

Fuzzy Relational Modeling of Cost and Affordability for Advanced Technology Manufacturing Environment

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Abstract

Relational representation of knowledge makes it possible to perform all the computations and decision making in a uniform relational way [46], by means of *special relational compositions* called triangle and square products. These were first introduced by Bandler and Kohout in 1977 [11],[5],[2] and are referred to as the BK-products in the literature [22],[17],[18]. Their theory and applications have made substantial progress since then.

BK-relational product can be used to compare relational structures. Relations so constructed might exhibit some important relational properties that reveal important characteristics and interrelationships of the source of information from which they were generated. Hence, methods for detecting various relational properties of given relations are important.

Collecting engineering data concerning various manufacturing processes, parts, subsystems and manufactured goods is usually done by physical measurements of such physical entities that serve as *cost drivers*. Because one of major concerns is to deal with affordability issues also in the situations when such “hard” data are not available, relational analysis on data and knowledge can be elicited by questioning engineers. A case study of this kind is described in Sec. 3. Here, instead of physical measurement devices we use psychometric tools invented by behavioral scientists called *repertory grids* (RPG). Our relational analysis can be used to analyze data (e.g. process parameters) collected by physical measurements as well as data obtained by knowledge elicitation from human experts.

Relational properties characterizing the structure of knowledge, such as reflexivity, symmetry, and transitivity, and classes such as tolerances, equivalences and partial orders can be extracted from the linguistic information elicited by repertory grids.

Testing the fuzzy relational structure for various relational properties allows us to discover dependencies, hierarchies, similarities, and equivalences of the attributes characterizing technological processes and manufactured artifacts in their relationship to costs and performance.

How to use our methods for ranking of various technologies with respect to affordability is shown in Sec. 4. In section 5, a more detailed study of cost drivers by means of fuzzy relational products is described.

A brief overview of mathematical aspects of BK-relational products is given in Appendix 1 together with further references in the literature.

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1 Importance of the Assessment of Cost and Affordability

1.1 Background, Goals, and Methods Used

In an advanced technology environment, the *key to achieving affordability goals* which are necessary to maintain a competitive position of the US industry in the domestic and world markets, is to deal with complicating uncertainties in materials, fabrication, and manufacturing.

Pratt&Whitney, our industrial partner cooperating with us in this project, is one of the companies that belongs to the United Technologies group, a diversified producer of consumer, commercial, and military products. Pratt&Whitney is one of the firms that are at the leading edge of high technology [19]. Such companies face a formidable problem, being often forced to make technological and business decisions based on *incomplete, uncertain information* about the product that is yet *to be* designed and manufactured. The industry needs affordability models applicable to such manufacturing problems. Scarcity of information concerning untried technologies and the lack of historical data base *are the main characteristics* of this problem.

Our current program addresses this objective. Working jointly with our industrial partner Pratt & Whitney, we have developed practical fuzzy relational techniques that can assist in affordability modeling interfaced with engineering design methods. Particularly important is identifying dependencies, hierarchies, similarities and equivalences of attributes characterizing processes and products in their relationship to cost and performances.

The importance of our techniques stems from the fact that they have been developed for the situations where often only incomplete information and small data sets are available. This is important for strategies for integration of cost into design at very early stages, and for advanced technological design of products never before manufactured.

1.2 The Long Term Objectives of Our Work

Jointly with our industrial partner Pratt&Whitney we have formulated the objectives for our **long term cooperation** that are listed below. Namely, we attempt to:

(LTO-1) Provide a systematic framework for integrating engineering design and manufacturing activities with the management, organizational, accounting and financial activities of the enterprise.

(LTO-2) Identify all technical, human and organizational contribution to costs.

(LTO-3) Deal systematically with incomplete, uncertain and conflicting information, constraints and consequences.

(LTO-4) Deal with uncertainty in estimates, and incorporate the estimates concurrently into engineering design.

(LTO-5) All the methodologies and techniques resulting from the objectives LTO-1 to LTO-4 have to represent data and knowledge in the form compatible with the framework needed for design of computer information systems – preferably computer based distributed Intelligent Systems for manufacturing and telemanufacturing.

(LTO-6) The techniques and methodologies should be compatible with a high level conceptual model of cooperating industrial firm, reflecting the the effect of interaction of cooperating firms on the cost and affordability of products. This should include the effect of procurement, purchasing and marketing.

To deal with LTO-1 and LTO-4 we use Fuzzy relational mathematics. This provides a framework for working with incomplete and/or conflicting information, constraints, consequences; and also with uncertainty of probabilistic as well as of non-probabilistic nature [63],[62],[25],[37].

To integrate human factors with technological ones (objective LTO-2) one has to take into account not only the technological design and production concepts and data, but also the psychological and linguistic constructs utilized by human participants. This requires special techniques we have developed [53],[37],[15],[1],[50],[49]. Value analysis method [83],[R12],[R15] has provided the bridge for incorporating management, financial and organizational activities (objective LTO-1).

Uniform data and knowledge representation equally applicable to objectives LTO-1, LTO-2, LTO-5 has been provided by the methodology of Activity Structures [34],[32] which includes relational data and knowledge representation as its integral part.

The much required unification of data analysis and computational methods we have achieved by combining relational mathematics [11],[8] and computational science [48],[27],[79],[80] within the framework of relational virtual computer architectures [44],[36],[35],[69]. In particular, fuzzy relations, BK-relational products¹ [11],[40],[14],[47],[21],[68] and fast fuzzy relational algorithms [12],[46],[13] have been consistently used for data analysis, knowledge elicitation, knowledge and data representation and further information processing. For recent results see publications² [R4],[R8],[R10] resulting from the grant support (NSF DMI 952 5991) listed in Appendix 2 .

Finally, integration of all the information and knowledge dealt with in our objectives into a global system that synthesize the information relevant to affordability analysis is based on Activity Structures methodology [77],[77],[67],[69], This methodology was created to give a unified platform for

¹BK-products is a term used in the literature on fuzzy sets to designate new relational compositions discovered by Bandler and Kohout in 1977.

²References starting with R appear in the list of publications originated from this grant listed in Appendix 2.

development of distributed intelligent systems [32], hence it has been used to achieve the objectives LTO-5 and LTO-6. For recent results see publications [R10] and [R12] resulting from the grant support (NSF DMI 9525991) listed in Appendix 2.

1.3 The Summary our work supported by the current grant NSF DMI 9525991

Within the framework of the long term objectives outlined in the previous section we have worked out more detailed objectives for our current projects. In this section we discuss the specific objectives our current NSF grant (NSF DMI 9525991) entitled *Decision-Making with Incomplete Information in an Integrated Product and Process Development Enterprise – A Management Decision Tool for Cost Modeling and Affordability Applications*. Our industrial partner for this work has been Pratt & Whitney.

The main objectives for the 3 years of the current grant for the period October 1995 – September 1998 are as follows:

1. Use of Fuzzy Relational Methods for data and knowledge elicitation and representation, and affordability modeling.
2. Value Analysis for Integration of Technology and Business.
3. Problems of Engineering Design: prototype software system for estimating product/process cost based on the fuzzy multi-attribute utility theory.
4. Comparison of Fuzzy and Probabilistic Methods in their applicability to affordability data.
5. Knowledge Transfer to Industry and Education.

As we are concerned in this paper with fuzzy relational knowledge representation techniques and data analysis with imprecise and incomplete data we discuss in the sequel objectives (1) and (2) in detail. For information on other objectives the reader is referred to the following papers: [52],[19],[65].

2 FRASMod Relational Affordability Knowledge Representation Structure

The contexts addressed in this project that are of particular interest to our industrial partner Pratt&Whitney [19] are depicted in Figure 1.

Out of 11 subsystems forming the Industrial context of Affordability modeling shown in this figure we identified six pivotal issues that have been addressed while developing our relational affordability representation scheme. These are as follows:

- Affordability;
- Management of Uncertainty;
- Cost interval and fuzzy modeling;
- Cost/Performance Trades,
- IPPD Environment activities
- Business Practices.

The Fuzzy Relational Affordability Systemic Model (FRASMod) we have developed is designed to capture and integrate the above listed 6 perspectives of manufacturing activities within a unified knowledge representation structure.

In the use of FRASMod the key entities of each perspective are identified using the exploratory knowledge elicitation and mapped into a relational subsystem and a relational coupling structure that shows potential interactions of the entities corresponding to different perspectives.

We have included the following conceptual categories (i.e. semiotic descriptors [32],[34],[37]) of relations in the knowledge representation structure [64] used in FRASMod:

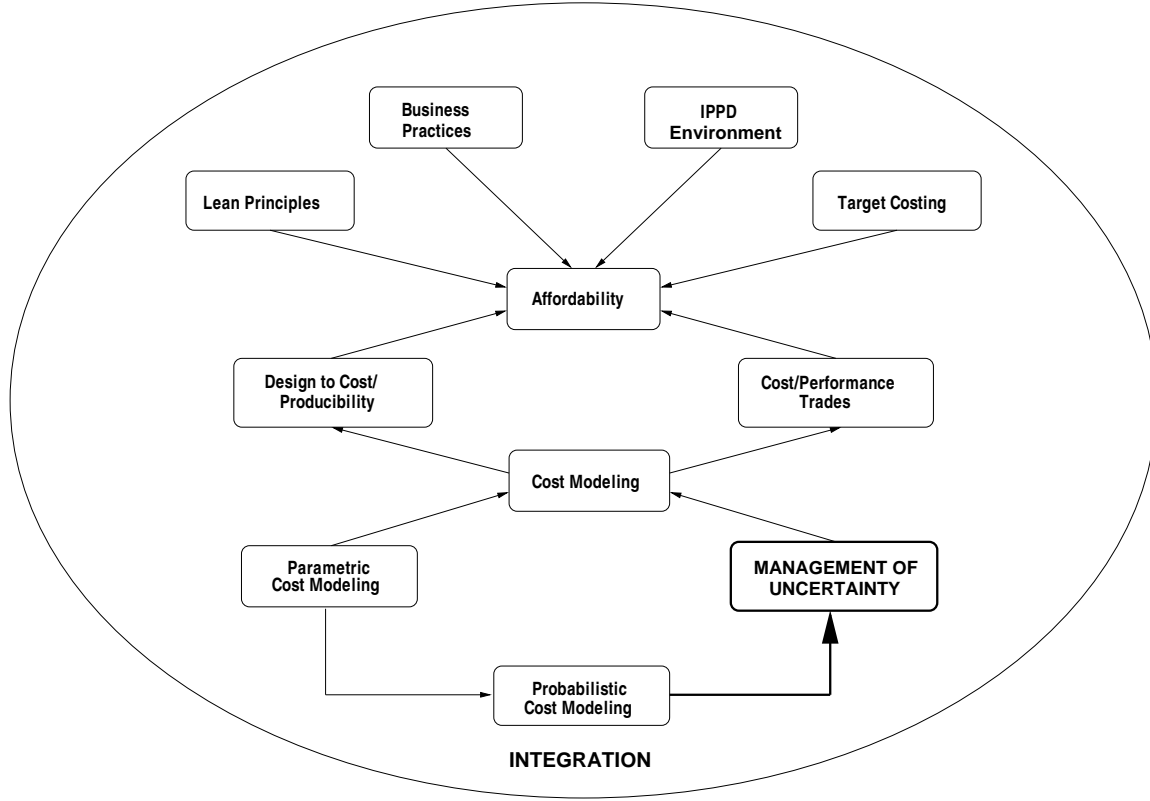


Figure 1:

- Objects,
- attributes,
- values,
- agents,
- perspectives,
- contexts,
- views.

Objects are e.g. components, parts or subsystems of a manufactured artifact, or even the whole technologies, depending on the resolution level of a specific snapshot (view) within the FRASMod.

Attributes are characterized by *linguistic descriptors* and/or physical or virtual measurement scales. Examples of linguistic descriptors are: `small_processing_windows`, `high_temperature`, `good_lubricity`, `low_variance_in_raw_material_costs`, etc. Examples of attributes that can also be characterized by *measurable* physical or fiscal parameters are:

temperature, lubricity, cost_reducing_potential, potential_investment, cost, etc.

Interactions (special kinds of relations), for example:

REL_1,5: `low_variance_in_raw_material_costs` \rightarrow common/standard material/alloy system

REL_2,3: `good_processing_control` \rightarrow low_raw_material_cost

REL_3,7: `low_raw_material_cost` \rightarrow common/standard material/alloy system.

Values. Values are assigned to linguistic variables or numerical variables, to express the magnitude of a physical or fiscal parameters of the attributes, or the truth-value (i.e. the degree to which an object possesses an attribute).

Perspectives. An object or a family of objects can be evaluated within different perspectives. For example, an LPT cover plate can be evaluated from the perspective of an engineer, or from the perspective of a business analyst performing value analysis of the part, or from the perspective of an accountant. Each perspective may employ attributes that are different from the attributes of

a different perspective for the same object. Some attributes may, however, be shared by different perspectives.

Contexts. Each object or family of objects can appear in several different contexts. For example an LPT cover plate may appear e.g. in context of ingot process, forging process, extrusion process, or other processes.

Views. Even in one particular perspective or context, different experts may assess the objects and situations in which objects appear differently. These differences of views of different experts can be captured by repertory grids and compared by relational methods using algorithms provided by TRYSIS.

Agents. In the context of this project, agents are the observers (e.g. engineers or accountants) assessing the degree to which an attribute is possessed by an object. For example, in [R4], [R8], [R10] describing the evaluation of an LPT cover plate the observers were engineers evaluating to what degree various attributes can be assigned to the LPT plate.

2.1 The Role of FRASMod Knowledge Representation Scheme in Affordability Studies

FRASMod has formed the backbone of the whole project, making it possible to link data collection, data analysis and evaluation in a unified framework that is computer representable. It also allows us to represent the cost and performance targets and other design criteria in the same framework. In this unified framework, we also perform analysis of uncertainty, fuzzy indeterminacy and evaluation of consistency of data and knowledge.

We cannot discuss here all the uses the FRASMod scheme was put to in this project in its entirety. Here we focus on relational data analysis and representation.

2.2 Application of Fuzzy Relational Methods in Evaluation of Affordability of a Manufacturing Process

This section is concerned with the work related to Objective 1 listed above in Sec. 1.3.

We have developed methods for Knowledge elicitation and relational representation [43],[46],[47] of the substantive knowledge (concepts, linguistic descriptors, physically measurable parameters and interactions) that are relevant to the affordability analysis and prediction and are applicable not only to *technical* but also to *human* and *organizational* subsystems of a total production system.

We have designed a set of repertory grids to capture the expertise of engineers [28],[19],[62] which is one of the most important sources of information in the situations where no historical data on the manufactured product are available. Repertory grids utilize verbal descriptors, thus making it possible to assign different levels of accuracy, precision, or certainty to each part and process, such as cost, material input, or processing condition [62]. Pratt & Whitney engineers found the repertory grids not difficult to comprehend and quick to fill in. This is important in a busy industrial environment.

The data has been collected and analysis performed so far at *three different resolution levels*:

- (1) the level of component parts of an aircraft jet engine: analyzing e.g. a γ -titanium Low Pressure Turbine (LPT) cover plate [R4],[R5],[R8],[R10]. Comparison with other parts (titanium rings) and materials (e.g. nickel) is in progress.
- (2) The level of integration components into a subsystem: developing a fuzzy model for computing interval bounds of the cost of the subsystem as a function of parts and values of the process attributes.
- (3) the level of cost estimation of competing technologies: [R5],[R16]. This also provides interval bounds. (See Sec. 4 below.)

In general, affordability modeling involves a variety of contexts and resolution levels, e.g. level of parts, processes, assembled artifacts, cost/performance tradeoffs, business practices, etc. (See Fig. 1 above).

For relating information concerning the structures of these different resolution levels, we have developed the technique of generalized morphisms [R3], [R7]. This makes it possible integrate sep-

arate models of different resolution levels into one multi-resolution global model, interrelating the relevant cost related features. Generalized morphisms (GMorphs) are also important for ensuring the correctness of scale measurements by repertory grids. Mathematically, GMorphs [8] are generalizations of homomorphisms that play an important role in the theory of measurement [78].

Collecting engineering data concerning various manufacturing processes, parts, subsystems and products is usually done by physical measurements of such physical entities that serve as *cost drivers*. Because one of our concerns is to deal with affordability issues when such “hard” data are not available, we shall, however, present here the results of relational analysis on data and knowledge elicited by questioning engineers. Here, instead of physical measurement devices we use psychometric tools invented by behavioral scientists called *repertory grids* (RPG). Our relational analysis can be used to analyze data (e.g. process parameters) collected by physical measurements as well as data obtained by knowledge elicitation from human experts.

A substantial effort in this project was devoted to exploratory knowledge elicitation that made it possible to develop such grids for problems relevant to the problem area of our industrial partner – integrating affordability into IPPD environment. Here is a brief summary of how RPGrids have been developed and utilized:

The entities of the processes were identified by exploratory knowledge elicitation and the cost drivers called process constructs (c_i) for each process selected. Using these results *repertory grids* (RPG) with bi-polar constructs were developed (Fig. 2 gives an example of a RPG). These RPGs were used to elicit information about relationships of process constructs by presenting these to Pratt & Whitney engineers.

By converting the grids to relational matrices and processing these by the TRYSIS system tests for various relational properties were performed. The computational tests for this purpose are based on BK-Products of relations and Fast Fuzzy Relational Algorithms [47]. These tests make explicit relational structures and properties intrinsically contained in data. Testing the fuzzy relational structure for various relational properties allows us to discover dependencies, hierarchies, similarities, and equivalences of the attributes characterizing technological processes and manufactured artifacts in their relationship to costs and performance.

The example of the ingot process shows dependences of the process constructs/cost-drivers represented as Hasse diagrams (see Fig. 3, Fig. 4 and Fig. 7) that are standard way of representing preorders.

3 LPT Cover Plate Relational Analysis: A demonstrator project

For the demonstration of our relational analysis method, our industrial partner selected a jet engine component, a *Low Pressure Turbine (LPT) Cover Plate* [28, 19].

Selecting a set of objects (e.g. engine parts), using repertory grids we have isolated technological attributes of these objects which are relevant to the cost and expressed as a fuzzy relational structures [R3]. Testing these structure for various relational properties yields dependencies, hierarchies, similarities, and equivalences of the attributes significant with respect to cost [R4]. Carry out this we had to develop the appropriate methodology.

Using a LPT cover plate as the appropriate object for a demonstration of our techniques, we have:

- Performed exploratory knowledge elicitation that resulted in selecting cost drivers for evaluating the affordability of LPT cover plate in all 5 processes involved in its manufacturing.
- Designed a set of Repertory grids for collecting data on LPT cover plate from engineers.
- Performed a set of experiments eliciting the values of process parameters for the LPT cover plate. (See description of the three scenarios for the use of repertory grids and results in Sec. 3.2 below.)

- Developed a method for comparison of different but similar parts with respect to process parameters and other attributes (see Sec. 3.2.2 below.)

3.1 The objective of the relational analysis of the LPT Cover Plate

A *Low Pressure Turbine (LPT) Cover Plate* is to be manufactured, using new material, namely, gamma titanium. Prior to any production characterization, the part is to be costed out, using the expert knowledge concerning manufacturing processes and available cost estimation that is available for other small gamma titanium parts.

This is a part with the limited characterization data in processes with little manufacturing base, for which only very limited empirical data are available. Hence elicitation of knowledge of human experts and further fuzzy relational extrapolation are necessary.

Figure 2: **Repertory Grid Analyzer (RPGA)**
A sample of input data for RPGA: LPT Cover Plate (Ingot process)

Pr.	Primary pole		3	2	1	0	-1	-2	-3	Secondary pole	
	description	range								description	range
1	Very low % of total cost	15%						✓		Fairly high % of total cost	30%
2	Low raw material costs	\$10/lb					✓			High raw material costs	\$40/lb
3	Low variability in raw material costs	±5%				✓				High variability in raw material costs	±20%
4	Good process control of raw materials	$C_{pk} \geq 1.3$				✓				Poor process control of raw materials	$C_{pk} \geq 0$
5	Standard size of ingot	24", 28", 30", 32"					✓			Non-standard size of ingot	
6	Small ingot weight	600lb				✓				Large ingot weight	2500lb
7	Short raw material lead time	2 months					✓			Long raw material lead time	12 months
8	Common/standard material/alloy system						✓			New material/alloy system	
9	Small variation in material properties					✓				Large variation in material properties	
10	Small numbers of defects					✓				Large numbers of defects	
11	100 % yield	100%				✓				25% yield	25%
13	Low cracking probability	5%						✓		High cracking probability	50%

The meaning of relational sorts (semiotic descriptors) in the Fuzzy Relational Affordability Systemic Model (FRASMod) is as follows.

Object: Low Pressure Turbine (LPT) Cover Plate.

Perspective: Dependency of the cost on the product – process relationship.

Contexts: Five processes during the manufacturing of the LPT Cover Plate, namely: ingot process, forging, extrusion, heat treatment, machining.

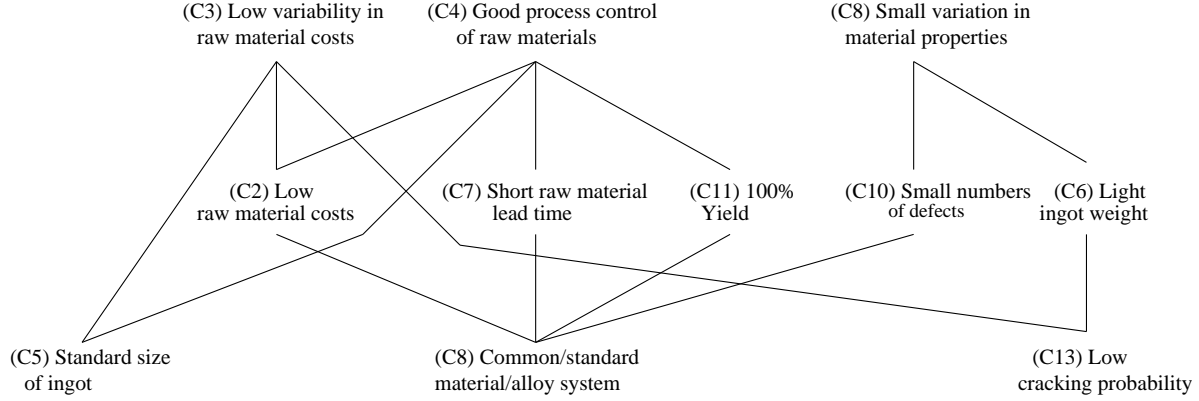
Agents. Agents are the respondents – 5 Pratt & Whitney engineers filling in the repertory grids, thus providing information about process' attributes of the nickel or γ -titanium LPT Cover Plate.

Attributes: Process' entities selected as cost drivers, represented as bipolar constructs, in the repertory grids that were presented to respondents – engineers.

Figure 3: Ingot
A sample of output results from RPGA package

Ingot : (Fuzzy Logic Operator, Alpha-Cut, Criteria)

$$\begin{aligned} (S.(H,HU,M).mean) &= (S.H_only.harsh) = (S*.(H,HU).(mean,harsh)) = (L.(H,HU).(mean,harsh)) \\ &= (G43.H.mean) = (G43.(H,HU).harsh) = (G43'.(H,HU).(mean,harsh)) \end{aligned}$$



The Evaluative Task Structure: A number of elicitation and evaluative schemes (*Scenarios*) can be formulated, capturing inter-process dependences, inter-observer dependences, etc.

3.2 Knowledge Elicitation and Data Analysis

There is a number of problems that can be solved by relational analysis of information obtained by repertory grids. In this section we outline 3 different scenario for evaluation of an LPT-plate, namely:

1. Discovering dependency structures of cost drivers.
2. Identification of characteristic similarities and differences between parts made of different materials.
3. Discovering interprocess differences between the meaning of cost drivers.

3.2.1 SCENARIO 1: Discovering dependency structures of cost drivers

1 object (LPT cover plate) and a group of respondents (5 engineers). Each respondent-engineer has assessed the object independently in the five processes involved in manufacturing of the part. The aim here is to find the dependences between process cost drivers, as well as the inter respondent consistency.

The resulting Hasse diagram computed from the RPGs (Fig.2) of the *ingot process* is shown in Fig. 3. Also the dependences between the judgments of engineers have been obtained.

From the Hasse diagrams computed for all the processes the necessary and possible fuzzy dependences have been derived.

We have to distinguish it necessary from *possible* dependences. This follows from the logic theory of BK-products and Fast Fuzzy Relational algorithms by which the Hasse diagrams are computed.

Let us briefly look at a sample of such dependences as they appear in the process of *machining*. Their Hasse diagram appears in Fig. 4. The verbal statement x is necessarily dependent on y and z we abbreviate by $x \stackrel{\square}{\Rightarrow} y \ \& \ z$. Similarly, $x \not\Rightarrow y$ reads x is independent of y . $x \stackrel{\diamond}{\Rightarrow} y \vee z$ reads x is

possibly dependent on y or z . For example, from Fig. 4 we can read the following.

- Necessary Dependencies of the Process Parameter C_8

$$C_8 : \text{Machining distortion/warpage} \stackrel{\square}{\Rightarrow} \begin{array}{l} C_2 : \text{Part size \&} \\ C_4 : \text{Material machinability \&} \\ C_{11} : \text{Machining data \&} \\ C_{14} : \text{Post-machining inspection} \end{array}$$

- Non-Dependencies of the Process Parameter C_8 :

C_1 : Init. part variability $\not\Rightarrow C_8$: Machining distortion $\vee C_2$: Part size $\vee C_{14}$: Post-machining inspection

Some of these dependencies and independencies appear fairly obvious to an engineer with some experience other dependences are less obvious but can be validated. The essential point here to realize is that all this inference has been obtained computationally from the repertory grids, each of which was filled by a different expert within a few minutes.

Machining : (Fuzzy Logic Operator, Alpha-Cut, Criteria)

$$\begin{aligned} (S.H.\text{only}.\text{mean,harsh}) &= (S^*.(H,HU).\text{mean,harsh}) = (G43.(H,HU).\text{mean,harsh}) \\ &= (G43'.(H,HU).\text{mean,harsh}) = (L.(H,HU).\text{mean,harsh}) = (KDL.HU.harsh) \end{aligned}$$

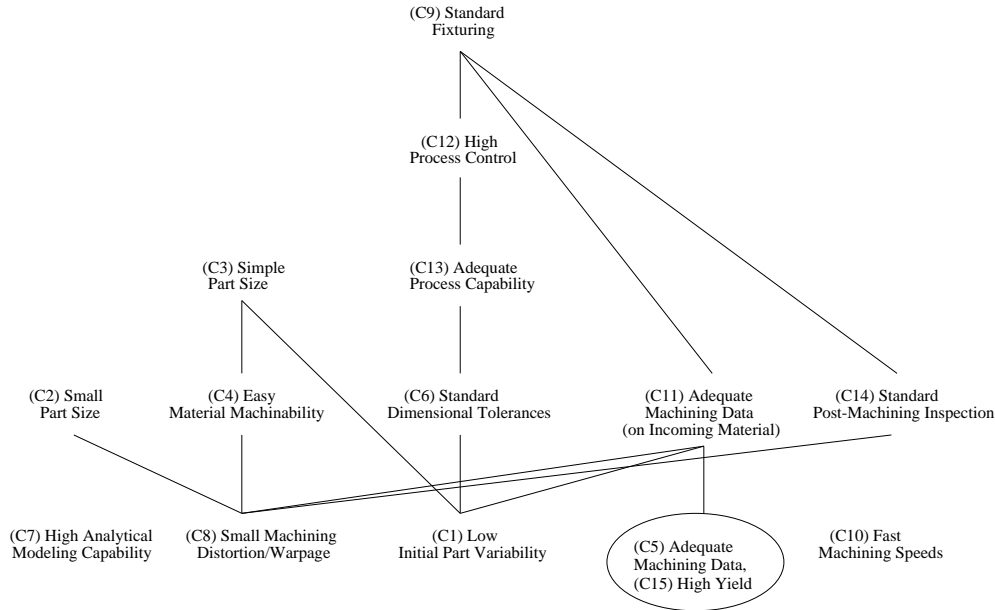


Figure 4: Hasse Diagrams of Machining

3.2.2 SCENARIO 2: Similarities and differences of parts made of different materials

Determining characteristic similarities and differences between parts made of different materials may involve one or more respondents (engineers) and a collection of objects (e.g. different LPT parts). Here, the aim is to detect characteristic similarities and differences between distinct objects.

We have chosen to compare the LPT cover plates made of two different materials, namely *nickel* and γ -*titanium* in all 5 manufacturing processes. The results are summarized in Table 1. Degrees of similarity were computed by the fuzzy logic using the the fuzzy equivalence operator based on the Lukasiewicz implication operator. Degrees of difference were computed by the operator dual to the fuzzy equivalence operator.

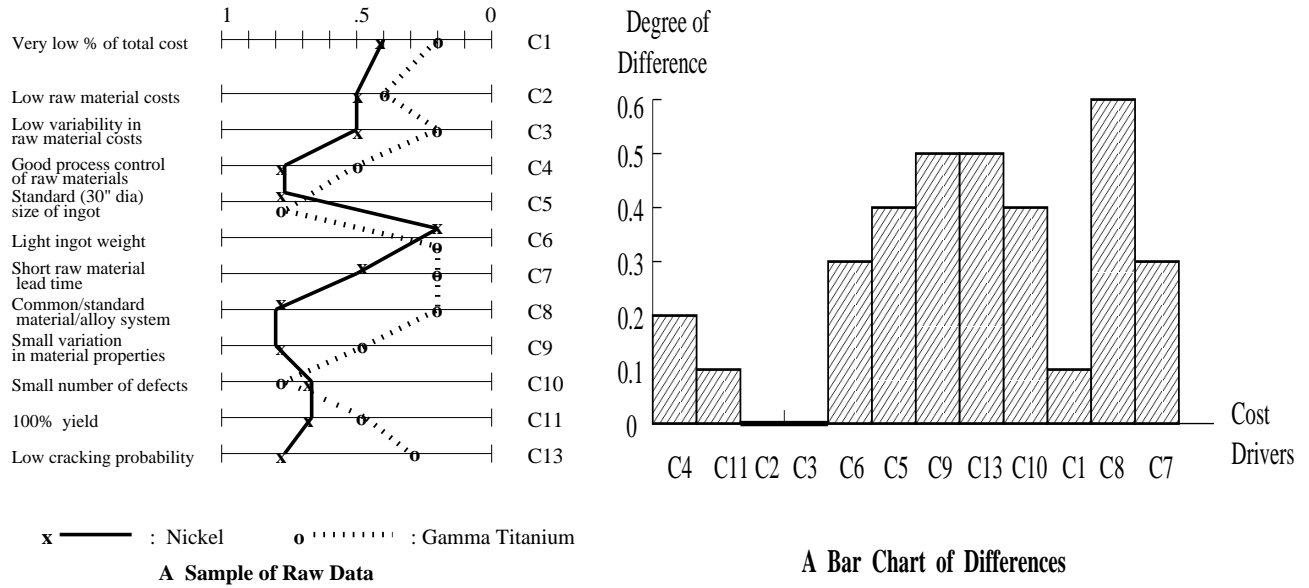


Figure 5:

The differences on a relative scale are plotted as bar-charts (for a sample see Fig. 5). The classivalence classes relating nickel and γ -titanium data were also computed [R19].

We have seen in Scenario 1 that testing for preorders reveals possible dependences of process entities (in particular those selected as the cost drivers). Knowing mutual dependencies allows for identifying these interrelationships. In the Scenario 2 another kind of generalized equivalence, namely *classivalence* [8] appears very useful. One may ask what classivalence, a generalized “equivalence” of two different sets is.

In general, equivalence may appear in a relation from a set to itself. Classivalence, related to bifunctionality can appear when two *different sets* are related by a relation.

Equivalence and classivalence classes identified in the data of Scenario 2 provide the information as to which process entities may have equivalent effect, hence can be treated as interchangeable in their impact on the other portions of affordability models. More detailed explanation of classivalence appears in [R19].

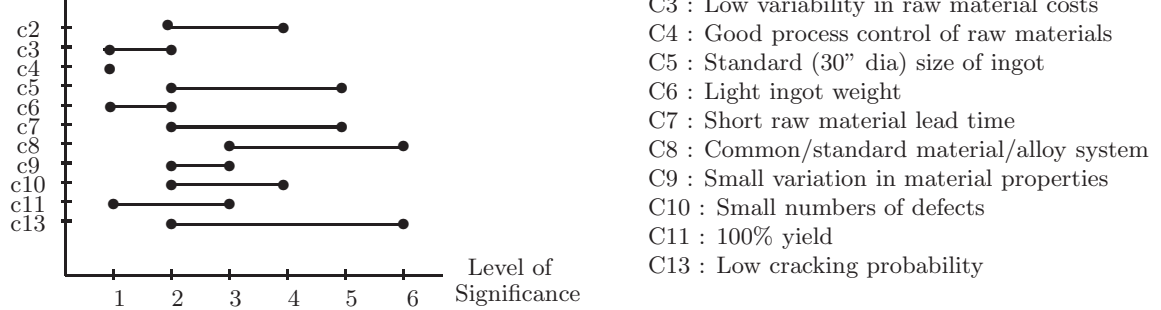


Figure 6: Interval ranking of cost drivers for LPT cover plate made of γ -titanium in process Ingot

3.2.3 SCENARIO 3: Interprocess differences between the meaning of cost drivers

This involves one or more respondents (engineers), 1 object, several situations or processes in which the object may appear. In this scenario, the primary goal is to detect similarities and dependences

Table 1: Summarization of differences of cost drivers of a LPT cover plate: Nickel material vs. γ =titanium

Measures	Processes					LPT global
	Ingot	Extrusion	Forging	Heat Treatment	Machining	
# of Cost Drivers	12	19	27	13	15	86
Mean Difference	0.28	0.185	0.24	0.23	0.2	0.23
Max Value of Difference	$< 0, 0.5 >$	$< 0, 0.4 >$	$< 0, 0.6 >$	$< 0, 0.4 >$	$< 0, 0.5 >$	$< 0, 0.48 >$
# of Cost Drivers in $< max, max - 10\% >$	3	9	4	7	5	28
# of Cost Drivers in $< min, min + 10\% >$	4	8	7	4	5	28
# of Cost Drivers with Similarity $\geq 70\%$	7	15	17	9	8	56
Percentage of Cost Drivers with Similarity $\geq 70\%$	58%	80%	63%	69%	53%	65%

between process attributes of different processes. See Fig. 7. for results of comparing cost drivers of the extrusion process with the cost drivers in the forging for the γ -titanium LPT cover plate.

Forging and extrusion are interesting from the point of view of investigating the effect of context on the meaning of process attributes. As we can see from Table 1 (which also shows the number of cost drivers in each process) *forging* has 27 cost drivers (RPG bipolar constructs), while *extrusion* has only 19 cost drivers. Out of this number, 9 cost drivers (RPG bipolar constructs) are overlapping; they have the same linguistic labels – names (see Fig. 7 for their names). In each process, although having the same name denoting the same general concept, they may have different minimum and maximum ranges assigned. The question then arises: do the overlapping bipolar constructs interact in each context in the same way? This is an empirical question answer to which can be provided amongst other things) by the experimental arrangement of Scenario 3.

We can see that the Hasse diagrams capturing the ordering (it is a pre-order) of cost drivers having the same linguistic label “name” are different. Hence their meaning is different, because the contexts, namely processes are different.

Comparing the Hasse diagrams for forging and extrusion in Fig. 7 we can see that only the constructs c_2 : *process window* c_7 : *Tooling* have a dependency in common $c_2 \Rightarrow c_7$ This link appears in both extrusion and forging.

This however does not mean that a specific list of repertory grid constructs having identical name have the same meaning in two different contexts.

Looking at equivalences in two different contexts we can see the following. Fig. 7 shows that in the context of *extrusion*, semiotic descriptors c_2 : *large process window* and c_9 : *long die life* lie within the equivalence class, while in the context of *forging* c_2 : *Large process window* is equivalent with c_5 : *air furnace atmosphere*. The equivalence of c_2 : *large process window* and c_9 : *long die life*, however, **does not hold** in the context of *forging* despite of the fact that it holds for extrusion.

Data can also be analyzed taking the negative side of bi-polar PRG constructs. The preorder depicted in Figure 7 on the left shows the property of *contrapositive symmetry*³.

We have e.g. $c_3 \Rightarrow c_4 \Rightarrow \{c_2, c_5\} \Rightarrow c_7 = \neg c_7 \Rightarrow \neg \{c_2, c_5\} \Rightarrow \neg c_4 \Rightarrow \neg c_3$.

The Hasse diagrams on the right side, however, do not have contrapositive property. Hence, the presence or absence of contrapositivity is an important characteristic of data that ought to be always tested.

³A logic proposition is contrapositive if $a \rightarrow b = \neg b \rightarrow \neg a$.

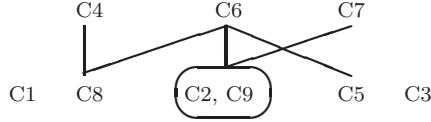
3.3 Integration of Perspectives and Resolution Levels of Relational Models and Summarization of Data

In general, affordability modeling involves a variety of contexts and resolution levels, e.g. level of parts, processes, assembled artifacts, cost/performance tradeoffs, business practices, etc. (See Fig. 1 above). In terms of fuzzy relational models we say that each resolution level represents different granularity [37],[64],[81] pp. 433-448 and [82]. These different perspectives and models at different resolution levels have to be appropriately integrated.

Positive Semiotic Descriptors		Negative Semiotic Descriptors	
Symbol	Meaning	Symbol	Meaning
C1	Capable Analytical Modeling	$\overline{C1}$	Limited Analytical Modeling
C2	Large Process Window	$\overline{C2}$	Small Process Window
C3	Low Temperature	$\overline{C3}$	High Temperature
C4	Good Lubricity	$\overline{C4}$	Low(or Difficult) Lubricity
C5	Air Furnace Atmosphere	$\overline{C5}$	Vacuum Furnace Atmosphere
C6	Good Process Control	$\overline{C6}$	Limited Process Control
C7	Available Tooling	$\overline{C7}$	New Tooling
C8	Flat Die Shape	$\overline{C8}$	Shaped Die Shape
C9	Long Die Life	$\overline{C9}$	Short Die Life

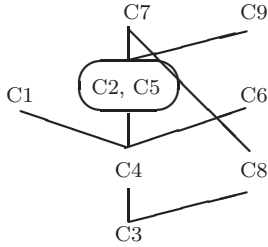
FRS of Extrusion

$(S,H,harsh) = (S,(H,HU,M), mean)$
 $= (S^*,(H,HU),harsh) = (S^*,(H,HU),mean)$
 $= (G43,(H,HU),harsh) = (G43, H,mean)$
 $= (G43',(H,HU),harsh) = (G43',(H,HU),mean)$
 $= (L,(H,HU),harsh) = (L,(H,HU),mean)$



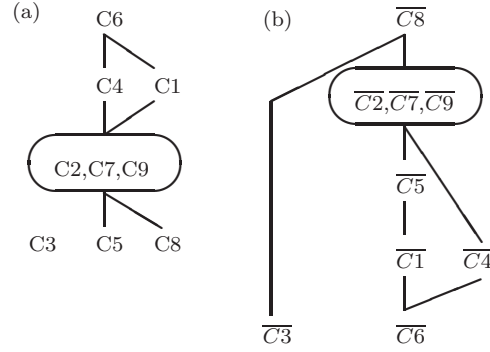
FRS of Forging

$(S,H,harsh) = (S,H,mean)$
 $= (S^*,(H,HU,M),harsh) = (S^*,(H,HU,M),mean)$
 $= (G43,(H,HU),harsh) = (G43,(H,HU),mean)$
 $= (G43',(H,HU),harsh) = (G43',(H,HU),mean)$
 $= (L,(H,HU),harsh) = (L,(H,HU),mean)$
 $= (KDL,HU,harsh)$



Extrusion

(G43, M, mean)



Forging

(G43, M, (harsh,mean))

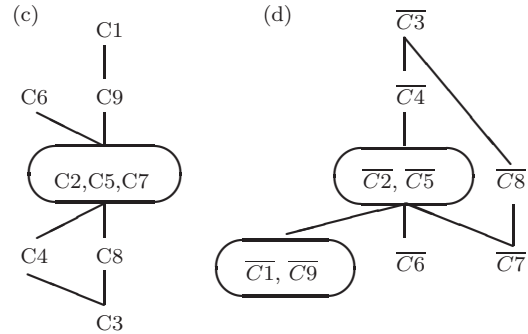


Figure 7: Comparison of overlapping cost drivers in Extrusion and Forging for γ -Titanium LPT Cover Plate

So far we have discussed some (but not all) results of analysis we have performed at the level of component parts of an aircraft jet engine. We have, however, also achieved significant results in developing new methods for the level of integration of components into a subsystem as well as integration and summarization of data creating the levels of coarser granularity. We shall survey some of these now.

The Fuzzy Relational Affordability Systemic Model (FRASMod) has been designed to capture and integrate diversity of contexts and perspectives of manufacturing activities within a unified

representation structure.

We have seen that in building of FRASMod the key entities of each perspective are identified using the exploratory knowledge elicitation and mapped into a relational subsystem and a relational coupling structure that shows potential interactions of the entities corresponding to different perspectives.

It is not only integration, but also information summarization that is essential at this level of knowledge representation. We have developed three techniques for this purpose. Namely,

- Interval Aggregation of costs: A fuzzy algorithm for computing interval bounds of the cost of the subsystem as a function of parts and values of the process attributes. (See Sec. C.3.4 below)
- The method of summarization of preorders to provide an interval ranking of objects or attributes.
- Generalized morphisms based comparison of structures: for relating information concerning the structures of resolution levels and correct aggregation of measurements.

3.4 Interval Aggregation of Costs

Based on the possibility measure and the plinth of fuzzy sets [4] we have developed an interval method for the computing the interval bounds of the affordability information to be used for its integration and summarization when moving from a lower resolution level to a higher one in our relational knowledge representation scheme FRASMod, just creating the levels of coarser granularity.

This method has been used for computing interval bounds of the aggregated cost that is the function of the values of the the 86 cost drivers of the five processes involved in manufacturing the LPT cover plate. The same procedure can be applied recursively, to yield the interval bound on the total cost of integrating the LPT cover plate with other parts of a Low Pressure Turbine. Further higher recursion is also possible. So the method applicable on any level, suitably using aggregated information from the lower levels.

4 Use of preorders to provide an interval ranking of competing technologies

Fig. 8 shows a demonstration example of fuzzy interval ranking of technologies using this set of data⁴.

Given parameters of selected technologies, *Investment priority* partial ordering of *technologies preferences* can be computed. The Hasse diagrams then express the partial ranking of technologies based on parameters such as *potential investment*, *Improvement of performance* and various *potential* and *benefit measures*. The evaluated objects are technologies T_1 to T_7 , that are characterized by seven attributes P_1 to P_7 as shown at the top of Fig. 8. The result of relational analysis are the Hasse diagrams displayed at the bottom of Fig. 8.

It can be seen from the Hasse diagrams that processing the data by different fuzzy logics yields different partial ordering of technologies. Hence, the input data fits several competing models. To reconcile the differences we have to collect more data or use interval fuzzy logics.

This interval method that we have developed uses one of the Checklist paradigm [48] based interval systems, a triple $\langle \text{Łukasiewicz, Reichenbach, Kleene-Dienes} \rangle$ logics combined with appropriate summarization procedure. The initial interval ranking obtained by applying the FIRE procedures to the sets of Hasse diagrams of technologies is displayed at the bottom of Fig. 8 at the right (for the α -cut with value 0.17).

In exploratory analysis of possible technological alternatives where only few global cost characteristics of the technologies are available, the preference is usually expressed by linear ranking done heuristically. If, however, the intrinsic order contained in the data is only a partial ordering, the linear order is usually enforced artificially, e.g by an accountant or economist disregarding whether or not it is this linear order is intrinsically present in the data. The case of such heuristic ranking

⁴This solves a problem proposed to us by our industrial partner Pratt & Whitney.

by an engineer is displayed in the right column of the table depicted in Fig. 8. The artificial unwarranted precision is introduced. Compare it with the ranking intervals computed by FIRE displayed in the same figure.

Clearly, FIRE does not impose linear ranking when it is not present in the given data. It will come out if it is there. But where there is only little information, with large “grey bands” of imprecision, our method does not artificially impose it, but works with intervals instead.

The importance of FIRE goes beyond just ranking technologies stems from the fact that the **FIRE method can be applied at any lower resolution level**: E.g. objects are not technologies but alternative manufacturing processes by which a component can be produced. The of the cost factors when integrating components into a subsystem.attributes may be cost drivers, performance measures, reliability measures etc.

Table 2: Original Data from Pratt & Whitney

$$ORG = T \times P$$

where $T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$, a set of technologies

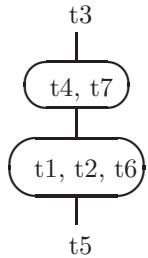
$P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$, a set of cost parameters

Technol.	Parameters							Investm. Priority
	Potential Investment	Cost Reducing Potential	Improvement in Performance	Performance Improvement Potential	Enabling Technology Benefits	Enabling Technology Potential	Non Weighted Economic, Performance & Enabling Technology Improvement Potential	
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	
	Indicator	Indicator	% & Fault	Indicator	% & A/C Cost	Indicator	Indicator	Ranking
T_1	65	90	1.00%	46	0.13%	56	64	4
T_2	90	45	1.20%	40	0.10%	32	39	7
T_3	30	186	1.20%	120	0.27%	263	190	2
T_4	60	3	2.00%	100	0.28%	124	75	3
T_5	25	14	0.80%	98	0.04%	45	52	6
T_6	100	14	1.80%	48	0.03%	9	24	8
T_7	75	5	2.00%	80	0.21%	80	55	5

Figure 8: Interval Ranks of Technologies and their HD structures

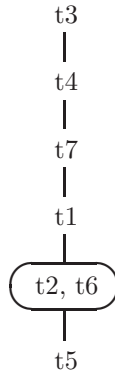
Technology : (Operator, α -cut, Criteria)

(L, M, mean)



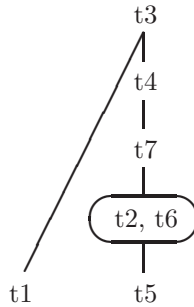
t1 : technology-1
t2 : technology-2
t3 : technology-3

(KDL, M, mean)



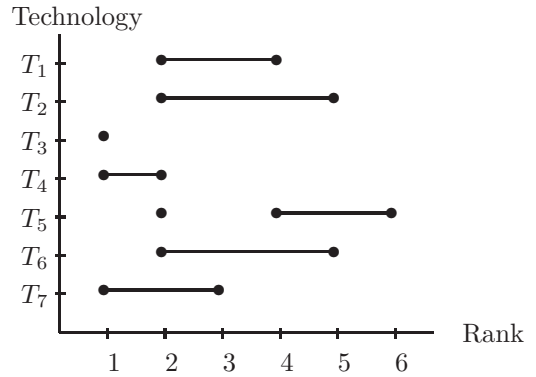
t4 : technology-4
t5 : technology-5
t6 : technology-6

(KD, M, mean)



t7 : technology-7
M : Mean α -cut

$\alpha = .17$



L : Łukasiewicz
KDL : Richenbach
KD : Kleene-Dienes

5 Value Analysis as a Tool for Identification of Unnecessary Costs

Value analysis [R12],[83] is the organized, systematic study of the function of a material, part, component, or system to identify areas of unnecessary cost used in any production or service. Value analysis (VA) consists of (1) analyzing the function of a product, (2) considering designs to accomplish this function, and (3) analyzing the costs of alternatives. Activity structures methodology unlike some other methods can combine analysis by activities with analysis by functions. This is made possible because it distinguishes substratum structures, system activity structures and functional activity structures [32],[43].

We have integrated the Value Analysis [83] and Activity Structures [34],[32] methodologies, and investigated the ways of building relational models for processing data generated by Value Analysis.

Relational model of Value Analysis activities using BK-relational products has been formulated [R12]. Such a model allows us to identify the *crucial technological and business factors* that influence the value of products and services in order to provide alternatives of better value. It also helps to integrate business factors with engineering factors and analyze these by relational computations for their similarity, equivalence and mutual dependence as described in objective 1 above. Currently, relational value analysis of the γ -titanium LPT cover plate is in progress, to supplement its engineering analysis by analysis of non-engineering factors influencing its cost.

The relational model [R12] has also been used to develop a fuzzy algorithm for cost generalized optimization [R15]. It makes it possible to optimize the cost of design of a system by choosing the best alternative with respect to cost, performance and undesirable side-effects.

Value analysis is an important method for reducing the cost of manufactured products. It is the organized, systematic study of the function of a material, part, component, or system to identify areas of unnecessary cost used in any production or service. Value analysis (VA) consists of (1) analyzing the function of a product, (2) considering designs to accomplish this function, and (3) analyzing the costs of alternatives.

VA allows us to identify the *crucial technological and business factors* that influence the value of products and services in order to provide alternatives of better value. It also helps to integrate business factors with engineering factors and analyze these by relational computations for their similarity, equivalence and mutual dependence.

Any formal model to be practically usable has to capture the great diversity of factors that influence the quality of the industrial product. The information and data for such an analysis model are drawn from a multiplicity of sources belonging to various company sections and personnel of different specialization. Typically, *“the purpose of Value Analysis is to bring together ... the combined talents of purchasing and its vendors as well as engineering, production, and other operating personnel to review the components and materials used by the organization on products or processes already in place. It is intended to provide a means of considering all possible alternatives in an atmosphere of open thinking and analysis.”* [83], p. 469.

Relational representation of Value Analysis data together with the compositions provided by triangle and square BK-products and further operations over these (such as fuzzy relational closures and interiors) can capture the great diversity of factors, investigate their similarity, equivalence and mutual dependence. This helps in identifying the crucial factors that influence the value of products and services and also in providing alternatives of better value.

Different sources and different knowledge domains entering the overall purchasing, design, manufacturing and marketing activities are in reality mixed together in a multiplicity of contexts. The modeling apparatus on which we base our computer support of Value Analysis must also possess the capability of dealing with a number of diverse contexts [43]. In each domain, appropriate contexts must be distinguished. In setting a relational model, one has to clearly understand what is the meaning of the key notions in individual contexts, and what role these play. In Value Analysis one wants to find regular phenomena and intrinsic dependencies of various factors within complex interrelationships of all factors and contexts into which the product enters. Within this framework we are specifically interested in detection of change to identify those trends that need to be encour-

Name: A set of:

- B Systems of functions.
- C Cost.
- G Processes.
- H Substratum units (physically related subsystems of components, etc.)
- I Investigations (quality tests, etc.).
- M Modifications.
- O Observation events (e.g. time indexing).
- P Part or Component.
- S Observable features, measurable properties, functional signs.
- U Usability measure.
- V Variant of a substratum unit/module (e.g. a part).
- Y Composed attributes, functional characteristics.

A number of meaningful relations between these entities can be formed.

Name/Type:	Definition:	Relation
VYC....	$\mathcal{R}(V \times Y \times C)$	between <i>variants of a part</i> , <i>functional features</i> and <i>cost</i> .
VYU....	$\mathcal{R}(V \times Y \times U)$	between <i>variants of a part</i> , <i>functional features</i> and <i>usability</i> .
BYS	$\mathcal{R}(B \times Y \times S)$	between <i>systems of functions</i> , <i>functional characteristics</i> and <i>properties</i> .
PVC....	$\mathcal{P}(P \times V \times C)$	between <i>parts</i> , <i>variants of parts</i> and <i>cost</i> .
PVY....	$\mathcal{P}(P \times V \times Y)$	between <i>parts</i> , <i>variants of parts</i> and <i>functional features</i> .
PVU....	$\mathcal{P}(P \times V \times U)$	between <i>parts</i> , <i>variants of parts</i> and <i>usability</i> .
PY	$\mathcal{R}(P \rightsquigarrow Y)$	from <i>parts</i> to <i>functional features</i>

Figure 9: The conceptual meaning of sets and relations used in the value analysis example below

aged or curtailed as the case may be. Interdependencies of various factors, parts, subsystems and observables characterizing the evaluated product and their links with cost and utility have to be established.

The following name-lists(see Fig.9) specify the concepts used in one of our VA relational models developed in this project. (In Knowledge Engineering these are called lists of 'ontologies' or sometimes 'semiotic descriptors').

The following examples show the use of relational models. For the explanation of the notation used, see Appendix 1. \bigoplus_j is an aggregation operator; its simple instance is e.g. $\frac{1}{n} \sum_{j=1}^n$.

Example of relational computations in value analysis: We wish to compare different parts with respect to their functional features using the entities listed in Fig.9.

Let PY be a relation from the set of parts P to the set Y of functional features. The triangle subproduct

$$(PY \triangleleft PY^T)_{ik} = \bigoplus_j (PY_{ij} \equiv PY_{jk}^T)$$

will give the degree to which the functional features of part p_i are included in the set of functional features of part p_k .

Let PYC be a ternary (3-place) relation between the set of parts P , the set Y of functional features and the set C of costs. The square product

$$(PYC \square PYC)_{ijlm} = \bigoplus_{k=n} (PYC_{ijk} \equiv PYC_{lmn})$$

will give the degree to which the cost of variant v_i of part p_i matches the cost of variant v_m of part p_l .

Let VYC be a ternary (3-place) relation between the set of variants of a part, the set Y of functional features and the set C of costs. The square product

$$(VYC \square VYC)_{ikln} = \bigoplus_{j=m} (VYC_{ijk} \equiv VYC_{lmn})$$

will give the degree to which variant v_i of a part costing c_k is exchangeable for variant v_l of the same part with respect to matching their functional features.

Many other relevant VA-questions can answered be by various combinations of fuzzy BK-products and the answers can be ranked by the degree of validity or relevance of the answer within a specific context.

The context addressed in this project that are of particular interest to our industrial partner Pratt&Whitney are depicted in the Figure 1 above, in Sec.1.3.

6 Appendix 1: A Survey of Theory and Applications of Fuzzy BK-Products

6.1 The Unifying Power of Relations

Relational representation of knowledge makes it possible to perform all the computations and decision making in a uniform relational way [46], by means of *special relational compositions* called triangle and square products. These were first introduced by Bandler and Kohout in 1977 [11],[5],[2] and are referred to as the BK-products in the literature [22],[17],[18]. Their theory and applications have made substantial progress since then.

Triangle relational products together with fast fuzzy relational algorithms [7],[12] have been applied to various practical problems in a number of scientific fields: computer protection and AI [32], medicine, information retrieval, handwriting classification, architecture and urban studies, investment and control fields [43]. See the survey in [46] with a list of 50 selected references on the theory and applications. The relational methods combining linguistic labels with BK-products give a natural conceptual framework for knowledge representation and inference from imprecise, incomplete, or not totally reliable information in a consistent manner. All these approaches may be enriched by *extending* these to the realm of **interval computations**. For example or knowledge-based medical system CLINAID combines fuzzy relations, with methods of interval inference [43].

There are several types of product used to produce product-relations [11], [46],[6].

Definition 1 For arbitrary fuzzy relations in $[0, 1]$, R from the set X to Y , S from Y to Z define:

1. $R \circ S = (\forall x)(\forall z)(\exists y)(xRy \& ySz)$;
2. $R \triangleleft S = (\forall x)(\forall z)(\forall y)(xRy \rightarrow ySz)$;
3. $R \triangleright S = (\forall x)(\forall z)(\forall y)(xRy \leftarrow ySz)$;
4. $R \square S = (\forall x)(\forall z)(\forall y)(xRy \equiv ySz)$

Only the conventional \circ is associative. The triangle and square products, on the other hand, have important properties that give the power and versatility to our methods of relational analysis. \square is not associative at all, and the the following pseudo-associativities hold: [8]:

1. $Q \triangleleft (R \triangleright S) = (Q \triangleleft R) \triangleright S$,
2. $Q \triangleleft (R \triangleleft S) = (Q \circ R) \triangleleft S$,
3. $Q \triangleright (R \triangleright S) = Q \triangleright (R \circ S)$.

On the abstract side of non-fuzzy (crisp) relational algebras (RA), Tarski and his school have investigated the interrelationship of various RAs. Namely, representable (RRA), semiassociative (SA), weakly associative (WA) and non-associative (NA) relational algebras. Maddux [71] gives the following result:

$RRA \subset RA \subset SA \subset WA \subset NA$. These results do not say anything about representations of these extended relational algebras.

The BK-products defined over relational calculi give the constructive realization of the non-associative products for both crisp and fuzzy relations. Hence, non-associative products have representations and that these products offer various computational advantages. For example, the following **universal representation of preorders** is given for all the relations that are in the lattice $\mathcal{R}(X \rightsquigarrow X)$:

Theorem 2 [8] (a) R is a preorder if and only if $R = R \triangleright R^{-1}$.
(b) Every preorder or relations can be expressed that way.
(c) $R = R \sqcap R^{-1}$ if and only if R is an equivalence.

$\triangleleft, \triangleright, \sqcap$ products add the expressive power to the mathematics of relations. Very important for distributed knowledge networking is a constructive generalization of conventional homomorphisms defined constructively by BK-products:

Definition 3 Let F, R, G, S be the relations between the sets A, B, C, D such that $R \in \mathcal{R}(A \rightsquigarrow B)$. The conditions that (for all $a \in A, b \in B, c \in C, d \in D$) aFc and aRb and bGd imply cSd , will be expressed in any of the following ways: (i) $FRG; S$ are **forward compatible** (ii) F, G are **generalized homomorphisms** from R to S .

Theorem 4 Compatibility [8] 1. $FRG; S$ are forward compatible if and only if $F^T \circ R \circ G \sqsubseteq S$.
2. Formulas for computing the explicit compatibility criteria for F and G are: $FRG; S$ are forward-compatible iff $F \sqsubseteq R \triangleleft (G \triangleleft S^T)$

Similarly, the *backward compatibility* is defined and constructive conditions for relations both-way compatible (i.e. forward and backward) given [8]. *Both-ways* compatibility subsumes the conventional homomorphisms.

6.2 Dealing With Incomplete and Uncertain Information

Expert reasoning, decision making and actions have to operate on the background of uncertainty, incompleteness of information and conflicting evidence. These activities involve conceptual structures and dispositions that the experts intuitively use. It also involves reference to linguistic structures and their capability to handle multiple contexts. Understanding these underlying processes is difficult, yet essential in our attempts to aid expert decision making with computing and information processing technology.

Reasoning with uncertainty, incompleteness and also with conflicting evidence (to be called *reasoning with imperfect information*) cannot be fully devoid of the conceptual structures upon which the phenomena of vagueness, uncertainty, incompleteness of information and conflicting evidence operate. Identification of relevant conceptual structures, meta-frameworks, frameworks and knowledge contents of individual knowledge domains therefore plays the crucial role in such reasoning with imperfect information.

It is not only the syntactic structure and logical form, but also the complete linguistic structure, including the semantic contents and other semiotic aspects that is important. For this reason, even partial attempts at capturing the essential features of expert's competence in our information processing technology require new tools and new architectures that are capable of dealing fully with these aspects. Otherwise, the richness of the conceptual and linguistic world of a competent expert would be distorted beyond recognition, with side effects on our everyday life that can be disastrous. Thus we have to face the problem of systematizing and formalizing the semantics of expert actions acquisition, representation and utilization of knowledge in a new way. This approach has to have special features: it is to be generally **context-dependent** where **localized** relevant fragments of knowledge, form a system; and within this system, reasoning with imperfect information ought to operate adequately.

Such a unification requires a formal descriptive and computational approach that would put on equal footing the conceptual, linguistic and semiotic part with the mathematical computational part. One has also face the problem of conceptual conflicts [72] and of their resolution. This may leads directly to paraconsistent logics [33]. This unification can be achieved by relational method using BK-products of relations.

6.3 A Brief Overview of Fuzzy BK-Products

Mathematical definitions. Where R is a relation from X to Y , and S a relation from Y to Z , a *product relation* $R * S$ is a relation from X to Z , determined by R and S . There are several types

of product used to produce product-relations [11], [46]. Each product type performs a **different logical action** on the intermediate sets, as *each logical type* of the product enforces a *distinct specific meaning* on the resulting product-relation $R * S$. We have the following definitions of the products. In these definitions, R_{ij}, S_{jk} represent the fuzzy degrees to which the respective statements $x_i R y_j, y_j S_{jk} z_k$ are true.

PRODUCT TYPE	SET-BASED DEFINITION	MANY-VALUED LOGIC FORMULA
Circle product:	$x(R \circ S)z \Leftrightarrow xR \text{ intersects } Sz$	$(R \circ S)_{ik} = \bigvee_j (R_{ij} \wedge S_{jk})$
Triangle Subproduct:	$x(R \triangleleft S)z \Leftrightarrow xR \overset{\sim}{\subseteq} Sz$	$(RS)_{ik} = \bigwedge_j (R_{ij} \rightsquigarrow S_{jk})$
Triangle Superproduct:	$x(R \triangleright S)z \Leftrightarrow xR \overset{\sim}{\supseteq} Sz$	$(R \triangleright S)_{ik} = \bigwedge_j (R_{ij} \leftarrow S_{jk})$
Square product:	$x(R \square S)z \Leftrightarrow xR \cong Sz$	$(R \square S)_{ik} = \bigwedge_j (R_{ij} \equiv S_{jk})$

The table of definitions given above contains two different notational forms: (1) The notation using the concept of set inclusion and equality [4],[5]. (2) Many-valued logic based notation, which uses the logic connectives \wedge and \leftarrow . These two different forms of relational compositions are algebraically equivalent, producing the same mathematical results. Distinguishing these forms is, however, important when constructing fast and efficient computational algorithms.

The logical symbols for the logic connectives *AND*, *OR*, both *implications* and the *equivalence* in the above formulas represent the connectives of some many-valued logic, **chosen** according to the properties of the products required. *Harsh* fuzzy products (defined above) are distinguished from the family of *mean* products. Given the general formula $(R @ S)_{ik} ::= \#(R_{ij} * S_{jk})$, a mean product is obtained by replacing the outer connective $\#$ by \sum and normalizing the resulting product appropriately. The details of choice of the appropriate many-valued connectives are discussed in [6], [9],[10],[55],[46].

6.4 From Abstract Relations to Conceptual Meaning of Fuzzy Relational Structures

To have abstract relations is not enough. Each relations must possess a clearly defined meaning giving it a concrete practical linguistic interpretation within the domain of its application. This interpretation is provided by means of interpretable *linguistic labels* of special kind that will be called **semiotic descriptors**. The difference between ordinary linguistic label and a semiotic descriptor is that the latter kind is subject to some constraints determined by the ontology of the specific domain of engineering, science or business practices. The assignment of semiotic descriptors also partially determines the linguistic meaning of the composed relation computed by the relational product.

A simple, but useful general relational model relates semiotic descriptors of two kinds: *objects* and *properties*. To provide a semantic interpretation of the relations involved, we have to select the appropriate concepts from the domain of our interest as the names of the sets that enter into a relationship. Let us look at a simple example from the medical domain using concepts everyone is familiar with. The objects can be concrete (e.g. patients) or abstract (diseases), the properties of these objects being signs, symptoms or clinical test results or constructs of some clinical psychological tests.

If R is the relation between *patients* and *individual symptoms*, and S a relation between *symptoms* and *diseases*, $R * S$ will be a relation between *patients and diseases*. The diagnostic clinical interpretation of each distinct logical type (e.g. the triangular square product types) of these product-relations has a **distinct clinical meaning**:

$x(R \circ S)z$: degree to which patient x has at least one symptom of illness z .

$x(R \triangleleft S)z$: degree to which x 's symptoms are among those which characterize z .

$x(R \triangleright S)z$: degree to which x 's symptoms include all those which characterize z .

$x(R \square S)z$: degree to which x 's symptoms are exactly those of illness z .

6.5 Comparison of Structures and Investigating Their Properties

BK-relational product can be used to compare relational structures. Thus, if R is any relation (perhaps itself a product of other relations) from X to Y $\mathcal{R}(X \rightsquigarrow Y)$ and R^T its *transpose*, then the product $R * R^T \in \mathcal{R}(X \rightsquigarrow X)$ (where $*$ $\in \{\circ, \triangleleft, \triangleright, \square\}$) might exhibit some relational properties that reveal important characteristics of the source of information from which they were generated.

Here is an example still from the medical fields using the specific relations mentioned above:

$x_i(R \triangleleft R^T)x_k$: patient x_i 's symptoms are among these of x_k

$x_i(R \square R^T)x_k$: patient x_i has exactly the same symptoms as x_k

$y_j(R^T \triangleleft R)y_l$: whenever symptom y_j occurs, so does y_l (in this group of patients)

$y_j(S \square S^T)y_l$: symptom y_j characterizes exactly exactly the same diseases as does y_l

Relations so constructed might exhibit some important relational properties that reveal important characteristics and interrelationships of the source of information from which they were generated. Hence, methods for detecting various relational properties of given relations are important.

Relational properties, such as reflexivity, symmetry, and transitivity, and classes such as tolerances, equivalences and partial orders can be extracted from the linguistic information elicited by repertory grids.

Closures and interiors of relations [12],[11] play an important role in design of fast fuzzy relational algorithms used in our approach. The idea of *comparison of a relation with its closure* and *comparison of a relation with its interior* leads to design and to validity proofs of fast fuzzy relational algorithms (FFRA) that can test various local properties and also automatically discover the cases when the tested properties hold not only locally, but also globally.

In the general terms, the abstract theoretical tools supporting identification and representation of relational properties are fuzzy closures and interiors [12],[7]. Having such means for testing relational properties opens the avenue to linking the empirical structures that can be observed and captured by fuzzy relations with their abstract, symbolic representations that have well defined mathematical properties.

Standard relational properties (both crisp and fuzzy), such as reflexivity, symmetry, and transitivity, and classes such as tolerances, equivalences and partial orders are well understood. One essential drawback that both the crisp (non-fuzzy) and standard fuzzy theories of relational properties share is that they are defined as global, i.e. the properties must be shared by all the elements of a relation. The contributions of Bandler and Kohout crucial for multi-level knowledge representation investigated in this project was to provide an adequate **definition of locality** for both crisp (non-fuzzy) and fuzzy relations [11],[12] and develop software tools for computational testing of local properties and comparing partial relational structures.

6.6 Multidisciplinary Work

BK-relational products and fast fuzzy relational algorithms [7], [12] were applied in numerous multidisciplinary application: medical AI, [3], [6],[59], [9], [16], [56]; information retrieval [24], [60], [58], [45], handwriting classification [54], natural language understanding [74],[73],[75], generating efficient search strategies for resolution-based theorem proving [66], [29], cognitive structure analysis and other areas [44], [57], [31]. A very promising recent application is concerned with generating efficient search strategies for resolution-based theorem proving [66], [29] and in engineering and manufacturing [23], [38],[61], [48], [30], [41], [26], [19],[52], [20], [39], [70], [63],[76], [65], [42], [64], [51]

Relational computations are inherently parallel, hence well suited for developing data analysis, design and decision making tools on distributed networked systems. This is a feature necessary e.g. for distributed manufacturing.

7 Appendix 2: Publications resulting from DMI 952 5991 Project

[R1] P. Hájek and L.J. Kohout. Fuzzy implications and generalized quantifiers. *Internat. Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, 4(3):225–233, 1996. [Also partially

supported by the National Research Council COBASE grant for collaborative projects with former East European countries.]

- [R2] L.J. Kohout. An invited lecture on *Fuzzy Sets in Data Analysis*. Joint Statistical Conferences, American Statistical Association. August 1996. (30 minutes)
- [R3] L.J. Kohout and E. Kim. Relational algorithms for theoretical and empirical evaluation of systems of fuzzy connectives. In D. Kraft, editor, *Proc. of IEEE Internat. Conf. on Fuzzy Systems*, IEEE, New York, 1996, vol. 2, pp. 918-923.
- [R4] E. Kim, L.J. Kohout, B. DuBrosky and W. Bandler. Use of fuzzy relations for affordability decisions in high technology. In *Applications of Artificial Intelligence in Engineering XI*. R.A. Addey, G. Rzevski and A.K. Sunol (eds.). Computational Mechanics Publications, Boston 1996.
- [R5] L.J. Kohout. Decision-Making with Incomplete Information in an Integrated Product and Process Development Enterprise – A Management Decision Tool for Cost Modeling and Affordability Applications. *A presentation given at MOTI/ManTech Meeting, Washington DC, October 19, 1996, 11.10am – 12.10pm*. Copies of transparencies and a report distributed to the participants of the meeting (Total 47 pages).
- [R6] L.J. Kohout. Tutorial materials prepared for 1996 Internat. Multidisciplinary Conference – Intelligent Systems: A Semiotic Perspective. National Institute of Standards and Technology, Gaithersburg, MD, 20-23 October 1996. 47 pages.
- [R7] E. Kim and L.J. Kohout. Generalized morphisms: A tool for evaluation of adequacy of logic connectives used in symbolic and soft computing. In *1st On-Line Workshop on Soft Computing (August 19-30, 1996)*. Electronic version: <http://www.bioele.nuee.nagoya-u.ac.jp/wsc1>. The Society of Fuzzy Theory and Systems (SOFT), 1996. Also appears in hardcopy Proceedings (published by "Nagoya University, Nagoya, 464-01, Japan) pp. 222-227.
- [R8] B. Dubrosky, L.J. Kohout, R.M. Walker, E. Kim, and H.P. Wang. Use of fuzzy relations for advanced technology cost modeling and affordability decisions. In *Proc. 35th AIAA Aerospace Sciences Meeting and Exhibit (Reno, Nevada, January 6-9, 1997)*. AIAA, January 1997, Document AIAA 97-0079, pp. 1-12.
- [R9] Kohout, L.J. and Dubrosky, B. and Wang, H.P. and Zenz, G. and Zhang, C. Decision-making with incomplete information in an integrated product and process development enterprise – A management decision tool for cost modeling and affordability applications. In: *Proc. of NSF Grantees Conference (Seattle, WA, January 7-10, 1997)*. T. Woo, (ed.), pages 401-402.
- [R10] E. Kim, L.J. Kohout and B.M. DuBrosky. Linguistic Models of Cost-Affordability for Aeronautics Industry based on Semiotic Descriptors and Fuzzy Relational Computations. In: *Proc. of 1997 Internat. Joint Conf. on Information Sciences JCIS'97, March 1-5, 1997, Research Triangle Park, NC*, vol. 2 – Computational Intelligence, Neural Network & Semiotics. P.P. Wang (ed.), pp. 241-244.
- [R11] Panel Discussion: *Probability, Possibility, and Fuzzy Logic Approaches to Affordability in the Aeronautic Industry*. Organized by L.J. Kohout. Presented at: *1997 Internat. Joint Conf. on Information Sciences JCIS'97*, March 1-5, 1997, Research Triangle Park, NC Monday, March 3, 14.10pm – 15.40pm. Co-chaired by: Prof. L.J. Kohout, FSU and Dr. Barbara DuBrosky, Pratt & Whitney.
- [R12] Kohout, L.J. and Zenz, G. Activity Structures and Triangle BK-Products of Fuzzy Relations – a Useful Modeling and Computational Tool in Value Analysis Studies. In: *Proc. of IFSA 1997 – The world Congress of Internat. Fuzzy Systems Association*, vol. IV, pp. 211-216.

- [R13] Kohout, L.J. and Kim, E. Group Transformations of Systems of Logic Connectives. In: *Proc. 6th IEEE International Conference on Fuzzy Systems*, vol. I, pp.157-162.
- [R14] Zhang, C., Wang, B. and Po-Kang Ting Cost Estimation in an Integrated Product and Process Development Enterprise. Techn. Report No. 97-01 (January 1997). Dept. of Industrial Engineering, FAMU-FSU College of Engineering.
- [R15] Noe C.S. Using BK-Products of Relations for Management of Granular Structures in Relational Architectures for Knowledge-Based Systems.
Ph.D. Dissertation, Dept. of Computer Science, Florida State University, May 1997.
Major professor: Dr. L.J. Kohout.
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An Invited paper presented at AIAA/SAE World Aviation Congress October 13-16, 1997, Anaheim, CA.
- [R17] Kohout, L.J. Relations and Their Products (A Wyllis Bandler Memorial Lecture). Invited plenary lecture. Presented at: *1997 Internat. Joint Conf. on Informat. Sci. JCIS'97*, March 1-5, 1997, Research Triangle Park, NC
- [R18] Kohout, L.J., Kim, E. The Role of Semiotic Descriptors in Relational Representation of Fuzzy Granular Structures.
An Invited paper presented at ISAS '97 Intelligent Systems and Semiotics: A Learning Perspective (September 22-25 1997) at National Institute of Standards and Technology, Gaithersburg.
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Proc. of ISIC/CIRA/ISAS '98 (IEEE International Symposium on Intelligent Control (ISIC), International Symposium on Computational Intelligence in Robotics and Automation (CIRA), Intelligent Systems and Semiotics (ISAS)). National Institute of Standards and Technology (NIST) Gaithersburg, Maryland U.S.A. (September 14-17, 1998).
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