

# Enhancing Sensor Network Lifetime Using Interactive Communication

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**Abstract**—We are concerned with maximizing the lifetime of a sensor network consisting of set of nodes directly communicating with a base-station. We model this scenario as the interactive communication between multiple correlated informants (sensor nodes) and a recipient (base-station). With this framework, we show that interactive communication can indeed enhance network lifetime. Both worst-case and average-case performances are considered.

## I. INTRODUCTION

Many future and extant sensor networks feature tiny sensor nodes with modest energy resources, processing power and communication ability. A key networking challenge is to devise protocols and architectures that can provide relatively long operational lifetimes, in spite of these limitations. Sensor nodes expend energy in sensing, computing and communication. In this paper, we are concerned with reducing the energy cost of communication. We neglect the energy consumed by the nodes in sensing and computing because sensing costs are independent of the communication strategy being deployed and computing costs are often negligible compared to communication costs.

The energy expended by a sensor node in communication has two components: reception energy and transmission energy. The energy consumed in reception depends on the number of bits received and the per bit energy cost required to keep the receiver circuitry energized. The transmission energy depends on a number of factors such as transmit power levels, receiver sensitivity, channel state (including path loss due to distance and fading) and the kind of channel coding employed. In this paper, we assume that the data rates are low and that optimal channel coding is employed. Both these assumptions allow us to assume that the transmit power is linearly proportional to the data rate. Therefore, the communication energy is minimized by transmitting and receiving as few bits as possible.

In this work, based on some ideas from the theory of communication complexity, we propose a formalism to minimize the number of bits communicated in a single-hop sensor network, hence maximizing the network lifetime. Assuming the correlation in sensor data, we model the communication between the base-station and sensor nodes as the interactive communication between multiple correlated informants and a recipient. To the best of our knowledge, our work for the first time, employs this approach to estimate the network lifetime.

## II. “MULTIPLE INFORMANTS - SINGLE RECIPIENT” COMMUNICATION COMPLEXITY

Let us consider a set of multiple correlated informants *interactively* communicating with a recipient, where the objective is that the recipient must learn about each informant’s data, but an informant may not learn about other informants’ data. We want to estimate the total number of bits exchanged in the worst and average cases for such communication scenarios.

Previously, the works such as [1]–[5] have attempted to bound the message complexity of “single informant - single recipient” communication. These works have shown that the number of bits exchanged depends on the number of messages exchanged. However, not much work has been done towards computing the message complexity of “ $N$  multiple correlated informants - single recipient” communication problem, which we have attempted to address in [6]. The correlation among informants’ data helps in reducing the problem of finding the optimal rates which minimize the communication complexity to the problem of finding an optimal schedule that minimizes the communication complexity [6]. However, for an arbitrary model of correlation in informants’ data, it is not straightforward to compute the optimal number of messages, which minimize the number of bits exchanged. So, in this work, we develop the formalism to compute the number of bits exchanged for a given number  $m$  of messages and an arbitrary model of correlation in informants’ data.

In this work, we set  $m = 2$  for three reasons. Firstly, it is shown in [1]–[3] that just two messages reduce the communication complexity exponentially compared to one message and at the same time, with just two messages, the number of bits exchanged is *at worst* four times the optimal number of bits. Secondly, two is the minimum number of messages to show how the interaction helps in reducing communication complexity. Thirdly, in interactive communication, two messages give most pessimistic estimates of the worst and average case communication complexities.

In the rest of this section, we propose and illustrate our thesis to use the notions of ambiguity and information entropy to compute the worst and average case communication complexities, respectively.

### A. Ambiguity and Entropy

This subsection extends the notions of *ambiguity set* and *ambiguity*, proposed in [1] and proves some of their properties.

Let  $(X_1, X_2)$  be a random pair,  $X_1 \in \mathcal{X}_1$  and  $X_2 \in \mathcal{X}_2$ , where  $\mathcal{X}_1, \mathcal{X}_2$  are discrete alphabets. Let  $S_{X_1, X_2}$  denote the support set of  $(X_1, X_2)$ . The support set of  $X_1$  is the set

$$S_{X_1} \stackrel{\text{def}}{=} \{x_1 : \text{for some } x_2, (x_1, x_2) \in S_{X_1, X_2}\} \in \mathcal{X}_1,$$

of possible  $X_1$  values. We also call  $S_{X_1}$  *unconditional ambiguity set* of  $X_1$ . The *ambiguity* in that case is defined as  $\mu_{X_1} = |S_{X_1}|$ , the number of possible  $X_1$  values and it is same as the *maximum ambiguity*  $\hat{\mu}_{X_1}$  of  $X_1$ .

The *conditional ambiguity set* when random variable  $X_2$  takes the value  $x_2 \in S_{X_2}$  is

$$S_{X_1|X_2}(x_2) \stackrel{\text{def}}{=} \{x_1 : (x_1, x_2) \in S_{X_1, X_2}\}, \quad (1)$$

the set of possible  $X_1$  values when  $X_2 = x_2$ . The *conditional ambiguity* in that case is

$$\mu_{X_1|X_2}(x_2) \stackrel{\text{def}}{=} |S_{X_1|X_2}(x_2)|, \quad (2)$$

the number of possible  $X_1$  values when  $X_2 = x_2$ . The *maximum conditional ambiguity* of  $X_1$  is

$$\hat{\mu}_{X_1|X_2} \stackrel{\text{def}}{=} \sup\{\mu_{X_1|X_2}(x_2) : x_2 \in S_{X_2}\}, \quad (3)$$

the maximum number of  $X_1$  values possible with any  $X_2$ .

Lemma 1: Conditioning reduces ambiguity, that is,  $\mu_{X_1|X_2}(X_2 = x_2) \leq \mu_{X_1}$ .

*Proof:* From the definitions of  $\mu_{X_1}$  and  $\mu_{X_1|X_2}(x_2)$ , the proof is immediately obvious. ■

It follows from above lemma that  $\hat{\mu}_{X_1|X_2} \leq \hat{\mu}_{X_1}$ . Contrast this with a similar statement about entropy:  $H_{X_1|X_2}(X_2 = x) \leq H_{X_1}$ , which may or not hold always.

Lemma 2:  $H(X_1|X_2 = x_2) \leq \log \mu_{X_1|X_2}(X_2 = x_2)$ .

*Proof:* The proof follows from the definitions of  $H(X_1|X_2 = x_2)$  and  $\log \mu_{X_1|X_2}(X_2 = x_2)$ . Note that equality is achieved in the statement of the lemma only when  $p(X_1|X_2 = x_2)$  is uniformly distributed. ■

Taking the expectation of the both side of the inequality in Lemma 2 with respect to  $p(x_2)$  gives:

$$H(X_1|X_2) \leq \sum_{x_2 \in \mathcal{X}_2} p(X_2 = x_2) \log \mu_{X_1|X_2}(X_2 = x_2). \quad (4)$$

Let us define *average ambiguity* of  $X_1$  as

$$\bar{\mu}_{X_1|X_2} \stackrel{\text{def}}{=} \sum_{x_2 \in S_{X_2}} p(X_2 = x_2) \mu_{X_1|X_2}(X_2 = x_2), \quad (5)$$

the average number of  $X_1$  values possible with all  $X_2$  values.

Let us consider two persons  $P_X$  and  $P_Y$  interactively communicating with each other.  $P_X$  observes the random variable  $X_1 \in \mathcal{X}_1$  and  $P_Y$  observes the random variable  $X_2 \in \mathcal{X}_2$ . Let the probability distribution  $p(x_1, x_2)$  model the correlation between  $X_1$  and  $X_2$ . Let us assume that only  $P_Y$  knows it. If  $P_Y$  observes that  $X_2 = x_2$ , then in the worst-case, it needs  $\lceil \log \hat{\mu}_{X_1|X_2} \rceil$  bits from  $P_X$  to learn about  $X_1$ . However, as  $P_X$  knows neither  $p(x_1, x_2)$ , nor that  $X_2 = x_2$ , it cannot compute  $\lceil \log \hat{\mu}_{X_1|X_2} \rceil$  and send its information in just these many bits. So,  $P_Y$  informs  $P_X$  about  $\lceil \log \hat{\mu}_{X_1|X_2} \rceil$  bits

in which  $P_X$  needs to send its information to  $P_Y$ . Similarly,  $P_Y$  does not need more than  $H(X_1|X_2)$  bits, on average, to learn about  $X_1$ . To help  $P_X$  send its information in  $H(X_1|X_2)$  its,  $P_Y$  needs to send  $H(X_1|X_2)$  bits to  $P_X$ , on average. A *meaningful* message from  $P_X$  to  $P_Y$  is the one that reduces the ambiguity of  $P_Y$  about  $P_X$ 's data. However, these worst and average case bounds are lowest possible, as soon we show that any realistic protocol will exchange more than these many bits, in the worst and average case, respectively.

In the following, we generalize this discussion to “ $N$  multiple correlated informants - single recipient” communication to compute the worst and average case communication complexities and give *almost* optimal communication protocols, with at most two messages exchanged. Let us assume that informants' data is described by the random vector  $(X_1, \dots, X_N) \sim p(x_1, \dots, x_N)$ , where each  $x_i$  takes values from the discrete sets  $\mathcal{X}_i$  with cardinality  $n$ . Assume that this distribution is *only* known to the recipient, so only it can compute conditional ambiguity and marginal distribution for every informant's data. Let  $\Pi$  denote the set of all  $N!$  schedules to poll  $N$  informants. Let  $S$  be the set of  $N$  informants. Let  $A$  be the set of informants who have already communicated their data to the recipient. So, it follows from above that

$$\begin{aligned} \hat{\mu}_{X_i|A} &\leq \hat{\mu}_{X_i}, \forall i \in S - A, \\ H_{X_i|A} &\leq H_{X_i}, \forall i \in S - A. \end{aligned}$$

### B. Worst-case communication complexity

An informant needs no more than  $\lceil \log n \rceil$  bits to communicate its data to the recipient. Let us consider a communication schedule  $\pi \in \Pi$ . Let us assume that the first informant  $\pi(1)$  in the schedule has communicated its data to the recipient in  $\lceil \log n \rceil$  bits. The conditional ambiguity of the recipient in informant  $\pi(2)$ 's data is  $\hat{\mu}_{\pi(2)|\pi(1)} \leq n$ . The recipient asks  $\pi(2)$  to send its data in  $\lceil \log \hat{\mu}_{\pi(2)|\pi(1)} \rceil$  bits and it responds by sending the index of its data. For this, the recipient sends  $\hat{\mu}_{\pi(2)|\pi(1)} \lceil \log n \rceil$  to  $\pi(2)$  to tell  $\pi(2)$  about which of its  $n$  possible data values actually belong to the conditional ambiguity set of the recipient and in another  $\lceil \log \hat{\mu}_{\pi(2)|\pi(1)} \rceil$  bits the recipient informs  $\pi(2)$  to send the index of its data in  $\lceil \log \hat{\mu}_{\pi(2)|\pi(1)} \rceil$  bits. So, the recipient transmits a total of  $\hat{\mu}_{\pi(2)|\pi(1)} \lceil \log n \rceil + \lceil \log \hat{\mu}_{\pi(2)|\pi(1)} \rceil$  bits to  $\pi(2)$ .

Following this protocol to poll all the informants, the total number of bits transmitted by recipient under schedule  $\pi$ , is

$$\begin{aligned} \hat{R}_\pi &= \sum_{i=1}^N \hat{B}_{\pi(i)}, \\ &= \sum_{i=1}^N \left( \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil + \lceil \log \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \rceil \right). \end{aligned} \quad (6)$$

The total number of bits transmitted by all the informants is

$$\hat{I}_\pi = \sum_{i=1}^N \lceil \log \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \rceil. \quad (7)$$

The objective is to find schedule  $\pi^*$  that solves:

$$\operatorname{argmin}_{\pi \in \Pi} (\widehat{R}_\pi + \widehat{I}_\pi). \quad (8)$$

Note that  $\widehat{R}_\pi$  is  $\sum_{i=1}^N \lceil \log \widehat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \rceil$  bits more than its optimum value, as we have striven to describe a simple protocol that illustrates our idea, than to describe a complicated protocol that gives the optimal number of transmitted bits.

### C. Average-case communication complexity

Using the marginal probabilities, recipient can construct Huffman tree for every informant. If the correlation among informants' data is not exploited, then every informant  $i$  sends  $H(X_i)$  bits on average to the recipient and the recipient sends a total of  $\mu(X_i) \lceil \log n \rceil + H(X_i)$  bits on average to query the informant  $i$ . However, if the correlation in the informants' data is exploited, then every informant sends only the number of bits equal to the entropy of its data conditioned on the data of the informants which have already communicated to the recipient. More formally, if the recipient queries the informants according to schedule  $\pi$ , then any informant  $i$ , sends no more than  $H(X_{\pi(i)} | X_{\pi(1)} \dots X_{\pi(i-1)})$  bits, on average to the recipient and the recipient sends no more than  $\bar{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil + H(X_{\pi(i)} | X_{\pi(1)} \dots X_{\pi(i-1)})$  bits, on average, to query the informant  $i$ .

Following this protocol to poll all the informants, the total number of bits transmitted by recipient under schedule  $\pi$ , is

$$\begin{aligned} \bar{R}_\pi &= \sum_{i=1}^N \bar{B}_{\pi(i)}, \\ &= \sum_{i=1}^N \left( \bar{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil \right. \\ &\quad \left. + H(X_{\pi(i)} | X_{\pi(1)} \dots X_{\pi(i-1)}) \right), \\ &\stackrel{(a)}{=} \left( \sum_{i=1}^N \bar{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil \right) + H(X_{\pi(1)}, \dots, X_{\pi(N)}) \\ &\stackrel{(b)}{=} \left( \sum_{i=1}^N \bar{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil \right) + H(X_1, \dots, X_N), \end{aligned} \quad (9)$$

where (a) follows from the chain-rule of conditional entropy and (b) follows from the fact that joint entropy is independent of the scheduling order. The total number of bits transmitted by all the informants is

$$\bar{I}_\pi = H(X_{\pi(1)}, \dots, X_{\pi(N)}). \quad (10)$$

The objective is to find schedule  $\pi^*$  that solves:

$$\operatorname{argmin}_{\pi \in \Pi} (\bar{R}_\pi + \bar{I}_\pi). \quad (11)$$

### III. SYSTEM MODEL

We consider a network of  $N$  battery operated sensor nodes strewn in a coverage area. The nodes are assumed to interactively communicate with the base-station in a single hop. Sensor node  $k, k \in \{1, \dots, N\}$  has  $E_k$  units of energy and the base-station has  $E_{BS}$  units of energy. The wireless

channel between sensor  $k$  and the base-station is described by a symmetrical path loss  $d_k$ , which captures various channel effects and is assumed to be constant. This is reasonable for static networks and also for the scenarios where the path loss varies slowly and can be accurately tracked.

The network operates in a time-division multiple access (TDMA) mode. Time is divided into slots and in each slot, every sensor communicates its data to the base-station. Let us assume that the sensor data at every time slot is described by a random vector  $(X_1, \dots, X_N) \sim p(x_1, \dots, x_N)$ ,  $X_i \in \mathcal{X}_i$ , where  $\mathcal{X}_i$  is the discrete alphabet set,  $|\mathcal{X}_i| = n$ . This distribution is *only* known to the base-station. We assume the spatial correlation in the sensor data and ignore temporal correlation, as it can easily be incorporated in our work for data sources satisfying the Asymptotic Equipartition Property.

We assume static scheduling, that is the base-station uses the same sensor polling schedule in every time slot, until the network dies. The worst-case lifetime of a sensor node (base-station) under schedule  $\pi \in \Pi$  is defined as the ratio of its total energy and its worst-case energy expenditure in a slot, under schedule  $\pi$ . However, as argued in Introduction, it is only the communication energy expenditure that we are here concerned with. The average lifetime of a sensor node (base-station) is similarly defined. We define network lifetime as the time until the first sensor node or the base-station runs out of the energy. This definition has the benefit of being simple, practical, and popular [7] and as shown below, provides a maximin formulation of the network lifetime in terms of the lifetimes of the sensor nodes and the base-station.

To model the transmit energy consumption at the base-station and the sensor nodes, we assume that transmission rate is linearly proportional to signal power. This assumption is motivated by Shannon's AWGN capacity formula which is approximately linear for low data rates. So, a node  $k$  under schedule  $\pi$  expends  $B_{\pi(k)} d_k$  units of energy to transmit  $B_{\pi(k)}$  units of information. Let  $E_r$  denote the energy cost of receiving one bit of information. For simplicity, let us assume that it is same for both the base-station and the sensor nodes.

The general problem is to find the optimal rates (the number of bits to transmit), which maximize network lifetime. However, the optimal rate-allocation is constrained to lie within the Slepian-Wolf achievable rate region. This makes the problem computationally challenging. We simplify the problem by introducing the notion of *instantaneous decoding* [8] and thus reduce the optimal rate allocation problem to computing the optimal scheduling order, albeit at some loss of optimality. This loss of optimality occurs because, in general, turning a multiple-access channel into an array of orthogonal channels by using a suitable MAC protocol (TDMA in our case) is well-known to be a suboptimal strategy, in the sense that the set of rates that are achievable with orthogonal access is strictly contained in the Ahlswede-Liao capacity region [9].

### IV. MAXIMIZING SENSOR NETWORK LIFETIME

Let us assume that the interaction between the base-station and the sensor nodes is not allowed. Then in the worst-case,

every node sends  $\lceil \log n \rceil$  bits to the base-station to convey its information. However, if every node knows  $p(x_1, \dots, x_N)$  and the data of all other nodes, then it only needs to send the bits describing its data conditioned on the data of the nodes already polled [10]. In the real single-hop sensor networks, neither it is possible that every node knows about all other nodes' data, given the limited communication capabilities of the sensor nodes; nor it is desired that the sensor nodes perform such computationally intense processing, given their limited computational and energy capabilities. However, if we allow the interaction between the base-station and sensor nodes, then the nodes can still send less than  $\lceil \log n \rceil$  bits, yet avoid above issues. In fact, this is precisely the "multiple correlated informants - single recipient" communication problem of section II. Using the results derived there, in the following, we show that even if we allow for just two messages exchanged between the base-station and a sensor node, the number of bits transmitted by the sensor nodes is greatly reduced, with concomitant reduction in their computational and communication burdens. In this situation, the base-station carries most of the burden of computation and communication in the network. This is reasonable in the scenarios where the base-station is computationally and energy-wise more capable than the sensor nodes. Still, it may not be infinitely more capable. So, in the network lifetime estimation problem, the base-station lifetime is also considered to include the situations where it is the base-station that runs out of the energy first.

In the following, we attempt to maximize the worst and average case lifetimes of the single-hop sensor networks, for the given model of energy consumption and spatial correlation in the sensor data. The base-station and a sensor node interactively communicate with each other by exchanging at most two messages. We estimate the communication energy expenditure at every sensor node as well as the base-station by explicitly including both, the transmission and reception energy expenditures.

#### A. Worst-case Network Lifetime

Let  $\hat{E}_{BS,i}$  denote the energy that the base-station spends in communicating with node  $i$  in the worst-case, that is, it denotes the energy that the base-station spends in transmitting and receiving the bits from node  $i$ , in the worst-case. So,

$$\hat{E}_{BS,i} = \hat{B}_{\pi(i)} d_i + \lceil \log \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \rceil E_r. \quad (12)$$

Similarly, let  $\hat{E}_{i,BS}$  denote the energy that the node  $i$  spends in communicating with the base-station. So,

$$\hat{E}_{i,BS} = \lceil \log \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \rceil d_i + \hat{B}_{\pi(i)} E_r. \quad (13)$$

On substituting for  $\hat{B}_{\pi(i)}$  from (6), these imply that

$$\hat{E}_{BS,i} - \hat{E}_{i,BS} = \hat{\mu}_{\pi(i)|\pi(1)\dots\pi(i-1)} \lceil \log n \rceil (d_i - E_r). \quad (14)$$

Assuming  $d_i \geq E_r$  implies that  $\hat{E}_{BS,i} - \hat{E}_{i,BS} \geq 0$ , that is, the base-station spends more energy in communicating with node  $i$  than that node  $i$  spends in communicating with the base-station.

Given our definitions of the sensor node, the base-station, and the network lifetimes, the worst-case lifetime  $\hat{L}$  of the network is the solution to the following optimization problem

$$\hat{L} = \max_{\pi \in \Pi} \min \left( \frac{E_{BS}}{\sum_{i=1}^N \hat{E}_{BS,i}}, \min_{i=1,\dots,N} \frac{E_i}{\hat{E}_{i,BS}} \right), \quad (15)$$

$$\hat{L}^{-1} = \min_{\pi \in \Pi} \max \left( \frac{\sum_{i=1}^N \hat{E}_{BS,i}}{E_{BS}}, \max_{i=1,\dots,N} \frac{\hat{E}_{i,BS}}{E_i} \right). \quad (16)$$

Before we discuss the nature of the general solution to this problem, let us consider its two special cases.

*Case 1:* Let  $E_{BS} = E_1 = \dots = E_N = E$ . Then, the problem in (16) reduces to

$$\hat{L}^{-1} = \frac{1}{E} \min_{\pi \in \Pi} \max \left( \sum_{i=1}^N \hat{E}_{BS,i}, \max_{i=1,\dots,N} \hat{E}_{i,BS} \right).$$

However, from (14), we know that  $\sum_{i=1}^N \hat{E}_{BS,i} \geq \max_{i=1,\dots,N} \hat{E}_{i,BS}$ , so above equation reduces to

$$\hat{L}^{-1} = \frac{1}{E} \min_{\pi \in \Pi} \sum_{i=1}^N \hat{E}_{BS,i}. \quad (17)$$

In the following, we prove that the *Minimum Cost Next* or *M CN* algorithm proposed in [8] computes the optimal lifetime for the optimization problem in (17).

The following pseudo-code of *M CN* algorithm computes the optimal schedule for *Case 1*.

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#### Algorithm: MCN

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- 1  $S$  : set of all  $N$  nodes.
  - 2  $A$  : ordered set of nodes whose polling order has been computed.
  - 3 Initialization:  $A = \phi$ ,  $k = 1$ .
  - 4 **while** ( $k \leq N$ )
  - 5      $\pi^{MCN}(k) = \operatorname{argmin}_{i \in S-A} \sum_{j \in A \cup i} \hat{E}_{BS,j}$
  - 6      $A = A \cup \pi^{MCN}(k)$ .
  - 7      $k = k + 1$ .
- 

*Case 2:* Let  $E_1 = \dots = E_N = E$ , but  $E_{BS} \gg E$ . Then, the problem in (16) reduces to

$$\begin{aligned} \hat{L}^{-1} &= \frac{1}{E_{BS}} \min_{\pi \in \Pi} \max \left( \sum_{i=1}^N \hat{E}_{BS,i}, \frac{E_{BS}}{E} \max_{i=1,\dots,N} \hat{E}_{i,BS} \right) \\ &= \frac{1}{E} \min_{\pi \in \Pi} \max_{i=1,\dots,N} \hat{E}_{i,BS}, \text{ for } E_{BS} \gg E. \end{aligned} \quad (18)$$

We can prove that the *Minimum Cost Next* algorithm above computes the optimal lifetime for the optimization problem in (18). The pseudo-code for *M CN* algorithm for *Case 2* is identical as above, except that RHS of line 5 is:  $\operatorname{argmin}_{i \in S-A} \frac{\hat{E}_{i,BS}}{E_i}$ .

**Lemma 3:** *M CN* schedule solves  $\min_{\pi \in \Pi} \sum_{i=1}^N \hat{E}_{BS,i}$ .

*Proof:* We describe a procedure to modify a given schedule into another schedule such that above sum does not increase. It will be apparent that iteratively applying this procedure on any schedule finally leads to the *M CN* schedule

$\pi^{MCN}$ . Let  $\pi^{OLD}$  be any schedule. Suppose it differs from  $\pi^{MCN}$  first in the  $m^{\text{th}}$  position, that is:

$$\begin{aligned}\pi^{OLD}(k) &= \pi^{MCN}(k), \quad 1 \leq k \leq m-1 \\ \pi^{OLD}(m) &\neq \pi^{MCN}(m).\end{aligned}\quad (19)$$

Then there exists a number  $l$  such that  $\pi^{OLD}(l) = \pi^{MCN}(m)$ ,  $l > m$ . We construct a new schedule  $\pi^{NEW}$  by modifying  $\pi^{OLD}$  as follows:

$$\begin{aligned}\pi^{NEW}(k) &= \pi^{MCN}(k), \quad 1 \leq k \leq m \\ \pi^{NEW}(k) &= \pi^{OLD}(k-1), \quad m < k \leq l \\ \pi^{NEW}(k) &= \pi^{OLD}(k), \quad l < k \leq N\end{aligned}\quad (20)$$

In words, in  $\pi^{NEW}$ , we poll  $\pi^{MCN}$  for first  $m$ -slots, followed by  $\pi^{OLD}$  for next  $N-m$  slots.

In order to establish that  $\pi^{NEW}$  is at least as good as  $\pi^{OLD}$ , we need to show that

$$\sum_{i=1}^N \hat{E}_{BS, \pi^{NEW}(i)} \leq \sum_{i=1}^N \hat{E}_{BS, \pi^{OLD}(i)}.\quad (21)$$

From (20), it follows that for  $1 \leq i \leq m-1$  and  $l+1 \leq i \leq N$

$$\hat{E}_{BS, \pi^{NEW}(i)} = \hat{E}_{BS, \pi^{OLD}(i)}.$$

So, it suffices to show that

$$\sum_{i=m}^l \hat{E}_{BS, \pi^{NEW}(i)} \leq \sum_{i=m}^l \hat{E}_{BS, \pi^{OLD}(i)}.\quad (22)$$

Using Lemma 1 (conditioning reduces ambiguity), we have

$$\sum_{i=m+1}^l \hat{E}_{BS, \pi^{NEW}(i)} \leq \sum_{i=m+1}^l \hat{E}_{BS, \pi^{OLD}(i)}.\quad (23)$$

Moreover, the  $MCN$  construction ensures that

$$\hat{E}_{BS, \pi^{NEW}(m)} \leq \hat{E}_{BS, \pi^{OLD}(m)}.\quad (24)$$

Equations (23) and (24), imply (22), proving the lemma. ■  
The proof that  $MCN$  schedule computes the optimal lifetime in (18) is almost identical.

The general problem in (15) or (16) turns out to be  $\mathcal{NP}$ -hard. However, here we omit the proof for the sake of brevity. As our discussion of two special cases above and the following theorem shows, the computational complexity of the problem in (15) depends on the variance  $\sigma^2$ , of the energies of the sensor nodes and the base-station.

**Theorem 1:** The computational complexity of the problem in (15) undergoes the “phase-transition”, with the order parameter  $\sigma^2 = \text{variance}(E_{BS}, E_1, \dots, E_N)$ .

*Proof:* Omitted for brevity. ■

The above two cases show that when  $\sigma^2 = 0$  (case 1) or  $\sigma^2 \gg 1$  (case 2), it is easy to solve the problem in (15). However, for the values of the order parameter  $\sigma^2$  between these two extremes, give the hardest instances of the problem. This behavior is well-known to be the characteristic of  $\mathcal{NP}$ -hard problems [11], [12].

## B. Average Network Lifetime

Let  $\bar{E}_{BS,i}$  denote the energy that the base-station spends in communicating with node  $i$ , on average. So,

$$\bar{E}_{BS,i} = \bar{B}_{\pi(i)} d_i + H(X_{\pi(i)} | X_{\pi(1)} \dots X_{\pi(i-1)}) E_r.\quad (25)$$

Similarly, let  $\bar{E}_{i,BS}$  denote the energy that the node  $i$  spends in communicating with base-station. So,

$$\bar{E}_{i,BS} = H(X_{\pi(i)} | X_{\pi(1)} \dots X_{\pi(i-1)}) d_i + \bar{B}_{\pi(i)} E_r.\quad (26)$$

Then, the average-case lifetime  $\bar{L}$  of the network is the solution to the following optimization problem

$$\bar{L} = \max_{\pi \in \Pi} \min \left( \frac{E_{BS}}{\sum_{i=1}^N \bar{E}_{BS,i}}, \min_{i=1, \dots, N} \frac{E_i}{\bar{E}_{i,BS}} \right).\quad (27)$$

Identifying conditional ambiguity in section IV-A as the conditional entropy and then following the same reasoning, all the discussion and results there hold true here too.

## V. CONCLUSIONS AND FUTURE WORK

We computed the worst and average case communication complexities for “multiple correlated informants - single recipient” communication, assuming that at most two messages are exchanged between an informant and the recipient. Then we applied these results to estimate the worst and average case lifetimes of the sensor networks. However, two message communication may not be optimal for every given model of correlation among informants’ data, so it is of interest to find the optimal number of messages for various popular models of correlation in sensor data and estimate the worst and average case lifetimes of the network then.

## REFERENCES

- [1] A. Orlitsky, “Worst-case interactive communication I: Two messages are almost optimal,” *IEEE Trans. Inform. Theory*, vol. IT-36, 1990.
- [2] —, “Worst-case interactive communication II: Two messages are not optimal,” *IEEE Trans. Inform. Theory*, vol. IT-37, 1991.
- [3] —, “Average-case interactive communication,” *IEEE Trans. Inform. Theory*, vol. IT-38, 1992.
- [4] Z. Zhang and X. -G. Xia, “Three messages are not optimal in worst case interactive communication,” *IEEE Trans. Inform. Theory*, vol. IT-40, 1994.
- [5] R. Ahlswede, N. Cai, and Z. Zhang, “On interactive communication,” *IEEE Trans. Inform. Theory*, vol. IT-43, 1997.
- [6] S. Agnihotri and P. Nuggehalli, “Energy conscious interactive communication for sensor networks,” [arXiv: cs.IT/0701048](#).
- [7] J. Chang and L. Tassiulas, “Energy conserving routing in wireless ad hoc networks,” *Proc. INFOCOM 2000*, Tel-Aviv, Israel, March 2000.
- [8] S. Agnihotri, P. Nuggehalli, and H. S. Jamadagni, “On maximizing lifetime of a sensor cluster,” *Proc. WoWMoM 2005*, Taormina, Italy, June 2005.
- [9] T. Cover and J. Thomas, *Elements of Information Theory*, John Wiley & Sons, 1991.
- [10] R. Cristescu, B. B. Lozano, and M. Vetterli, “On network correlated data gathering,” *Proc. IEEE INFOCOM 2004*, Hong Kong, March 2004.
- [11] P. Cheeseman, B. Kanefsky, and W. M. Taylor, “Where the really hard problems are,” *Proc. IJCAI 1991*, San Mateo, CA, 1991.
- [12] B. Hayes, “Can’t get no satisfaction,” *American Scientist*, vol. 85, 1997.