree satisfying $2g - 2 < \deg(\max\{U_1, \dots, U_m\}) < n$ and $\text{supp}(\max\{U_1, \dots, U_m\}) \cap P = \emptyset$ (resp. $2g' - 2 < \deg(\max\{U'_1, \dots, U'_m\}) < n'$, $\text{supp}(\max\{U'_1, \dots, U'_m\}) \cap P' = \emptyset$). Suppose $\deg(U_i) = \deg(U'_i)$ for $i = 1, \dots, m$. We have another $GF(2^{w'})$ -rational effective divisor H on X' satisfying $\deg(H) + \deg(\max\{U'_1, \dots, U'_m\}) < n'$ and $w'(\deg(H) - g' + 1) \geq m$. It is assumed that U'_1, \dots, U'_m, H are disjoint divisors (that is, their supports are disjoint). Then we have $(wn + w'n', m)$ vectorial t -resilient $PC(l)$ of order k Boolean functions with $wn + w'n'$ variables, where*

$$\begin{aligned} l &= \min\{\deg(\max\{U_1, \dots, U_m\}) - 2g + 1, \deg(\max\{U'_1, \dots, U'_m\}) - 2g' + 1\} \\ k &= \min\{n - \deg(\max\{U_1, \dots, U_m\}) - 1, n' - \deg(\max\{U'_1, \dots, U'_m\}) - 1\} \\ t &= n' - \deg(\max\{U'_1, \dots, U'_m, H\}) - 1. \end{aligned}$$

If the curves, the bases of the linear space $L(U_i)$'s and $\Omega(U_i)$'s (resp. $L(U'_i)$'s, $L(H)$ and $\Omega(U'_i)$'s) are explicitly given, the $(wn + w'n', m)$ vectorial t -resilient PC(l) of order k Boolean functions can be explicitly given.

Proof. We consider the $D_1^i = C_L(P, U_i)$, $D_2^i = C_L(P', U'_i)$, then $(D_1^i)^\perp = C_\Omega(P, U_i)$, $(D_2^i)^\perp = C_\Omega(P', U'_i)$. Let B and B' be self dual bases of $GF(2^w)$ and $GF(2^{w'})$ over $GF(2)$. We will use the linear binary codes $C_1^i = B(D_1^i)$, $C_2^i = B'(D_2^i)$. From Proposition 1 $(C_1^i)^\perp = B(C_\Omega(P, U_i))$, $(C_2^i)^\perp = B'(C_\Omega(P', U'_i))$. The code parameters of C_1^i and C_2^i are $[wn, w(deg(U_i) - g + 1), \geq n - deg(U_i)]$ and $[w'n', w'(deg(U'_i) - g' + 1), \geq n' - deg(U'_i)]$. The code parameters of $(C_1^i)^\perp$ and $(C_2^i)^\perp$ are $[wn, w(n - deg(U_i) + g - 1), \geq deg(U_i) - 2g + 2]$ and $[w'n', w'(n' - deg(U'_i) + g' - 1), \geq deg(U'_i) - 2g' + 2]$.

Let Q_i and R_i be the generator matrices of the binary linear code C_1^i and C_2^i respectively, for $i = 1, \dots, m$. Here we note that Q_i 's (resp R_i 's) are $w(deg(U_i) - g + 1) \times wn$ matrices (resp. $w'(deg(U'_i) - g' + 1) \times w'n'$ matrices). Since $w'(deg(H) - g' + 1) \geq m$, we can find m linear independent vectors v_1, \dots, v_m in the binary linear code $B(C_L(H, P'))$. Set $\phi_i(y) = (R_i)^T Q_i(y) + v_i$, $y \in GF(2)^{wn}$ for $i = 1, \dots, m$, in Maiorana-MacFarland construction we get our $(wn + w'n', m)$ Boolean function $\mathbf{f} = (f_1, \dots, f_m)$. Here ϕ_i 's are mappings from $GF(2)^{wn}$ to $GF(2)^{w'n'}$. The image of ϕ_i is in the coset $v_i + C_2^i$ for $i = 1, \dots, m$.

For any nonzero linear combination $a_1 f_1 + \dots + a_m f_m$, we set $\phi(y) = \sum_i a_i \phi_i(y) + \sum_i a_i v_i$. Then it is clear that $\sum_i a_i \phi_i(y)$ is in the binary linear code $B(C_L(P', \max\{U'_1, \dots, U'_m\}))$ and $\sum_i a_i v_i$ is in the binary linear code $B(C_L(P', H))$. Because $\max\{U'_1, \dots, U'_m\}$ and H are disjoint, so $\sum_i a_i \phi_i(y) + \sum_i a_i v_i$ is not zero. On the other hand this is a nonzero code word in $B(C_L(P', \max\{U'_1, \dots, U'_m, H\}))$, its weight is at least $n' - deg(\max\{U'_1, \dots, U'_m, H\})$. Hence \mathbf{f} is t -resilient.

From the above argument it is also known that $\phi(y) = \sum_i a_i \phi_i(y) + \sum_i a_i v_i$ takes over all codewords in the coset of the binary linear code $B(C_L(P', \max\{U'_1, \dots, U'_m\}))$, when y takes over all vectors in $GF(2)^{wn}$. Thus the sum of arbitrary j (where, $1 \leq j \leq l$) coordinates of this function $\phi(y)$ is a nonzero function, since l is less than the Hamming distance of the code $B(C_\Omega(P', \max\{U'_1, \dots, U'_m\})) = (B(C_L(P', \max\{U'_1, \dots, U'_m\})))^\perp$. We

can check that $\phi^{-1}(c)$ has the same cardinality for all $c \in GF(2)^{w'n'}$ since it is an affine mapping. Because it is a subcode of the coset code of $B(C_L(P', \max\{U'_1, \dots, U'_m\}))$, the dual distance of $\phi^{-1}(c)$ is at least $k + 1$. From Corollary 14 in [1], $\phi(y)$ is a k -resilient function. The 1st condition of Carlet Theorem is satisfied.

For any $b \in GF(2)^{wn}$, $\phi(y + b) + \phi(y) = \phi(b)$. If the weight of b has its weight smaller than or equal to l , it is not in $B(C_\Omega(P, \max\{U_1, \dots, U_m\}))$, thus $Q_i b$ can not be zero for all $i = 1, \dots, m$. Thus at least one $(R_i)^\tau Q_i b$ is not zero. From the condition U'_1, \dots, U'_m are disjoint effective divisors on X' , we know that $\phi(b) = \sum_i a_i (R_i)^\tau Q_i b$ is a nonzero codeword in $B(C_L(P', \max\{U'_1, \dots, U'_m\}))$. Thus $\phi(b)$ has its weight at least $k + 1$. The 2nd condition of Carlet Theorem is satisfied. The conclusion is proved.

It is well known in the theory of algebraic curves over finite fields, there are many curves over $GF(2^w)$ (see [12], [13] and [14]) with various numbers of rational points and genres. Thus when we use Theorem 1 for constructing vectorial t -resilient $PC(l)$ of order k functions, we have very flexible choices of parameters on l, k and the number of variables $wn + w'n'$. This is quite similar to the role of the algebraic curves in the theory of error-correcting codes. Therefore the algebraic-geometric method offer us numerous vectorial t -resilient $PC(l)$ of order k functions. Moreover the supports of the divisors $U_1, \dots, U_m, U'_1, \dots, U'_m, H$ need no to be the $GF(2^w)$ (or $GF(2^{w'})$) rational points, it is sufficient for the constructions in Theorem 1 that the divisors are $GF(2^w)$ (or $GF(2^{w'})$) rational. Thus we can easily choose the sets of points P, P' and divisors to construct vectorial resilient $PC(l)$ of order k Boolean functions.

III. Constructions

In this section some examples of vectorial t -resilient $PC(l)$ of order k Boolean functions are constructed by Theorem 1. Comparing our constructions with the previously-known $PC(l)$ of order k functions in [1] and [8], it seems our constructed vectorial t -resilient $PC(l)$ of order k functions are quite good.

We take $X = X'$ genus g curve which is defined over $GF(2^w)$, $U_i = U'_i$,

$i = 1, \dots, m$ are m disjoint effective divisors rational over $GF(2^w)$, in the case m is small and $\deg(U_i) = \deg(U'_i) = t$ is not 1, we can always choose the supports of U_i 's outside all $GF(2^w)$ rational points on X , for example, we can choose their supports $GF(2^{2w})$ -rational points of X . In the following example, $P = P'$ are n $GF(2^w)$ points of X . So the only restriction is the upper bound of $n \leq N(X)$, the number of $GF(2^w)$ -rational points of X . Thus $\max\{U_1, \dots, U_m\} = U_1 + \dots + U_m$. We take H another degree t' $GF(2^w)$ -rational effective divisor which is disjoint to U_1, \dots, U_m such that $w(t' - g + 1) \geq m$. In this construction we have $(2wn, m)$ vectorial $(n - mt - t' - 1)$ -resilient Boolean functions satisfying $PC(mt - 2g + 1)$ of order $n - mt - 1$.

Example 1. We use the genus 0 curve over $GF(4)$ in the construction. Then $(20, 2)$ vectorial $PC(5)$ function is constructed if we take $m = 2, t = 2, n = 5$.

Example 2. We use the genus 1 curve over $GF(4)$ in the construction, then $n \leq 9$. We have $(4n, m)$ vectorial $(n - mt - t' - 1)$ -resilient functions satisfying $PC(mt - 1)$ of order $n - mt - 1$, where $2t' \geq m$. Thus $(36, 4)$ vectorial $PC(7)$ Boolean functions are constructed, $(36, 3)$ vectorial Boolean functions satisfying $PC(5)$ of order 1 are constructed, $(24, 2)$ vectorial Boolean functions satisfying $PC(3)$ of order 1 are constructed.

When $m = 1, t = 2$ we have $(n - 5)$ -resilient $SAC(n - 3)$ functions of $4n$ variables for $n = 5, 6, 7, 8, 9$.

Example 3 We use the genus 4 curve over $GF(4)$ in the construction, then $n \leq 15$ (see [14]). The $(4n, m)$ vectorial $(n - mt - t' - 1)$ -resilient Boolean functions satisfying $PC(mt - 7)$ of order $n - mt - 1$ are constructed, where $2(t' - 3) \geq m$. Thus we have $(60, 7)$ vectorial $PC(7)$ and $(44, 5)$ vectorial $PC(3)$ Boolean functions, $(48, 5)$ vectorial Boolean functions satisfying $PC(3)$ of order 1, $(60, 6)$ vectorial Boolean functions satisfying $PC(5)$ of order 2.

When $m = 4, t = 2$ we have $(4n, 4)$ vectorial $(n - 14)$ -resilient $SAC(n - 9)$ Boolean functions. For example, $(60, 4)$ vectorial 1-resilient $SAC(6)$ Boolean functions are constructed.

Example 4. We use the Klein quartic X , an algebraic curve over $GF(8)$ of genus 3, then $n \leq 24$, $(6n, m)$ vectorial $(n - mt - t' - 1)$ -resilient $PC(mt - 5)$ of order $n - mt - 1$ Boolean functions are constructed for $n = 7, 8, \dots, 24$, where $3(t' - 2) \geq m$. There are at least 19 degree 2 $GF(8)$ -rational divisors on X (see [14]). Thus we have $(90, 7)$ vectorial $PC(9)$ Boolean functions, $(90, 6)$ vectorial $PC(7)$ of order 4 Boolean functions. When $n = 10, \dots, 24$, we have $(6n, 3)$ vectorial $(n - 10)$ -resilient $SAC(n - 7)$ Boolean functions.

Corollary 2. *Let X be an algebraic curve over $GF(2^w)$ with genus g and n $GF(2^w)$ rational points and there are at least $2g$ $GF(2^{2w})$ -rational points on X . Then we have $(2wn, g)$ vectorial $(n - \lceil \frac{7g}{2} \rceil - 1)$ -resilient $SAC(n - 2g - 1)$ Boolean functions.*

Applying Theorem 1 to Garcia-Stichtenoth curves [5] over $GF(2^{2w})$, we have the following result.

Corollary 3. *For a positive integer $w \geq 1$, we have $(4wn, m)$ vectorial Boolean functions satisfying $PC(mt - 2^{wh+1} + 1)$ of order $(n - mt - 1)$ for $2^{wh+1} + 1 \leq n \leq (2^w - 1)2^{2wh}$ and $m \leq n$.*

Comparing with the constructions in [1] and [8] we can see our method based on AG-codes offer more flexibility for the parameters n, m, t, k, l . The main result is more suitable for constructing *vectorial* Boolean functions satisfying propagation criteria and resiliency, because there are many $GF(2^w)$ -rational divisors on the algebraic curves to choose.

IV. Conclusion

In this paper we presented a method based on AG-codes for constructing (n, m) vectorial t -resilient Boolean functions satisfying $PC(l)$ of order k functions. The parameters n, m, t, k, l in our constructions can be chosen quite flexibly. Many such functions of less than 100 variables have been given in our examples.

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