

On Interference of Signals and Generalization in Feedforward Neural Networks

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Abstract

This paper studies how the generalization ability of neurons can be affected by mutual processing of different signals. This study is done on the basis of a feedforward artificial neural network, that is used here as a model of the very basic processes in a network of biological neurons. The mutual processing of signals, called here an interference of signals, can possibly be a good model of patterns in a set generalized either by a biological network of neurons or by an artificial feedforward neural network, and in effect may improve generalization. In this paper it is discussed that the interference may however also cause highly random generalization. Adaptive activation functions in the studied model are discussed as a way of reducing that type of generalization. A test of a feedforward neural network is performed that shows the discussed random generalization. Hypotheses about ways of preventing the described random generalization in the biological neural networks are discussed.

keywords: modeling biological neural network, feedforward neural networks, generalization, interference of signals, overfitting

1 INTRODUCTION

Generalization can be viewed as one of the basic properties of the brain. Since the very low-level processing of visual information, where the color of the region not seen by the yellow spot on retina is ‘filled’ with the color of the surrounding region, till imagining how one would feel on an alien planet on basis of its terrestrial experiences, the brain estimates the missing information on the foundation of a similar information that it already knows. Generalization is also a term in machine learning – one of the reasons of building nonlinear regressors like feedforward neural networks is their ability to generalize to unknown data, after being trained with the learning data.

A feedforward artificial neural network, further denoted by FNN, is studied in this paper as a very simple model of the generalization in a biological neural network. A FNN can be viewed as a rather ‘unconstrained’ structure – in a typical multilayered architecture an output of a neuron in one layer is simply connected to all inputs in the succeeding layer, and the weights of connections can just be initialized randomly. The combination function of an artificial neuron of the McCulloch and Pitts (1943) type treats all its arguments as equivalent, simply adding them. In the process of training, attributes of the training observations are propagated through such a relatively generic structure, possibly in a

random order. It may rise several questions. How that somewhat unconstrained structure of an artificial neural network copes with generalization, especially when there are several ‘competing’ stimuli, that simultaneously want to be ‘extrapolated’ onto ‘regions’ in the inputs space of the FNN not covered by the training data. How such conflicts can possibly destroy the ability of generalization, and what can be the ways to reduce such phenomena? And lastly, how the similar phenomena are resolved in the brain?

2 RANDOM GENERALIZATION

The summing of signals in the combination function of an artificial neuron, called here an *interference* of signals, may improve generalization. For example, in the case of a multi-dimensional data set, processing of values from one input of a neural network can be influenced by values at another input of the neural network, what may model well the patterns in the training set. The error-minimizing learning process can prevent harmful interference if the interference would increase the neural network error of approximation of the training set. The signals propagated from attributes of observations that are absent in the training set, however, can be interfered with no effect on the error. Therefore, the interference can decrease the generalization ability of the network. A decrease of generalization quality in neural networks can also be an effect of overfitting (Schaffer, 1991; Rosin and Fierens, 1995; Lawrence et al., 1997; Lawrence and Giles, 2000). Yet the worsening of generalization caused by the discussed interference can be very different from that caused by overfitting. While excessive fitting of the neural network function to the training set means only that some particular patterns of the set are memorized, the discussed interference of signals may introduce *highly random* changes to the generalizing function of the neural network.

Let us further discuss such a type of a random generalization in more detail.

3 STRONG PROPAGATION REGIONS

In this section the so-called strong propagation regions in the input spaces of neurons will be discussed. The notion will be used further in this paper to describe the discussed interference of signals.

A neuron with linear weight functions and a hyperbolic tangent activation function has its output value equal to a given value r for its input values that, in the neuron input space, create a hyperplane P_r , except of the special case where all weights in the neuron are equal to 0. Specifically, there is a hyperplane P_0 for the neuron output value equal to 0. Because the hyperbolic tangent activation functions have the greatest value of its derivative at 0, the hyperplane P_0 is the region in the neuron input space for which there is the strongest propagation of signals through the neuron. As the distance from this hyperplane increases, the derivative of the activation function decreases and in effect the propagation becomes weaker. Let us call the region with relatively strong level of propagation a strong propagation region. Let the region consist of points whose distances to P_0 in the input space of the neuron do not exceed a certain value.

Let there be two fully connected subsequent layers L_i and L_{i+1} in a feedforward neural network. Let there be N_i and N_{i+1} neurons in the layers, respectively. Let us discuss the

input spaces of the neurons in the layer L_{i+1} . Each of the neurons in the layer L_{i+1} has $N_i + 1$ inputs, N_i of which are from the neurons in the preceding layer and a single input is from the bias element. Therefore, the transformation made in the layer L_{i+1} can be represented by parameterized N_{i+1} N_i -dimensional input spaces of the neurons in L_{i+1} , where the parameters in the spaces are the values of functions of the respective neurons in L_{i+1} .

An example of input spaces of neurons in L_{i+1} is shown in Fig. 1. The lines represent

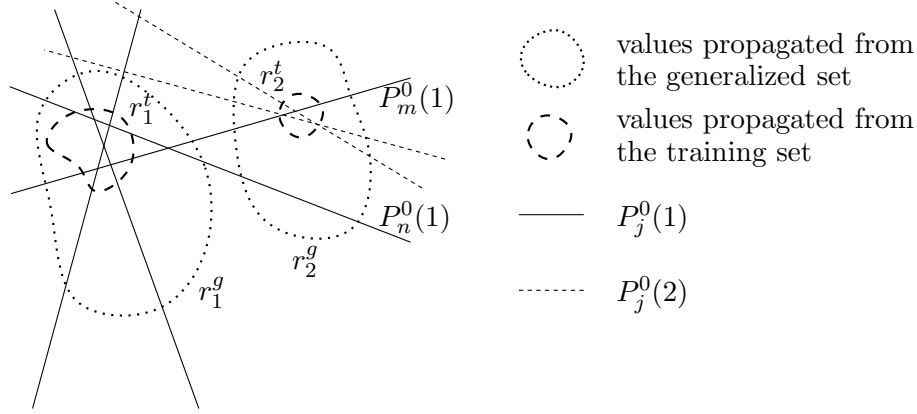


Figure 1: An example diagram of input spaces of neurons in a layer.

the hyperplanes P_0 , denoted by P_j^0 , $j = 0, 1, \dots, N_{i+1} - 1$, where j denotes a respective neuron in the layer L_{i+1} . This is not a full representation of the input spaces of the neurons in the discussed layer, because the values of functions of the neurons are not given, yet this diagram shows the regions with the strong propagation of signals, being on and near the hyperplanes P_j^0 . The values propagated to the neurons in the layer L_{i+1} are either the direct values of attributes of observations if L_{i+1} is the first hidden layer, or images of the attributes if L_{i+1} is any of the succeeding layers. Anyway, the region r_t of values propagated from the observations in the training set and the region r_g of values propagated from the observations in the generalized set can be shown in the input spaces of the neurons, as it is done in Figure 1. In the example diagram, the region r_t consists of two regions r_p^t , $p = 1, 2$, and the region r_g consists of another two regions r_q^g , $q = 1, 2$. The regions are schematically shown by solid regions in the diagrams, but they are sets of discrete points, where each point corresponds to one or more observations.

Let each observation has its input attributes, that is these that are propagated from the inputs of a neural network, and its output attributes, that is these that are compared to values at the outputs of the network. The hyperplanes P_j^0 in the example diagram generally concentrate in or near the regions r_p^t . This may happen during the training process if there are relatively large differences between the values of output attributes of observations whose input attributes are propagated through r_p^t . Thus, relatively high values of derivatives of functions of the neurons in L_{i+1} may correspond to relatively large differences between the output attributes of observations in the training set. The hyperplanes P_j^0 , by extending infinitely in the space, may allow for generalization to the points outside r_t , including the points that are relatively far from r_t .

4 INTERFERENCE OF SIGNALS

Let us discuss again the diagram of input spaces of neurons in Figure 1. Let there be several hyperplanes P_j^0 , denoted by $P_j^0(i)$, where j determines a respective neuron and $i = 1, 2$, that were placed during the learning process near r_t , to minimize the component of ξ_t caused by the observations in the training set, whose attributes propagate through r_t . They are marked in the diagram by solid lines for $i = 1$ and by dotted lines for $i = 2$. Let the regions r_1^g and r_2^g be overlapping or be near to r_1^t or r_2^t , respectively. Let the observations whose input attributes are propagated through the regions r_1^g and r_2^g be generalized well because of the hyperplanes $P_j^0(1)$ and $P_j^0(2)$, respectively. This is possible because the hyperplanes $P_j^0(1)$ extend from r_1^t and the hyperplanes $P_j^0(2)$ extend from r_2^t , thus ‘extrapolating’ the patterns in the region r_t .

Now, if a hyperplane $P_j^0(i)$, that normally is generalizing patterns in r_i^t , would by a chance ‘intersect’ r_{3-i}^t , like $P_0^m(1)$ does, it could possibly increase the training error ξ_t , and thus in a possible further training the intersecting hyperplane $P_j^0(i)$ could, for example, be driven out of r_{3-i}^t . Yet if the hyperplane would intersect r_{3-i}^g , like $P_0^n(1)$ does, it could intervene the generalization from r_{3-i}^t to r_{3-i}^g without any reaction in the training process. More, a region r_i^t could, during the training, be placed itself in r_{3-i}^g , thus causing all $P_j^0(i)$, associated with generalization of r_i^t , to intervene the generalization to r_{3-i}^g .

The interference of signals, causing a possibly high randomness of generalization, could be reduced if the strong propagation region of a neuron would not extend itself infinitely in space. This is like in the radial basis function neural networks (Broomhead and Lowe, 1988; Moody and Darken, 1989; Poggio and Girosi, 1989). On the other hand, such forms of finite strong propagation regions like in the radial basis function networks could worsen the ability of generalization of a neural network for sets where long strong propagation regions are needed for good generalization. A possible method of finding a good trade-off between infinite and finite strong propagation regions could be using adaptive activation functions. Such adaptive activation functions could, during training with a special learning algorithm, smoothly adapt their form, for example in the range between a radial basis function and a hyperbolic tangent.

5 TESTS

Because in some relatively simple generalization problems that were conducted the discussed random generalization seemed to be rather rarely observed – usually the trained neural networks after some time began only to overfit the data, showing only some randomness connected with a limited flexibility – in this test a relatively complex training set will be used. Then some analogies to biological neural networks will be discussed.

Let there be two three-dimensional sets θ_l and θ_c , as illustrated in Figures 2(a) and 2(b), respectively. The sets are 64×64 images, whose pixel coordinates determine the neural network input vector values, a single value for each dimension, and the pixels brightnesses determine corresponding values in the neural network output vectors. The pixel at the lower left corner has the coordinates $(-0.5, -0.5)$ and the pixel at the upper right corner has the coordinates $(0.5, 0.5)$. The brightness of the pixels represents the range from -0.5 for black to 0.5 for white. Feedforward layered networks with two inputs, a single neuron in the output layer and two hidden layers of 16 neurons each, were trained

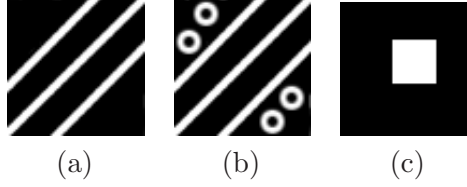


Figure 2: The data sets (a) θ_l , (b) θ_c and (c) the training subsets mask.

by the training subsets of either θ_l or θ_c . The neural networks had hyperbolic tangent activation functions. There was a weight decay at a rate of $2 \cdot 10^{-7}$ to improve generalization (Krogh and Hertz, 1992). An online training was used with a learning step of 0.02. The training subsets are represented by the image in Figure 2(c). Black pixels in the image mean that the corresponding pixels in Figures 2(a) and 2(b) represent the training subsets of the respective generalized sets.

There were four neural networks \mathcal{N}_i^l , $i = 0 \dots 3$, trained with the subset of θ_l , and four another neural networks \mathcal{N}_i^c , $i = 0 \dots 3$, trained with the subset of θ_c . The generalizing functions of the networks were sampled and the weights of the neurons in the first input layer were saved at each of the iterations 10000000th, 31622777th and 100000000th. The results are illustrated in Fig. 3. There is a table for each iteration in the figure, with sampled generalization functions in the upper row and diagrams representing input spaces of neurons in the first hidden layer in the lower row. The representation of the generalization functions is analogous to that of the sets θ_l and θ_c . Each of the input space diagrams shows with translucent lines the zeroes of the outputs of the first hidden layer neurons, that is it shows the hyperplanes P_j^0 , against the common input values from the input layer. The lower left corner of the dotted rectangles drawn within the diagrams represents input values $(-0.5, -0.5)$ and the upper right corner of the rectangles represents input values $(0.5, 0.5)$. Therefore, the input attributes of the observations in the sets θ_l and θ_c are propagated into the space marked in the diagrams by the dotted rectangles. The propagation to the first hidden layer is without any transformation of course, because the nodes in the input layer only pass signals to the first hidden layer.

Let us look at the diagrams of the input spaces of the neurons in the first hidden layer. Because of the direct relation between the space of the input attributes of the observations and the input spaces of the first hidden layer neurons it can be said that in the cases of both \mathcal{N}_i^l and \mathcal{N}_i^c the hyperplanes P_j^0 generally concentrate as it was discussed in Sec. 3. In particular, in \mathcal{N}_i^c , generally some hyperplanes concentrate near the linear features f_l and some concentrate near the circular features f_c . In effect, the lines in the diagrams concentrated near f_c cross these concentrated near f_l . Additionally, the crossings occur partially in the region not covered by the training set. These are exactly the conditions prone to the random generalization, discussed in Sec. 4. In fact, unlike \mathcal{N}_i^l , where the hyperplanes finely ‘extrapolate’ the regions in the training file, in the functions of \mathcal{N}_i^c a highly random generalization can be seen.

Let us study an example situation of a biological organism, living in conditions similar to that expressed by the case of the set θ_c . Let us imagine an animal in a world, where the switch between warm and cold temperatures is very rapid, i. e. there is a thin atmosphere and when sun rises then there gets instantly warm, and when sun sets it gets fast cold.

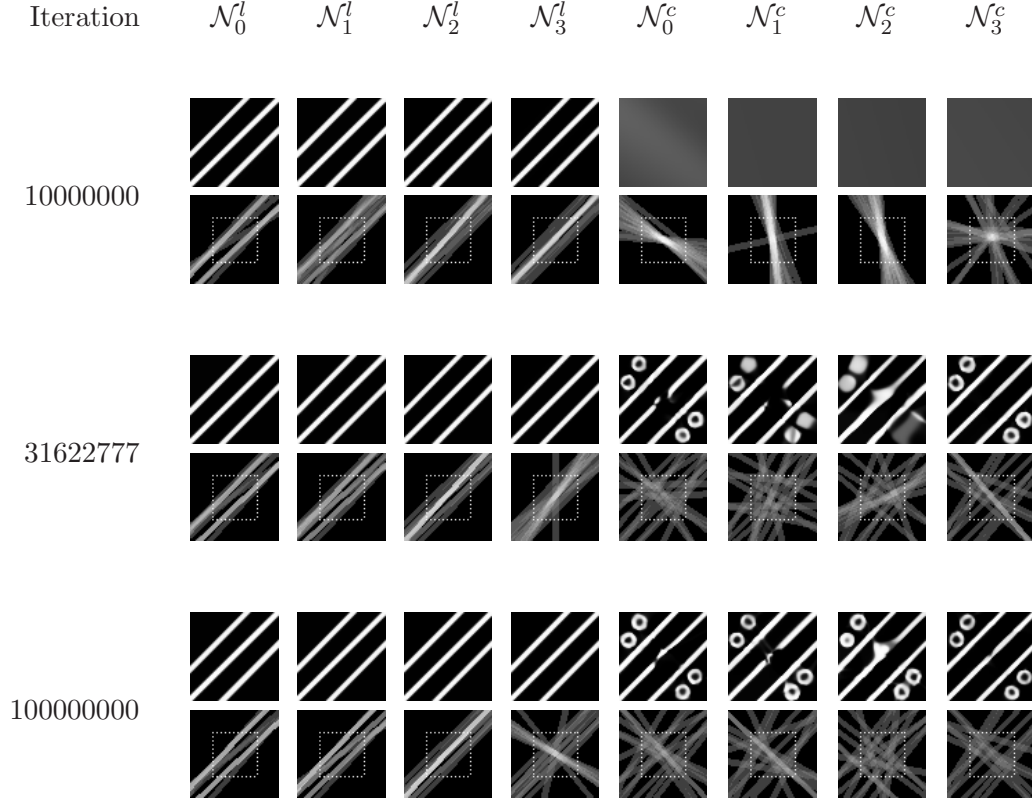


Figure 3: The generalizing functions and diagrams of the zeroes of the first hidden layer neurons.

The one exception is a water pool, that has moderate temperature during both night and day. The animal, for which the world is a natural environment, knows therefore well the situations marked in Fig. 4. But one day there occurred a very unusual situation –

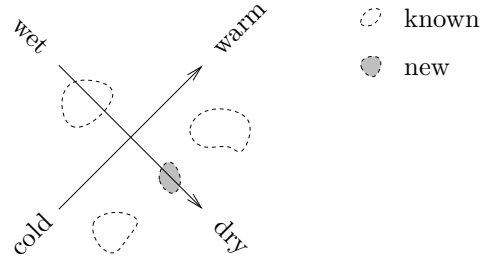


Figure 4: A ‘knowledge map’ of an imaginary animal.

a thick layer of clouds appeared and the animal found itself in the situation marked in Fig. 4 as the grayed region. Does the animal decide whether to walk or to swim only by sensing temperature, what would be enough only in the usual conditions, or is the animal developed enough to understand that swimming is clearly associated with wetness only?

We may imagine the brain of the animal acting in either way. And that the situations similar to those that could ‘destabilize’ both the tested simple FNNs and the brain of the imaginary animal may obviously occur on various level in a biological neural system.

6 CONCLUSIONS

It was discussed that the interference of signals within a FNN, while being possibly one of its strengths, may cause a substantially random generalization. In some cases the randomness may be treated as an unpredictable and spurious artifact that lowers the quality of the generalization, but on the other hand the randomness may be beneficial – it may increase the diversity of ‘behavior’ in a population, be the population a set of genetically evolved learning machines or a colony of biological organisms. And maintaining the diversity may be an advantage of the population (Kuo et al., 1997; Morrison and Oppacher, 1998).

Let us hypothesize how a biological neural network could prevent such a highly random generalization. A neuron or a system of several neurons could more ‘tune’ itself only to a specific stimulus. It may resemble the finite strong propagation regions described in Sec. 4. But again, such limiting of reception may decrease generalization ability just as it could decrease it in the case of the FNNs with the limited strong propagation regions. The discussed method may also remind of the receptive fields in neurons in the visual cortex – a group of neurons may be ‘tuned’ to a specific type of a visual stimulation. The further way of reducing the highly random generalization could be an association of a stimulus with the corresponding actions only. I. e. in the case of the imaginary animal the ‘wetness’ stimulus would be associated only with the ‘swim’ action. But first, the ‘tuning’ may be a hard task – the separation of ‘temperature’ and ‘wetness’ stimuli could be easy, but consider the separation of the two types of features in the set θ_c – such a task could possibly require more complicated structures than the FNNs that were trained with the set in Sec. 5 themselves. Secondly, it may be in some cases undesired – it is sometimes the joint processing of high dimension data that gives the highest quality of generalization.

Concluding, modeling a biological neural network with a relatively ‘unconstrained’ and generic artificial neural network, may possibly lead to substantial disparities between the model network, possibly prone to the interference-based random generalization, and the modeled network, in which perhaps specialized mechanisms exist that control such a, possibly undesired, random generalization.

References

- Broomhead, D. S. and Lowe, D. (1988). Multivariable functional interpolation and adaptive networks. *Complex Systems*, 2:321–355.
- Krogh, A. and Hertz, J. A. (1992). A simple weight decay can improve generalization. In Moody, J. E., Hanson, S. J., and Lippmann, R. P., editors, *Advances in Neural Information Processing Systems*, volume 4, pages 950–957. Morgan Kaufmann Publishers, Inc.

- Kuo, T., Shu-Yuen, and Hwang (1997). Using disruptive selection to maintain diversity in genetic algorithms. *Applied Intelligence*, 7(3):257–267.
- Lawrence, S. and Giles, C. L. (2000). Overfitting and neural networks: Conjugate gradient and backpropagation. In *Proceedings of the IEEE International Conference on Neural Networks*, pages 114–119. IEEE Press.
- Lawrence, S., Giles, C. L., and Tsoi, A. C. (1997). Lessons in neural network training: Overfitting may be harder than expected. In *Proceedings of the Fourteenth National Conference on Artificial Intelligence, AAAI-97*, pages 540–545. AAAI Press, Menlo Park, California.
- McCulloch, W. S. and Pitts, W. H. (1943). A logical calculus of the ideas immanent in nervous activity. *Bulletin of Mathematical Biophysics*, 5:115–133.
- Moody, J. and Darken, C. (1989). Fast learning in networks of locally tuned units. *Neural Computations*, 1(2):281–294.
- Morrison, J. and Oppacher, F. (1998). Maintaining genetic diversity in genetic algorithms through co-evolution. In *Canadian Conference on AI*, pages 128–138.
- Poggio, T. and Girosi, F. (1989). A theory of networks for approximation and learning. Technical Report AIM-1140.
- Rosin, P. and Fierens, F. (1995). Improving neural network generalisation.
- Schaffer, C. (1991). Overfitting avoidance as bias. In *IJCAI-91 Workshop on Evaluating and Changing Representation in Machine Learning*, Sydney.