Predicting the Expected Behavior of Agents that Learn About Agents: The CLRI Framework

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Abstract

We describe a framework and equations used to model and predict the behavior of multi-agent systems (MASs) with learning agents. A difference equation is used for calculating the progression of an agent's error in its decision function, thereby telling us how the agent is expected to fare in the MAS. The equation relies on parameters which capture the agent's learning abilities, such as its change rate, learning rate and retention rate, as well as relevant aspects of the MAS such as the impact that agents have on each other. We validate the framework with experimental results using reinforcement learning agents in a market system, as well as with other experimental results gathered from the AI literature. Finally, we use PAC-theory to show how to calculate bounds on the values of the learning parameters.

Multi-Agent Systems, Machine Learning, Complex Systems.

1 Introduction

With the steady increase in the number of multi-agent systems (MASs) with learning agents (Durfee et al. [1997], Chavez and Maes [1996], Etzioni [1996], Stone and Velos [1997]) the analysis of these systems is becoming increasingly important. Some of the research in this area consists of experiments where a number of learning agents are placed in a MAS, then different learning or system parameters are varied and the results are gathered and analyzed in a effort to determine how changes in the individual agent behaviors will affect the system behavior. We have learned about the dynamics of market-based MASs using this approach (Vidal and Durfee [1998]). However, in this article we will take a step beyond these observation-based experimental results and describe a framework that can be used to model and predict the behavior of MASs with
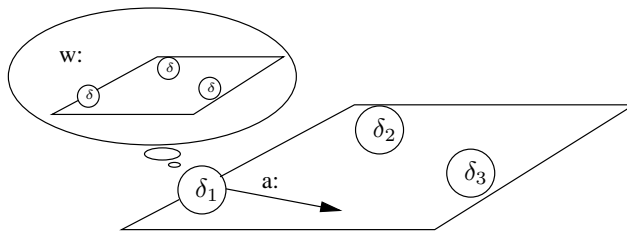
Figure 1: The agents in a MAS.

learning agents. We give a difference equation that can be used to calculate the progression of an agent's error in its decision function. The equation relies on the values of parameters which capture the agents' learning abilities and the relevant aspects of the MAS. We validate the framework by comparing its predictions with our own experimental results and with experimental results gathered from the AI literature. Finally, we show how to use probably approximately correct (PAC) theory to get bounds on the values of some of the parameters.

The types of MAS we study are exemplified by the abstract representation shown in Figure 1. We assume that the agents observe the physical state of the world (denoted by w in the figure) and take some action (a) based on their observation of the world state. An agent's mapping from states to actions is denoted by the decision function (δ) inside the agent. Notice that the "physical" state of the world includes everything that is directly observable by the agent using its sensors. It could include facts such as a robot's position, or the set outstanding bids in an auction, depending on the domain. The agent does not know the decision functions of the other agents. After taking action an agent will sometimes receive some reward or feedback which will help it modify its decision function.

We have a situation where agents are changing their decision function based on the effectiveness of their actions. However, the effectiveness of their actions depends on the other agents' decision functions. This scenario leads to the immediate problem: if all the agents are changing their decisions functions then it is not clear what will happen to the system as a whole. Will the system settle to some sort of equilibrium where all agents stop changing their decision functions? How long will it take to converge? How do the agents' learning abilities influence the system's behavior and possible convergence? These are some of the questions we address in this article.

Section 2 presents our framework for describing an agent's learning abilities and the error in its behavior. Section 3 presents an equation that can be used to predict an agent's expected error, as a function of time, when the agent is in a MAS composed of other learning agents. This equation is simplified in Section 4 by making some assumptions about the type of MAS being modeled. Section 5 then defines the last few parameters used in our framework—volatility and impact. Section 6 gives an illustrative example of the use of the framework.

The predictions made by our framework are verified by our own experiments, as shown in Section 7, and with the experiments of others, as shown in Section 8. The use of PAC theory for determining bounds on the learning parameters is detailed in Section 9. Finally Section 10 describes some of the related work and Section 11 summarizes our claims.

2 A Framework for Modeling MASs

In order to analyze the behaviors of agents in MASs composed of learning agents, we must first construct a formal framework for describing these agents. The framework must state any assumptions and simplifications it makes about the world, the agents, and the agents' behaviors. It must also be mathematically precise, so as to allow us to make quantitative predictions about expected behaviors. Finally, the simplifications brought about because of the need for mathematical precision should not be so constraining that they prevent the applicability of the framework to a wide variety of learning algorithms and different types of MASs. We now describe our framework and explain the types of MASs and learning behaviors that it can capture.

2.1 The World and its Agents

A MAS consists of a finite number of agents, actions, and world states. We let N denote the finite set of agents in the system. W denotes the finite set of world states. Each agent is assumed to have a set of perceptors (e.g., a camera, microphone, bid queue) with which it can perceive the world. An agent uses is sensors to "look" at the world and determine which world state w it is in; the set of all these states is W. A_i, where $|A_i| \geq 2$, denotes the finite set of actions agent $i \in N$ can take.

We assume discrete time, indexed in the various functions by the superscript t, where t is an integer greater than or equal to 0. The assumption of discrete time is made, for practical reasons, by a number of learning algorithms. It means that, while the world might be continuous, the agents perceive and learn in separate discrete time steps.

We also assume that there is only one world state w at each time, which all the agents can perceive in its completeness. That is, we asume the enviroment is accessible (as defined in [Russell and Norvig, 1995, p46]). This assumption holds in market systems where all the actions of all the agents are perceived by all the agents, or in software agent domains where all the agents have access to the same information. However, it might not hold for robotic domains where one agent's view of the world might be obscured by some physical obstacle. Even in such domains, it is possible that there is a strong correlation between the states perceived by each agent. These correlations could be used to create equivalency classes over the agents' perceived states, and these classes could then be used as the states in W.
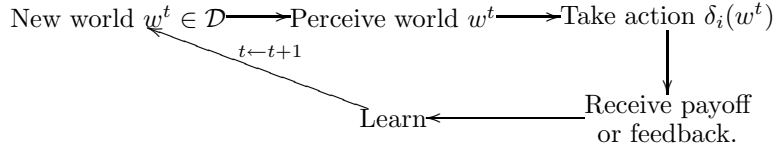
New world $\underline{w}^t \in \mathcal{D}$ ⟶ Perceive world w^t ⟶ Take action $\delta_i(w^t)$

$t \leftarrow t+1$

Learn ⟵ Receive payoff or feedback.

Figure 2: Action/Learn loop for an agent.

Finally, we assume the environment is determistic [Russell and Norvig, 1995, p46]. That is, the agents' combined actions will always have the expected effect. Of course, agent i might not know what action agent j will take so i might not know the eventual effect of its own individual action.

2.2 A Description of Agent Behavior

In the types of MASs we are modeling, every agent i perceives the state of the world w and takes an action a_i, at each time step. We assume that every agent's **behavior**, at each moment in time, can be described with a simple state-to-action mapping. That is, an agent's choice of action is solely determined by its current state-to-action mapping and the current world w.

Formally, we say that agent i's behavior is represented by a **decision function** (also known as a "policy" in control theory and a "strategy" in game theory) , given by $\delta_i^t : W \to A_i$. This function maps each state $w \in W$ to the action $a_i \in A_i$ that agent i will take in that state, at time t. This function can effectively describe any agent that deterministically chooses its action based on the state of the world. Notice that the decision function is indexed with the time t. This allows us to represent agents that change their behavior.

The action agent i *should* take in each state w is given by the **target function** $\Delta_i^t : W \to A_i$, which also maps each state $w \in W$ to an action $a_i \in A_i$. The agent does not have direct access to its target function. The target function is used to determine how well an agent is doing. That is, it represents the "perfect" behavior for a given agent. An agent's learning task is to get its decision function to match its target function as much as possible.

Since the choice of action for agent i often depends on the actions of other agents, the target function for i needs to take these actions into account. That is, in order to generate Δ_i^t, one would need to know $\delta_j^t(w)$ for all $j \in N_{-i}$ and $w \in W$. These $\delta_j^t(w)$ functions tell us the actions that all the other agents will take in every state w. For example, in order for one to determine what an agent should bid in every world w of an auction-based market system, one will need to know what the other agents will bid in every world w. One can use these actions, along with the state w, in order to identify the best action for i to take.

An agent's $\delta_i^t(w)$ can change over time, so that $\delta_i^{t+1} \neq \delta_i^t$. These changes in an agent's decision function reflect its learned knowledge. The agents in the MASs we consider are engaged in the discrete action/learn loop shown in Figure 2. The loop works as follows: At time t the agents perceive a world $w^t \in W$ which is drawn from a fixed distribution $\mathcal{D}(w)$. They then each take

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the action dictated by their δ_i^t functions; all of these actions are assumed to be taken effectively in parallel. Lastly, they each receive a payoff which their respective learning algorithms use to change the δ_i^t so as to (hopefully) better match Δ_i^t. By time $t+1$, the agents have new δ_i^{t+1} functions and are ready to perceive the world again and repeat the loop. Notice that, at time t, an agent's Δ_i^t is derived by taking into account the δ_j^t of all other agents $j \in N_{-i}$.

We assume that $\mathcal{D}(w)$ is a fixed probability distribution from which we take the worlds seen at each time. This assumption is not unreasonably limiting. For example, in an economic domain where the new state is the new good being offered, or in an episodic domain where the agents repeatedly engage in different games (e.g. a Prisoner's Dilemma competition) there is no correlation between successive world states or between these states and the agents' previous actions. However, in a robotic domain one could argue that the new state of the world will depend on the current state of the world; after all, the agents probably move very little each time step.

Our measure of the correctness of an agent's behavior is given by our **error** measure. We define the error of agent i's decision function $\delta_i^t(w)$ as

$$
\begin{aligned}
e(\delta_i^t) &= \sum_{w \in W} \mathcal{D}(w) \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \\
&= \mathbf{Pr}_{w \in \mathcal{D}}[\delta_i^t(w) \neq \Delta_i^t(w)].
\end{aligned}
\tag{1}
$$

$e(\delta_i^t)$ gives us the probability that agent i will take an incorrect action; it is in keeping with the error definition used in computational learning theory (Kearns and Vazirani [1994]). We use it to gauge how well agent i is performing. An error of 0 means that the agent is taking all the actions dictated by its target function. An error of 1 means that the agent never takes an action as dictated by its target function. All the notation from this section is summarized in Figure 3.

2.3   The Moving Target Function Problem

The learning problem the agent faces is to change its $\delta_i^t(w)$ so that it matches $\Delta_i^t(w)$. If we imagine the space of all possible decision functions, then agent i's δ_i^t and Δ_i^t will be two points in this space, as shown in Figure 4. The agent's learning problem can then be re-stated as the problem of moving its decision function as close as possible to its target function, where the distance between the two functions is given by the error $e(\delta_i^t)$. This is the traditional machine learning problem.

However, once agents start to change their decision functions (i.e., change their behaviors) the problem of learning becomes more complicated because these changes might cause changes in the other agents' target functions. We end up with a moving target function, as seen in Figure 5. In these systems, it is not clear if the error will ever reach 0 or, more generally, what the expected error will be as time goes to infinity. Determining what will happen to an agent's error in such a system is what we call the **moving target function problem**,

N the set of all agents, where $i \in N$ is one particular agent.

W the set of possible states of the world, where $w \in W$ is one particular state.

A_i the set of all actions that agent i can take.

$\delta_i^t : W \to A_i$ the **decision** function for agent i at time t. It tells which action agent i will take in each world.

$\Delta_i^t : W \to A_i$ the **target** function for agent i at time t. It tells us what action agent i should take. It takes into account the actions that other agents will take.

$e(\delta_i^t) = \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w) \,|\, w \in \mathcal{D}]$ the **error** of agent i at time t. It is the probability that i will take an incorrect action, given that the worlds w are taken from the fixed probability distribution $\mathcal{D}$.

Figure 3: Summary of notation used for describing a MAS and the agents in it.
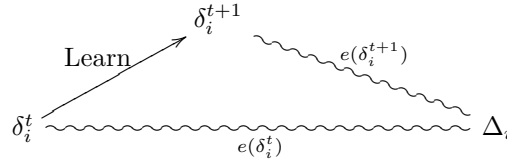

Figure 4: The traditional learning problem.

which we address in this article. However, we will first need to define some parameters that describe the capabilities of an agent's learning algorithm.
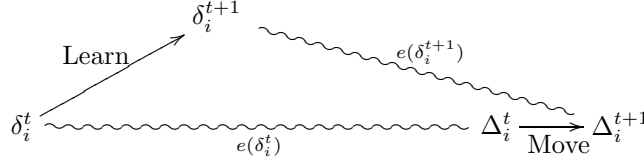

Figure 5: The learning problem in learning MASs.

2.4   A Model of Learning Algorithms

An agent's learning algorithm is responsible for changing δ_i^t into δ_i^{t+1} so that it is a better match of Δ_i^t. Different machine learning algorithms will achieve this match with different degrees of success. We have found a set of parameters that can be used to model the effects of a wide range of learning algorithms. The parameter are: Change rate, Learning rate, Retention rate, and Impact; and they will be explained in this section, except for Impact which will be introduced in Section 5. These parameters, along with the equations we provide, form the **CLRI** framework (the letters correspond to the first letter of the parameters' names).

After agent i takes an action and receives some payoff, it activates its learning algorithm, as we showed in Figure 2. The learning algorithm is responsible for using this payoff in order to change δ_i^t into δ_i^{t+1}, making δ_i^{t+1} match Δ_i^t as much as possible. We can expect that for some w it was true that $\delta_i^t(w) = \Delta_i^t(w)$, while for some other w this was not the case. That is, some of the $w \to a_i$ mappings given by $\delta_i^t(w)$ might have been incorrect. In general, a learning algorithm might affect both the correct and incorrect mappings. We will treat these two cases separately.

We start by considering the incorrect mappings and define the **change rate** of the agent as the probability that the agent will change one of its incorrect mappings. Formally, we define the change rate c_i for agent i as

$$\forall_w \ c_i = \mathbf{Pr}[\delta_i^{t+1}(w) \neq \delta_i^t(w) \,|\, \delta_i^t(w) \neq \Delta_i^t(w)]. \tag{2}$$

The change rate tells us the likelihood of the agent changing an incorrect mapping into something else. This "something else" might be the correct action, but it could also be another incorrect action. The probability that the agent changes an incorrect mapping to the correct action is called the **learning rate** of the agent. It is defined as l_i where

$$\forall_w \ l_i = \mathbf{Pr}[\delta_i^{t+1}(w) = \Delta_i^t(w) \,|\, \delta_i^t(w) \neq \Delta_i^t(w)]. \tag{3}$$

When determining the value of l_i, for a particular agent, one must remember to take into account the fact that the worlds seen at each time step are taken from $\mathcal{D}(w)$.

There are two constraints which must always be satisfied by these two rates. Since changing to the correct mapping implies that a change was made, the value of l_i must be less than or equal to c_i, that is, $l_i \leq c_i$ must always be true. Also, if $|A_i| = 2$ then $c_i = l_i$ since there are only two actions available, so the one that is not wrong must be right.

The complementary value for the learning rate is $1 - l_i$ and refers to the probability that an incorrect mapping does not get changed to a correct one. An example learning rate of $l_i = .5$ means that, if agent i initially has all mappings wrong, it will get half of them right after the first iteration.

We now consider the agent's correct mappings and define the **retention rate** as the probability that a correct mapping will stay correct in the next iteration. The retention rate is given by r_i where

$$\forall_w \; r_i = \mathbf{Pr}[\delta_i^{t+1}(w) = \Delta_i^t(w) \,|\, \delta_i^t(w) = \Delta_i^t(w)]. \qquad (4)$$

We propose that the behavior of a wide variety of learning algorithms can be captured (or at least approximated) using appropriate values for c_i, l_i, and r_i. Notice, however, that these three rates claim that the $w \rightarrow a$ mappings that change are independent of the w that was just seen. We can justify this independence by noting that most learning algorithms usually perform some form of generalization. That is, after observing one world state w and the payoff associated with it, a typical learning algorithm is able to generalize what it learned to some other world states. This generalization is reflected in the fact that the change, learning, and retention rates apply to all w's. However, a more precise model would capture the fact that, in some learning algorithms, the mapping for the world state that was just seen is more likely to change than the mapping for any other world state.

The rates are not time dependent because we assume that agents use one learning algorithm during their lifetimes. The rates capture the capabilities of this learning algorithm and, therefore, do not need to vary over time.

Finally, we define **volatility** to mean the probability that the target function will change from time t to time $t+1$. Formally, volatility is given by v_i where

$$\forall_w \; v_i = \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)]. \qquad (5)$$

In Section 5, we will show how to calculate v_i in terms of the error of the other agents. We will then see that volatility is not a constant but, instead, varies with time.

3 Calculating the Agent's Error

We now wish to write a difference equation that will let us calculate the agent's expected error, as defined in Eq. (1), at time $t+1$ given the error at time t and the other parameters we have introduced. We can do this by observing that there are two conditions that determine the new error: whether $\Delta_i^{t+1}(w) = \Delta_i^t(w)$ or not, and whether $\delta_i^t(w) = \Delta_i^t(w)$ or not. If we define $a \equiv \Delta_i^{t+1}(w) = \Delta_i^t(w)$,

and $b \equiv \delta_i^t(w) = \Delta_i^t(w)$, we can then say that we need to consider the four cases where: $a \wedge b$, $a \wedge \neg b$, $\neg a \wedge b$, and $\neg a \wedge \neg b$. Formally, this implies that

$$
\begin{aligned}
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w)] = \\
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \wedge a \wedge b] + \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \wedge a \wedge \neg b] + \\
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \wedge \neg a \wedge b] + \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \wedge \neg a \wedge \neg b],
\end{aligned}
\tag{6}
$$

since the four cases are exclusive of each other. Applying Bayes Theorem, we can rewrite each of the four terms in order to get

$$
\begin{aligned}
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w)] = \mathbf{Pr}[a \wedge b] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, a \wedge b] + \\
\mathbf{Pr}[a \wedge \neg b] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, a \wedge \neg b] + \\
\mathbf{Pr}[\neg a \wedge b] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \neg a \wedge b] + \\
\mathbf{Pr}[\neg a \wedge \neg b] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \neg a \wedge \neg b].
\end{aligned}
\tag{7}
$$

We can now find values for these conditional probabilities. We start with the first term where, after replacing the values of a and b, we find that

$$
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \Delta_i^{t+1}(w) = \Delta_i^t(w) \wedge \delta_i^t(w) = \Delta_i^t(w)] = 1 - r_i.
\tag{8}
$$

Since the target function does not change from time t to $t+1$ and the agent was correct at time t, the agent will also be correct at time $t+1$; *unless* it changes its correct $w \to a$ mapping. The agent changes this mapping with probability $1 - r_i$.

The value for the second conditional probability is

$$
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \Delta_i^{t+1}(w) = \Delta_i^t(w) \wedge \delta_i^t(w) \neq \Delta_i^t(w)] = 1 - l_i.
\tag{9}
$$

In this case the target function still stays the same but the agent was incorrect. If the agent was incorrect then it will change its decision function to match the target function with probability l_i. Therefore, the probability that it will be incorrect next time is the probability that it does not make this change, or $1 - l_i$.

The third probability has a value of

$$
\begin{aligned}
\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \Delta_i^{t+1}(w) \neq \Delta_i^t(w) \wedge \delta_i^t(w) = \Delta_i^t(w)] \\
= (r_i + (1 - r_i) \cdot B)
\end{aligned}
\tag{10}
$$

In this case the agent was correct and the target function changes. This means that if the agent retains the same mapping, which it does with probability r_i, then the agent will definitely be incorrect at time $t+1$. If it does not retain the same mapping, which happens with probability $1 - r_i$, then it will be incorrect with probability B, where

$$
\begin{aligned}
B = \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) | \delta_i^t(w) = \Delta_i^t(w) \wedge \Delta_i^{t+1}(w) \neq \Delta_i^t(w) \\
\wedge \delta_i^{t+1}(w) \neq \Delta_i^t(w)].
\end{aligned}
\tag{11}
$$

Finally, the fourth conditional probability has a value of

$$\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) \,|\, \Delta_i^{t+1}(w) \neq \Delta_i^t(w) \wedge \delta_i^t(w) \neq \Delta_i^t(w)]$$
$$= (1 - c_i)D + l_i + (c_i - l_i)F, \tag{12}$$

where

$$D = \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^{t+1}(w) | \delta_i^t(w) \neq \Delta_i^t(w) \wedge \Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \tag{13}$$

$$F = \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w) | \delta_i^t(w) \neq \Delta_i^t(w) \wedge \Delta_i^{t+1}(w) \neq \Delta_i^t(w) \tag{14}$$
$$\wedge\, \delta_i^{t+1}(w) \neq \Delta_i^t(w) \wedge \delta_i^{t+1}(w) \neq \delta_i^t(w)].$$

This is the case where the target function changes and the agent was wrong. We have to consider three possibilities. The first possibility is for the agent not to change its decision function, which happens with probability $1 - c_i$. The probability that the agent will be incorrect in this case is given by D. The second possibility, when the agent changes its mapping to the correct function, has a probability of l_i and ensures that the agent will be incorrect the next time. The third possibility happens, with probability $c_i - l_i$ when the agent changes its mapping to an incorrect value. In this case, the probability that it will be wrong next time is given by F.

We can substitute Eqs. (8), (9), (10), and (12) into Eq. (7), substitute the values of a and b, and expand $\mathbf{Pr}[a \wedge b]$ into $\mathbf{Pr}[a \,|\, b] \cdot \mathbf{Pr}[b]$, in order to get

$$E[e(\delta_i^{t+1})] = E[\sum_{w \in W} \mathcal{D}(w)\mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w)]] = \sum_{w \in W} \mathcal{D}(w)($$
$$\mathbf{Pr}[\Delta_i^{t+1}(w) = \Delta_i^t(w)|\delta_i^t(w) = \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot (1 - r_i)$$
$$+ \mathbf{Pr}[\Delta_i^{t+1}(w) = \Delta_i^t(w)|\delta_i^t(w) \neq \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot (1 - l_i)$$
$$+ \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)|\delta_i^t(w) = \Delta_i^t(w)]$$
$$\cdot \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot (r_i + (1 - r_i) \cdot B) \tag{15}$$
$$+ \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)|\delta_i^t(w) \neq \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)]$$
$$\cdot (1 - c_i)D + l_i + (c_i - l_i)F.$$

Equation (15) will model any MAS whose agent learning can be described with the parameters presented Section 2.4 and whose action/learn loop is the same as we have described. We can use Eq. (15) to calculate the successive expected errors for agent i, given values for all the parameters and probabilities. In the next section we show how this is done in a simple example game.

3.1   The Matching game

In this matching game we have two agents i and j each of whom, in every world w, wants to play the same action as the other one. Their set of actions is $A_i = A_j$, where we assume $|A_i| > 2$ (for $|A_i| = 2$ the equation is simpler). After every time step, the agents both learn and change their decision functions in

accordance to their learning rates, retention rates, and change rates. Since the agents are trying to match each other, in this game it is always true that $\Delta_i^t(w) = \delta_j^t(w)$ and $\Delta_j^t(w) = \delta_i^t(w)$. Given all this information, we can find values for some of the probabilities in Eq. (15) (including values for Equations (11) (13) (14)) and rewrite (see Appendix A for derivation) it as:

$$
\begin{aligned}
E[e(\delta_i^{t+1})] = \sum_{w \in W} \mathcal{D}(w) \{ & r_j \cdot \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot (1 - r_i) \\
& + (1 - c_j) \cdot \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot (1 - l_i) \\
& + (1 - r_j) \cdot \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot \left(r_i + (1 - r_i) \cdot \left(\frac{|A_i| - 2}{|A_i| - 1} \right) \right) \\
& + c_j \cdot \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot \left(1 - l_j + \frac{c_i l_j(|A_i| - 1) + l_i(1 - l_j) - c_i}{|A_i| - 2} \right) \}
\end{aligned}
\tag{16}
$$

We can better understand this equation by plugging in some values and simplifying. For example, lets assume that $r_i = r_j = 1$ and $l_i = l_j = 1$, which implies that $c_i = c_j = 1$. This is the case where the two agents always change all their incorrect mappings so as to match their respective target functions at time t. That is, if we had $\delta_i^t(w_1) = x$ and $\delta_j^t(w_1) = y$, then at time $t + 1$ we will have $\delta_i^{t+1}(w_1) = y$ and $\delta_j^{t+1}(w_1) = x$. This means that agent i changes all its incorrect mappings to match j, while j changes to match i, so all the mappings stay wrong after all (i.e., i ends up doing what j did before, while j does what i did before). The error, therefore, stays the same. We can see this by plugging the values into Eq. (16). The first three terms will become 0 and the fourth term will simplify to the definition of error, as given by Eq. (1). Since the fourth term is the only one that is non-zero, we end up with $E[e(\delta_i^{t+1})] = e(\delta_i^t)$.

We can also let c_i and l_i (keeping $c_j = l_j = 1$) be arbitrary numbers, which gives us $E[e(\delta_i^{t+1})] = c_i e(\delta_i^t)$. This tells us that the error will drop faster for a smaller change rate c_i. The reason is that i's learning (remember $l_i \leq c_i$) in this game is counter-productive because it is always made invalid by j's learning rate of 1. That is, since j is changing all its mappings to match i's actions, i's best strategy is to keep its actions the same (i.e., $c_i = 0$).

4 Further Simplification

We can further simplify Eq. (15) if we are willing to make two assumptions. The first assumption is that the new actions chosen when either $\delta_i^t(w)$ changes (and does not match the target), or when $\Delta_i^t(w)$ changes, are both taken from flat probability distributions over A_i. By making this assumption we can find values for B, D, and F, namely:

$$
B = D = \frac{|A_i| - 2}{|A_i| - 1} \qquad F = \frac{|A_i| - 3}{|A_i| - 2}
\tag{17}
$$

The second assumption we make is that the probability of $\Delta_i^t(w)$ changing, for a particular w, is independent of the probability that $\delta_i^t(w)$ was correct.

In Section 3.1 we saw that in the matching game the probabilities of $\Delta_i^t(w)$ and $\delta_i^t(w)$ changing were correlated since, if $\delta_i^t(w)$ was wrong then $\delta_j^t(w)$ was also wrong, which meant j would probably change $\delta_j^t(w)$, which would change $\Delta_i^t(w)$.

However, the matching game is a degenerate example in exhibiting such tight coupling between the agents' target functions. In general, we can expect that there will be a number of MASs where the probability that any two agents i and j are correct is uncorrelated (or loosely correlated). For example, in a market system all sellers try to bid what the buyer wants, so the fact that one seller bids the correct amount says nothing about another seller's bid. Their bids are all uncorrelated. In fact, the Distributed Artificial Intelligence literature is full of systems that try to make the agents' decisions as loosely-coupled as possible (Lesser and Corkill [1981], Liu and Sycara [1995]).

This second assumption we are trying to make can be formally represented by having Eq. (18) be true for all pairs of agents i and j in the system.

$$
\begin{aligned}
&\mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w) \wedge \delta_j^t(w) = \Delta_j^t(w)] \\
&= \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_j^t(w) = \Delta_j^t(w)]
\end{aligned}
\tag{18}
$$

Once we make these two assumptions we can rewrite Eq. (15) as:

$$
\begin{aligned}
E[e(\delta_i^{t+1})] = \sum_{w \in W} \mathcal{D}(w)(&\mathbf{Pr}[\Delta_i^{t+1}(w) = \Delta_i^t(w)] \cdot (\mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot (1 - r_i) \\
&+ \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot (1 - l_i)) \\
+ \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \cdot (&\mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot \left(r_i + (1 - r_i) \cdot \left(\frac{|A_i| - 2}{|A_i| - 1} \right) \right) \\
&+ \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot \left(\frac{|A_i| - 2 - c_i + 2l_i}{|A_i| - 1} \right)))
\end{aligned}
\tag{19}
$$

Some of the probabilities in this equation are just the definition of v_i, and others simplify to the agent's error. This means that we can simplify Eq. (19) to:

$$
\begin{aligned}
E[e(\delta_i^{t+1})] = 1 - r_i + v_i &\left(\frac{|A_i| r_i - 1}{|A_i| - 1} \right) \\
&+ e(\delta_i^t) \left(r_i - l_i + v_i \left(\frac{|A_i|(l_i - r_i) + l_i - c_i}{|A_i| - 1} \right) \right)
\end{aligned}
\tag{20}
$$

Eq. (20) is a difference equation that can be used to determine the expected error of the agent at any time by simply using $E[e(\delta_i^{t+1})]$ as the $e(\delta_i^t)$ for the next iteration. While it might look complicated, it is just the function for a line $y = mx + b$ where $x = e(\delta_i^t)$ and $y = e(\delta_i^{t+1})$. Using this observation, and the fact that $e(\delta_i^{t+1})$ will always be between 0 and 1, we can determine that the final convergence point for the error is the point where Eq. (20) intersects the line $y = x$. The only exception is if the slope equals -1, in which case we will see the error oscillating between two points.
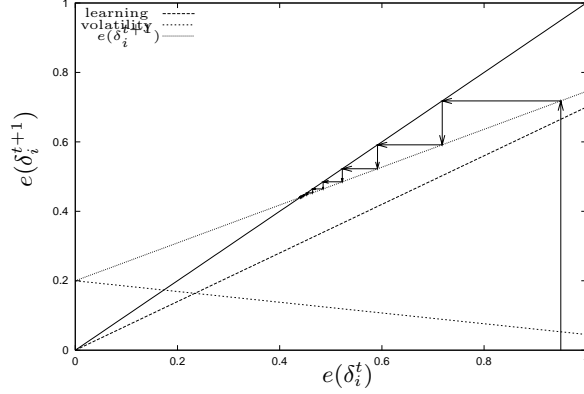
12

Figure 6: Error progression for agent i, assuming a fixed volatility $v_i = .2$, $c_i = 1$, $l_i = .3$, $r_i = 1$, $|A_i| = 20$. The error converges to .44.

By looking at Eq. (20) we can also determine that there are two "forces" acting on the agent's error: volatility and the agent's learning abilities. The volatility tends to increase the agent's error past its current value while the learning reduces it. We can better appreciate this effect by separating the v_i terms in Eq. (20) and plotting the v_i terms (volatility) and the rest of the terms (learning) as two separate lines. By definition, these will add up to the line given by Eq. (20). We have plotted these three lines and traced a sample error progression in Figure 6. The error starts at .95 and then decreases to eventually converge to .44. We notice the learning curve always tries to reduce the agent's error, as confirmed by the fact that its line always falls below $y = x$. Meanwhile, the volatility adds an extra error. This extra error is bigger when the agent's error is small since, any change in the target function is then likely to increase the agent's error.

5 Volatility and Impact

Equation (20) is useful for determining the agent's error when we know the volatility of the system. However, it is likely that this value is not available to us (if we knew it we would already know a lot about the dynamics of the system). In this section we determine the value of v_i in terms of the other agents' changes in their decision functions. That is, in terms of $\mathbf{Pr}[\delta_j^{t+1} \neq \delta_j^t]$, for all other agents j.

In order to do this we first need to define the **impact** I_{ji} that agent j's changes in its decision function have on i's target function.

$$\forall_{w \in W} \ I_{ji} = \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w) \,|\, \delta_j^{t+1}(w) \neq \delta_j^t(w)] \tag{21}$$

We can now start to define volatility by first determining that, for two agents i and j

$$
\begin{aligned}
\forall_{w \in W} \; v_i^t &= \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \\
&= \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w) \,|\, \delta_j^{t+1}(w) \neq \delta_j^t(w)] \cdot \mathbf{Pr}[\delta_j^{t+1}(w) \neq \delta_j^t(w)] \\
&\quad + \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w) \,|\, \delta_j^{t+1}(w) = \delta_j^t(w)] \cdot \mathbf{Pr}[\delta_j^{t+1}(w) = \delta_j^t(w)].
\end{aligned}
\tag{22}
$$

The reader should notice that volatility is no longer constant; it varies with time (as recorded by the superscript). The first conditional probability in Eq. (22) is just I_{ji}. The second one we will set to 0, since we are specifically interested in MASs where the volatility arises *only* as a side-effect of the other agents' learning. That is, we assume that agent i's target function changes only when j's decision function changes. For cases with more than two agents, we similarly assume that one agent's target function changes only when some other agent's decision function changes. That is, we ignore the possibility that outside influences might change an agent's target function.

We can simplify Eq. (22) and generalize it to N agents, under the assumption that the other agents' changes in their decision functions will not cancel each other out, making Δ_i^t stay the same as a consequence. v_i^t then becomes

$$
\begin{aligned}
\forall_{w \in W} \; v_i^t &= \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \\
&= 1 - \prod_{j \in N_{-i}} (1 - I_{ji}\mathbf{Pr}[\delta_j^{t+1}(w) \neq \delta_j^t(w)]).
\end{aligned}
\tag{23}
$$

We now need to determine the expected value of $\mathbf{Pr}[\delta_j^{t+1}(w) \neq \delta_j^t(w)]$ for any agent. Using i instead of j we have

$$
\begin{aligned}
\forall_{w \in W} \; \mathbf{Pr}[\delta_i^{t+1}(w) \neq \delta_i^t(w)] \\
= \mathbf{Pr}[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^t(w) \,|\, \delta_i^t(w) \neq \Delta_i^t(w)] \quad (24) \\
+ \mathbf{Pr}[\delta_i^t(w) = \Delta_i^t(w)] \cdot \mathbf{Pr}[\delta_i^{t+1}(w) \neq \Delta_i^t(w) \,|\, \delta_i^t(w) = \Delta_i^t(w)],
\end{aligned}
$$

where the expected value is:

$$
E[\mathbf{Pr}[\delta_i^{t+1}(w) \neq \delta_i^t(w)]] = c_i e(\delta_i^t) + (1 - r_i) \cdot (1 - e(\delta_i^t)).
\tag{25}
$$

We can then plug Eq. (25) into Eq. (23) in order to get the expected volatility

$$
E[v_i^t] = 1 - \prod_{j \in N_{-i}} 1 - I_{ji}(c_j e(\delta_j^t) + (1 - r_j) \cdot (1 - e(\delta_j^t))).
\tag{26}
$$

We can use this expected value of v_i^t in Eq. (20) in order to find out how the other agents' learning will affect agent i. In MASs that have identical learning agents (i.e., their c, l, r, and I rates are all the same and they start with the same initial error) we can replace the multiplier in Eq. (26) with an exponent of $|N| - 1$. We use this simplification later in Section 8.2.
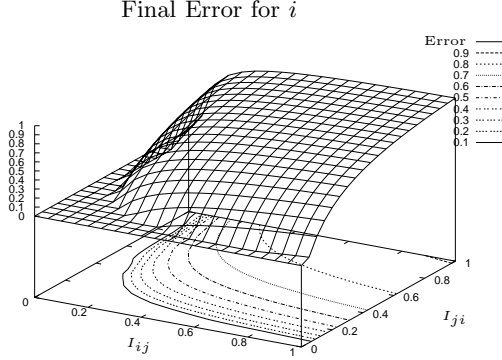
Final Error for i

Figure 7: Plot of Final Error for agent i, given $l_i = l_j = .2$, $r_i = r_j = 1$, $c_i = c_j = 1$, $|A_j| = |A_i| = 20$.

6   An Example with Two Agents

In a MAS with just two agents i and j, we can use Eq. (26) to rewrite Eq. (20) as

$$
\begin{aligned}
E[e(\delta_i^{t+1})] = 1 - r_i &+ I_{ji}(c_j e(\delta_j^t) + (1 - r_j) \cdot (1 - e(\delta_j^t))) \left(\frac{|A_i| r_i - 1}{|A_i| - 1} \right) \\
+ e(\delta_i^t) \{ r_i - l_i &+ I_{ji}(c_j e(\delta_j^t) + (1 - r_j) \cdot (1 - e(\delta_j^t))) \\
&\cdot \left(\frac{|A_i|(l_i - r_i) + l_i - c_i}{|A_i| - 1} \right) \}.
\end{aligned}
\tag{27}
$$

We can now use Eq. (27) to plot values for one particular example. Let us say that $l_i = l_j = .2$, $c_i = c_j = 1$, $r_i = r_j = 1$, $|A_j| = |A_i| = 20$ and we let the impacts I_{ij} and I_{ji} vary between zero and one. Figure 7 shows the final error, after convergence, for this situation. It shows an area where the error is expected to be below .1, corresponding to low values for either I_{ij}, I_{ji} or both. This area represents MASs that are loosely coupled, i.e., one agent's change in behavior does not significantly affect the other's target function. In these systems we can expect that the error will eventually[1] reach a value close to zero. We see that as the impact increases the final error also increases, with a fairly abrupt transition between a final error of 0 and bigger final errors. This abrupt transition is characteristic of these types of systems where there are tendencies for the system to either converge or diverge, and both of them are self-enforcing behaviors. Notice also that the graph is not symmetric—I_{ij} has more weight in determining i's final error than I_{ji}. This result seems counterintuitive, until we realize that it is j's error that makes it hard for i to converge to a small error.

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[1]Notice that we are not representing how long it takes for the error to converge. This can easily be done and is just one more of the parameters our theory allows us to explore.
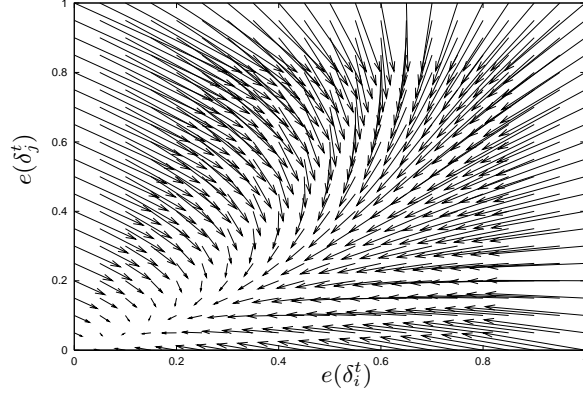
Figure 8: Vector plot for $e(\delta_i^t)$ and $e(\delta_j^t)$, where $|A_i| = |A_j| = 20$, $l_i = l_j = .2$, $r_i = r_j = 1$, $c_i = .5$, $c_j = 1$, $I_{ij} = .1$, $I_{ji} = .3$.

If I_{ij} is high then, if i has a large error then j's error will increase, which will make j change its decision function often and make it hard for i to reduce its error. If I_{ij} is low then, even if I_{ji} is high, j will probably settle down to a low error and as it does i will also be able to settle down to a low error.

   If we were about to design a MAS we would try to build it so that it lies in the area where the final error is zero. This way we can expect all agents to eventually have the correct behavior. We note that a substantial percentage of the research in DAI and MAS deals with taking systems that are not inherently in this area of near-zero error and designing protocols and rules of encounter so as to move them into this area, as in Rosenschein and Zlotkin [1994].

   The fact that the final error is 1 for the case with $I_{ij} = I_{ji} = 1$ can seem non-intuitive to readers familiar with game theory. In game theory there are many games, such as the "matching game" from Section 3.1, where two agents have an impact of 1 on each other. However, it is known (Binmore [1988]) that, in these games, two learning agents will eventually converge to one of the equilibria (if there are any), making their final error equal to 0. This is certainly true, and it is exactly what we showed in Section 3.1. The same result is not seen in Figure 7 because the figure was plotted using our simplified Equation. (20), which makes the simplifying independence assumption given by Eq. (18). This assumption cannot be made in games such as the matching game because, in these games, there is a correlation between the correctness of each of the agents actions. Specifically, in the matching game it is always true that both agents are either correct, or incorrect, but it is never true that one of them is correct while the other one is incorrect, i.e., either they matched, or they did not match.

   Another view of the system is given by Figure 8 which shows a vector plot of the agents' errors. We can see how the bigger errors are quickly reduced but the pace of learning decreases as the errors get closer to the convergence point.

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$\alpha_j = .1$
$\alpha_j = .3$
$\alpha_j = .9$

error

time

$l_j = .005$
$l_j = .04$
$l_j = .055$

error

time
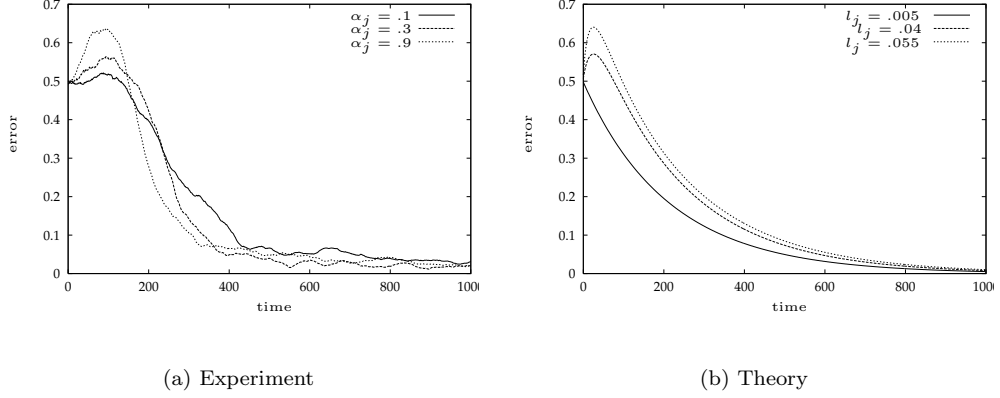
(a) Experiment

(b) Theory

Figure 9: Comparison of observed and predicted error.

Notice also that an agent's error need not change in a monotonic fashion. That is, an agent's error can get bigger for a while before it starts to get smaller.

7 A Simple Application

In order to demonstrate how our theory can be used, we tested it on a simple market-based MAS. The game consists of three agents, one buyer and two seller agents i and j. The buyer will always buy at the cheapest price—but the sellers do not know this fact. The sellers can bid any one of 20 prices in an effort to maximize their profits. In this system we had one good being sold ($|W| = 1$).

As predicted by economic theory, the price in this system settles to the sellers' marginal cost, but it takes time to get there due to the learning inefficiencies. The sellers we use are 0-level reinforcement learning agents (Vidal and Durfee [1998]). We experimented with different α_j rates[2] for the reinforcement learning of agent j, while keeping $\alpha_i = .1$ fixed, and plotted the running average of the error of agent i. A comparison is shown in Figure 9. Figure 9(a) gives the experimental results for three different values of α_j. It shows i's average error, over 100 runs, as a function of time. Since both sellers start with no knowledge, their initial actions are completely random which makes their error equal to .5. Then, depending on α_j, i's error will either start to go down from there or will first go up some and then down. Eventually, i's error gets very close to 0, as the system reaches a market equilibrium.

We can predict this behavior using Eq. (27). Based on the game description, we set $|A_i| = |A_j| = 20$, since there were 20 possible actions. We let $r_i =$

---

[2]α is the relative weight the algorithm gives to the most recent payoff. $\alpha = 1$ means that it will forget all previous experience and use only the latest payoff to determine what action to take.

$r_j = 1$ because reinforcement learning with fixed payoffs enforces the condition that once an agent is taking the correct action it will never change its decision function to take a different action. The agent might, however, still take a wrong action but only when its exploration rate dictates it.

We then let $I_{ij} = I_{ji} = .17$ based on the rough calculation that each agent has an equal probability of bidding any one of the 20 prices. If $\Delta_i^t = 20$ then I_{ji} for this situation is the probability that j was also bidding 20 or above, i.e., $1/20$, times the probability that j's new price is lower than 20, i.e. $19/20$. Similarly, if $\Delta_i^t = 19$ then I_{ji} is equal to $2/20$ times $18/20$. The average of all of these probabilities is .17. A more precise calculation of the impact would require us to find it via experimentation by actually running the system.

Finally, we chose $l_i = l_j = c_i = c_j = .005$ for the first curve (i.e., the one that compares with $\alpha_j = .1$). We knew that for such a low α_j the learning and change rate should be the same. The actual value was chosen via experimentation. The resulting curve is shown in Figure 9(b). At this moment, we do not possess a formal way of deriving learning and change rates from α-rates.

For the second curve ($\alpha_j = .3$) we knew that, since only α_j had changed from the first experiment, we should only change l_j and c_j. In fact, these two values should only be increased. We found their exact values, again by experimentation, to be $l_j = .04$, $c_j = .4$. For the third curve we found the values to be $l_j = .055$, $c_j = .8$.

One difference we notice between the experimental and the theoretical results is that the experimental results show a longer delay before the error starts to decrease. We attribute this delay to the agent's initially high exploration rate. That is, the agents initially start by taking all random actions but progressively reduce this rate of exploration. As the exploration rate decreases the discrepancy between our theoretical predictions and experimental results is reduced.

In summary, while it is true that we found l_j and c_j by experimentation, all the other values were calculated from the description of the problem. Even the relative values of l_j and c_j follow the intuitive relation with α_j that, as α_j increases so does l_j and (even more) c_j. Section 9 shows how to calculate lower bounds on the learning rate. We believe that this experiment provides solid evidence that our theory can be used to approximately determine the quantitative behaviors of MASs with learning agents.

8   Application of our Theory to Experiments in the Literature

In this section we show how we can apply our theory to experimental results found in the AI and MAS literature. While we will often not be able to completely reproduce the authors' results exactly, we believe that being able to reproduce the flavor and the main quantitative characteristics of experimental results in the literature shows that our theory can be widely applied and used by practitioners in this area of research.
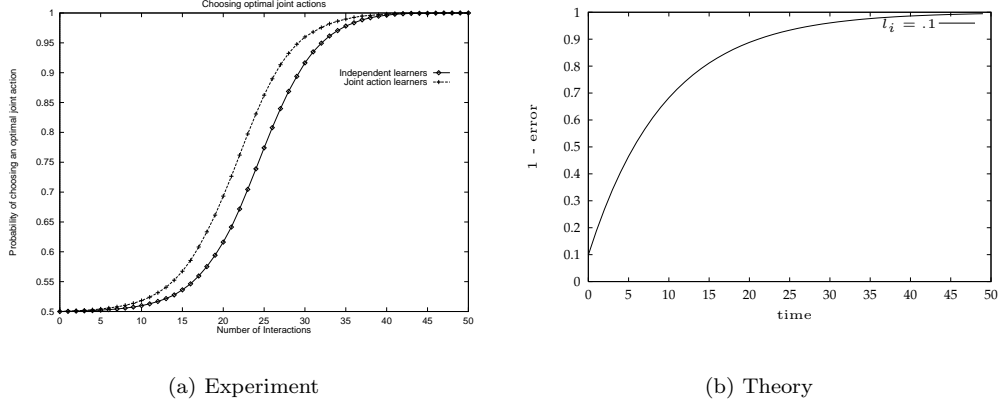
Figure 10: Comparing theory (b) with results from Claus and Boutilier [1997] (a).

8.1 Claus and Boutilier

Claus and Boutilier [1997] study the dynamics of a system that contains two reinforcement learning agents. Their first experiment puts the two agents in a matching game exactly like the one we describe in Section 3.1 with $|A_i| = |A_j| = 2$. Their results show the probability that both agents matched (i.e., 1 - $e(\delta_i^t)$) as time progressed. Since they were using two reinforcement learning agents, it was not surprising that the curve they saw, seen in Figure 10(a), was nearly identical to the curve we saw in our experiments with the two buying agents (Figure 9(a) with $\alpha_j = \alpha_i = .1$, except upside-down).

We can reproduce their curve using our equation for the matching game Eq. (16). The results can be seen in Figure 10(b). Our theory again fails to account for the initial exploration rate. We can, however, confirm that by time 15 their Boltzmann temperature (the authors used Boltzmann exploration) had been reduced from an initial value of 16 to 3.29 and would keep decreasing by a factor of .9 each time step. This means that by time 15 the agents were, indeed, starting to do more exploitation (i.e., reduce their error) while doing little exploration.

8.2 Shoham and Tennenholtz

Shoham and Tennenholtz [1997] investigate how learning agents might arrive at social conventions. The authors introduce a simple learning algorithm (strategy-selection rule) called *highest cumulative reward* (HCR) which their agents use for learning these conventions. Shoham and Tennenholtz also provide the results of a series of experiments using populations of learning agents. We try to reproduce the results they present in their Section 4.1 where they study the "coordination

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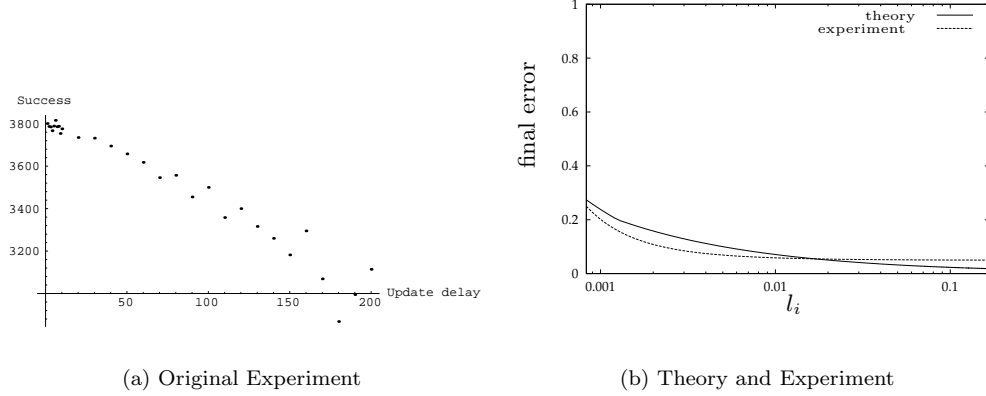(a) Original Experiment (b) Theory and Experiment

Figure 11: Comparing theory (b) with results from Shoham and Tennenholtz [1997] (a).

game" which is similar to our matching game, but with only two actions.

The experiment in question involves 100 agents, all of them identical and all of them using HCR. At each time instant the agents take one of two available actions. The aim is for every pair of chosen agents to take the same action as each other. The authors do not state how the agents are paired up so we will assume that they are all randomly matched. The agents update their behavior (i.e., apply HCR) after a given delay. The authors try a series of delays (from 0 to 200) and show that increasing the update delay decreases the percentage of trials where, after 1600 iterations, at least 95% of the agents reached a convention. The authors show surprise at finding this phenomenon. Their results are reproduced in Figure 11(a) (cf. Figure 1 in their article). The number of actions for all agents is easily set to $|A_i| = 2$, which implies that we must have $l_i = c_i$. By examining HCR, it is easy to determine that $r_i = 1$ (i.e if an agent took the right action, it will only get more support for it). At first intuition, one's impulse is to set $I_{ij} = 1$ for every pair of agents i and j. However, since there are 100 of them and only pairs of them interact at every time instant, the real impact is $I_{ij} = 1/99$.

We will now convert from their units of measurement into ours. In Figure 11(a) we can see that their x-axis is called the *update delay*, which we will refer to as d. This value is the number of time units that pass before the agent is allowed to learn. For $d = 0$ the agent learns after every interaction (i.e., on every time t), while for $d = 200$ the agent takes the same action for 200 time instances and only learns after every 200 iterations. This means that we must set $l_i = \frac{1}{p(d+1)}$ where $p > 0$. The value of p depends on their learning algorithm's performance, but we know that it must be a small number (< 50) greater than 0. Through some experimentation we settled on $p = 6$ (other values close to

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this one give similar results). Since in their graph they look at $0 \leq d \leq 200$, we must then look at l_i where $\frac{1}{1206} \leq l_i \leq \frac{1}{6}$ Finally, we find the value of d in terms of l_i to be

$$d = \frac{1}{pl_i} - 1 \tag{28}$$

The y-axis of Figure 11(a) is the *success*, i.e., number of trials, out of 4000, where at least 95% of the agents reached a convention. We will refer to this value as s. We know that in $s/4000$ of the trials *at least* 95% of the agents have error close to 0 (i.e., reaching a convention means that the agents take the right action almost all the time), and for the rest of the trials the error was greater. We can approximately map this to an error by saying that in $s/4000$ of the trials the error was 0 (a slight underestimate), while in $1 - s/4000$ of the trials the error was 1 (a slight overestimate). We add these two up (the 0 makes the first term disappear) and arrive at an equation that maps s to $e(\delta_i^t)$.

$$e(\delta_i^t) \approx \left(\frac{4000 - s}{4000} \right) \tag{29}$$

The mapping from d to s is given by their actual data. Their data can be fit by the following function:

$$s = 3900 - 4d - \frac{(d - 100)^2}{100} \tag{30}$$

Plugging Eq. (28) into Eq. (30), and the result into Eq. (29), we finally arrive at a function that maps their experimental results into our units:

$$\text{Final error} = \frac{4000 - \left(3900 - 4(1/pl_i - 1) - \left(\frac{(1/pl_i - 1 - 100)^2}{100} \right) \right)}{4000} \tag{31}$$

for the range $\frac{1}{1206} \leq l_i \leq \frac{1}{6}$.

Now that we have values for c_i, l_i, r_i, I_{ij}, $|A_i|$, a range for l_i and an equation that maps their experimental results into our units, we can plot both functions, as seen in Figure 11(b). The x-axis was plotted on a log-scale in order to better show the shape of the experiment curve, otherwise it would appear mostly as a straight line. For our theory curve we used Equations (20) and (26), and iterated for 1600 time units, just like in the experiment, and plotted the error at that point. For the experiment curve we used Eq. (31). We plotted both of these curves in the specified range for l_i. The reader will notice that our theory was able to make precise quantitative predictions. The maximum distance from our theory curve to the experimental curve is .05, which means that our predictions for the final error were, at worst, within 5% of the experimental values. Also, an error of about 5% was introduced when mapping from their success percentage s to our error.

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8.3   Others

There are several other examples in the literature where we believe our theory can be successfully applied. [Ishida, 1997, chapter 3.7] gives results of an experiment where two agents try to find each other in a 100 by 100 grid. He shows that if the grid has few obstacles it is faster if both agents move towards each other, while if there are many obstacles it is faster if one of the agents stays still while the other one searches for it. We believe that the number of obstacles is proportional to the change rate that the agents experience and, perhaps, to the impact that they have on each other. When there are no obstacles the agents never change their decision functions (because their initial Manhattan heuristics lead them in the correct path). As the number of obstacles increases, the agents will start to change their decision functions as they move, which will have an impact on the other agent's target function. If, however, one of them stays put, this means that his change rate is 0 so the other agent's target function will stay still and he will be able to reach his target (i.e., error 0) quicker.

Notice that the problem of a moving target that Ishida studies is different from the problem of a moving target function which we study. It is, however, interesting to note their similarities and how our theory can be applied to some aspects of that domain.

Another possible example is given by Sen et al. [1994]. They show two Q-learning agents trying to cooperate in order to move a block. The authors show how different α rates (β in their article) affect the quality of the result that the agents converge to. This quality roughly corresponds to our error, except for the fact that their measurements implicitly consider some actions to be better than others, while we consider an action to be either correct or incorrect. This discrepancy would make it harder to apply our theory to their results but we still believe that a rough approximation is possible. Our future work includes the extension of the CLRI framework to handle a more general definition of error—one that attaches a utility to each state-action pair, rather than the simple correct/incorrect categorization we use.

9   Bounding the Learning Rate with Sample Complexity

In the previous examples we have used our knowledge of the learning algorithms to determine the values of the agent's c_i, l_i, and r_i parameters. However, there might be cases where this is not possible—the learning algorithm might be too complicated or unknown. It would be useful, in these cases, to have some other measure of the agent's learning abilities, which could be used to determine some bounds on the values of these parameters.

One popular measure of the complexity of learning is given by Probably Approximately Correct (PAC) theory (Kearns and Vazirani [1994]), in the form of a measure called the *sample complexity*. The sample complexity gives us a loose upper bound on the number of examples that a consistent learning agent

must observe before arriving at a PAC hypothesis.

There are two important assumptions made by PAC-theory. The first assumption is that the agents are consistent learners[3]. Using our notation, a consistent learner is one who, once it has learned a correct $w \rightarrow a$ mapping does not forget it. This simply means that the agent must have $r_i = 1$. The second assumption is that the agent is trying to learn a fixed concept. This assumption makes $\Delta_i^{t+1} = \Delta_i^t$ true for all t.

The sample complexity m of an agent's learning problem is given by

$$m \geq \frac{1}{\epsilon} \left(\ln \frac{|H|}{\gamma} \right), \tag{32}$$

where $|H|$ is the size of the hypothesis space for the agent. In other words, $|H|$ is the total number of different $\delta_i(w)$ functions that the agent will consider. For an agent with no previous knowledge we have $|H| = |A_i|^{|W|}$. However, agents with previous knowledge might have smaller $|H|$, since this knowledge might be used to eliminate impossible mappings. If a consistent learning agent has seen m examples then, with probability at least $(1 - \gamma)$, it has error at most ϵ.

While we cannot map the sample complexity m to a particular learning rate l_i, we can use it to put a lower bound on the learning rate for a consistent learning agent. That is, we can find a lower bound for the learning rate of an agent who does not forget anything it has seen, and who is trying to learn a fixed target function. Since the agent does not forget anything it has seen, we can deduce that its retention rate must be $r_i = 1$. Since the target function is not changing, we know that $\Pr[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] = 0$ and $\Pr[\Delta_i^{t+1}(w) = \Delta_i^t(w)] = 1$. We can plug these values into Eq. (19) and simplify in order to get:

$$E[e(\delta_i^{t+1})] = e(\delta_i^t) \cdot (1 - l_i). \tag{33}$$

We can solve the difference Eq. (33), for any time n, in order to get:

$$E[e(\delta_i^n)] = e(\delta_i^0) \cdot (1 - l_i)^n. \tag{34}$$

We now remember that after m time steps we expect, with probability $(1 - \gamma)$, the error to be less than ϵ. Since Eq. (34) only gives us an expected error, not a probability distribution over errors, we cannot use it to calculate the likelihood of the agent having that expected error. That is, we cannot calculate the "probably" (γ) part of probably approximately correct. We will, therefore, assume that the γ chosen for m is small enough so that it will be safe to say that, after m time steps, the error is less than ϵ. In a typical application one uses a small γ because it guarantees a high degree of certainty on the upper bound of the error.

Since we can now safely say that, after m time steps, the error is less than ϵ, we can then deduce that the l_i for this agent should be small enough such that, if $n = m$, then $E[e(\delta_i^n)] \leq \epsilon$. This is expressed mathematically as:

$$e(\delta_i^0) \cdot (1 - l_i)^m \leq \epsilon. \tag{35}$$

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[3]See [Mitchell, 1997, p162] for a formal definition of a consistent learner.

We solve this equation for l_i in order to get:

$$l_i \geq 1 - \left(\frac{\epsilon}{e(\delta_i^0)} \right)^{1/m} . \tag{36}$$

This equation is not defined for $e(\delta_i^0) = 0$. However, given our assumption of a fixed target function and $r_i = 1$, we already know, from Eq. (33), that if an agent starts with an error of 0 it will maintain this error of 0 for any future time $t > 0$. Therefore, in this case, the choice of a learning rate has no bearing on the agent's error, which will always be 0.

Equation (36) gives us a lower bound on the learning rate that a consistent learner must have, given that it has sample complexity m, and based on an error ϵ and a sufficiently small γ. A designer of an agent that uses a reasonable learning algorithm can expect that, if his agent has sample complexity m (for ϵ error), then his agent will have a learning rate of at least l_i, as given by Eq. (36). Furthermore, if a designer is comparing two possible agent designs, each with a different sample complexity but both with similarly powerful learning algorithms, he can calculate bounds on the learning rates of both agents and compare their relative performance.

10 Related Work

The topic of agents learning about agents arises often in the studies of complexity (Cowan et al. [1995]). In fact, systems where the agents try to adapt to endogenously created dynamics are being widely studied (Hübler and Pines [1994], Arthur et al. [1997b]). In these systems, like in ours, the agents co-create their expectations as they learn and change their behaviors. Complexity research uses simulated agents in an effort to understand the complex behaviors of these systems as observed in the real world.

One example is the work of Arthur et al. [1997b], who arrive at the conclusion that systems of adaptive agents, where the agents are allowed to change the complexity of their learning algorithms, end up in one of two regimes: a stable/simple regime where it is trivial to predict an agent's future behavior, and a complex regime where the agents' behaviors are very complex. It is this second regime that interests complexity researchers the most. In it, the agents are able to reach some kind of "equilibrium" point in model building complexity. These same results are echoed by Darley and Kauffman [1997] in a similar experiment. In this article we have not allowed the agents to dynamically change the complexity of their learning algorithms. Therefore, our dynamics are simpler. Allowing the agents to change their complexity amounts to allowing them to change the values of their c, l, and r parameters while learning.

However, while complexity research is very important and inspiring, it is only partially relevant to our work. Our emphasis is on finding ways to predict the behavior of MASs composed of machine-learning agents. We are only concerned with the behavior of simpler artificial programmable agents, rather than the complex behavior of humans or the unpredictable behavior of animals.

The dynamics of MASs have also been studied by Kephart et al. [1990]. In this work the authors show how simple predictive agents can lead to globally cyclic or chaotic behaviors. As the authors explain, the chaotic behaviors were a result of the simple predictive strategies used by the agents. Unlike our agents, most of their agents are not engaged in learning, instead they use simple fixed predictive strategies, such as "if the state of the world was x ten time units before, then it will be x next time so take action a". The authors later show how learning can be used to eliminate these chaotic global fluctuations.

Matarić [1995] has studied reinforcement learning in multi-robot domains. She notes, for example, how learning can give rise to social behaviors (Matarić [1997]). The work shows how robots can be individually programmed to produce certain group behaviors. It represents a good example of the usefulness and flexibility of learning agents in multi-agent domains. However, the author does not offer a mathematical justification for the chosen individual learning algorithms, nor does she explain why the agents were able to converge to the global behaviors. Our research hopes to provide the first steps in this direction.

One particularly interesting approach is taken by Carmel and Markovitch [1997]. They work on model-based learning, that is, agents build models of other agents via observations. They use models based on finite state machines. The authors show how some of these models can be effectively learned via observation of the other agent's actions. The authors concentrate on the development of learning algorithms that would let one agent learn a finite-state machine model of another agent. They have not considered the case where two or more agents are simultaneously learning about each other, which we study in this article. However, their work is more general in the sense that they model agents as state machines, rather than the state-action pairs we use.

Finally, a lot of experimental work has been done in the area of agents learning about agents (Sen [1996], Weiß [1997]). For example, Sen and Sekaran [1998] show how learning agents in simple MAS converge to system-wide optimal behavior. Their agents use Q-learning or modified classifier systems in order to learn. The authors implement these agents and compare the performance of the different learning algorithms for developing agent coordination. Hu and Wellman [1998b, 1996] have studied reinforcement learning in market-base MASs, showing how certain initial learning biases can be self-fulfilling, and how learning can be useful but is affected by an agent's models of other agents. Claus and Boutilier [1997] have also carried out experimental studies of the behavior of reinforcement learning agents. We have been able to use the CLRI framework to predict some of their experimental results Vidal [1998]. Other researchers such as Stone and Veloso [1999], Littman [1994], and Hu and Wellman [1998a] have extended the basic Q-learning Watkins and Dayan [1992] algorithm for use with MASs in an effort to either improve or prove convergence to the optimal behavior.

We have also successfully experimented with reinforcement learning simulations (Vidal and Durfee [1998]), but we believe that the formal treatment elucidated in these pages will shed more light into the real nature of the problem and the relative importance of the various parameters that describe the

capabilities of an agent's learning algorithm.

11   Summary

We have presented a framework for studying and predicting the behavior of MASs composed of learning agents. We believe that this framework captures the most important parameters that describe an agents' learning and the system's rules of encounter. Various comparisons between the framework's predictions and experimental results were given. These comparisons showed that the theoretical predictions closely match our experimental results and the experimental results published by others. Our success in reproducing these results allows us to confidently state the effectiveness and accuracy of our theory in predicting the expected error of machine learning agents in MASs.

Since our theory describes an agent's behavior at a high-level (i.e., the agent's error), it is not capable of making system-specific predictions (e.g., predicting the particular actions that are favored). These types of system-specific predictions can only be arrived at by the traditional method of implementing populations of such agents and testing their behaviors. However, we expect that there will be times when the predictions from our theory will be enough to answer a designer's questions. A MAS designer that only needs to determine how "good" the agent's behavior will be could probably use the CLRI framework. A designer that needs to know which particular emergent behaviors will be favored by his agents will need to implement the agents.

Finally, while we have given some examples as to how learning rates can be determined for particular machine learning implementations, we do not have any general method for determining these rates. However, we showed how to use the sample complexity of a learning problem to determine a lower bound on the learning rate of a consistent learning agent. This bound is useful for quickly ruling out the possibility of having agents with high expected errors and of stating that an agent's expected error will be, at most, a certain constant value. Still, if the agent's learning algorithm is much better than the one assumed by a consistent learner (e.g., the agent is very good at generalizing from one world state to many others), then these lower bounds could be significantly inaccurate.

A   Derivation for Matching Game

If we can assume that the action chosen when an agent changes $\delta_i^t(w)$ and the result does not match $\Delta_i^t(w)$ (for some specific w) is taken from a flat probability distribution, then we can say that:

$$B = \frac{|A_i| - 2}{|A_i| - 1}. \tag{37}$$

We will now show how to calculate the fourth term in (16). For the matching game we find that we can set:

$$D = 1 - l_j \tag{38}$$

$$F = l_j + (1 - l_j)\left(\frac{|A_i| - 3}{|A_i| - 2}\right). \tag{39}$$

Having $|A_i| = 2$ implies that $c_i = l_i$, this means that for this case we have

$$(1 - c_i)D + l_i + (c_i - l_i)F = l_i + (1 - c_i)(1 - l_j). \tag{40}$$

For the case where $|A_i| > 2$, which is the case we are interested in, we can plug in the values for D and F and simplify, in order to get the fourth term:

$$(1 - c_i)D + l_i + (c_i - l_i)F = 1 - l_j + \frac{c_i l_j(|A_i| - 1) + l_i(1 - l_j) - c_i}{|A_i| - 2}. \tag{41}$$

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