

Energy-Efficient Power Control in Impulse Radio UWB Wireless Networks

Giacomo Bacci, Marco Luise, H. Vincent Poor and Antonia M. Tulino

Abstract

In this paper, a game-theoretic model for studying power control for wireless data networks in frequency-selective multipath environments is analyzed. The uplink of an impulse-radio ultrawideband system is considered. The effects of self-interference and multiple-access interference on the performance of generic Rake receivers are investigated for synchronous systems. Focusing on energy efficiency, a noncooperative game is proposed in which users in the network are allowed to choose their transmit powers to maximize their own utilities, and the Nash equilibrium for the proposed game is derived. It is shown that, due to the frequency selective multipath, the noncooperative solution is achieved at different signal-to-interference-plus-noise ratios, depending on the channel realization and the type of Rake receiver employed. A large-system analysis is performed to derive explicit expressions for the achieved utilities. The Pareto-optimal (cooperative) solution is also discussed and compared with the noncooperative approach.

Index Terms

Energy efficiency, game theory, Nash equilibrium, power control, large-system analysis, impulse-radio (IR), ultra-wide band (UWB), Rake receivers, frequency-selective multipath.

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G. Bacci and M. Luise are with the Dipartimento Ingegneria dell'Informazione, Università di Pisa - 56122 Pisa, Italy (email: giacomo.bacci@iet.unipi.it, marco.luise@iet.unipi.it).

H. V. Poor is with the Department of Electrical Engineering, Princeton University - 08544 Princeton, NJ, USA (email: poor@princeton.edu).

A. M. Tulino is with the Dipartimento Ingegneria Elettronica e Telecomunicazioni, Università degli Studi di Napoli Federico II - 80125 Napoli, Italy (email: atulino@princeton.edu).

I. INTRODUCTION

As the demand for wireless services increases, the need for efficient resource allocation and interference mitigation in wireless data networks becomes more and more crucial. A fundamental goal of radio resource management is transmitter power control, which aims to allow each user to achieve the required quality of service (QoS) at the uplink receiver without causing unnecessary interference to other users in the system. Another key issue in wireless system design is energy consumption at user terminals, since, in many scenarios, the terminals are battery-powered.

Recently, game theory has been used as an effective tool to study noncooperative power control in data networks [1]–[8]. The advantages of noncooperative (distributed) approaches with respect to a cooperative (centralized) approach are mainly due to the scalability of the network. Many of the problems to be solved in a communications system are in fact known to be NP-hard. As a consequence, real-time solution of these optimization problems in a centralized fashion becomes infeasible as the network size increases and as the number of users varies [1]. Game theory is the natural framework for modeling and studying these kinds of interactions. A model for power control as a noncooperative game is proposed in [2]. In this scenario, the users choose their transmit powers to maximize their utilities, defined as the ratio of throughput to transmit power. In [3], a network-assisted power-control scheme is proposed to improve the overall utility of a direct-sequence code-division multiple access (DS-CDMA) system. In [4], [5], the authors use pricing to obtain a more efficient solution for the power control game. Joint network-centric and user-centric power control are discussed in [6]. In [7], the authors propose a power control game for multicarrier CDMA (MC-CDMA) systems, while in [8] the effects of the receiver have been considered, particularly extending the study to multiuser detectors and multiantenna receivers.

This work considers power control in ultrawideband (UWB) systems. UWB technology is considered to be a potential candidate for next-generation short-range high-speed data transmission, due to its large spreading factor (which implies large multiuser capacity) and low power spectral density (which allows coexistence with incumbent systems in the same frequency bands). Commonly, impulse-radio (IR) systems, which transmit very short pulses with a low duty cycle, are employed to implement UWB systems [9], [10]. In an IR system, a train of pulses is sent and the information is usually conveyed by the position or the polarity of the pulses, which correspond to pulse position modulation (PPM) and binary phase shift keying (BPSK), respectively. To prevent catastrophic collisions among different users and, thus, to provide robustness against multiple access interference (MAI), each information symbol is represented by a sequence of pulses; the positions of the pulses within that sequence are determined by a pseudo-

random time-hopping (TH) sequence that is specific to each user [10]. In “classical” impulse radio, the polarity of those pulses representing an information symbol is always the same, whether PPM or BPSK is employed [10], [11]. Recently, pulse-based polarity randomization was proposed [12], where each pulse has a random polarity code in addition to the modulation scheme, providing additional robustness against MAI [13] and helping to optimize the spectral shape according to US Federal Communications Commission (FCC) specifications [14]. Due to the large bandwidth, UWB signals have a much higher temporal resolution than conventional narrowband or wideband signals. Hence, channel fading cannot be assumed to be flat [15], and self-interference (SI) must be taken into account [16]. To the best of our knowledge, this paper is the first to study the problem of radio resource allocation in a frequency-selective multipath environment using a game-theoretic approach. Previous work in this area has assumed flat fading [17]–[19].

Our focus throughout this work is on energy efficiency. In this kind of applications it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput. We first propose a noncooperative (distributed) game in which users are allowed to choose their transmit powers according to a utility-maximization criterion. Focusing on Rake receivers [20] at the base station, we derive the Nash equilibrium for the proposed game, also proving its existence and uniqueness. Using a large-system analysis, we obtain an approximation of the interference which is suitable for any kind of fading channels, including both small- and large-scale statistics. We also compute explicit expressions for the utility achieved at the equilibrium in a particular scenario and compare the performance of our noncooperative approach with the optimal cooperative (centralized) solution. It is shown that the difference between these two approaches is not significant for typical networks.

The remainder of this paper is organized as follows. Some background for this work is given in Sect. II. In Sect. III, we describe our power control game and analyze the Nash equilibrium for this game. In Sect. IV, we use the game-theoretic framework along with a large-system analysis to evaluate the performance of the system in terms of transmit powers and achieved utilities. The Pareto-optimal (cooperative) solution to the power control game is discussed in Sect. V, and its performance is compared with that of the noncooperative approach. Numerical results are discussed in Sect. VI, where also an iterative and distributed algorithm for reaching the Nash equilibrium is presented. Some conclusions are drawn in Sect. VII.

II. PROBLEM STATEMENT

A. Formulation

Consider the uplink of an IR-UWB data network, where every user wishes to locally and selfishly choose its action to maximize its own utility function. The strategy chosen by a user affects the performance of the other users in the network through MAI. Furthermore, since a realistic IR-UWB transmission takes place in frequency-selective multipath channels, the effect of SI cannot be neglected.

To pose the power control problem as a noncooperative game, a suitable definition of a utility function is needed to measure energy efficiency for wireless data applications. A tradeoff relationship exists between obtaining high signal-to-interference-plus-noise ratio (SINR) levels and consuming low energy. These issues can be quantified [2] by defining the utility function of the k th user to be the ratio of its throughput T_k to its transmit power p_k , i.e.

$$u_k(\mathbf{p}) = \frac{T_k}{p_k}, \quad (1)$$

where $\mathbf{p} = [p_1, \dots, p_K]$ is the vector of transmit powers, with K denoting the number of users in the network. Throughput, here referred to as the net number of information bits that are received without error per unit time, can be expressed as

$$T_k = \frac{D}{M} R_k f_s(\gamma_k), \quad (2)$$

where D and M are the number of information bits and the total number of bits in a packet, respectively; R_k and γ_k are the transmission rate and the SINR for the k th user, respectively; and $f_s(\gamma_k)$ is the efficiency function representing the packet success rate (PSR), i.e., the probability that a packet is received without an error. Our assumption is that a packet will be retransmitted if it has one or more bit errors. The PSR depends on the details of the data transmission, including its modulation, coding, and packet size. To prevent the mathematical anomalies described in [2], we replace PSR with an *efficiency function* $f(\gamma_k)$ when calculating the throughput for our utility function. In the case of BPSK TH-IR systems in multipath fading channels, a reasonable approximation to the PSR for moderate-to-large values of M is $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$.

However, our analysis throughout this paper is valid for any efficiency function that is increasing, S-shaped,¹ and continuously differentiable, with $f(0) = 0$, $f(+\infty) = 1$, and $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$.

¹An increasing function is S-shaped if there is a point above which the function is concave, and below which the function is convex.

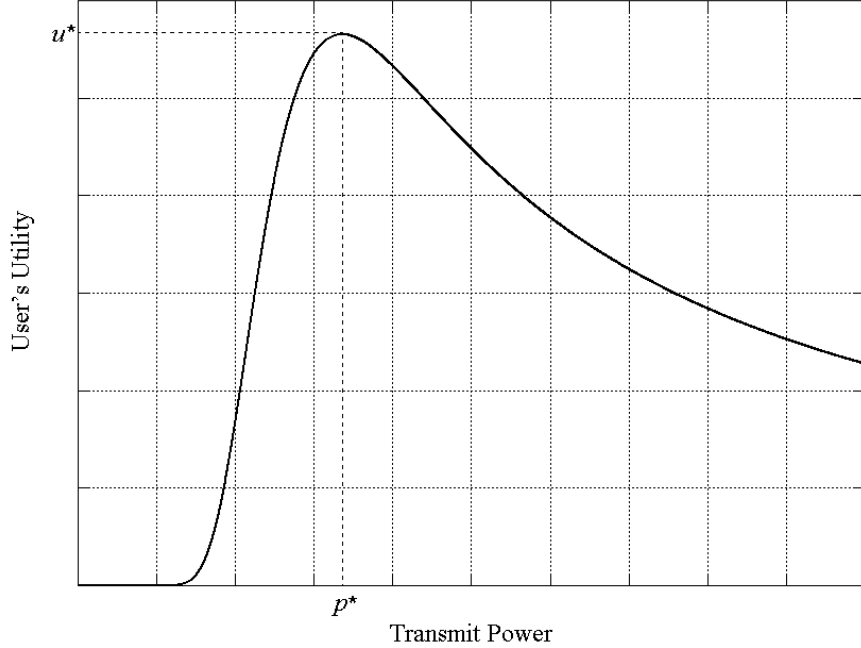


Fig. 1. User's utility as a function of transmit power for fixed interference.

These assumptions are valid in many practical systems. Furthermore, we assume that all users have the same efficiency function. Generalization to the case where the efficiency function is dependent on k is straightforward. Note that the throughput T_k in (2) could also be replaced with the Shannon capacity formula if $u_k(\mathbf{p})$ in (1) is appropriately modified to ensure $u_k(\mathbf{p}) = 0$ when $p_k = 0$.

Combining (1) and (2), and replacing the PSR with $f(\gamma_k)$,

$$u_k(\mathbf{p}) = \frac{D}{M} R_k \frac{f(\gamma_k)}{p_k}. \quad (3)$$

This utility function, which has units of bits/Joule, represents the total number of data bits that are delivered to the destination without an error per Joule of energy consumed, capturing the tradeoff between throughput and battery life. For the sake of simplicity, we assume that the transmission rate is the same for all users, i.e., $R_1 = \dots = R_K = R$. All the results obtained here can easily be generalized to the case of unequal rates. Fig. 1 shows the shape of the utility function in (3) as a function of transmit power keeping other users' transmit power fixed (the meaning of p^* and u^* will be provided in the remainder of the paper).

B. System Model

We consider a BPSK random TH-IR system² with K users in the network transmitting to a receiver at a common concentration point. The processing gain of the system is assumed to be $N = N_f \cdot N_c$, where N_f is the number of pulses that represent one information symbol, and N_c denotes the number of possible pulse positions in a frame [9]. The transmission is assumed to be over *frequency selective channels*, with the channel for user k modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^L \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \quad (4)$$

where T_c is the duration of the transmitted UWB pulse, which is the minimum resolvable path interval; L is the number of channel paths; $\alpha_k = [\alpha_1^{(k)}, \dots, \alpha_L^{(k)}]^T$ and τ_k are the fading coefficients and the delay of user k , respectively. Considering a chip-synchronous scenario, the symbols are misaligned by an integer multiple of the chip interval T_c : $\tau_k = \Delta_k T_c$, for every k , where Δ_k is uniformly distributed in $\{0, 1, \dots, N-1\}$. In addition we assume that the channel characteristics remain unchanged over a number of symbol intervals [16].

Especially in indoor environments, multipath channels can have hundreds of multipath components due to the high resolution of UWB signals. In such cases, linear receivers such as matched filters (MFs), pulse-discarding receivers [21], and multiuser detectors (MUDs) [22] cannot provide good performance, since more collisions will occur through multipath components. In order to mitigate the effect of multipath fading as much as possible, we consider a base station where K Rake receivers [20] are used.³ The Rake receiver for user k is composed of L fingers, where the vector $\beta_k = \mathbf{G} \cdot \alpha_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$ represents the combining weights for user k , and the $L \times L$ matrix \mathbf{G} depends on the type of Rake receiver employed. In particular, if $\mathbf{G} = \mathbf{I}$, an all-Rake (ARake) receiver is considered.

The SINR of the k th user at the output of the Rake receiver can be well approximated⁴ by [16]

$$\gamma_k = \frac{h_k^{(\text{SP})} p_k}{h_k^{(\text{SI})} p_k + \sum_{\substack{j=1 \\ j \neq k}}^K h_{kj}^{(\text{MAI})} p_j + \sigma^2}, \quad (5)$$

where σ^2 is the variance of the additive white Gaussian noise (AWGN) at the receiver, and the gains are

²Throughout all the paper, we consider IR-UWB systems with polarity code randomization [12].

³For ease of calculation, perfect channel estimation is considered throughout the paper.

⁴This approximation is valid for large N_f (typically, at least 5).

expressed by

$$h_k^{(\text{SP})} = \beta_k^H \cdot \alpha_k, \quad (6)$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2}{\beta_k^H \cdot \alpha_k}, \quad (7)$$

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\|\mathbf{B}_k^H \cdot \alpha_j\|^2 + \|\mathbf{A}_j^H \cdot \beta_k\|^2 + |\beta_k^H \cdot \alpha_j|^2}{\beta_k^H \cdot \alpha_k}, \quad (8)$$

where the matrices

$$\mathbf{A}_k = \begin{pmatrix} \alpha_L^{(k)} & \cdots & \cdots & \alpha_2^{(k)} \\ 0 & \alpha_L^{(k)} & \cdots & \alpha_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_L^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad (9)$$

$$\mathbf{B}_k = \begin{pmatrix} \beta_L^{(k)} & \cdots & \cdots & \beta_2^{(k)} \\ 0 & \beta_L^{(k)} & \cdots & \beta_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \beta_L^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad (10)$$

$$\Phi = \text{diag} \{\phi_1, \dots, \phi_{L-1}\}, \quad \phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}}, \quad (11)$$

have been introduced for convenience of notation.

By considering frequency selective channels, the transmit power of the k th user, p_k , does appear not only in the numerator of (5), but also in the denominator, owing to the SI due to multiple paths. In the following sections, we extend the approach of game theory to multipath channels, accounting for the SI in addition to MAI and AWGN. The problem is more challenging than with a single path since every user achieves a different SINR at the output of its Rake receiver.

III. THE NONCOOPERATIVE POWER CONTROL GAME

In this section, we propose a noncooperative power control game (NPCG) in which every user seeks to maximize its own utility by choosing its transmit power. Let $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ be the proposed noncooperative game where $\mathcal{K} = \{1, \dots, K\}$ is the index set for the terminal users; $P_k = [\underline{p}_k, \bar{p}_k]$ is the strategy set, with \underline{p}_k and \bar{p}_k denoting minimum and maximum power constraints, respectively; and

$u_k(\mathbf{p})$ is the payoff function for user k [5]. Throughout this paper, we assume $\underline{p}_k = 0$ and $\bar{p}_k = \bar{p} > 0$ for all $k \in \mathcal{K}$.

Formally, the NPCG can be expressed as

$$\max_{p_k \in P_k} u_k(\mathbf{p}) = \max_{p_k \in P_k} u_k(p_k, \mathbf{p}_{-k}) \quad \text{for } k = 1, \dots, K, \quad (12)$$

where \mathbf{p}_{-k} denotes the vector of transmit powers of all terminals except terminal k . The latter notation is used to emphasize that the k th user has control over its own power p_k only. Assuming equal transmission rate for all users, (12) can be rewritten as

$$\max_{p_k \in P_k} \frac{f(\gamma_k(p_k, \mathbf{p}_{-k}))}{p_k} \quad \text{for } k = 1, \dots, K, \quad (13)$$

where we have explicitly shown that γ_k is a function of \mathbf{p} , as expressed in (5).

The solution that is most widely used for noncooperative game theoretic problems is the *Nash equilibrium* [1]. A Nash equilibrium is a set of strategies such that no user can unilaterally improve its own utility. Formally, a power vector $\mathbf{p} = [p_1, \dots, p_K]$ is a Nash equilibrium of $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ if, for every $k \in \mathcal{K}$, $u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k})$ for all $p'_k \in P_k$.

The Nash equilibrium concept offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point where no player wishes to deviate. However, such a point does not necessary exist. First, we investigate the existence of an equilibrium in the NPCG.

Theorem 1: A Nash equilibrium exists in the NPCG $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$. Furthermore, the unconstrained maximization of the utility function occurs when each user k achieves an SINR γ_k^* that is a solution of

$$f'(\gamma_k^*)\gamma_k^*(1 - \gamma_k^*/\gamma_{0,k}) = f(\gamma_k^*), \quad (14)$$

where

$$\gamma_{0,k} = \frac{h_k^{(\text{SP})}}{h_k^{(\text{SI})}} = N \cdot \frac{(\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k)^2}{\|\boldsymbol{\Phi} \cdot (\mathbf{B}_k^H \cdot \boldsymbol{\alpha}_k + \mathbf{A}_k^H \cdot \boldsymbol{\beta}_k)\|^2} \geq 1 \quad (15)$$

and $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma_k^*}$.

The proof of Theorem 1 can be found in App. I-A.

The Nash equilibrium can be seen from another point of view. The power level chosen by a *rational* self-optimizing user constitutes a *best response* to the powers chosen by other players. Formally, terminal

k 's best response $r_k : P_{-k} \rightarrow P_k$ is the correspondence that assigns to each $\mathbf{p}_{-k} \in P_{-k}$ the set

$$r_k(\mathbf{p}_{-k}) = \{p_k \in P_k : u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k})\} \quad \text{for all } p'_k \in P_k, \quad (16)$$

where P_{-k} is the strategy space of all users excluding user k .

With the notion of a terminal's best response correspondence, the Nash equilibrium can be restated in a compact form: the power vector \mathbf{p} is a Nash equilibrium of the NPCG $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ if and only if $p_k \in r_k(\mathbf{p}_{-k})$ for all $k \in \mathcal{K}$.

Prop. 1: Using the above definition in the NPCG, with a slight abuse of notation, terminal k 's best response to a given interference vector \mathbf{p}_{-k} is [5]

$$r_k(\mathbf{p}_{-k}) = \min(\bar{p}, p_k^*), \quad (17)$$

where

$$\begin{aligned} p_k^* &= \arg \max_{p_k \in \mathbb{R}^+} u_k(p_k, \mathbf{p}_{-k}) \\ &= \frac{\gamma_k^* \left(\sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^* / \gamma_{0,k})} \end{aligned} \quad (18)$$

is the unconstrained maximizer of the utility in (3) (see Fig. 1). Furthermore, p_k^* is unique. The proof of Prop. 1 can be found in App. I-B.

It is worth noting that, at any equilibrium of the NPCG, a terminal either attains the utility maximizing SINR γ_k^* or it fails to do so and transmits at maximum power \bar{p} .

Theorem 2: The NPCG has a unique Nash equilibrium.

The proof of Theorem 2 can be found in App. I-C.

IV. ANALYSIS OF THE NASH EQUILIBRIUM

In the previous section, we have proven that a Nash equilibrium for the NPCG exists and is unique. In the following, we study the properties of this equilibrium. It is worth emphasizing that, unlike the previous work in this area, γ_k^* is dependent on k , because of the SI in (5). Hence, each user attains a different level of SINR. More importantly, the only term dependent on k in (14) is $\gamma_{0,k}$, which is affected only by the channel of user k . This means that γ_k^* can be assumed constant when the channel characteristics remain unchanged, irrespectively of the transmit powers \mathbf{p} and the channel coefficients of the other users. For convenience of notation, we can express γ_k^* as a function of $\gamma_{0,k}$:

$$\gamma_k^* = \Gamma(\gamma_{0,k}). \quad (19)$$

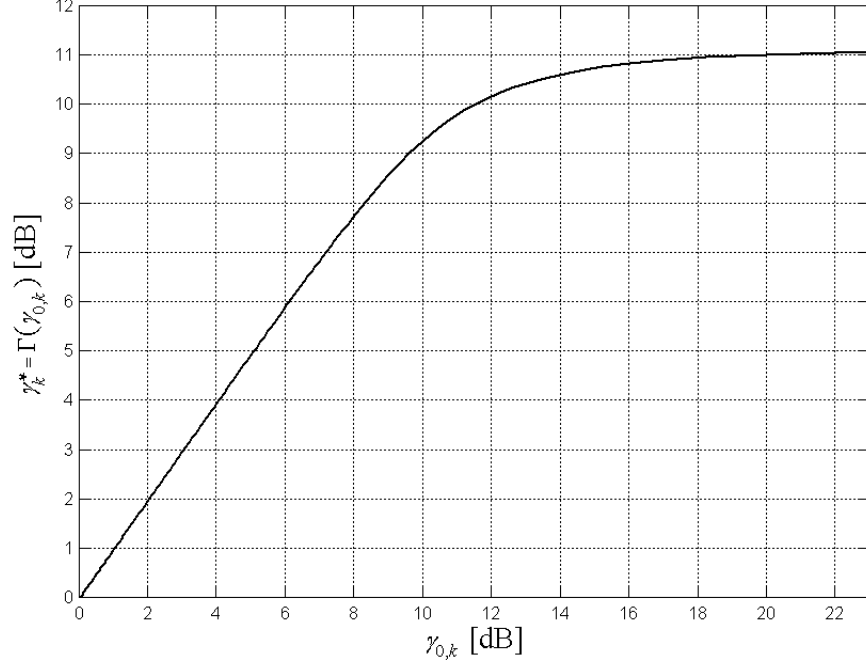


Fig. 2. Shape of γ_k^* as a function of $\gamma_{0,k}$ ($M = 100$).

Fig. 2 shows the shape of γ_k^* as a function of $\gamma_{0,k}$, where the efficiency function is taken as $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$, with $M = 100$. Even though γ_k^* is shown for values of $\gamma_{0,k}$ approaching 0 dB, it is worth emphasizing that $\gamma_{0,k} > 10$ dB in most practical situations.

As can be noticed, the NPCG proposed herein represents a generalization of the power control games discussed thoroughly in literature [2]–[8]. If $L = 1$, i.e. in a flat-fading scenario, we obtain from (7) and (15) that $\gamma_{0,k} = \infty$ for all k . This implies that γ_k^* is the same for every $k \in \mathcal{K}$, and thus it is possible to apply the approach proposed, e.g., in [5].

Assumption 1: To simplify the analysis, let us assume the typical case of multiuser UWB systems, where $N \gg K$. In addition, \bar{p} is considered sufficiently large that $p_k < \bar{p}$ for those users who achieve γ_k^* . In particular, when $N \gg K$, at the Nash equilibrium the following property holds:

$$h_k^{(\text{SP})} p_k^* \simeq q > 0 \quad \forall k \in \mathcal{K}. \quad (20)$$

The heuristic derivation of (20) can be justified by SI reduction due to the hypothesis $N \gg K > 1$. Using (7), $\gamma_{0,k} \gg 1$ for all k . Hence, the noncooperative solution will be similar to that studied, e.g., in [8]. The validity of this assumption will be shown in Sect. VI through simulations.

The following proposition helps identify the Nash equilibrium for a given set of channel realizations.

Prop. 2: A necessary and sufficient condition for a desired SINR γ_k^* to be achievable is

$$\gamma_k^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1}) < 1 \quad \forall k \in \mathcal{K}, \quad (21)$$

where $\gamma_{0,k}$ is defined as in (15), and

$$\zeta_k^{-1} = \sum_{\substack{j=1 \\ j \neq k}}^K \frac{h_{kj}^{(\text{MAI})}}{h_j^{(\text{SP})}}. \quad (22)$$

When (21) holds, each user can reach the optimum SINR, and the minimum power solution to do so is to assign each user k a transmit power

$$p_k^* = \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \gamma_k^*}{1 - \gamma_k^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1})}. \quad (23)$$

When (21) does not hold, the users cannot achieve γ_k^* simultaneously, and some of them would end up transmitting at the maximum power \bar{p} . The proof of Prop. 2 can be found in App. I-D.

Based on Prop. 2, the amount of transmit power p_k^* required to achieve the target SINR γ_k^* will depend not only on the gain $h_k^{(\text{SP})}$, but also on the SI term $h_k^{(\text{SI})}$ (through $\gamma_{0,k}$) and the interferers $h_{kj}^{(\text{MAI})}$ (through ζ_k). In order to derive some quantitative results for the utility function and for the transmit powers independent of SI and MAI terms, it is possible to resort to a large systems analysis. For convenience of notation, we introduce the following definitions, with $\text{Var}[\cdot]$ denoting the variance of a random variable:

- let \mathbf{D}_j^α be a diagonal matrix whose elements are

$$\{\mathbf{D}_j^\alpha\}_l = \sqrt{\text{Var}[\alpha_j^{(l)}]}; \quad (24)$$

- let \mathbf{D}_k^β be a diagonal matrix whose elements are

$$\{\mathbf{D}_k^\beta\}_l = \sqrt{\text{Var}[\beta_k^{(l)}]}; \quad (25)$$

- let \mathbf{C}_j^α be an $L \times (L-1)$ matrix whose elements are

$$\{\mathbf{C}_j^\alpha\}_{li} = \sqrt{\frac{\text{Var}[\{\mathbf{A}_j\}_{li}]}{L}}; \quad (26)$$

- let \mathbf{C}_k^β be an $L \times (L-1)$ matrix whose elements are

$$\{\mathbf{C}_k^\beta\}_{li} = \sqrt{\frac{\text{Var}[\{\mathbf{B}_k\}_{li}]}{L}}; \quad (27)$$

- let $\varphi(\cdot)$ be the matrix operator

$$\varphi(\cdot) = \lim_{L \rightarrow \infty} \frac{1}{L} \text{Tr}(\cdot), \quad (28)$$

where $\text{Tr}(\cdot)$ is the trace operator.

Theorem 3: Assume that $\alpha_k^{(l)}$ are zero-mean random variables independent across k and l , and \mathbf{G} is a deterministic diagonal matrix (thus implying that $\alpha_k^{(l)}$ and $\beta_j^{(m)}$ are dependent only when $j = k$ and $m = l$). In the asymptotic case where K and N_f are finite,⁵ while $L, N_c \rightarrow \infty$, the term ζ_k^{-1} converges almost surely (a.s.) to

$$\zeta_k^{-1} \xrightarrow{a.s.} \frac{1}{N} \sum_{\substack{j=1 \\ j \neq k}}^K \frac{\varphi(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha) + \varphi(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta)}{\varphi(\mathbf{D}_j^\alpha \mathbf{D}_j^\beta) \cdot \varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta)}. \quad (29)$$

Theorem 4: Assume $\alpha_k^{(l)}$ and \mathbf{G} as in Theorem 3. In the asymptotic case where K and N_f are finite, while $L, N_c \rightarrow \infty$,

$$\gamma_{0,k}^{-1} \xrightarrow{a.s.} \frac{1}{N} \frac{\lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L+l-i)}{\left(\varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta)\right)^2}, \quad (30)$$

where ϕ_i is defined as in (11) and

$$\begin{aligned} \theta_k(l, L+l-i) &= \{\mathbf{D}_k^\alpha\}_l \{\mathbf{D}_k^\beta\}_{L+l-i} \\ &\quad + \{\mathbf{D}_k^\beta\}_l \{\mathbf{D}_k^\alpha\}_{L+l-i}. \end{aligned} \quad (31)$$

The proofs of Theorems 3 and 4 can be found in App. II.

It is worth emphasize that the results above can be applied to any kind of fading models, since only the second-order statistics are required. Furthermore, due to the symmetry of (29) and (30), it is easy to verify that the results are independent of large-scale fading models. Hence, Theorems 3 and 4 apply to any kind of channel, which may include both large- and small-scale statistics.

For ease of calculation, in the following we derive the asymptotic values when considering a flat Power Delay Profile (PDP) [23] for the channel coefficients. In addition, the variance of $\alpha_k^{(l)}$ is assumed dependent on the user k , but independent of l , i.e., $\mathbb{V}\text{ar}[\alpha_k^{(l)}] = \sigma_k^2$ for all l .

Prop. 3: Under the above mentioned hypotheses, when using an ARake, and thus $\mathbf{G} = \mathbf{I}$,

$$\zeta_k^{-1} \xrightarrow{a.s.} \frac{K-1}{N}, \quad (32)$$

$$\gamma_{0,k}^{-1} \xrightarrow{a.s.} \frac{\nu(\rho)}{N}, \quad (33)$$

⁵In order for the analysis to be consistent, and also considering regulations by the FCC [14], it is worth noting that N_f could not be smaller than a certain threshold ($N_f \geq 5$).

where $\rho = N_c/L$, $0 < \rho < \infty$, and

$$\nu(\rho) = \begin{cases} \frac{2}{3}(3 - 3\rho + \rho^2), & \rho \leq 1, \\ 2/(3\rho), & \rho > 1. \end{cases} \quad (34)$$

Using definitions (11) and (24) – (28), and applying Theorems 3 and 4, after some algebraic manipulations, the proof is straightforward. As already noticed to justify Ass. 1, but also from (33), $\gamma_{0,k} \gg 1$ for all k . Thus, $\gamma_k^* = \Gamma(\gamma_{0,k})$ approaches $\bar{\gamma}^* = \Gamma(\infty)$, leading to a nearly SINR-balancing scenario.

An immediate consequence of Prop. 3 is the expression for transmit powers and utility functions at the Nash equilibrium, which are independent of the channel realizations of the other users, and of the SI:⁶

$$p_k^* \simeq \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \bar{\gamma}^*}{1 - \bar{\gamma}^* \cdot [K - 1 + \nu(\rho)]/N}, \quad (35)$$

$$u_k^* \simeq h_k^{(\text{SP})} \cdot \frac{D}{M} R_k \frac{f(\bar{\gamma}^*) (1 - \bar{\gamma}^* \cdot [K - 1 + \nu(\rho)]/N)}{\sigma^2 \bar{\gamma}^*}, \quad (36)$$

where u_k^* is the utility function of the user k at the Nash equilibrium (see Fig. 1), and where the condition (21) translates into

$$N_f \geq \left\lceil \bar{\gamma}^* \cdot \frac{K - 1 + \nu(\rho)}{N_c} \right\rceil. \quad (37)$$

The validity of the claims above is verified in Sect. VI using simulations. To show that the results (32) – (33) represent good approximations not only for the model with flat PDP and equal variances, which has been used only for convenience of calculation, simulations are carried out using the exponential decaying PDP described in [24], which provides a more realistic channel model for the UWB scenario.

V. SOCIAL OPTIMUM

The solution to the power control game is said to be Pareto-optimal if there exists no other power allocation \mathbf{p} for which one or more users can improve their utilities without reducing the utility of any of the other users. It can be shown that the Nash equilibrium presented in the previous section is not Pareto-optimal. This means that it is possible to improve the utility of one or more users without harming other users. On the other hand, it can be shown that the solution to the following social problem gives the Pareto-optimal frontier [8]:

$$\mathbf{p}_{opt} = \arg \max_{\mathbf{p}} \sum_{k=1}^K \xi_k u_k(\mathbf{p}), \quad (38)$$

⁶Of course, the amount of transmit power p_k^* needed to achieve γ_k^* is dependent on the channel realization.

for $\xi_k \in \mathbb{R}^+$ (the set of positive real numbers). Pareto-optimal solutions are, in general, difficult to obtain. Here, we conjecture that the Pareto-optimal solution occurs when all users achieve the same SINRs, γ_{opt} . This approach is chosen not only because SINR balancing ensures fairness among users in terms of throughput and delay [8], but also because, for large systems, the Nash equilibrium is achieved when all SINRs are similar. We also consider the hypothesis $\xi_1 = \dots = \xi_K = 1$, suitable for a scenario without priority classes. Hence, the maximization (38) can be written as

$$\mathbf{p}_{opt} = \arg \max_{\mathbf{p}} f(\gamma) \sum_{k=1}^K \frac{1}{p_k}. \quad (39)$$

In a network where the hypotheses of Ass. 1, Theorem 3 and Theorem 4 are fulfilled, and where ARake receivers are employed, at the Nash equilibrium all users achieve a certain output SINR γ with $h_k^{(SP)} p_k \simeq q(\gamma)$, where

$$q(\gamma) = \frac{\sigma^2 \gamma}{1 - \gamma \cdot [K - 1 + \nu(\rho)]/N}, \quad (40)$$

with $\rho = N_c/L$. Therefore, (39) can be expressed as

$$\gamma_{opt} \simeq \arg \max_{\gamma} \frac{f(\gamma)}{q(\gamma)} \sum_{k=1}^K h_k^{(SP)}, \quad (41)$$

since there exists a one-to-one correspondence between γ and \mathbf{p} . It should be noted that, while the maximizations in (13) consider no cooperation among users, (39) assumes that users cooperate in choosing their transmit powers. That means that the relationship between the user's SINR and transmit power will be different from that in the noncooperative case.

Prop. 4: In a network where $L, N_c \rightarrow \infty$ and $N \gg K$, using ARake receivers, the Nash equilibrium approaches the Pareto-optimal solution.

Proof: The solution γ_{opt} to (41) must satisfy the condition $d(f(\gamma)/q(\gamma))/d\gamma|_{\gamma=\gamma_{opt}} = 0$. Using this fact, combined with (40), gives us the equation that must be satisfied by the solution of the maximization problem in (41):

$$f'(\gamma_{opt})\gamma_{opt} \left[1 - \gamma_{opt} \cdot \frac{K - 1 + \nu(\rho)}{N} \right] = f(\gamma_{opt}). \quad (42)$$

We see from (42) that the Pareto-optimal solution differs from the solution (14) of the noncooperative utility-maximizing method, since (42) also takes into account the contribution of the interferers. In particular,

$$\gamma_{opt} = \Gamma \left(\frac{K - 1 + \nu(\rho)}{N} \right). \quad (43)$$

Since the function $\Gamma(\cdot)$ is increasing with its argument for any S-shaped $f(\gamma)$ (as can also be seen in Fig. 2), and since $N/[(K-1+\nu(\rho))] \leq \gamma_{0,k}$ for all k (from (32) and (33)),

$$\gamma_{opt} \leq \gamma \leq \bar{\gamma}^*, \quad (44)$$

due to (19) and (43). On the other hand, assuming $N \gg K$ and $0 < \rho < \infty$ implies $\gamma_{opt} \rightarrow \bar{\gamma}^*$. From (44), it is apparent that $\gamma \rightarrow \bar{\gamma}^*$ as well. This means that, in almost all typical scenarios, the target SINR for the noncooperative game, γ , is close to the target SINR for the Pareto-optimal solution, γ_{opt} . Consequently, the average utility provided by the Nash equilibrium is close to the one achieved according to the Pareto-optimal solution. ■

The validity of the above claims will be verified in Sect. VI using simulations.

VI. SIMULATION RESULTS

A. Implementation

In this subsection, we present an iterative and distributed algorithm for reaching a Nash equilibrium of the proposed power control game. This algorithm is applicable to all types of Rake receivers, as well as to any kind of channel model. The description of the algorithm is as follows.

The Best-Response Power-Control (BRPC) Algorithm

Consider a network with K users, a processing gain $N = N_f \cdot N_c$, a channel with L fading paths, and a maximum transmit power \bar{p} .

- 1) Simulate the channel fading coefficients α_k for all users according to the chosen channel model.
- 2) Set the Rake receivers coefficients β_k for all users according to the chosen receiver.
- 3) Compute the SP term, $h_k^{(SP)}$, the SI term, $h_k^{(SI)}$, the MAI term, $h_{kj}^{(MAI)}$, according to (6) – (8), and the optimum SINR, γ_k^* , solution of (14), for all users.
- 4) Initialize randomly the transmit powers of all users within the range $[0, \bar{p}]$.
- 5) Compute the received SINR γ_k at the base station for each user according to (5).
- 6) Set $k = 1$.
- 7) Adjust the transmit power for user k according to (17) and to

$$p_k^* = p_k \cdot \frac{\gamma_k^*}{\gamma_k} \cdot \frac{1 - \gamma_k/\gamma_{0,k}}{1 - \gamma_k^*/\gamma_{0,k}}, \quad (45)$$

where $\gamma_{0,k}$ is defined as in (15).

- 8) $k = k + 1$.
- 9) If $k \leq K$, then go back to Step 7.
- 10) Stop if the powers have converged; otherwise, go to Step 5.

This is a best-response algorithm, since at each stage a user decides to transmit at a power that maximizes its own utility (i.e., its best-response strategy), given the current conditions of the system. Looking at Step 7, it may appear from (18) that each user should know its own transmit power p_k and ratio $\gamma_{0,k}$, as well as some other quantities (p_j and $h_{kj}^{(\text{MAI})}$ for $j \neq k$) relevant to all of the other users in the network. On the contrary, it turns out that user k only needs to know its own received SINR at the base station γ_k . In fact, the term due to interference-plus-noise in (18) can be obtained from (5) as

$$\sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 = h_k^{(\text{SP})} p_k \cdot \frac{1 - \gamma_k / \gamma_{0,k}}{\gamma_k}, \quad (46)$$

where p_k is the transmit power of user k at the previous iteration. Therefore, after straightforward manipulation, (18) translates into the noncooperative update (45). The received SINR γ_k can be fed back to the user terminal from the access point, along with SP and SI terms.

It is clear that the above algorithm converges to a Nash equilibrium, whose existence and uniqueness are proven in App. I. The convergence of the BRPC algorithm is also validated in Sect. VI-B using extensive simulations.

B. Numerical Results

In this subsection, we present numerical results for the analysis presented in the previous sections. We assume that each packet contains 100 b of information and no overhead (i.e., $D = M = 100$). The transmission rate is $R = 100$ kb/s, the thermal noise power is $\sigma^2 = 5 \times 10^{-16}$ W, and the maximum transmit power is $\bar{p} = 1$ μ W. We use the efficiency function $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$. Using $M = 100$, $\bar{\gamma}^* = \Gamma(\infty) = 11.1$ dB. To model the UWB scenario [15], channel gains are simulated following [24], where both small- and large-scale statistics are taken into account. The distance between the users and the base station is assumed to be uniformly distributed between 3 and 20 m.

Before showing the numerical results for both the noncooperative and the cooperative approaches, some simulations are provided to verify the validity of Ass. 1 introduced in Sect. IV. Table I reports the ratio σ_q^2 / η_q^2 of the variance σ_q^2 to the squared mean value η_q^2 of the values $q = h_k^{(\text{SP})} p_k^*$, obtained averaging 10 000 realizations of channel coefficients for different values of network parameters using ARake receivers. We can see that, when the processing gain is much greater than the number of users, $\sigma_q^2 / \eta_q^2 \ll 1$. Hence, (20) can be used to carry out the theoretical analysis of the Nash equilibrium.

TABLE I
RATIO σ_q^2/η_q^2 FOR DIFFERENT NETWORK PARAMETERS.

	(L, K)			
(N_c, N_f)	$(20, 8)$	$(20, 16)$	$(50, 8)$	$(50, 16)$
$(30, 10)$	9.4E-4	3.2E-3	4.8E-4	1.7E-3
$(30, 50)$	2.9E-5	6.4E-5	1.6E-5	3.4E-5
$(50, 10)$	2.9E-4	6.8E-4	1.5E-4	3.7E-4
$(50, 50)$	1.0E-5	2.2E-5	0.6E-5	1.2E-5
$(100, 10)$	6.7E-5	1.5E-4	3.7E-5	7.8E-5
$(100, 50)$	0.3E-5	0.6E-5	0.1E-5	0.3E-5

Fig. 3 shows the utilities achieved at the Nash equilibrium as functions of the channel gains $h_k = \|\alpha_k\|^2$. These results have been obtained using random channel realizations for $K = 16$ users. The number of possible pulse positions is $N_c = 100$, while the number of paths is $L = 60$, in order to satisfy the large system assumption with $\nu(\rho) = 0.4$. The number of frame is $N_f = 10$, thus leading to a processing gain $N = 1000 \gg K$. The line represents the theoretical values of (36) when using an ARake, whereas the square markers correspond to the simulations achieved with the BRPC algorithm. We can see that the simulations match closely with the theoretical results.

Fig. 4 shows the utility as a function of the channel gain when the processing gain N is constant, but the ratio N_c/N_f is variable.⁷ The results have been obtained for a network with $K = 16$ users, $L = 100$ channel paths and processing gain $N = 1000$, using ARake receivers at the base station. The solid line corresponds to $N_c/N_f = 40$, the dotted line represents $N_c/N_f = 2.5$, and the dashed line shows $N_c/N_f = 0.1$. As expected, higher N_c/N_f ratios (and thus higher N_c , when N is fixed) correspond to higher utility, as described in (36). In fact, $\nu(\rho)$ decreases as ρ increases, i.e., as N_c increases (when L is fixed). This result complies with theoretical analysis of UWB systems [16], since, for a fixed total processing gain N , increasing the number of chips per frame, N_c , will decrease the effects of SI, while the dependency of the expressions on the MAI remains unchanged. Hence, a system with a higher N_c achieves better performance.

Fig. 5 shows the probability $P_o = \Pr\{\max_k p_k = \bar{p} = 1 \mu\text{W}\}$ of having at least one user transmitting

⁷To avoid a too busy graph, only the theoretical values are reported.

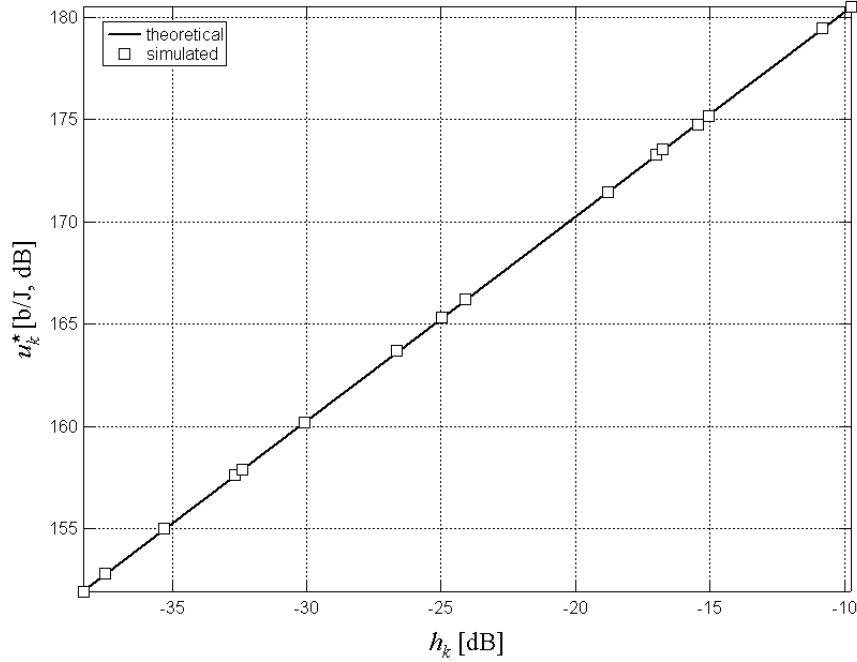


Fig. 3. Achieved utility versus channel gain at the Nash equilibrium for the ARake receiver.

at the maximum power, as a function of the number of frames N_f . We consider 10 000 realizations of the channel gains, using a network with ARake receivers at the base station, $K = 32$ users, $N_c = 50$, and $L = 100$ (thus $\rho = 0.5$ and $\nu(\rho) \simeq 1.17$). From (37), the minimum value of N_f that allows all K users to achieve the optimum SINRs is $N_f = \lceil 8.33 \rceil = 9$. The simulations thus agree with the analytical results of Sect. IV.

We now analyze the performance of the system when using a Pareto-optimal solution instead of the Nash equilibrium. Fig. 6 shows the normalized utility u_k/h_k as a function of the load factor ρ . We consider a network with $K = 5$ users, $N_f = 20$ frames and ARake receivers at the base station. The lines represent theoretical values of Nash equilibrium (dotted line), using (36), and of the social optimum solution (solid line), using (36) again, but substituting $\bar{\gamma}^*$ with the numerical solution of (42), γ_{opt} . The markers correspond to the simulation results. In particular, the circles represent the averaged solution of the BRPC algorithm, while the square markers show averaged numerical results (through a complete search) of the maximization (38), with $\xi_k = 1$. As stated in Sect. V, the difference between the noncooperative approach and the Pareto-optimal solution is not significant. Fig. 7 compares the target SINRs of the noncooperative solutions with the target SINRs of the Pareto-optimal solutions. As before, the lines correspond to the theoretical values (dashed line for the noncooperative solution, solid

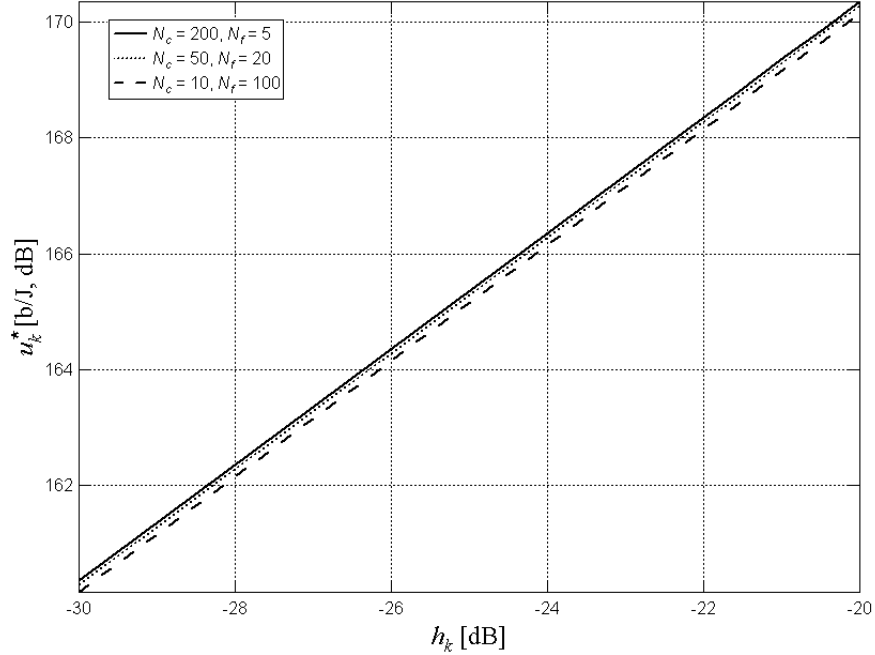


Fig. 4. Achieved utility versus channel gain at the Nash equilibrium for different ratios N_c/N_f .

line for the social optimum solution), while the markers represent the simulation results (circles for the noncooperative solutions, square markers for the Pareto-optimal solution). It is seen that, in both cases, the averaged target SINRs for the Nash equilibrium, γ , and the averaged target SINRs for the social optimum solution, γ_{opt} , are very close to $\bar{\gamma}^*$, as shown in Sect. V.

VII. CONCLUSION

In this paper, we have used a game-theoretic approach to study power control for a wireless data network in frequency-selective environments, where the user terminals transmit IR-UWB signals and the common concentration point employs Rake receivers. A noncooperative game has been proposed in which users are allowed to choose their transmit powers according to a utility-maximizing criterion, where the utility function has been defined as the ratio of the overall throughput to the transmit power. For this utility function, we have shown that there exists a unique Nash equilibrium for the proposed game, but, due to the frequency selective multipath, this equilibrium is achieved at a different output SINR for each user, depending on the channel realization and the kind of Rake receiver used. Resorting to a large system analysis, we have obtained a general characterization for the terms due to multiple access interference and self-interference, suitable for any kind of channel model and for different types

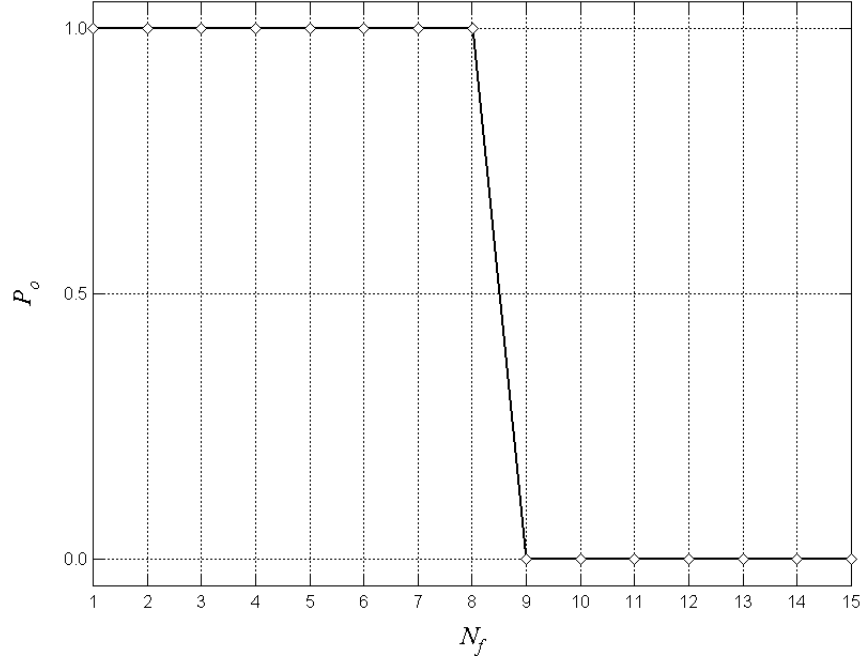


Fig. 5. Probability of having at least one user transmitting at maximum power versus number of frames.

of Rake receiver. Furthermore, explicit expressions for the utilities achieved at the equilibrium have been derived for the case of an ARake receiver. It has also been shown that, under these hypotheses, the noncooperative solution leads to a nearly SINR-balancing scenario. In order to evaluate the efficiency of the Nash equilibrium, we have studied an optimum cooperative solution for the case of ARake receivers, where the network seeks to maximize the sum of the users' utilities. It has been shown that the difference in performance between Nash and cooperative solutions is not significant for typical values of network parameters.

APPENDIX I

A. Proof of Theorem 1

The following results are obtained from [5], [25]–[27].

A Nash equilibrium exists in the noncooperative game $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ if, for all $k = 1, \dots, K$:

- 1) P_k is a nonempty, convex, and compact subset of some Euclidean space \mathbb{R}^K ; and
- 2) $u_k(\mathbf{p})$ is continuous in \mathbf{p} and quasi-concave in p_k .

Each user has a strategy space that is defined by a minimum power \underline{p}_k and a maximum power \bar{p}_k , and all power values in between. We also assume that $\bar{p}_k \geq \underline{p}_k$. Thus, the first condition is satisfied.

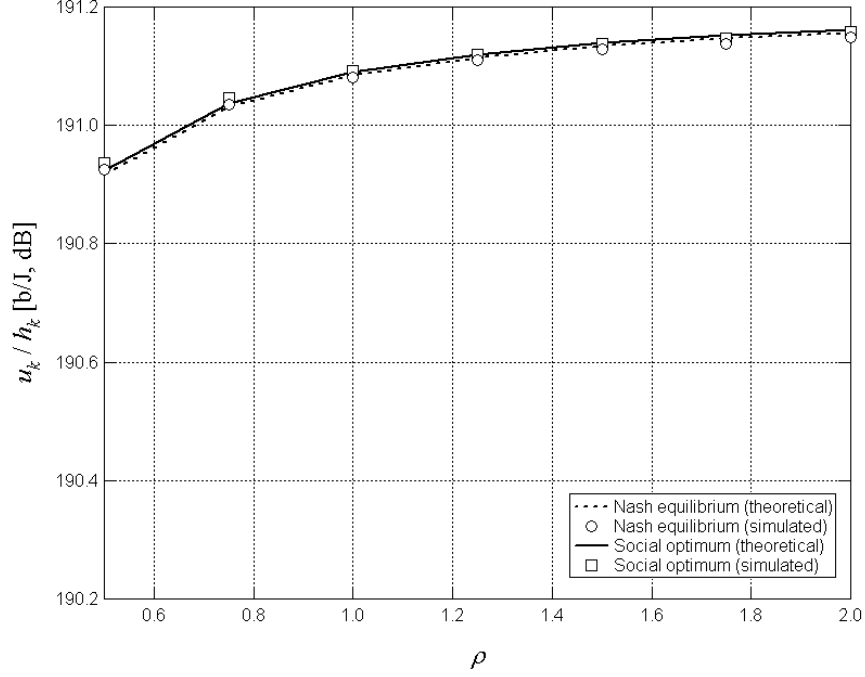


Fig. 6. Comparison of the normalized utility versus load factor for the noncooperative and Pareto-optimal solutions.

Moreover, since $p_k \geq 0$, it is apparent from (3) and (5) that $u_k(\mathbf{p})$ is continuous in \mathbf{p} . To show that the utility function $u_k(\mathbf{p})$ is quasi-concave in p_k for all k in the NPCG, it is sufficient to prove that the local maximum of $u_k(\mathbf{p})$ is at the same time a global maximum [28], [29].

For a differentiable function, the first-order necessary optimality condition is given by $\partial u_k(\mathbf{p})/\partial p_k = 0$. Recalling (3) and (5), the partial derivative of $u_k(\mathbf{p})$ with respect to p_k is

$$\frac{\partial u_k(\mathbf{p})}{\partial p_k} = \frac{DR}{Mp_k^2} (f'(\gamma_k)\gamma_k(1 - \gamma_k/\gamma_{0,k}) - f(\gamma_k)), \quad (47)$$

where $\gamma_{0,k}$ is defined as in (15) and $f'(\gamma_k) = df(\gamma_k)/d\gamma_k$. For the sake of simplicity, we do not explicitly show the dependence of γ_k on p_k . Since $p_k \geq 0$ for the NPCG, we examine only positive real numbers. Evaluating (47) at $p_k = 0$, we get $\partial u_k(p_k, \mathbf{p}_{-k})/\partial p_k = 0$. Therefore, $p_k = 0$ is a stationary point and the value of utility at this point is $u_k(\mathbf{p}) = 0$. If we evaluate utility in the ε -neighborhood of $p_k = 0$, where ε is a small positive number, we notice that utility is positive, which implies utility is increasing at $p_k = 0$. Hence, $p_k = 0$ cannot be a local maximum. For nonzero values of the transmit power, we examine the values of $\gamma_k^* = \gamma_k(p_k^*)$ such that $\partial u_k(p_k, \mathbf{p}_{-k})/\partial p_k|_{p_k=p_k^*} = 0$, thus satisfying the first-order necessary optimality condition. In other words, we evaluate γ_k^* such that

$$\gamma_k^* (1 - \gamma_k^*/\gamma_{0,k}) = f(\gamma_k^*)/f'(\gamma_k^*), \quad (48)$$

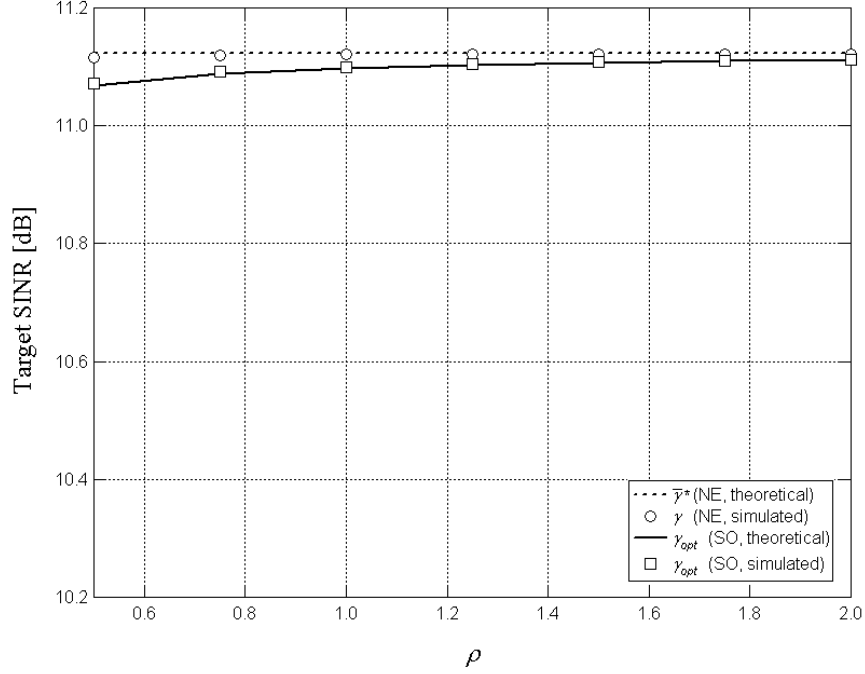


Fig. 7. Comparison of the target SINRs versus load factor for the noncooperative and Pareto-optimal solutions.

as shown in (14).

We observe that the left-hand side of the above equation is a concave parabola with its vertex in $\gamma_k = \gamma_{0,k}/2 > 0$, and $d(\gamma_k(1 - \gamma_k/\gamma_{0,k}))/d\gamma_k|_{\gamma_k=0} = 1$. The right-hand side is an increasing function, with $d(f(\gamma_k)/f'(\gamma_k))/d\gamma_k|_{\gamma_k=0} = 1/i' < 1$ when $f'(0) = 0$, where $i' = \min\{i \in \mathbb{N} : d^i f(\gamma_k)/d\gamma_k^i|_{\gamma_k=0} \neq 0\}$. Furthermore, the equation is satisfied at $\gamma_k = 0$. Therefore, there is a single value γ_k^* that satisfies (14) for $\gamma_k > 0$. The second-order partial derivative of the utility with respect to the power reveals that this point is a local maximum and therefore a global maximum. Hence, the utility function of user k is quasi-concave in p_k for all k . The same conclusion applies also if $p_k^* > \bar{p}_k$ for some k , even though γ_k^* cannot be achieved. In fact, by applying the previous considerations to (47), it is easy to verify that $u_k(p_k, \mathbf{p}_{-k})$ is strictly increasing in $\gamma_k \in [0, \gamma_k(p_k^*) = \gamma_k^*)$, which in turn, from (5), is strictly increasing in $p_k \in [0, p_k^*)$. Since $p_k \in [\underline{p}_k, \bar{p}_k]$, which is a subset of $[0, p_k^*)$, $u_k(p_k = \bar{p}_k, \mathbf{p}_{-k})$ represents both the local and the global maximum of the utility function. ■

Lemma 1: The solution γ_k^* of (14) satisfies the condition

$$0 \leq \gamma_k^* < \gamma_{0,k}. \quad (49)$$

Proof: As $f(\gamma_k^*)$ is an increasing function of γ_k^* , $f'(\gamma_k^*) \geq 0$ for every γ_k^* . Since the existence

of the solution is ensured by Theorem 1 and γ_k^* and $f(\gamma_k^*)$ are both greater than zero, the condition $(1 - \gamma_k^*/\gamma_{0,k}) > 0$ must hold. ■

B. Proof of Prop. 1

Using Theorem 1, for a given interference, the SINR γ_k^* corresponds to the transmit power p_k^* as in (18). Since γ_k^* is the unique maximizer of the utility, the correspondence between the transmit power and the SINR must be studied. As can be verified, (18) represents the equation of a hyperbola passing through the origin, with the asymptotes parallel to the Cartesian axes. In particular, the vertical asymptote is $\gamma_k^* = \gamma_{0,k}$. Therefore, using Lemma 1 presented in App. I-A, there exists a one-to-one correspondence between the transmit power, $p_k^* \in [0, +\infty)$, and the SINR, $\gamma_k^* \in [0, \gamma_{0,k})$. Thus, the transmit power p_k^* is also unique. If $p_k^* \notin P_k$ for some user k , since it is not a feasible point, then p_k^* cannot be the best response to a given \mathbf{p}_{-k} . In this case, we observe that $\partial u_k(\mathbf{p})/\partial p_k \leq 0$ for any $\gamma_k \leq \gamma_k^*$, and hence for any $p_k \leq p_k^*$. This implies that the utility function is increasing in that region. Since \bar{p} is the largest power in the strategy space, it yields the highest utility among all $p_k \leq \bar{p}$ and thus is the best response to \mathbf{p}_{-k} . ■

C. Proof of Theorem 2

By Theorem 1, we know that there exists an equilibrium in the NPCG. Let \mathbf{p} denote the Nash equilibrium in the NPCG. By definition, the Nash equilibrium must satisfy $\mathbf{p} = \mathbf{r}(\mathbf{p})$, where $\mathbf{r}(\mathbf{p}) = [r_1(\mathbf{p}), \dots, r_K(\mathbf{p})]$. The fixed point $\mathbf{p} = \mathbf{r}(\mathbf{p})$ is unique if the correspondence $\mathbf{r}(\mathbf{p})$ is a *standard* function [30], i.e., if it satisfies the following properties:

- 1) positivity: $\mathbf{r}(\mathbf{p}) > 0$;
- 2) monotonicity: if $\mathbf{p} > \mathbf{p}'$, then $\mathbf{r}(\mathbf{p}) > \mathbf{r}(\mathbf{p}')$;
- 3) scalability: for all $\mu > 1$, $\mu \mathbf{r}(\mathbf{p}) > \mathbf{r}(\mu \mathbf{p})$.

It is apparent that $r_k(\mathbf{p}) = r_k(\mathbf{p}_{-k})$. Taking into account (17) and (18), the first condition translates into $p_k^* > 0$ for all $k \in \mathcal{K}$. Using (5), (8) and (49), the proof is straightforward. Recalling (17) and (18), the second and the third condition are also apparent, since \mathbf{p}_{-k} modifies only the numerator of (18). Therefore, since $\mathbf{r}(\mathbf{p})$ is a standard function, the Nash equilibrium of the NPCG is unique. ■

D. Proof of Prop. 2

Based on Prop. 1, when all users reach the Nash equilibrium, their transmit powers are

$$p_k^* = \frac{\gamma_k^* \left(\sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j^* + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^*/\gamma_{0,k})}. \quad (50)$$

Using Ass. 1 in (50), it is straightforward to obtain:

$$q \cdot \left[1 - \gamma_k^* \cdot \left(\gamma_{0,k}^{-1} + \zeta_k^{-1} \right) \right] = \sigma^2 \gamma_k^* > 0, \quad (51)$$

which implies $\gamma_k^* \cdot \left(\gamma_{0,k}^{-1} + \zeta_k^{-1} \right) < 1$, proving necessity. It is also straightforward to show that, if each terminal k uses transmit power p_k^* as in (23), all terminals will achieve the SINR requirement, finishing the proof of sufficiency. Finally, consider any other joint distribution of powers and channel realizations, and let $q' = \inf_{k \in \mathcal{K}} \left\{ h_k^{(\text{SP})} p_k^* \right\}$. Then, by exactly the same argument as was used in the proof of necessity,

$$q' \geq \frac{\sigma^2 \gamma_k^*}{1 - \gamma_k^* \cdot \left(\gamma_{0,k}^{-1} + \zeta_k^{-1} \right)} = q. \quad (52)$$

This means that assigning powers according to (23) does indeed give the minimal power solution. \blacksquare

APPENDIX II

In order to prove Theorems 3 and 4, it is worth introducing the following results.

Lemma 2: [31] Consider an n -dimensional vector $\mathbf{r}_n = [R_1, \dots, R_n]$ with independent and identically distributed (i.i.d.) standardized (complex) entries (i.e. $\mathbb{E}[R_i] = 0$ and $\mathbb{E}[|R_i|^2] = 1$, with $\mathbb{E}[\cdot]$ denoting expectation), and let \mathbf{C}_n be an $n \times n$ (complex) matrix independent of \mathbf{r}_n . For any integer p ,

$$\begin{aligned} \mathbb{E}[|\mathbf{r}_n^H \mathbf{C}_n \mathbf{r}_n - \text{Tr}(\mathbf{C}_n)|^p] &= K_p ((\mathbb{E}[|R_1|^4] \text{Tr}(\mathbf{C}_n \mathbf{C}_n^H))^{p/2} \\ &\quad + \mathbb{E}[|R_1|^{2p}] \text{Tr}(\mathbf{C}_n \mathbf{C}_n^H)^{p/2}), \end{aligned} \quad (53)$$

where the constant K_p does not depend either on n or on \mathbf{C}_n .

Theorem 5: Consider an n -dimensional vector $\mathbf{x}_n = \frac{1}{\sqrt{n}}[X_1, \dots, X_n]$ with i.i.d. standardized (complex) entries with finite eighth moment, and let \mathbf{C}_n be an $n \times n$ (complex) matrix independent of \mathbf{x}_n with uniformly bounded spectral radius for all n . Under these hypotheses,

$$\mathbf{x}_n^H \mathbf{C}_n \mathbf{x}_n \xrightarrow{a.s.} \frac{1}{n} \text{Tr}(\mathbf{C}_n). \quad (54)$$

Proof: Using Lemma 2 and Markov's inequality,

$$\begin{aligned} \Pr[|\mathbf{x}_n^H \mathbf{C}_n \mathbf{x}_n - \frac{1}{n} \text{Tr}(\mathbf{C}_n)| > \epsilon] &\leq \frac{\mathbb{E}[|\mathbf{x}_n^H \mathbf{C}_n \mathbf{x}_n - \frac{1}{n} \text{Tr}(\mathbf{C}_n)|^4]}{\epsilon^4} \\ &\leq \kappa \cdot \frac{1}{n^2 \cdot \epsilon^4}, \end{aligned} \quad (55)$$

where $\kappa < \infty$ is a constant value independent of n . Thus,

$$\sum_{n=1}^{\infty} \Pr[|\mathbf{x}_n^H \mathbf{C}_n \mathbf{x}_n - \frac{1}{n} \text{Tr}(\mathbf{C}_n)|^4 > \epsilon] < \infty. \quad (56)$$

Using the Borel-Cantelli lemma [32], the result (54) is straightforward. \blacksquare

Theorem 6: Suppose $\mathbf{x}_n = [X_1, \dots, X_n]$ and $\mathbf{y}_n = [Y_1, \dots, Y_n]$ are n -dimensional independent vectors with i.i.d. standardized (complex) entries with finite eighth moment, and \mathbf{C}_n is an $n \times n$ matrix (complex) independent on \mathbf{x}_n and \mathbf{y}_n with uniformly bounded spectral radius for all n . Then,

$$\mathbf{x}_n^H \mathbf{C}_n \mathbf{y}_n \xrightarrow{a.s.} 0. \quad (57)$$

Proof: The proof can be obtained using the same steps as that of Theorem 5. ■

A. Proof of Theorem 3

To prove that $N\zeta_k^{-1}$ converges a.s. to non-random limits, we focus on the ratio

$$\begin{aligned} N \frac{h_{kj}^{(\text{MAI})}}{h_j^{(\text{SP})}} &= \frac{||\mathbf{B}_k^H \cdot \boldsymbol{\alpha}_j||^2 + ||\mathbf{A}_j^H \cdot \boldsymbol{\beta}_k||^2 + |\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_j|^2}{(\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k) \cdot (\boldsymbol{\beta}_j^H \cdot \boldsymbol{\alpha}_j)} \\ &= \frac{\frac{1}{L^2} [||\mathbf{B}_k^H \cdot \boldsymbol{\alpha}_j||^2 + ||\mathbf{A}_j^H \cdot \boldsymbol{\beta}_k||^2 + |\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_j|^2]}{\frac{1}{L} (\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k) \cdot \frac{1}{L} (\boldsymbol{\beta}_j^H \cdot \boldsymbol{\alpha}_j)}. \end{aligned} \quad (58)$$

It is sufficient to show that both numerator and denominator of (58) converge a.s. to a non-random limit.

Let

$$\mathbf{w}^{(k)} = (\mathbf{D}_k^\alpha)^{-1} \boldsymbol{\alpha}_k \quad (59)$$

and

$$\begin{aligned} \boldsymbol{\beta}_k &= \mathbf{G} \boldsymbol{\alpha}_k \\ &= (\mathbf{G} \circ \mathbf{D}_k^\alpha) \mathbf{w}^{(k)} \\ &= \mathbf{D}_k^\beta \mathbf{w}^{(k)}, \end{aligned} \quad (60)$$

where \mathbf{D}_k^α and \mathbf{D}_k^β are defined as in (24) and (25), respectively; the operator \circ denotes the Hadamard (element-wise) product; and the matrix \mathbf{G} is dependent on the type of Rake receiver employed. Using (59) and (60), by Theorem 5, we obtain

$$\begin{aligned} \frac{1}{L^2} ||\mathbf{B}_k^H \cdot \boldsymbol{\alpha}_j||^2 &\xrightarrow{a.s.} \varphi \left(\frac{1}{L} \mathbf{D}_j^\alpha \mathbf{B}_k \mathbf{B}_k^H \mathbf{D}_j^\alpha \right) \\ &= \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^L \{\mathbf{D}_j^\alpha\}_i^2 \sum_{l=i+1}^L (\beta_l^{(k)})^2 \\ &= \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{l=1}^{L-1} (\beta_{l+1}^{(k)})^2 \sum_{m=1}^l \{\mathbf{D}_j^\alpha\}_m^2 \\ &= \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{l=1}^{L-1} \chi_l, \end{aligned} \quad (61)$$

where $\varphi(\cdot)$ is defined as in (28) and

$$\chi_l = \frac{1}{L} (\beta_{l+1}^{(k)})^2 \sum_{m=1}^l \{\mathbf{D}_j^\alpha\}_m^2 \quad (62)$$

are independent random variables, with

$$\mathbb{E}[\chi_l] = \frac{1}{L} \{\mathbf{D}_k^\beta\}_{l+1}^2 \sum_{m=1}^l \{\mathbf{D}_j^\alpha\}_m^2 \quad (63)$$

and

$$\begin{aligned} \mathbb{V}\text{ar}[\chi_l] &= \frac{1}{L^2} \mathbb{V}\text{ar}[(\beta_{l+1}^{(k)})^2] \left(\sum_{m=1}^l \{\mathbf{D}_j^\alpha\}_m^2 \right)^2 \\ &\leq \mathbb{V}\text{ar}[(\beta_{l+1}^{(k)})^2] \cdot \left(\frac{\text{Tr}((\mathbf{D}_j^\alpha)^2)}{L} \right)^2 \\ &< \infty. \end{aligned} \quad (64)$$

Using the weak version of the law of large numbers for non-i.i.d. random variables,

$$\begin{aligned} \frac{1}{L} \sum_{l=1}^{L-1} \chi_l &\xrightarrow{a.s.} \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{l=1}^{L-1} \mathbb{E}[\chi_l] \\ &= \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{l=1}^{L-1} \{\mathbf{D}_k^\beta\}_{l+1}^2 \sum_{m=1}^l \{\mathbf{D}_j^\alpha\}_m^2 \\ &= \varphi \left(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha \right), \end{aligned} \quad (65)$$

where \mathbf{C}_k^α and \mathbf{C}_k^β are defined as in (26) and (27), respectively. Similar arguments yield

$$\frac{1}{L^2} \|\mathbf{A}_j^H \cdot \beta_k\|^2 \xrightarrow{a.s.} \varphi \left(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta \right). \quad (66)$$

Then applying Theorem 6, from (60) we obtain

$$\frac{1}{L} \beta_k^H \cdot \alpha_j \xrightarrow{a.s.} 0, \quad (67)$$

since β_k is independent of α_j . Analogously, using Theorem 5, from (59) we obtain

$$\frac{1}{L} \beta_k^H \cdot \alpha_k \xrightarrow{a.s.} \varphi \left(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta \right). \quad (68)$$

Using (65) – (68), the result (29) is straightforward. ■

B. Proof of Theorem 4

In order to prove that $N/\gamma_{0,k}$ converges a.s. to a non-random limit, it is sufficient to show that both the numerator and the denominator converge to non-random limits. Note that

$$\begin{aligned}
 & \left\| \Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k) \right\|^2 = \\
 &= \sum_{i=1}^{L-1} \phi_i^2 \cdot \left(\sum_{l=1}^i \alpha_l^{(k)} \beta_{L+l-i}^{(k)} + \sum_{l=1}^i \beta_l^{(k)} \alpha_{L+l-i}^{(k)} \right)^2 \\
 &= \sum_{i=1}^{L-1} \phi_i^2 \cdot \left(\sum_{l=1}^i \theta_k(l, L+l-i) \cdot \mathbf{w}_l^{(k)} \cdot \mathbf{w}_{L+l-i}^{(k)} \right)^2,
 \end{aligned} \tag{69}$$

where $\theta_k(l, L+l-i)$ is defined as in (31).

Following the same steps as in the proof of Theorem 3, after some algebraic manipulation, it can be proven that

$$\frac{1}{L^2} \left\| \Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k) \right\|^2 \xrightarrow{a.s.} \lim_{L \rightarrow \infty} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \frac{\theta_k^2(l, L+l-i)}{L^2}. \tag{70}$$

Using (70), in conjunction with (68), the result (30) is straightforward. ■

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