

# A Counterexample to a Proposed Proof of P=NP

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In [1], the claim is put forth that  $P=NP$ ; the form of this claim is an algorithm which purportedly can solve the 3SAT problem in  $O(n^4)$  time.

The 3SAT problem (or “3-SAT problem,” as it is referred to in [1]) is to determine if the formula

$$d_1 \wedge d_2 \wedge \cdots \wedge d_m \quad (1)$$

is satisfiable, where each clause  $d_k$  with  $1 \leq k \leq m$  is a disjunction of at most three variables or their negations from the set

$$B = \{b_1, b_2, \dots, b_n\}. \quad (2)$$

The validity of the algorithm rests on the following claim:

**Claim 1** *Let (1) and (2) be the given instance of 3SAT. Let  $C$  be the set of clauses of the instance:*

$$C = \{d_1, d_2, \dots, d_m\}.$$

*The instance is non-satisfiable if and only if at least one of the following is true:*

**Pattern 1.** *There is  $\alpha \in B$ :*

$$\{\alpha, \bar{\alpha}\} \subseteq C;$$

**Pattern 2.** *There are different  $\alpha, \beta \in B$ :*

$$\{\alpha \vee \beta, \alpha \vee \bar{\beta}, \bar{\alpha} \vee \beta, \bar{\alpha} \vee \bar{\beta}\} \subseteq C;$$

**Pattern 3.** *There are different  $\alpha, \beta, \gamma \in B$ :*

$$\{\alpha \vee \beta \vee \gamma, \alpha \vee \beta \vee \bar{\gamma}, \alpha \vee \bar{\beta} \vee \gamma, \alpha \vee \bar{\beta} \vee \bar{\gamma}, \\ \bar{\alpha} \vee \beta \vee \gamma, \bar{\alpha} \vee \beta \vee \bar{\gamma}, \bar{\alpha} \vee \bar{\beta} \vee \gamma, \bar{\alpha} \vee \bar{\beta} \vee \bar{\gamma}\} \subseteq C.$$

This claim is incorrect. The proof supplied in [1] only addresses the “if” direction; that is, the following Proposition is proved, which *is* true.

**Proposition 1** *Let (1) and (2) be the given instance of 3SAT. Let  $C$  be the set of clauses of the instance:*

$$C = \{d_1, d_2, \dots, d_m\}.$$

*The instance is non-satisfiable if any of the following are true:*

1. *There is  $\alpha \in B$ :*

$$\{\alpha, \bar{\alpha}\} \subseteq C;$$

2. *There are different  $\alpha, \beta \in B$ :*

$$\{\alpha \vee \beta, \alpha \vee \bar{\beta}, \bar{\alpha} \vee \beta, \bar{\alpha} \vee \bar{\beta}\} \subseteq C;$$

3. *There are different  $\alpha, \beta, \gamma \in B$ :*

$$\{\alpha \vee \beta \vee \gamma, \alpha \vee \beta \vee \bar{\gamma}, \alpha \vee \bar{\beta} \vee \gamma, \alpha \vee \bar{\beta} \vee \bar{\gamma}, \\ \bar{\alpha} \vee \beta \vee \gamma, \bar{\alpha} \vee \beta \vee \bar{\gamma}, \bar{\alpha} \vee \bar{\beta} \vee \gamma, \bar{\alpha} \vee \bar{\beta} \vee \bar{\gamma}\} \subseteq C.$$

The supposition is a sufficient but not necessary condition for a given formula to lack a solution. It is easy to find a counterexample.

**Counterexample 1** *The formula*

$$\bar{a} \wedge \bar{b} \wedge \bar{c} \wedge (a \vee b) \wedge (a \vee c) \wedge (b \vee c)$$

*is not satisfiable, even though it does not meet either condition 1, 2, or 3 of Claim 1.*

## References

- [1] Sergey Gubin. A polynomial time algorithm for 3-sat. <http://arxiv.org/list/cs/new>, January 2006.