

Joint Beamforming and Scheduling for SDMA Systems with Limited Feedback

Kaibin Huang, Robert W. Heath, Jr and Jeffrey G. Andrews

Wireless Networking and Communication Group

Dept. of Electrical & Computer Engineering

University of Texas at Austin

1 University Station C0803

Austin, TX 78712-0240

Phone: +1-512-921-8582

{khuang, rheath, jandrews}@ece.utexas.edu

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Abstract

On a multi-antenna broadcast (downlink) channel, simultaneous transmission to multiple users is capable of achieving very high throughput. Unfortunately, the optimal approach for such multi-user transmission, namely dirty paper coding, is highly complicated. Therefore, this paper proposes a practical joint beamforming and scheduling scheme that is enabled by feedback of quantized channel state information. In this approach, the base station generates multiple sets of orthogonal beamforming vectors and associates each vector with a specific user. Next, the base station selects one set of beamforming vectors and the associated users for downlink transmission such that the throughput is maximized. Unlike the optimal approach, the proposed scheme has computational complexity that increases only linearly with the number of users since it does not require an exhaustive search over all possible subsets of users. Compared with two recently proposed schemes by Sharif *et al.* and Choi *et al.*, the present scheme achieves higher downlink throughput, avoids multi-user conflicts when selecting beamforming vectors, requires no broadcast of beamforming vectors, and facilitates feedback compression.

I. INTRODUCTION

In multi-antenna broadcast (downlink) channels, simultaneous transmission to multiple users, known as *space division multiple access* (SDMA), is capable of achieving much higher throughput than other

multiple-access schemes such as *time division multiple access* (TDMA) [1]. Due to this advantage, SDMA has been included recently in the IEEE 802.16e standard. The optimal SDMA strategy is known as dirty paper coding [2], which is highly complicated and non-causal. Therefore, more practical SDMA schemes based on transmit beamforming have been designed using different criteria and methods, including zero forcing [3]–[6], a signal-to-interference-plus-noise-ratio (SINR) constraint [7], minimum mean squared error (MMSE) [8], and channel decomposition [9]. These SDMA schemes can be combined with multi-user scheduling to further increase the sum capacity by exploiting *multi-user diversity*, which refers to scheduling only a subset of users with good channels for each transmission [10]–[16]. In general, to realize high throughput for a SDMA system, the base station must perform joint scheduling and beamforming as well as acquiring the required channel state information (CSI), which form the theme of this paper.

A. Related Work and Motivation

In this paper, we consider a practical scenario where partial CSI is acquired by the base station through quantized CSI feedback, known as *limited feedback* [17]. Quantized CSI feedback for point-to-point communications has been extensively studied recently and see e.g. [17], [18] and the references therein. The effects of CSI quantization on a SDMA system have been studied recently in [13], [19], [20]. In [19], combined quantized CSI feedback and zero-forcing dirty paper coding are shown to attain most of the capacity achieved by perfect CSI feedback. In [20], it is shown that for a small number of users the number of CSI feedback bits must increase with signal-to-noise ratio (SNR) to ensure that the sum capacity grows with SNR. The key result of [13] is that the number of CSI feedback bits can be reduced by multi-user diversity.

This paper is focused on joint beamforming and scheduling for SDMA systems with continuous traffic for all users and the objective of maximizing sum capacity. The similar scenario but with bursty traffic and the objective of meeting quality-of-service (QoS) for different users is addressed in [21] and references therein. The optimal approach for our (full queue) scenario involves an exhaustive search, where for each possible subset of users a corresponding set of beamforming vectors is designed using algorithms such

as that proposed by Schubert and Boche in [7]. Note that unlike the proposed scheme, the Schubert and Boche algorithm by itself includes no scheduling and results in non-orthogonal beamforming vectors. The main drawback of the optimal approach is its complexity, which increases exponentially with the number of users. A more practical scheme, *opportunistic SDMA* (OSDMA), is proposed in [15]. As shown in [15], for a large number of users, an arbitrary set of orthogonal beamforming vectors ensures that the sum capacity increases with the number of users at the optimal rate. Nevertheless, for a small number of users, such arbitrary beamforming vectors are highly sub-optimal due to excessive interference between scheduled users.

One simple method for improving the beamforming vectors for OSDMA is *beam and user selection*, in which a set of orthogonal beamforming vectors is chosen from multiple sets (beam selection) and assigned to a subset of users (user selection) for downlink transmission. The key advantage of beam and user selection lies in its low complexity that increases linearly with the number of users rather than exponentially. In [14], an extension called *OSDMA with beam selection* (OSDMA-S) is proposed, where each mobile iteratively selects beamforming vectors broadcast by the base station and sends back its choices. Due to distributed beam selection, numerous iterations of broadcast and feedback are required for implementing OSDMA-S, which incurs significant downlink overhead and more feedback. As a result, capacity gains of OSDMA-S over OSDMA are marginal.

To overcome the drawback of OSDMA-S due to distributed beam selection, we propose OSDMA with centralized beam and user selection, which is enabled by limited feedback. The proposed scheme is named *limited feedback OSDMA* (LF-OSDMA). Specifically, using quantized CSI feedback from the users, the base station iteratively searches for an optimal set of scheduled users and their optimal beamforming vectors for downlink transmission. The search criterion is maximizing the downlink sum capacity under a transmit beamforming constraint.

B. Contributions and Organization

The main contributions are the LF-OSDMA scheme and the analysis of its sum capacity and capacity gain with respect to OSDMA. First, in the limit of a large number of users, an upper bound for the sum capacity is obtained, showing beam and user selection has the virtual effect of expanding the user set. Second, also for the case of a large number of users, an upper bound for the capacity gain contributed by beam and user selection is derived. As illustrated by numerical results, this upper bound closely approximates the actual capacity gain even for a small to moderate number of users. Furthermore, this upper-bound shows that the capacity gain increases logarithmically with the number of iterations for beam and user selection but decreases also logarithmically with the number of users U . Third, it is proven that increasing the number of CSI feedback bits in proportion to U guarantees optimal growth in sum capacity with U . Numerical results show that LF-OSDMA achieves significant gains in sum capacity with respect to OSDMA and OSDMA-S, if both the numbers of CSI feedback bits and iterations for beam and user selection increase with U following specific rates derived.

We now summarize the advantages and differences of LF-OSDMA with respect to OSDMA [15] and OSDMA-S [14]. First, LF-OSDMA achieves higher downlink throughput than both OSDMA and OSDMA-S at the expense of more computation at the base station. Second, in contrast to OSDMA and OSDMA-S, LF-OSDMA eliminates the possibility that different users select identical beamforming vectors since their assignments are centralized. Third, the present scheme does not require any broadcast of beamforming vectors from the base station, which are essential to OSDMA and OSDMA-S, and hence reduces the feedback delay that degrades the downlink throughput [22]. Lastly, the multi-user feedback content for LF-OSDMA is quantized CSI while for OSDMA and OSDMA-S it is indices of selected beamforming vectors. The former facilitates feedback compression by exploiting channel temporal correlation [22], which, however, is inapplicable to the latter because consecutive feedbacks are independent.

The remainder of this paper is organized as follows. The system model is described in Section II. The algorithms used in LF-OSDMA for CSI quantization, and beam and user selection are presented in

Section III. The capacity gain of and the requirement on CSI quantization for LF-OSDMA are analyzed in Section IV. The performance of LF-OSDMA is evaluated using Monte Carlo simulations in Section V, followed by concluding remarks in Section VI.

II. SYSTEM MODEL

The downlink system illustrated in Fig. 1 is described as follows. The base station with N_t antennas transmits data simultaneously to N_t active users chosen from a total of U users, each with one receive antenna. The base station separates the multi-user data streams by beamforming, i.e. assigning a beamforming vector to each of the N_t active users. The beamforming vectors $\{\mathbf{w}_n\}_{n=1}^{N_t}$ are selected from multiple sets of unitary orthogonal vectors following the beam and user selection algorithm described in Section III-B. The received signal of the n th scheduled user is expressed as

$$y_n = \sqrt{P}\mathbf{h}_n^\dagger \mathbf{w}_n x_n + \nu_n, \quad n = 1, \dots, N_t, \quad (1)$$

where we use the following notation

N_t	number of transmit antennas and also	\mathbf{w}_n	$(N_t \times 1$ vector) beamforming vector
	number of scheduled users;		with $\ \mathbf{w}_n\ ^2 = 1$;
\mathbf{h}_n	$(N_t \times 1$ vector) downlink channel;	P	transmit SNR; and
x_n	transmitted symbol with $ x_n = 1$;	ν_n	AWGN sample with $\nu_n \sim \mathcal{CN}(0, 1)$.
y_n	received symbol;		

For the purpose of asymptotic analysis of LF-OSDMA, we make the following assumption:

AS 1: *The downlink channel $\mathbf{h}_u \forall u = 1, 2, \dots, U$ is an i.i.d. vector with $\mathcal{CN}(0, 1)$ coefficients.*

Given this assumption, which is commonly made in the literature of SDMA and multi-user diversity [11], [12], [14], [15], [20], the channel direction vector $\mathbf{h}_u/\|\mathbf{h}_u\|$ of each user follows a uniform distribution, which greatly simplifies the analysis of LF-OSDMA in Section IV.

III. ALGORITHMS

The proposed LF-OSDMA scheme is comprised of (i) CSI quantization at the mobiles and (ii) beam and user selection at the base station. The algorithms for performing these two functions are discussed in Section III-A and Section III-B, respectively. LF-OSDMA is compared with OSDMA-S and OSDMA in Section III-C.

A. CSI Quantization

Without loss of generality, the discussion in this section is focused on the u th user and the same algorithm for CSI quantization is used by other users. For simplicity, we assume:

AS 2: *The u th user has perfect receive CSI \mathbf{h}_u .*

This assumption allows us to neglect channel estimation error at the u th mobile. For convenience, the CSI, \mathbf{h}_u , is decomposed into two components: the *gain* and the *shape*, which are quantized separately.

Hence,

$$\mathbf{h}_u = g_u \mathbf{s}_u, \quad u = 1, \dots, U, \quad (2)$$

where $g_u = \|\mathbf{h}_u\|$ is the gain and $\mathbf{s}_u = \mathbf{h}_u / \|\mathbf{h}_u\|$ is the shape. Since the channel gain is a scalar and hence easy to quantize, we make the following assumption:

AS 3: *The channel gain g_u is perfectly known to the base station through feedback.*

The same assumption is also made in [14], [15]. Given AS 3, we can focus our discussion on quantization of the channel shape \mathbf{s}_u .

The *random vector quantization* (RVQ) method [20] is applied for quantizing the channel shape \mathbf{s}_u . First, this method requires random generation of N unitary vectors, which follow the uniform distribution. These vectors as a group are called a *codebook*, denoted as \mathcal{F} . Second, RVQ selects from the codebook \mathcal{F} a member that forms the smallest angle with the channel shape \mathbf{s}_u as in [23], [24]. The selected member gives the quantized channel shape, denoted as $\hat{\mathbf{s}}_u$. Mathematically,

$$\hat{\mathbf{s}}_u = \mathcal{Q}(\mathbf{s}_u) = \arg \max_{\mathbf{f} \in \mathcal{F}} |\mathbf{f}^\dagger \mathbf{s}_u|, \quad (3)$$

where the function \mathcal{Q} represents the CSI quantization process. We define the *quantization error* as the angle between $\hat{\mathbf{s}}_u$ and \mathbf{s}_u , denoted as $\angle(\hat{\mathbf{s}}_u, \mathbf{s}_u)$. It is clear that the quantization error is zero if $\hat{\mathbf{s}}_u = \mathbf{s}_u$. Again, due to the ease of sending back a scalar quantity, we make the following assumption similar to AS 3:

AS 4: *The quantization error $\angle(\hat{\mathbf{s}}_u, \mathbf{s}_u)$ is perfectly known to the base station through feedback.*

The quantized channel shape $\hat{\mathbf{s}}_u$ is sent back to the base station through a finite-rate feedback channel [17], [23]. Since the quantization codebook \mathcal{F} can be known *a priori* to both the base station and mobiles, only the index of $\hat{\mathbf{s}}_u$ needs to be sent back. Therefore, the number of feedback bits per user is $\log_2 N$ since $|\mathcal{F}| = N$.

B. Centralized Beam and User Selection

Having collected quantized CSI from all U users¹, the base station schedules N_t users for transmission and computes their beamforming vectors. To maximize the sum capacity, N_t scheduled users must be selected through an exhaustive search, which is infeasible for a large user pool. Therefore, we propose a more practical scheduling algorithm whose complexity is scalable with the base station's computational power. The proposed algorithm consists of I independent *iterations*, each of which involves randomly generating N_t orthogonal beamforming vectors and selecting N_t associated users. Among the I iterations, the one that yields the largest *expected*² sum capacity is chosen. The results of the chosen iteration, namely the generated beamforming vectors and the scheduled users, are used for the downlink transmission.

Before discussing the procedure for each iteration, we derive the expression of SINR at a mobile terminal. Consider a user whose channel power is $\rho = \|\mathbf{h}\|^2$ and channel shape \mathbf{s} . Moreover, the user's beamforming vector \mathbf{w} is assigned by the base station from a set of orthogonal vectors \mathcal{W} . From (1), the

¹For simplicity, we assume that the number of feedback bits per user is limited but not the total number of feedback bits from all users. Nevertheless, the sum feedback from all users can be reduced by allowing only a small subset of users for feedback, which is an topic currently under investigation.

²Throughout this paper, we use the word "expected" to refer to values of some quantities, for example sum capacity, computed at the base station. Due to CSI quantization error, the expected value differs from the actual ones.

received SINR is obtained as

$$\text{SINR}(\rho, \mathbf{s}, \mathbf{w}) = \frac{P\rho|\mathbf{s}^\dagger\mathbf{w}|^2}{1 + P\rho \sum_{\mathbf{w}' \in \mathcal{W}/\{\mathbf{w}\}} |\mathbf{s}^\dagger\mathbf{w}'|^2}, \quad (4)$$

where P is the transmission SNR. Since the orthogonal vectors in the set $\mathcal{W}/\{\mathbf{w}\}$ span the null space of \mathbf{w} , the interference term in (4) can be simplified as

$$\sum_{\mathbf{w}' \in \mathcal{W}/\{\mathbf{w}\}} |\mathbf{s}^\dagger\mathbf{w}'|^2 = 1 - |\mathbf{s}^\dagger\mathbf{w}|^2. \quad (5)$$

By substituting (5) into (4), a simplified SINR expression is obtained as

$$\text{SINR}(\rho, \mathbf{s}, \mathbf{w}) = \frac{|\mathbf{s}^\dagger\mathbf{w}|^2}{1/(P\rho) + 1 - |\mathbf{s}^\dagger\mathbf{w}|^2}. \quad (6)$$

Equivalently, by substituting $\theta = \angle(\mathbf{s}, \mathbf{w})$.

$$\text{SINR}(\rho, \theta) = \frac{1 + P\rho}{1 + P\rho \sin^2(\theta)} - 1. \quad (7)$$

The criterion for beam and user selection is maximizing SINR lower bounds of scheduled users. Ideally, actual SINRs should be maximized, but they cannot be determined due to quantization error in CSI feedback. Nevertheless, the SINR lower bounds to be obtained shortly serve as a good substitute. Consider a user with channel power ρ , channel shape \mathbf{s} and its quantized version $\hat{\mathbf{s}}$. Assuming this user is assigned a beamforming vector \mathbf{w} , we have the following inequalities:

$$|\angle(\mathbf{s}, \hat{\mathbf{s}}) - \angle(\hat{\mathbf{s}}, \mathbf{w})| \leq \angle(\mathbf{s}, \mathbf{w}) \leq \angle(\mathbf{s}, \hat{\mathbf{s}}) + \angle(\hat{\mathbf{s}}, \mathbf{w}). \quad (8)$$

Since the function $\text{SINR}(\rho, \theta)$ in (7) is monotonically decreasing in θ with θ being small, it follows from (8) that

$$\text{SINR}(\rho, \angle(\mathbf{s}, \mathbf{w})) \geq \text{SINR}(\rho, \angle(\mathbf{s}, \hat{\mathbf{s}}) + \angle(\hat{\mathbf{s}}, \mathbf{w})). \quad (9)$$

The beam and user selection process described shortly avoids an exhaustive search, namely computation of optimal beamforming vectors for every possible set of scheduled users. Consequently, the complexity of beam and user selection increases linearly rather than exponentially with the number of users U . The

procedure of the i th iteration for beam and user selection is described as follows. N_t unitary beamforming vectors $\{\mathbf{w}_n^{(i)}\}$ are randomly generated. For convenience, each of the UN_t possible combinations of users and beamforming vectors is represented by a two-element index (u, n) where u is the user index with $1 \leq u \leq U$ and n the beamforming vector index with $1 \leq n \leq N_t$. We group all indices into a single set represented as $\mathcal{J} = \{(u, n)\}$. Furthermore, we define a subset $\mathcal{J}^* \subset \mathcal{J}$, used for containing N_t combinations selected sequentially as follows. Following the criterion of maximizing the SINR lower-bound in (9), the first combination of user and beamforming vector is selected as follows

$$(u^*, n^*) = \arg \max_{(u, n) \in \mathcal{J}} \text{SINR}(\rho_u, \angle(\mathbf{s}_u, \hat{\mathbf{s}}_u) + \angle(\hat{\mathbf{s}}_u, \mathbf{w}_n)). \quad (10)$$

Next, the index sets \mathcal{J} and \mathcal{J}^* are updated by transferring the selected combination (u^*, n^*) from \mathcal{J} to \mathcal{J}^* as follows:

$$\mathcal{J} = \mathcal{J} / \{(u^*, n^*)\}, \quad \mathcal{J}^* = \mathcal{J}^* \cup \{(u^*, n^*)\}. \quad (11)$$

After performing N_t iterations of the steps in (10) and (11), the i th iteration is completed with N_t beamforming vectors $\{\mathbf{w}_n^{(i)}\}$ and N_t scheduled users whose indices are

$$\mathcal{K}^{(i)} = \{1 \leq u \leq U \mid (n, u) \in \mathcal{J}^* \quad 1 \leq n \leq N_t\}.$$

Moreover, the i th iteration yields an expected sum capacity given as

$$\mathcal{R}^{(i)} = \sum_{(u, n) \in \mathcal{J}^*} \log_2 [1 + \text{SINR}(\rho_u, \angle(\mathbf{s}_u, \hat{\mathbf{s}}_u) + \angle(\hat{\mathbf{s}}_u, \mathbf{w}_n))]. \quad (12)$$

After I iterations, the expected sum capacity is hence given as

$$\mathcal{R}^* = \max_{1 \leq i \leq I} \mathcal{R}^{(i)}, \quad (13)$$

where $\mathcal{R}^{(i)}$ is obtained in (12). The gain in sum capacity contributed by beam and user selection is analyzed in Section IV-A. As we know, \mathcal{R}^* in (13) differs from the actual sum capacity due to the channel quantization error. In Section IV-B, we analyze the requirement on CSI quantization for ensuring optimal growth rate of the actual sum capacity with the number of users.

C. Algorithm Comparison

The key differences between the proposed LF-OSDMA algorithm and existing algorithms, namely OSDMA-S and OSDMA, are described as follows:

- 1) LF-OSDMA centralizes assignments of beamforming vectors at the base station and avoids conflicts between different users who choose the same beamforming vectors, which occur for OSDMA and OSDMA-S with a significant probability if the user pool is small.
- 2) The proposed scheme requires no broadcast of beamforming vectors from the base station in contrast to OSDMA and OSDMA-S. In particular, the multiple rounds of broadcast required for OSDMA-S cause feedback delay that has negative impact on the sum capacity [22].
- 3) While OSDMA and OSDMA-S require feedback of each user's choice of a beamforming vector, LF-OSDMA demands feedback of quantized CSI from each user. Therefore, the feedback for LF-OSDMA can be compressed by exploiting channel temporal correlation [22], which is however inapplicable for OSDMA and OSDMA-S.
- 4) Compared with OSDMA and OSDMA-S, LF-OSDMA requires more computation at the base station, reflected in the multiple (I) iterations. This may be a desirable tradeoff given the higher computational power at the base station.

We summarize the differences between LF-OSDMA, OSDMA-S and OSDMA in Table I.

IV. CAPACITY ANALYSIS

For LF-OSDMA, the beam and user selection (cf. Section III-B) at the base station increases the sum capacity but the quantization error in the CSI feedback decreases it. The gain in sum capacity due to beam and user selection is analyzed in Section IV-A. In Section IV-B, we analyze the requirement on CSI quantization for ensuring that the sum capacity increases with the number of users following the optimal rate.

A. Capacity Gain for Beam and User Selection

The analysis in this section is focused on the capacity gain for beam and user selection in LF-OSDMA with respect to OSDMA [15], which can be considered as the special case of LF-OSDMA with only one iteration for beam and user selection. For simplicity, CSI quantization error is neglected and its analysis is postponed to Section IV-B. In other words, we make the following assumption just for Section IV-A:

AS 5: *In the capacity gain analysis of beam and user selection for LF-OSDMA, CSI feedback is assumed perfect. Hence, $\hat{\mathbf{s}}_u = \mathbf{s}_u$ for $u = 1, \dots, U$.*

Roughly speaking, the effect of CSI quantization is to offset the capacity gain of beam and user selection. The analysis in this section is carried out by first obtaining an upper-bound for the sum capacity of LF-OSDMA and subsequently an upper-bound for the capacity gain of LF-OSDMA due to beam and user selection.

To simplify notation, let $\text{SINR}_{u,n}^{(i)}$ denote the SINR for the u th user who is assigned the n th beamforming vector generated in the i th iteration:

$$\text{SINR}_{u,n}^{(i)} = \text{SINR}(\rho_u, \angle(\mathbf{s}_u, \mathbf{w}_n^{(i)})), \quad 1 \leq u \leq U, \quad 1 \leq i \leq I, \quad 1 \leq n \leq N_t, \quad (14)$$

where the function $\text{SINR}(\cdot)$ is given in (7). As shown in [15], the CDF of each $\text{SINR}_{n,m}^{(i)}$ is

$$F_s(S) = 1 - e^{-S/P}(1 + S)^{-(N_t-1)}, \quad S \geq 0. \quad (15)$$

With AS 5 and beam and user selection as described in Section III-B, the sum capacity for LF-OSDMA can be written as

$$\mathcal{C}_U(I) = E \left[\max_{1 \leq i \leq I} \sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i)} \right) \right]. \quad (16)$$

Several lemmas useful for deriving the upper bound of the sum capacity in (16) are provided as follows. Lemma 1 shows that the angle between the channel shape and the beamforming vector for each scheduled user converges to zero.

Lemma 1: Let $\hat{\mathbf{s}}_{n,U}^*$ and $\mathbf{w}_{n,U}^*$ denote the channel shape and beamforming vector of the n th of the N_t scheduled users with a total of U users. Then the angle $\angle(\hat{\mathbf{s}}_{n,U}^*, \mathbf{w}_{n,U}^*)$ converges to zero:

$$\lim_{U \rightarrow \infty} \angle(\hat{\mathbf{s}}_{n,U}^*, \mathbf{w}_{n,U}^*) = 0. \quad (17)$$

The proof is given in Appendix A. The result in Lemma 2 is that switching the expectation and the maximization operations in (16) increases the sum capacity, leading to an upper bound.

Lemma 2: Asymptotically, the ergodic capacity in (16) is bounded as

$$\mathcal{C}_U \leq N_t E \left[\log_2 \left(1 + \max_{1 \leq i \leq I} \max_{1 \leq u \leq U} \text{SINR}_u^{(i)} \right) \right], \quad U \rightarrow \infty, \quad (18)$$

where $\{\text{SINR}_u^{(i)}\}$ are random variables having the CDF in (15).

The proof is given in Appendix B. Last, Lemma 3 concerns the asymptotic distribution of the term $\max_i \max_u \text{SINR}_u^{(i)}$ in (18).

Lemma 3: Let $\{\text{SINR}_u\}$ denote a sequence of i.i.d. random variables having the CDF in (15). The term $\max_i \max_u \text{SINR}_u^{(i)}$ in (18) converges in distribution to the maximum of $\{\text{SINR}_u\}$,

$$\max_{1 \leq i \leq I} \max_{1 \leq u \leq U} \text{SINR}_u^{(i)} \sim \max_{1 \leq u \leq UI} \text{SINR}_u, \quad \text{as } U \rightarrow \infty, \quad (19)$$

where \sim indicates equivalence in distribution.

The proof is given in Appendix C. Using these lemmas, we obtain an asymptotic upper bound for the sum capacity of LF-OSDMA as shown in Theorem 1.

Theorem 1: Let the number of iterations for beam and user selection I be fixed and the CSI feedback be perfect. Then the sum capacity for LF-OSDMA is bounded as

$$N_t \log_2 U \leq \lim_{U \rightarrow \infty} \mathcal{C}_U(I) \leq N_t \log_2 \log_2(UI). \quad (20)$$

The proof is given in Appendix D. The upper bound in (20) is equal to the sum capacity of OSDMA with UI users. By treating OSDMA as the special case of LF-OSDMA with one iteration for beam and user selection, this upper bound implies that to some extent, the beam and user selection in LF-OSDMA has the virtual effect of expanding the size of user pool from U to UI .

Next, we investigate the capacity gain of beam and user selection with respect to no beam selection, hence OSDMA. In [15], the sum capacity for OSDMA and a large number of users is derived as

$$\lim_{U \rightarrow \infty} \frac{C_U(1)}{N_t \log_2 \log_2 U} = 1. \quad (21)$$

Therefore, we define the capacity gain of beam and user selection as

$$\Delta \mathcal{C}_U(I) = \mathcal{C}_U(I) - N_t \log_2 \log_2 U, \quad (22)$$

where $\mathcal{C}_U(I)$ is given in (16). An upper-bound for $\Delta \mathcal{C}_U(I)$ for a large number of users is shown in the following corollary of Theorem 1.

Corollary 1: *There exist an integer U_0 such that for $U \geq U_0$, the capacity gain due to beam and user selection is bounded as*

$$0 \leq \Delta \mathcal{C}_U(I) \leq N_t \frac{\log_2 I}{\log_2 U}. \quad (23)$$

The proof is given in Appendix E. Two remarks are in order:

- 1) From (23), for a fixed number of iterations for beam and user selection, the upper bound of the capacity gain $\Delta \mathcal{C}_U(I)$ decreases logarithmically with the number of users U .
- 2) For a fixed number of users, the upper bound of $\Delta \mathcal{C}_U(I)$ increases with the number of iterations also logarithmically.

In Fig. ??, we show that the upper-bound derived in (23) for the actual capacity gain due to beam and user selection is close to the capacity gain even for a moderate number of users.

B. Requirement on CSI Quantization

An interesting question to ask is: *how much CSI feedback is sufficient for LF-OSDMA?* In this section, we answer this question by showing that increasing CSI feedback bits with the number of users ensures the optimal sum capacity scaling law [15]:

$$\lim_{U \rightarrow \infty} \frac{\mathcal{C}(U)}{N_t \log_2 \log_2 U} = 1.$$

Since a direct proof is difficult, we adopt an indirect approach of proving this result for an alternative scheme inferior to LF-OSDMA. This approach facilitates the use of an analytical result obtained in [25], addressing multi-user scheduling in a SDMA system with perfect CSI feedback.

As a tool for obtaining our main result, the alternative scheduling and beamforming scheme is described as follows. Unlike LF-OSDMA, this alternative scheme has no beam selection and uses only a single set of orthogonal vectors for generating beamforming vectors. Furthermore, the alternative scheme schedules users by applying thresholds rather than maximizing scheduled users' SINR lower bounds. Specifically, the alternative scheme applies an angular threshold and a pair of lower and upper thresholds for channel power, which are defined as

$$\varphi = \arcsin \left[\left(\frac{\log_2 \log_2 U}{2N_t \log_2 U} \right)^{1/(N_t-1)} \right], \quad (24)$$

$$\rho^- = \log_2 U - \log_2(\log_2 U + 1), \quad (25)$$

$$\rho^+ = \log_2 U. \quad (26)$$

Given a randomly generated set of orthogonal beamforming vectors $\{\mathbf{w}_n\}_{n=1}^{N_t}$, by applying the thresholds in (24)-(26), the alternative scheme associate each beamforming vector with a sub-set of users, specified by the following index set:

$$\mathcal{K}_n = \{1 \leq u \leq U \mid \angle(\mathbf{w}_n, \mathbf{s}_u) \leq \varphi, \quad \rho^- \leq \rho_u \leq \rho^+, \quad n = 1, \dots, N_t. \quad (27)$$

For downlink transmission, the alternative scheme schedules N_t users by randomly selecting a single user from each subset, say \mathcal{K}_n , and assign the associated beamforming vector, hence \mathbf{w}_n .

To obtain our main result, a useful lemma given in [25] is provided as follows:

Lemma 4 ([25]): *Given a total of U users, the number of users who meet the power thresholds in (25) and (26) is asymptotically bounded as:*

$$2N_t \log_2 U \leq \lim_{U \rightarrow \infty} U \Pr\{\rho^- \leq \rho \leq \rho^+\} \leq 2N_t(\log_2 U + 1). \quad (28)$$

The main result of this section is written as the following theorem.

Theorem 2: *Consider LF-OSDMA with finite iterations for beam and user selection. If the quantization codebook size N increases with the number of users U as*

$$N = \alpha(\log_2 U)^{N_t-1} + \beta, \quad (29)$$

with $0 < \alpha < \infty$ and $|\beta| < \infty$, the sum capacity $C_U(I)$ in (16) converges as:

$$\lim_{U \rightarrow \infty} \frac{C_U(I)}{N_t \log_2 \log_2 U} = 1. \quad (30)$$

The proof is given in Appendix F. Even though the values of α and β in (29) have no effect on the asymptotic value of the sum capacity in (30) as shown by the above theorem, they should be small, e.g. $\alpha = 1$ and $\beta = 0$, to give reasonable values for the quantization codebook size N . For a large number of users ($U \rightarrow \infty$) and perfect CSI feedback, since the capacity gain of LF-OSDMA over OSDMA converges to zero (cf. Corollary 1), the sum capacity of LF-OSDMA increases with the number of users at the same optimal rate as OSDMA, namely $N_t \log_2 \log_2 U$ [15]. Theorem 2 shows that as long as the size of the quantization codebook increases following (29), asymptotically CSI quantization does not affect the optimal growth rate of the sum capacity C_U with U .

Fig. ?? compares the curves of sum capacity vs. the number of users for the cases of perfect and quantized CSI feedback to illustrate the result in Theorem 2. The number of iterations for beam and user selection increases as $I = U$ to maintain a constant capacity gain contributed by beam and user selection (cf. Corollary 1). We can observe that with the codebook size increasing at the rate of $N = 5(\log_2 U)^{N_t-1}$ (cf. Theorem 2), the sum capacity scales up with the number of users ($U \geq 15$) with the same slope as for the case of perfect CSI feedback. In contrast, fixing the codebook size N reduces the growth rate of sum capacity and incurs a larger loss in sum capacity.

V. PERFORMANCE COMPARISON

In this section, the performance of LF-OSDMA is compared with that of OSDMA and OSDMA-S using Monte Carlo simulations.

The sum capacities of LF-OSDMA with different numbers of iterations for beam and user selection are compared with those of OSDMA and OSDMA-S in Fig. ??(a). The number of transmit antenna is $N_t = 4$ and the SINR is 10 dB. For fair comparison, following the setup for OSDMA-S in [14], a CSI feedback overhead factor of $\lambda = 5\%$ is applied, which reduces the sum capacity by the factor λ for each round of CSI feedback. Hence, the sum capacity with feedback overhead is $\mathcal{C}_\lambda = (1 - K\lambda)\mathcal{C}$ where K is the number of rounds for CSI feedback³ and \mathcal{C} is the sum capacity without considering feedback overhead. For OSDMA-S, the number of feedback rounds, hence the beam and user selection iterations, follows the optimal values obtained in [14]. For LF-OSDMA, the codebook size and the iterations scale with the number of users as $N = 5(\log_2 U)^{N_t-1}$ and $I = U$, respectively. As observed, LF-OSDMA achieves a capacity gain ranging from 0.8 to 1.6 b/s/Hz with respect to OSDMA-S and 1.1 to 1.7 b/s/Hz with respect to OSDMA. Fig. ??(b) shows the CSI feedback bits for the SDMA schemes under consideration. Fig. ??(b) shows that the capacity gain of LF-OSDMA over OSDMA-S and OSDMA comes at the price of a few additional bits of feedback per user, which seems affordable in a practical system.

Fig. ?? shows the effect of the feedback overhead factor λ on the sum capacities achieved by LF-OSDMA, OSDMA and OSDMA-S. We observe that OSDMA-S is more sensitive to the value of λ than the other two schemes. A smaller λ value does narrow the capacity gap between OSDMA-S and LF-OSDMA but also increases the feedback bits for OSDMA-S significantly while those for LF-OSDMA and OSDMA remain unchanged. Consequently, for $U \leq 13$, OSDMA-S requires more CSI feedback than LF-OSDMA.

Fig. ?? shows the effect of fixing the amount of CSI feedback for LF-OSDMA. We observe that each bit of reduction on CSI feedback decreases the sum capacity of LF-OSDMA by an approximately constant value. Furthermore, one bit of feedback reduction has deeper impact on the sum capacity when the CSI feedback bits are fewer. This is reflected by the loss in sum capacity of 0.6 b/s/Hz for CSI feedback reduction from 7 to 6 bits compared with merely 0.2 b/s/Hz for reduction from 9 to 8 bits, where the

³ $K = 1$ for LF-OSDMA and OSDMA and $K > 1$ for OSDMA-S.

number of users is $U = 25$.

VI. CONCLUSION

This paper proposes a new scheme for SDMA downlink with centralized beamforming and scheduling, called LF-OSDMA. We derived an upper bound for its capacity gain due to beam selection with respect to no beam selection. Moreover, we showed that increasing the quantization codebook size ensures that the sum capacity grows optimally with the number of users. Numerical results showed that LF-OSDMA can achieve significant gains in sum capacity with respect to OSDMA-S and OSDMA at the cost of modest computational complexity (base station) and additional feedback.

A common problem for LF-OSDMA, OSDMA-S and OSDMA is that the sum feedback rate increases linearly with the number of users. We are currently investigating the effect of a constraint on the sum feedback rate. The random quantization method used in this paper for CSI quantization is sub-optimal. Additional gains by using optimal quantization methods is also a topic for future investigation.

APPENDIX

A. Proof of Lemma 1

Denote $\angle(\mathbf{s}_{n,U}^*, \mathbf{w}_{n,U}^*)$ as $\varphi_{n,U}^*$. From (7),

$$\text{SINR}(\rho_n^*, \varphi_{n,U}^*) \leq \frac{P\rho_n^* \cos^2(\varphi_{n,U}^*)}{P\rho_n \sin^2(\varphi_{n,U}^*)} \leq \frac{1}{\sin^2(\varphi_{n,U}^*)}. \quad (31)$$

By using Lemma 4 in [15],

$$\lim_{U \rightarrow \infty} \frac{\text{SINR}(\rho_n^*, \varphi_{n,U}^*)}{P \log_2 n} = 1. \quad (32)$$

From (31) and (32), $\lim_{U \rightarrow \infty} \sin^2(\varphi_{n,U}) P \log_2 U \leq 1$. Therefore, $\lim_{U \rightarrow \infty} \sin^2(\varphi_{n,U}) = 0$. Hence, (17) follows.

B. Proof of Lemma 2

Let j^* denote the index of the selected iteration, hence

$$i^* = \arg \max_{1 \leq i \leq I} \sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i)} \right). \quad (33)$$

From (33) and (16),

$$\mathcal{C}_U(I) = E \left[\sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i^*)} \right) \right]. \quad (34)$$

The summation term in (34) can be bounded as

$$\log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i^*)} \right) \leq \max_{1 \leq i \leq I} \log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i)} \right). \quad (35)$$

By substituting (35) into (34) and since $\log_2(\cdot)$ is a monotonically increasing function, (18) follows.

C. Proof of Lemma 3

First, we show that the probability that the same user is selected in two different iterations for beam and user selection reduces to zero for a larger number of users. Consider the event, represented by $\mathbb{E}(U)$, that the same user with a channel shape \mathbf{s} is selected in two different beam and user selection iterations, say the a th and the b th iterations. As the result, this user is assigned two beamforming vectors $\mathbf{w}^{(a)}$ and $\mathbf{w}^{(b)}$. Define the angles $\varphi_U^{(a)} = \angle(\mathbf{s}, \mathbf{w}^{(a)})$ and $\varphi_U^{(b)} = \angle(\mathbf{s}, \mathbf{w}^{(b)})$. The probability of the above even \mathbb{E} is bounded as

$$\Pr\{\mathbb{E}(U)\} \leq E[1 - (1 - \sin^{N_t-1}(\varphi_a + \varphi_b))^I]. \quad (36)$$

Therefore,

$$\lim_{U \rightarrow \infty} \Pr\{\mathbb{E}(U)\} \leq \lim_{U \rightarrow \infty} E[1 - (1 - \sin^{N_t-1}(\varphi_U^{(a)} + \varphi_U^{(b)}))^I], \quad (37)$$

$$\leq E[1 - (1 - \sin^{N_t-1}(\lim_{U \rightarrow \infty} \varphi_U^{(a)} + \lim_{U \rightarrow \infty} \varphi_U^{(b)}))^I] \stackrel{(a)}{\leq} 0, \quad (38)$$

where (a) is obtained by applying Lemma 1. By combining (38) and $\Pr\{\mathbb{E}(U)\} \geq 0$, we have

$$\lim_{U \rightarrow \infty} \Pr\{\mathbb{E}(U)\} = 0. \quad (39)$$

Let $\cup\mathbb{E}(U)$ denote the event that a user is selected twice in two different iterations for beam and user selection. Then

$$\cup\mathbb{E}(U) \leq \binom{I}{2} \Pr\{\mathbb{E}(U)\}. \quad (40)$$

From (39) and (40),

$$\lim_{U \rightarrow \infty} \cup\mathbb{E}(U) = 0. \quad (41)$$

Second, by applying Lemma 2 and since $\log_2(\cdot)$ is a monotonically increasing function,

$$\mathcal{C} \leq E \left[\sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq i \leq I} \max_{1 \leq u \leq U} \text{SINR}_{u,n}^{(i)} \right) \right]. \quad (42)$$

From (41), The SINRs in the above equation are independent. It follows that

$$\mathcal{C} \leq N_t E \left[\log_2 \left(1 + \max_{1 \leq u \leq UI} \text{SINR}_u \right) \right], \quad (43)$$

where $\{\text{SINR}_i\}$ is a sequence of i.i.d. random variables whose CDFs are given by (15).

D. Proof of Theorem 1

Define

$$\mathcal{C}_U^+ = E \left[\sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq u \leq UI} \text{SINR}_{u,n} \right) \right], \quad \mathcal{C}_U^- = E \left[\sum_{n=1}^{N_t} \log_2 \left(1 + \max_{1 \leq u \leq U} \text{SINR}_{u,n} \right) \right]. \quad (44)$$

It follows from (16) and (43) that

$$\mathcal{C}_U^- \leq \mathcal{C}_U(I) \leq \mathcal{C}_U^+. \quad (45)$$

By applying Theorem 1 of [15],

$$\lim_{U \rightarrow \infty} \frac{\mathcal{C}_U^+}{N_t \log_2 \log_2(UI)} = 1, \quad \lim_{U \rightarrow \infty} \frac{\mathcal{C}_U^-}{N_t \log_2 \log_2 U} = 1. \quad (46)$$

We can infer from (45) and (46) that there exists an integer U_0 such that for $U \geq U_0$,

$$N_t \log_2 \log_2 U \leq \mathcal{C}_U(I) \leq N_t \log_2 \log_2(UI). \quad (47)$$

E. Proof of Corollary 1

Since $N_t \log_2 \log_2(UI) \leq N_t \log_2 \log_2 U + N_t \log_2 I / \log_2 U$, we have

$$N_t \log_2 \log_2 U \leq \mathcal{C}_U(I) \leq N_t \log_2 \log_2 U + N_t \frac{\log_2 I}{\log_2 U}, \quad U \geq U_0.$$

Therefore,

$$0 \leq \mathcal{C}_U(I) - N_t \log_2 \log_2 U \leq N_t \frac{\log_2 I}{\log_2 U}, \quad U \geq U_0. \quad (48)$$

Using the definition in (22), (23) follows from (48).

F. Proof of Theorem 2

By using definitions in (25) and (27) and the SINR lower bound in (9), we can lower bound the sum capacity of the alternative scheduling and beamforming scheme in Section IV-B as follows:

$$\mathcal{C}_U \geq LE \left[\log_2 \left(\frac{1 + P\rho^-}{1 + P\rho^- \sin^2(\varphi + \theta)} \right) \right] (1 - \Pr\{|\mathcal{I}| = 0\}) \quad (49)$$

where the angle threshold φ is defined in (24) and θ the channel shape quantization error. For $U \rightarrow \infty$, by applying Lemma 4,

$$\Pr\{|\mathcal{I}| = 0\} \leq [1 - (\sin(\varphi))^{N_t-1}]^{2N_t \log_2 U}, \quad (50)$$

$$\leq \exp(-(\sin(\varphi))^{N_t-1} \cdot 2N_t \log_2 U), \quad (51)$$

$$\stackrel{(b)}{=} (\log_2 U)^{-\log_2 e}, \quad (52)$$

where (b) follows by substituting (24). Therefore,

$$\lim_{U \rightarrow \infty} \Pr\{|\mathcal{I}| = 0\} = 0. \quad (53)$$

Furthermore, it follows from the definition in (24) that

$$\lim_{U \rightarrow \infty} \varphi = 0. \quad (54)$$

From (49), (53) and (54),

$$\lim_{U \rightarrow \infty} \frac{\mathcal{C}_U}{N_t \log_2 \log_2 U} \geq \lim_{U \rightarrow \infty} \frac{E[-\log_2 \sin^2(\theta)]}{\log_2 \log_2 U}, \quad (55)$$

$$\stackrel{(c)}{\geq} \lim_{U \rightarrow \infty} \frac{\log_2 N}{(N_t - 1) \log_2 \log_2 U} = 1, \quad (56)$$

where (c) follows from the following result obtained in [20],

$$E[-\log_2 \sin^2(\theta)] \geq \frac{\log_2 N}{(N_t - 1)}. \quad (57)$$

Similarly, we can obtain an upper bound for the sum capacity as follows:

$$C_U \leq LE \left[\log_2 \left(\frac{1 + P\rho^+}{1 + P\rho^+ \sin^2(\theta)} \right) \right] (1 - \Pr\{|I| = 0\}), \quad (58)$$

$$\leq N_t \log_2 (1 + P\rho^+). \quad (59)$$

Therefore,

$$\lim_{U \rightarrow \infty} \frac{C_U}{N_t \log_2 \log_2 U} \leq \lim_{U \rightarrow \infty} \frac{E[\log_2(1 + P\rho^+)]}{\log_2 \log_2 U} \stackrel{(d)}{=} 1, \quad (60)$$

where (d) follows by substituting (26). By combining (56) and (60),

$$\lim_{U \rightarrow \infty} \frac{C_U}{N_t \log_2 \log_2 U} = 1. \quad (61)$$

Since LF-OSDMA is more robust than the alternative scheme using in the above derivation, (61) also holds for LF-OSDMA and hence the result of Theorem 2 follows.

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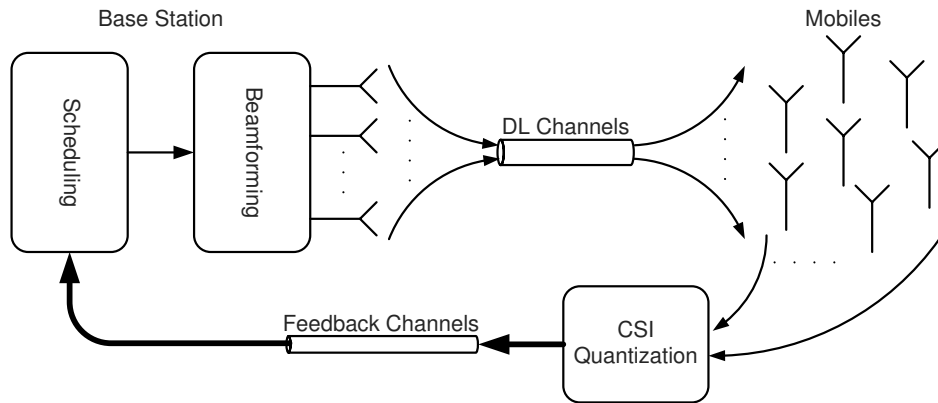


Fig. 1. Downlink system with limited feedback

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TABLE I
COMPARISON OF LF-OSDMA, OSDMA-S AND OSDMA

	LF-OSDMA	OSDMA-S	OSDMA
Feedback/User (bits) ^a	$2B + \log_2 N$	$B + I \log_2 N_t$	$B + \log_2 N_t$
Sum Capacity (bits/s/Hz) ^b	largest (7.5)	moderate (6.4)	smallest (6.2)
Base Station Computation	largest	moderate	smallest
beam and user selection	centralized	distributed	N/A
User Conflict ^c	No	Yes	Yes

^a Assume B bits are required for quantizing a channel gain and the quantization error of the channel shape .

^b Sum capacity is computed for $U = 20$, $N_t = 4$ and $\text{SNR} = 10\text{dB}$. Following [14], the sum capacity is reduced by the feedback overhead factor $\lambda = 5\%$ for each round of CSI feedback.

^c Refer to possibility that different users select a same beamforming vector.

