Chapter 12

Omnidirectional Vision

MRGCV Computer Vision

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1542

Omnidirectional Vision

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2. Distortion models
3. Radially symmetric models
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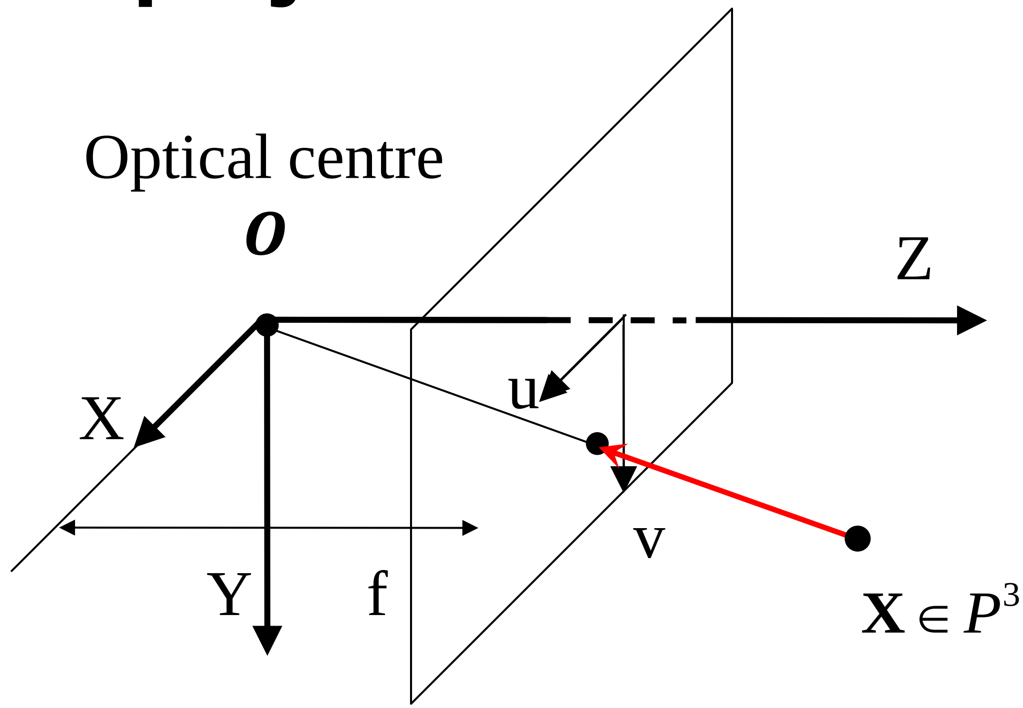
PinHole linear projection

$$\mathbf{u} = \begin{pmatrix} u \\ v \\ s \end{pmatrix} = \lambda \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}}_{{}^{c}\mathbf{T}_{abs}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\mathbf{X}_{abs}}$$

$$\mathbf{K} = \begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$



c_x, c_y principal point

$\alpha_x = \dfrac{f}{d_x}$ horizontal focal length (pixels)

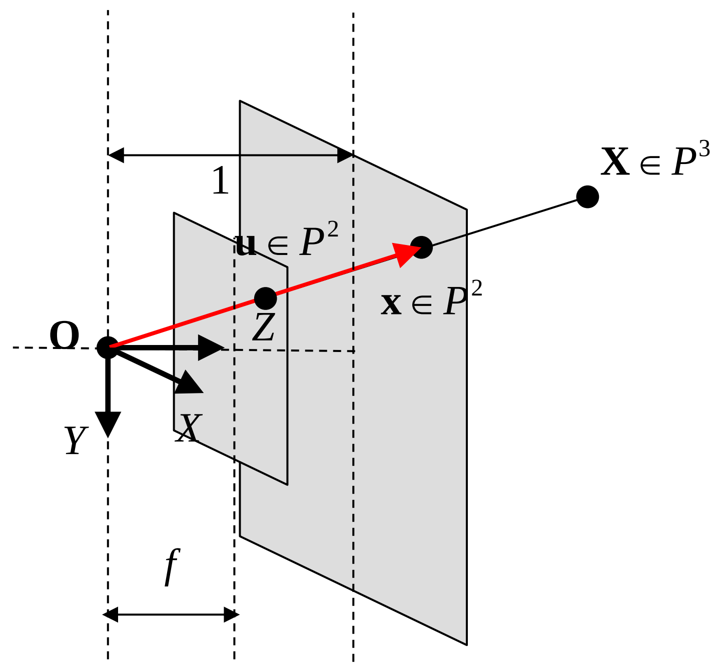
$\alpha_y = \dfrac{f}{d_y}$ vertical focal length (pixels)

Pinhole linear unprojection

$$\mathbf{x} = \mathbf{K}^{-1}\begin{bmatrix} u \\ v \\ s \end{bmatrix} \text{, such that } \mathbf{x} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} \in P^2$$

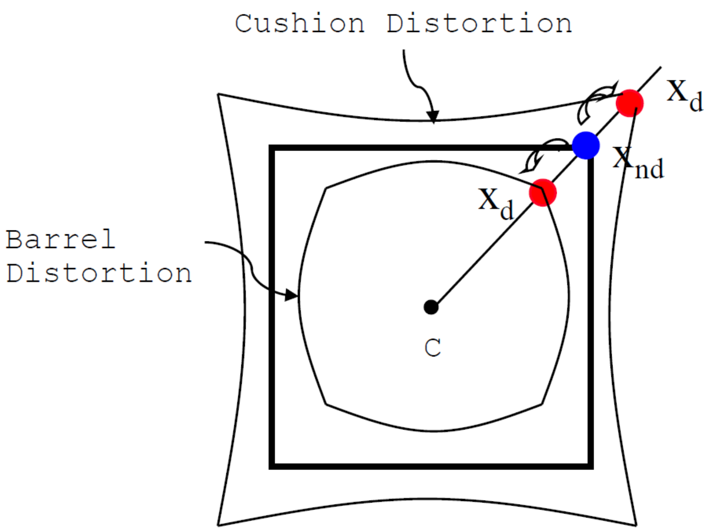
$$\mathbf{K} = \begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{x} \sim \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \mathbf{K}^{-1}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad \mathbf{x} \parallel \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \qquad \mathbf{x} \sim Z\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



Lens distortions models

- Real lenses do not exactly follow the pinhole model
- The main component is radial distortion



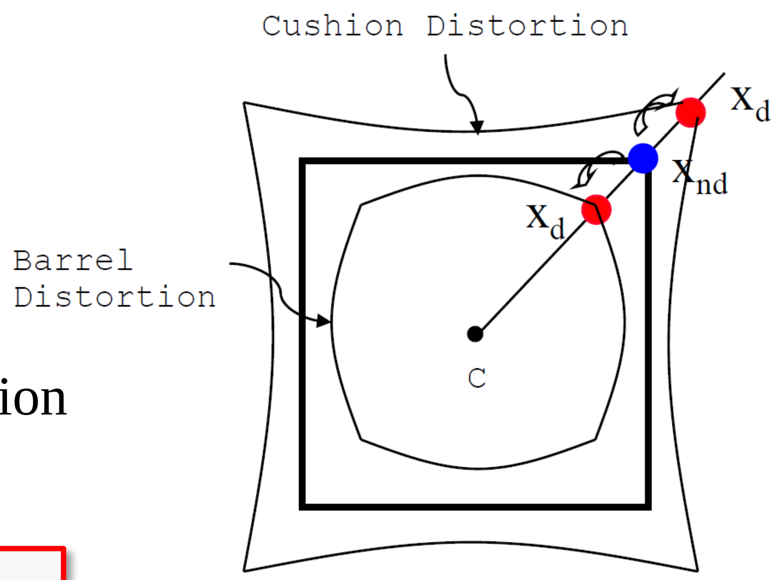
Lens distortions forward model

- From undistorted to distorted coordinates a polynomial expression
- From distorted to undistorted → solving a system of non-linear equations
- Bouget model/OpenCV model

$$
\left.
\begin{aligned}
r_u^2 &= x_u^2 + y_u^2 \\
d_{ry} &= y_u\left(k_1 r_u^2 + k_2 r_u^4 + k_3 r_u^6\right) \\
d_{rx} &= x_u\left(k_1 r_u^2 + k_2 r_u^4 + k_3 r_u^6\right)
\end{aligned}
\right\}\text{Radial distortion}
$$

$$
\left.
\begin{aligned}
d_{py} &= p_2\left(r_u^2 + 2y_u^2\right) + 2p_1 x_u y_u \\
d_{px} &= p_1\left(r_u^2 + 2x_u^2\right) + 2p_2 x_u y_u
\end{aligned}
\right\}\text{Tangential distortion}
$$

$$\boxed{x_d = x_u + d_{rx} + d_{px} \qquad y_d = y_u + d_{ry} + d_{py}}$$



Lens distortions forward model

- From undistorted to distorted coordinates a polynomial expression
- Cheap for bundle adjustment, expensive for triangulation
- OpenCV model

$$\begin{pmatrix} x_u \\ y_u \\ 1 \end{pmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}}_{{}^{c}\mathbf{T}_{abs}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\mathbf{X}_{abs}}$$

$$r_u^2 = x_u^2 + y_u^2$$

$$\left.\begin{aligned} d_{ry} &= y_u\left(k_1 r_u^2 + k_2 r_u^4 + k_3 r_u^6\right) \\ d_{rx} &= x_u\left(k_1 r_u^2 + k_2 r_u^4 + k_3 r_u^6\right) \end{aligned}\right\}\text{Radial distortion}$$

$$\left.\begin{aligned} d_{py} &= p_2\left(r_u^2 + 2y_u^2\right) + 2p_1 x_u y_u \\ d_{px} &= p_1\left(r_u^2 + 2x_u^2\right) + 2p_2 x_u y_u \end{aligned}\right\}\text{Tangential distortion}$$

$$x_d = x_u + d_{rx} + d_{px} \qquad y_d = y_u + d_{ry} + d_{py}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d \\ y_d \\ 1 \end{pmatrix}$$

Lens distortions Tsai backward model

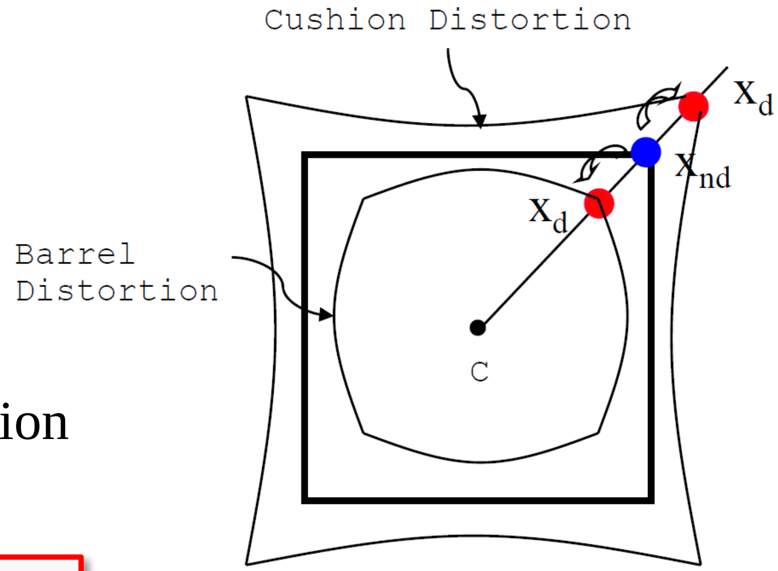
- From distorted to undistorted coordinates a polynomial expression
- From undistorted to distorted → solving a system of non-linear equations
- Distortion applied on image plane in millimeters
- Tsai, Photomodeler

$$r_d^2 = x_d^2 + y_d^2$$

$$\left.\begin{aligned} d_{rx} &= x_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \\ d_{ry} &= y_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \end{aligned}\right\}\text{Radial distortion}$$

$$\left.\begin{aligned} d_{px} &= p_1\left(r_d^2 + 2x_d^2\right) + 2p_2 x_d y_d \\ d_{py} &= p_2\left(r_d^2 + 2y_d^2\right) + 2p_1 x_d y_d \end{aligned}\right\}\text{Tangential distortion}$$

$$\boxed{x_u = x_d + d_{rx} + d_{px} \qquad y_u = y_d + d_{ry} + d_{py}}$$



Lens distortions Tsai backward model

- From distorted to undistorted coordinates a polynomial expression
- Cheap for triangulation, expensive for bundle adjustment

$$\begin{pmatrix} x_d \\ y_d \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{d_y} & 0 & c_x \\ 0 & \frac{1}{d_y} & c_y \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

$$r_d^2 = x_d^2 + y_d^2$$

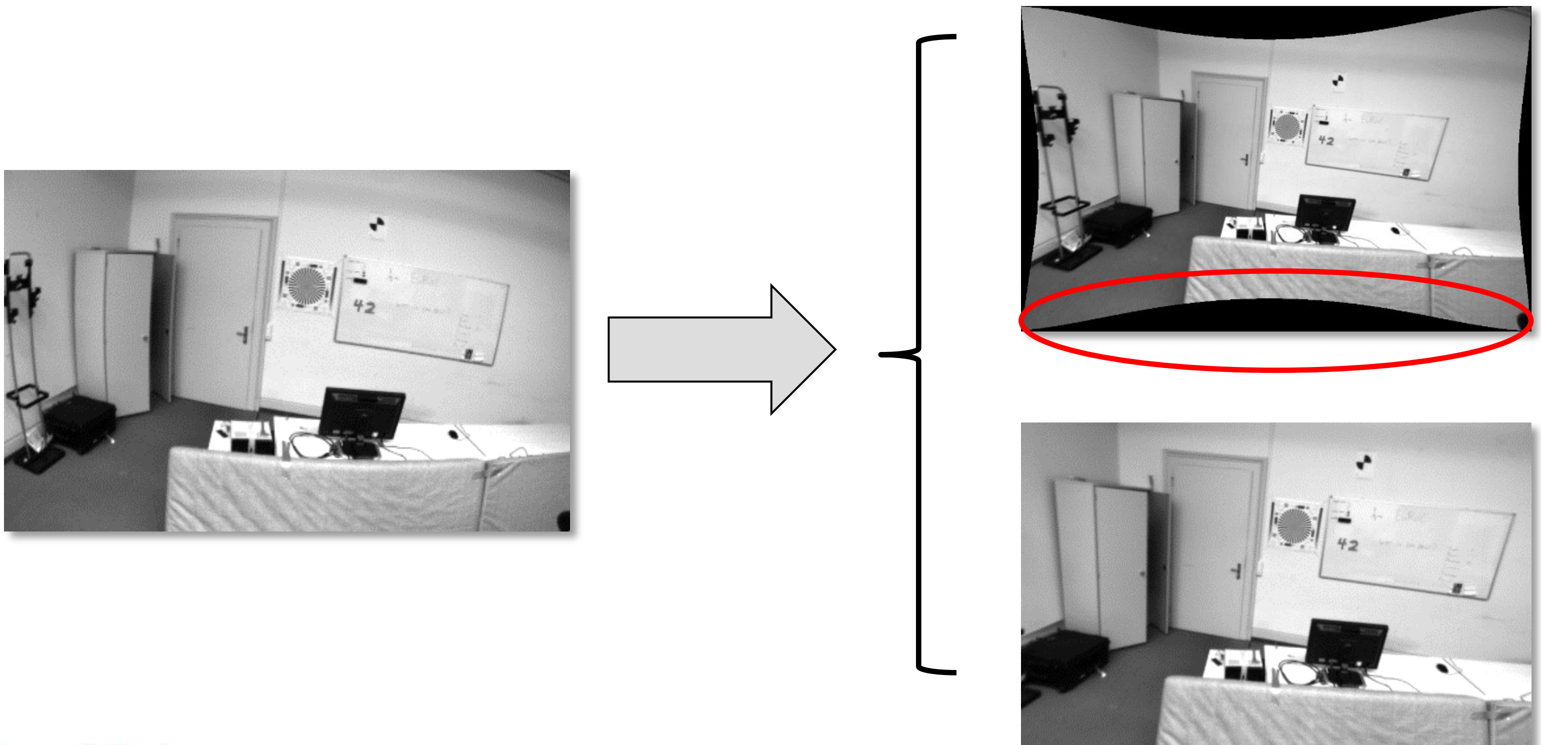
$$\left.\begin{aligned} d_{rx} &= x_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \\ d_{ry} &= y_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \end{aligned}\right\}\text{Radial distortion} \qquad \left.\begin{aligned} d_{px} &= p_1\left(r_d^2 + 2x_d^2\right) + 2p_2 x_d y_d \\ d_{py} &= p_2\left(r_d^2 + 2y_d^2\right) + 2p_1 x_d y_d \end{aligned}\right\}\text{Tangential distortion}$$

$$x_u = x_d + d_{rx} + d_{px} \qquad y_u = y_d + d_{ry} + d_{py}$$

$$\mathbf{x} \sim \begin{pmatrix} \frac{1}{f} & 0 & 0 \\ 0 & \frac{1}{f} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_u \\ y_u \\ 1 \end{pmatrix}$$

Undistorting images

- Distorted images can be undistorted before applying computer vision algorithms.
- If we want to take advantage of the full field of view of the camera we have to deal with masks.



Limitations of undistorting images

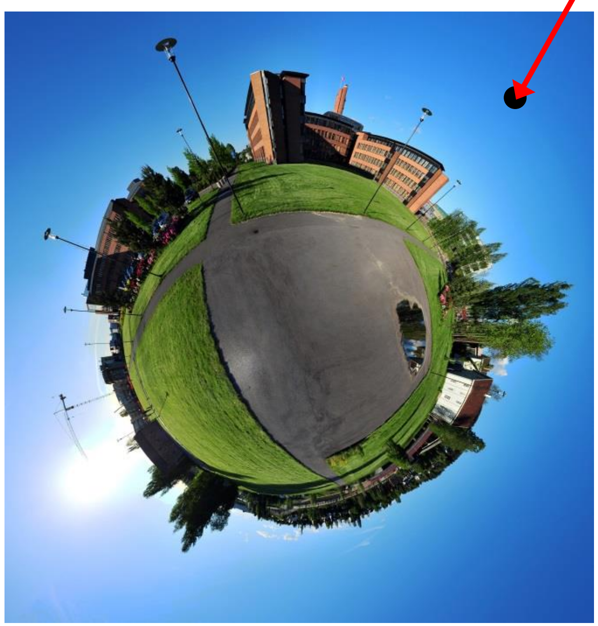
- In the peripheral region (high FOV) the size of the objects is highly deformed.

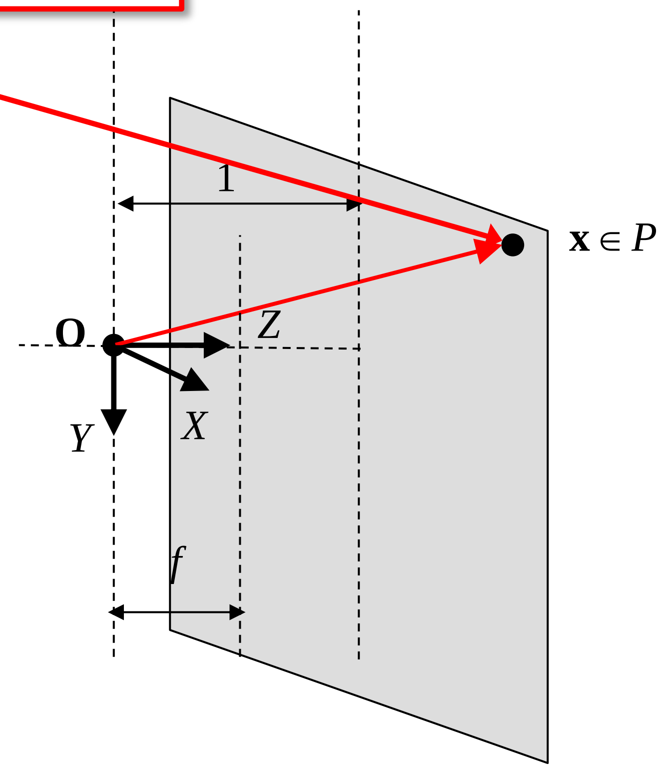
Figure courtesy of Juan José Gómez Rodríguez

Limitations of distortion models

- They are not able to manage fields of view greater than 180 degrees.

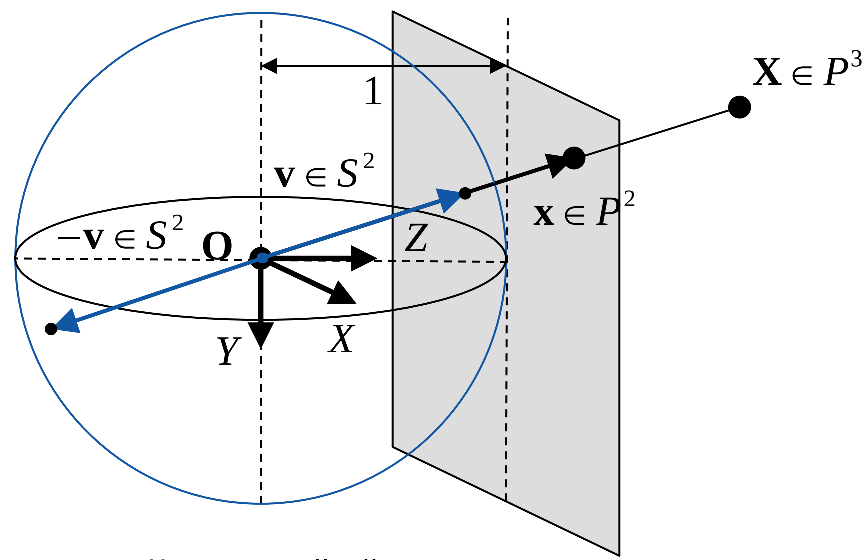
This point is looking at the back of the camera but the model is assuming that is in front of the camera





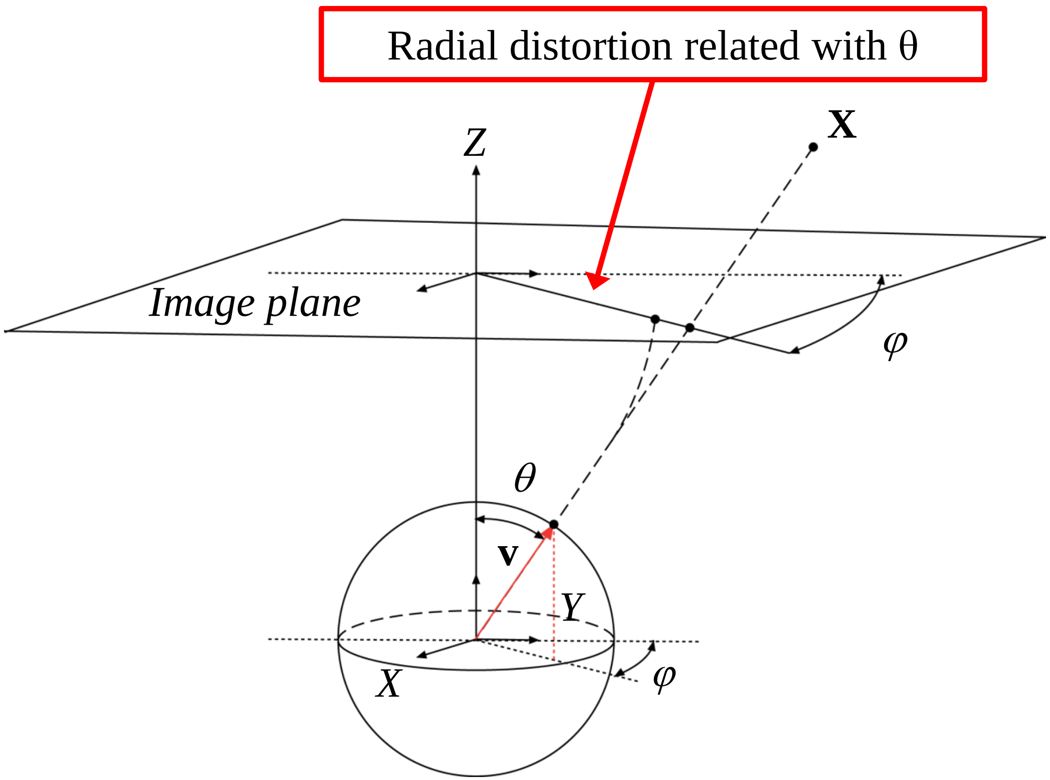
Spherical projection

- Projection and unprojection on the unitary sphere
- **x** and –**x** are the same homogenous points and correspond to the same pixel.
- **v** and –**v** correspond to a different pixel.
- Sense of direction vector v has meaning.
- Norm of **v** can be greater than 1.
- Can model omnidirectional imaging.



$\mathbf{v} \parallel \mathbf{x}$ $\quad$ $\|\mathbf{v}\| = 1$

Radially symmetric models



In a pinhole $r = f\tan\theta$

Projection

$$\mathbf{X} \in P^3$$

$$(\varphi, \theta)$$

$$r = f(\theta)$$

$$\mathbf{x}_c = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ 1 \end{pmatrix}$$

$$\mathbf{u} \sim \mathbf{K}_c \mathbf{x}_c$$

Unprojection

$$\mathbf{x}_c \sim \mathbf{K}_c^{-1}\mathbf{u}$$

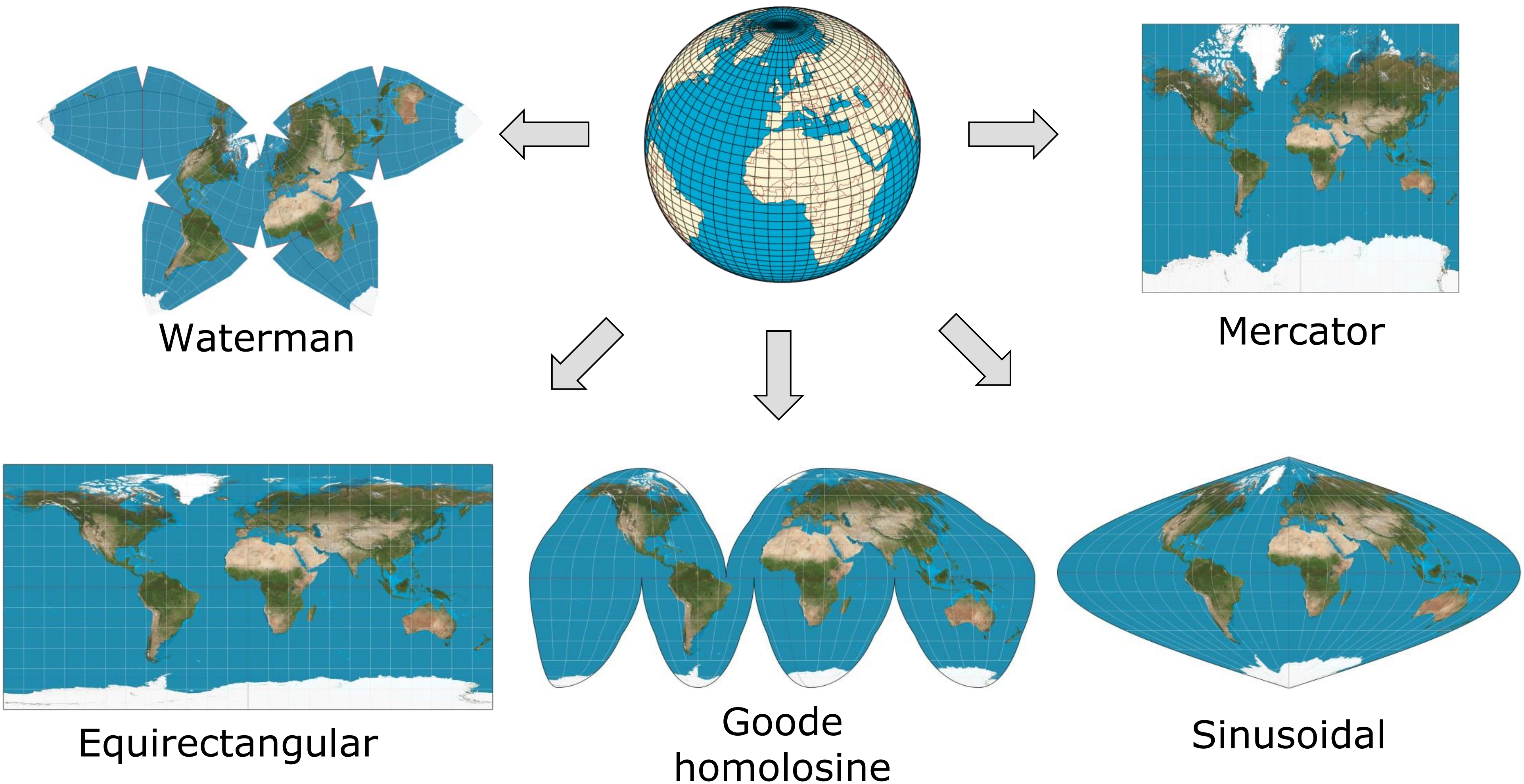
$$r = \sqrt{\frac{x_c^2 + y_c^2}{z_c^2}}$$

$$\varphi = \arctan 2(y_c, x_c)$$

$$\theta = f^{-1}(r)$$

$$\mathbf{v} = \begin{pmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ \sin\theta \end{pmatrix} \in S^2$$

Non-radially symmetric models



Omnidirectional cameras: Technologies

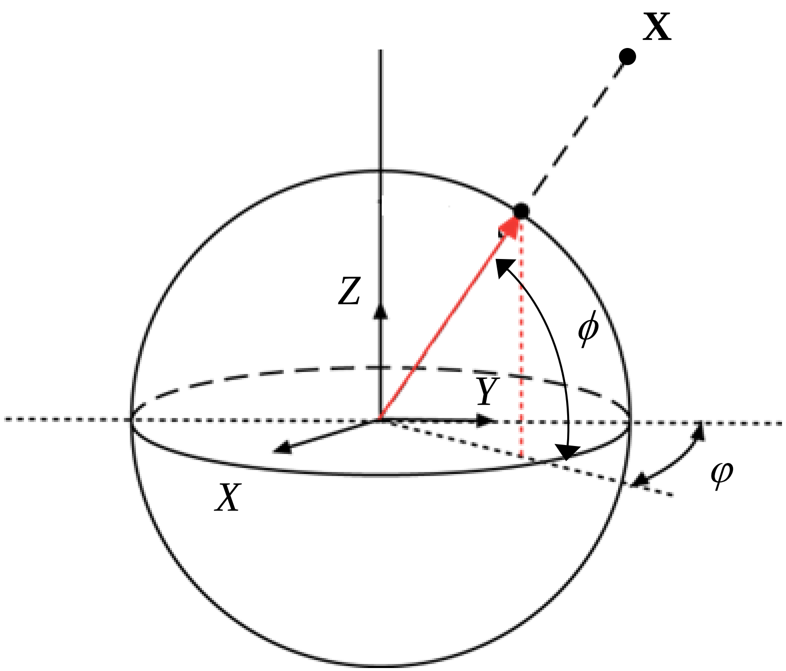
Dioptric

Catadioptric

Multi-camera

Panoramas: Equirectangular projection

$$\mathbf{X} \approx \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T, \quad \mathbf{X} \in P^3$$



$$\varphi = \arctan(Y, X) \qquad \phi = \arcsin\left(\frac{Z}{\sqrt{X^2 + Y^2}}\right)$$

$$u = u_0 + \frac{(\varphi + \pi)n}{2\pi} \qquad v = v_0 + \frac{\left(\phi - \pi/2\right)m}{\pi}$$

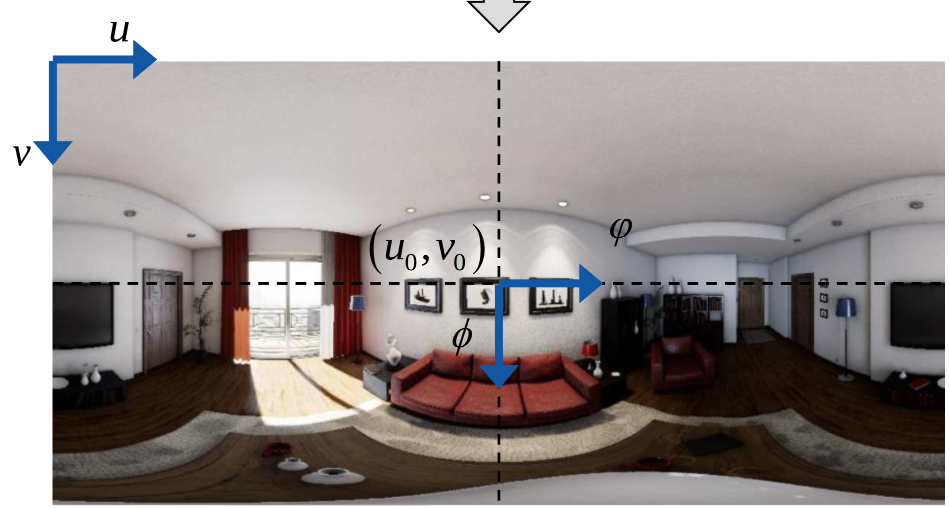
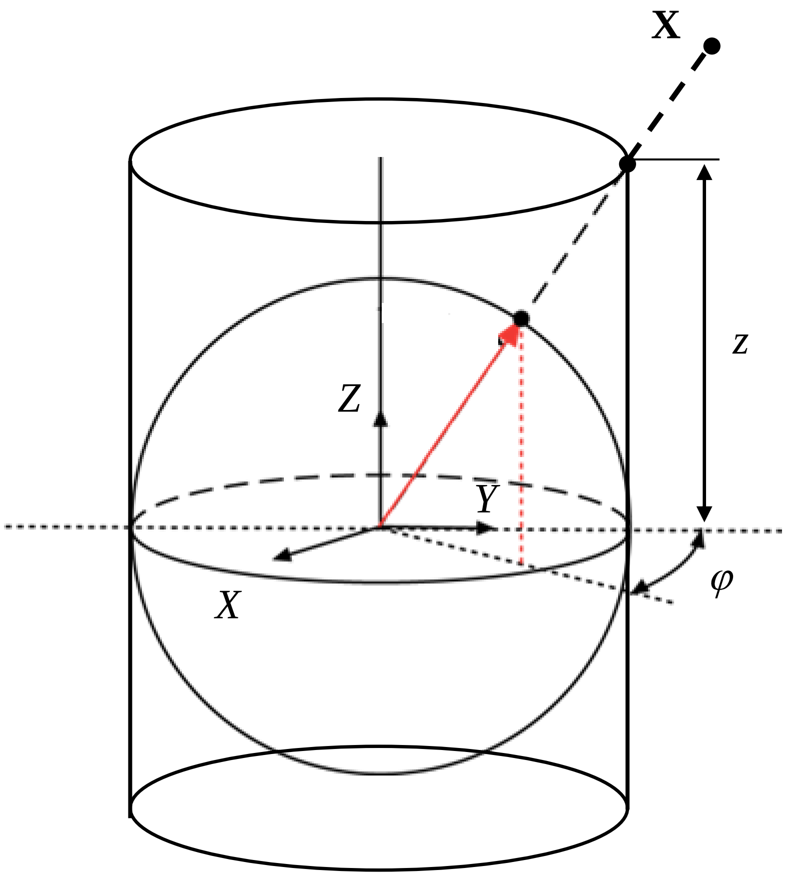


Figure courtesy of Bruno Berenguel

Panoramas: Cylindrical projection



$$\mathbf{X} \approx \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T, \quad \mathbf{X} \in P^3$$

$$\varphi = \arctan(Y, X) \qquad z = \frac{Z}{\sqrt{X^2 + Y^2}}$$

$$u = u_0 + \frac{(\varphi + \pi)\, n}{2\pi} \qquad v = v_0 + f_v z$$

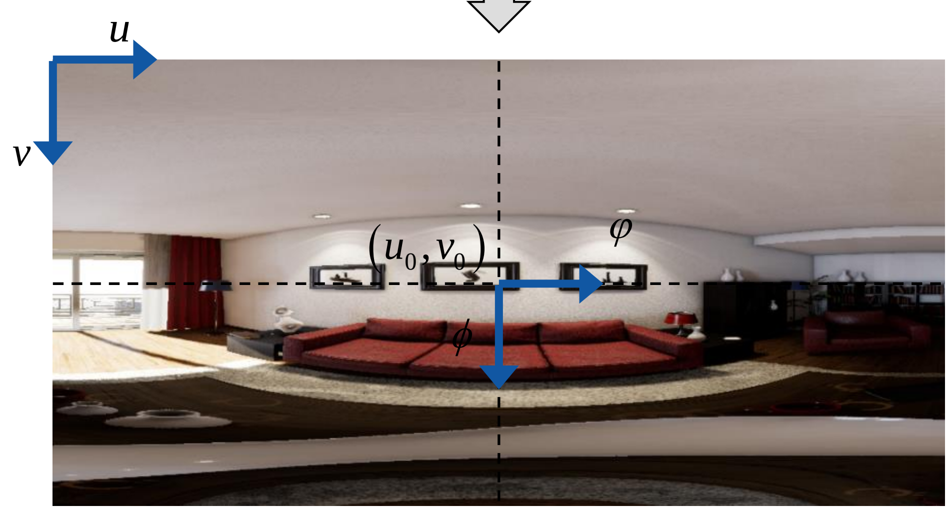
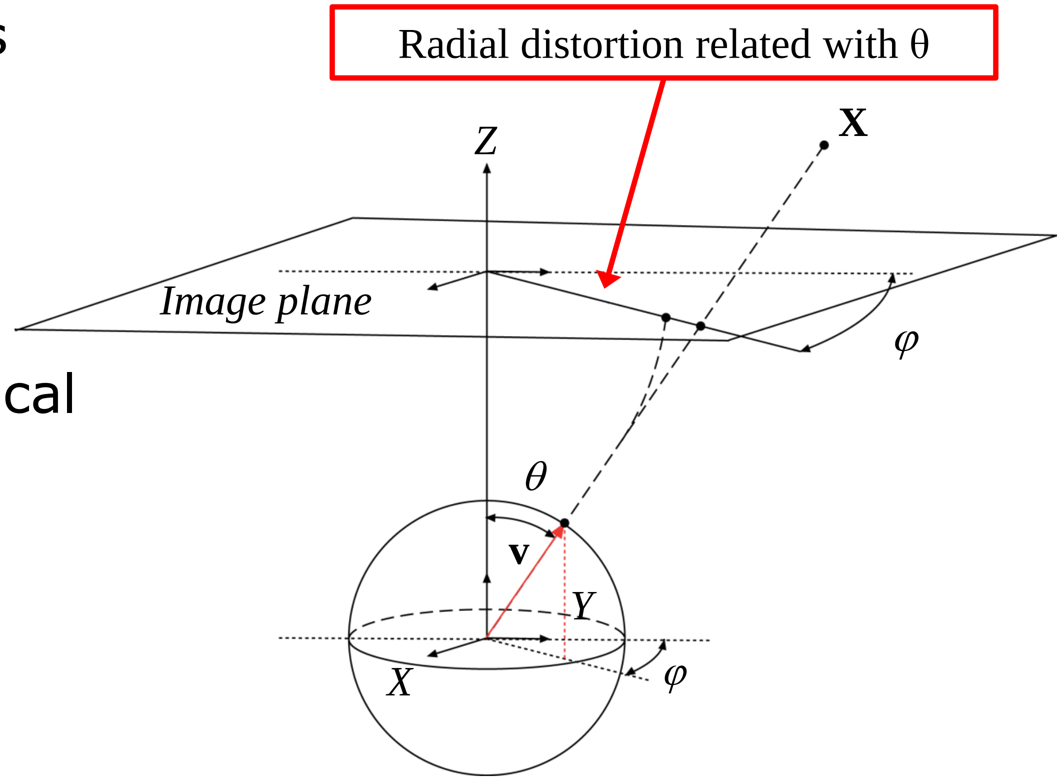
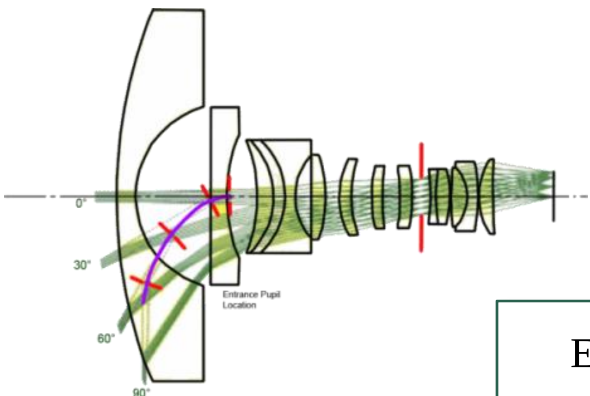


Figure courtesy of Bruno Berenguel

Dioptric systems: Classical models

- Radially symmetric models
- Invertible models
- Manufactures design the fisheyes in order to follow these models
- Almost Central (single optical center)

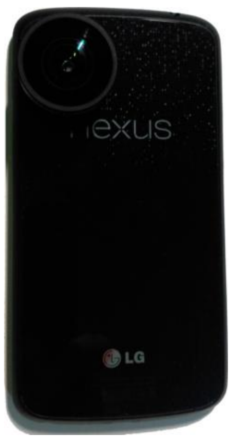




Equiangular Fisheye	Stereographic Fisheye	Orthogonal Fisheye	Equisolid Fisheye
$r = f\theta$	$r = 2f\tan\left(\frac{\theta}{2}\right)$	$r = f\sin(\theta)$	$r = 2f\sin\left(\frac{\theta}{2}\right)$

Dioptric systems: Classical models

- Equiangular fisheye



$$r = f\theta$$

$$2\theta_{\max} = FOV$$

Stereographic projection

- Classical mapping: Planisphaerium
- Maps S2\[0,0,1] on a plane

$$r = 2f\tan\left(\frac{\theta}{2}\right)$$

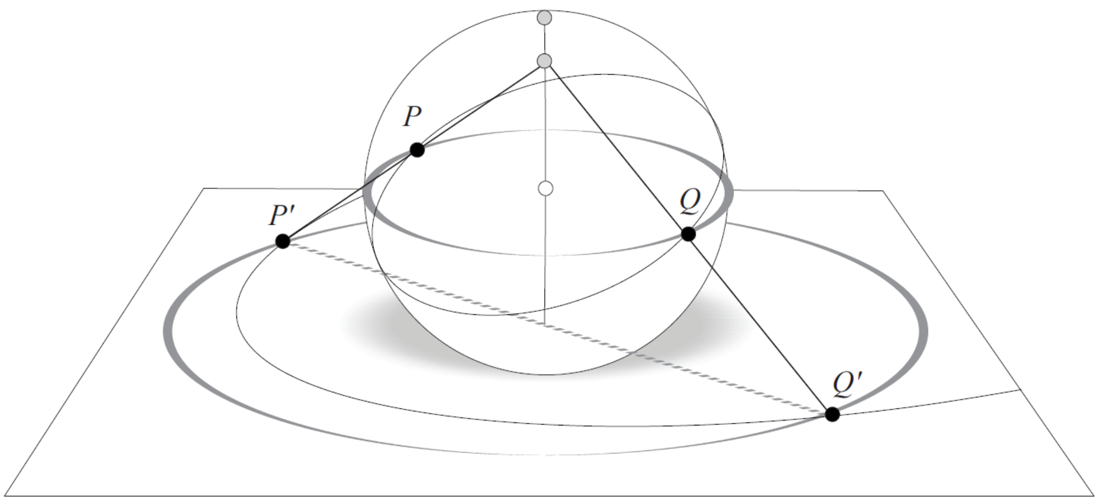


Figure courtesy of Micusik Two-View Geometry of Omnidirectional Cameras PhD Thesis, Branislav Micusık, 2004

Dioptric systems: Classical models

- Orthographic fisheye $r = f \sin(\theta)$

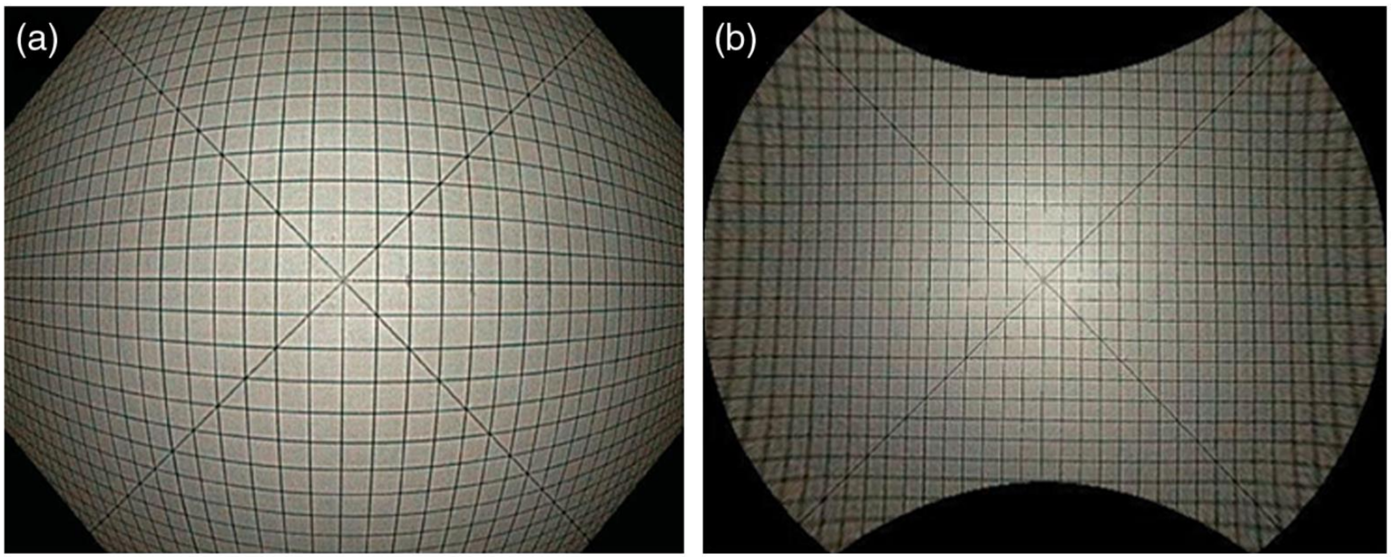


Fig. 15 Original and corrected images: (a) Original distorted image taken with the endoscope and (b) the corrected image according to M_{LR}.

Figure courtesy of Wang, Q., Cheng, W. C., Suresh, N., & Hua, H. (2016). Development of the local magnification method for quantitative evaluation of endoscope geometric distortion. *Journal of Biomedical Optics*,

Dioptric systems: Classical models

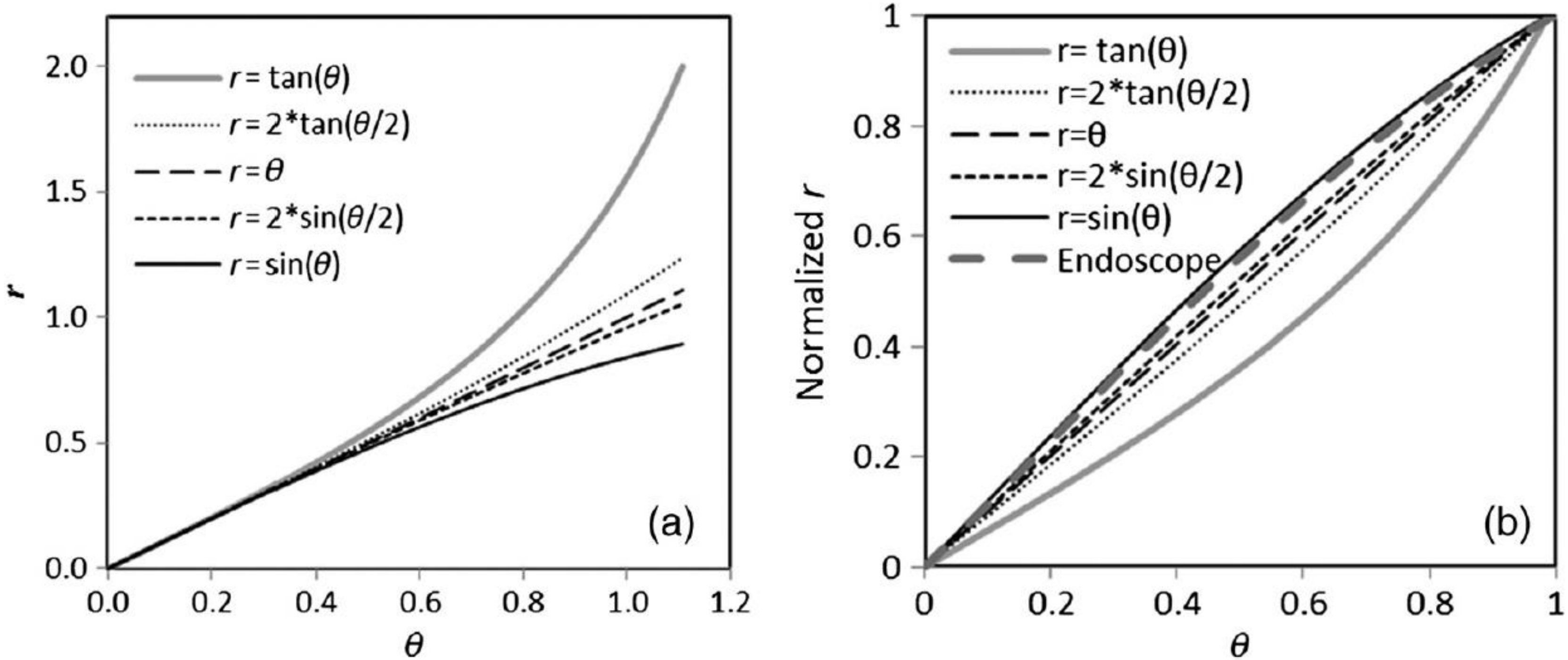


Fig. 17 Some projection methods for lenses with a wide FOV (assuming f is 1): (a) r versus θ, and (b) normalized r versus θ.

Figure courtesy of Wang, Q., Cheng, W. C., Suresh, N., & Hua, H. (2016). Development of the local magnification method for quantitative evaluation of endoscope geometric distortion. *Journal of Biomedical Optics*,

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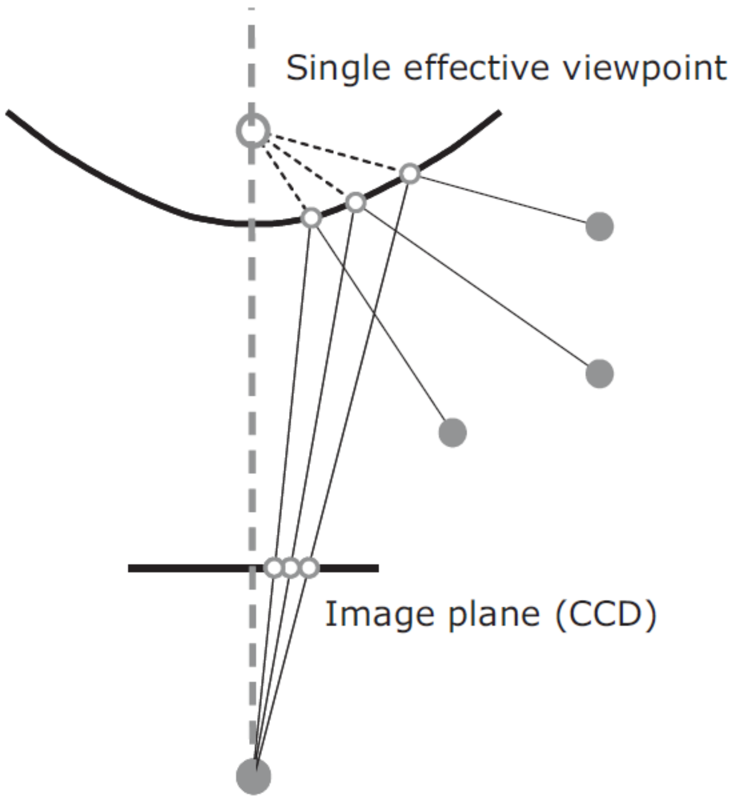
Catadioptric Cameras
Central vs Non-Central



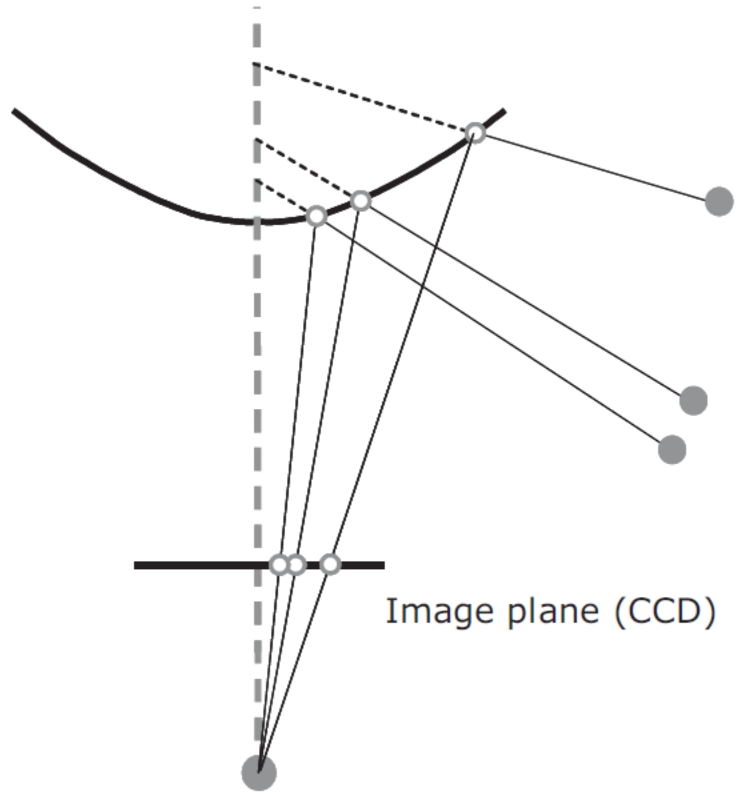
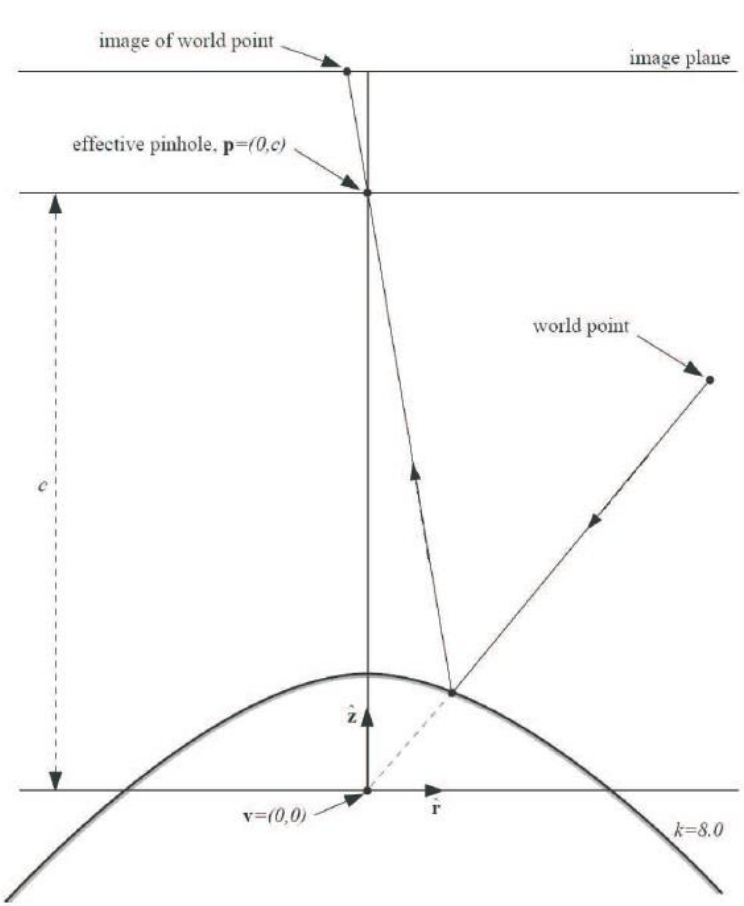
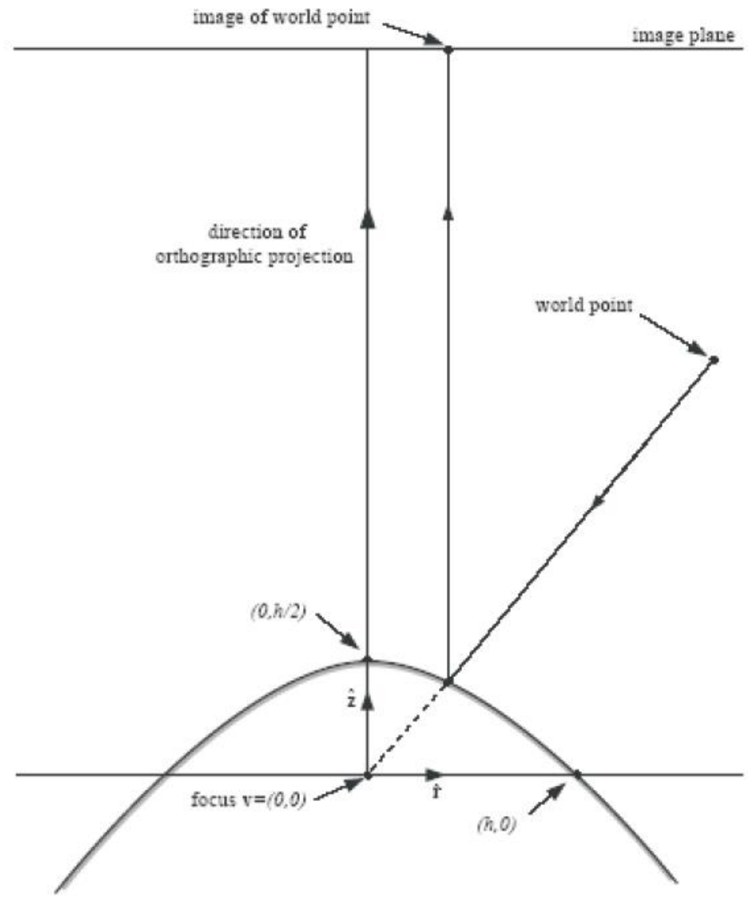


Figure cortesy of Davide Scaramuzza, Omnidirectional vision: From Calibration To Robot Estimation. Thesis, 2004

Central Catadioptric Systems



Hyperboloidal mirror



Paraboloidal mirror

Figure cortesy of Davide Scaramuzza, Omnidirectional vision: From Calibration To Robot Estimation. Thesis, 2004

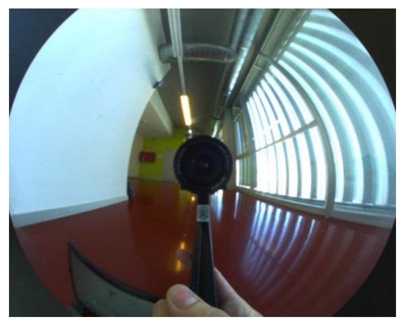
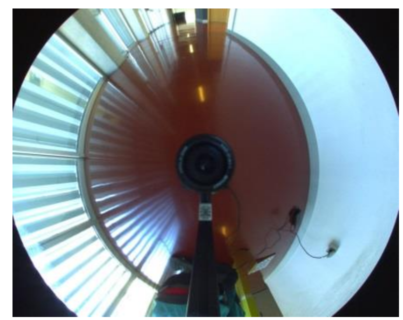
Central Catadioptric Systems: Technologies

- Para-catadioptric system: Composed by an orthographic camera and a parabolic mirror



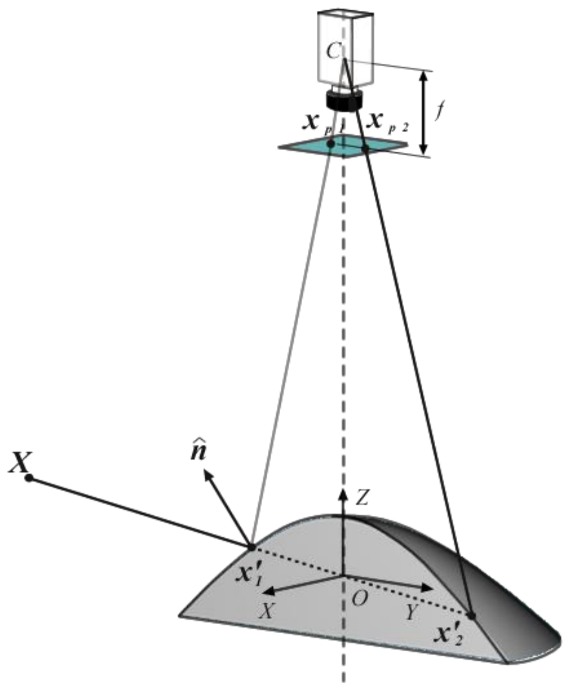
- Hyper-catadioptric system: Composed by a perspective camera and an hyperbolic mirror.



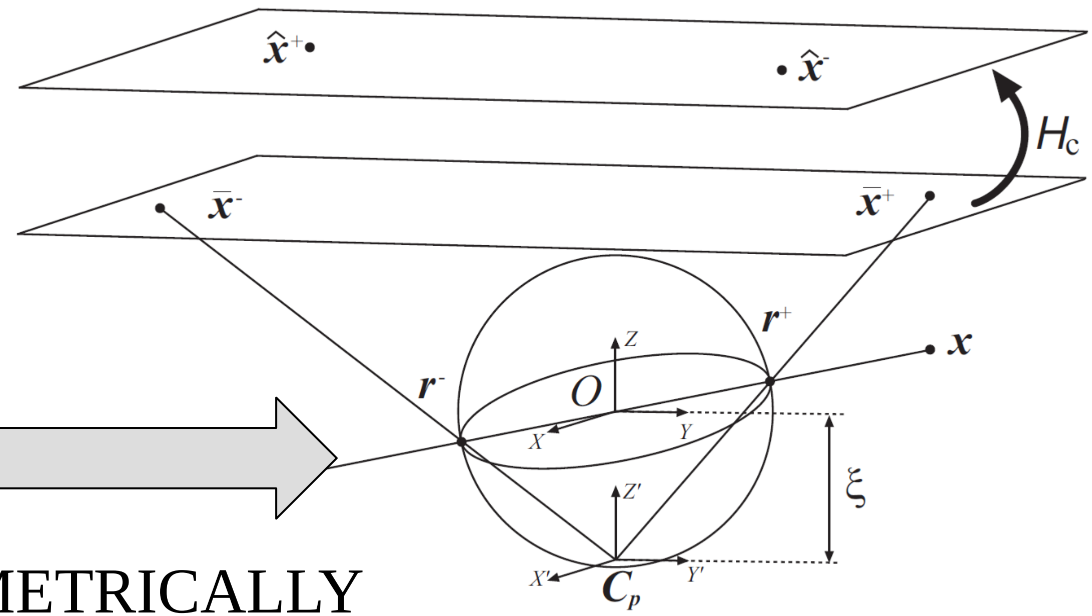



Physic model and sphere model

Reflection model
based on Snell's law

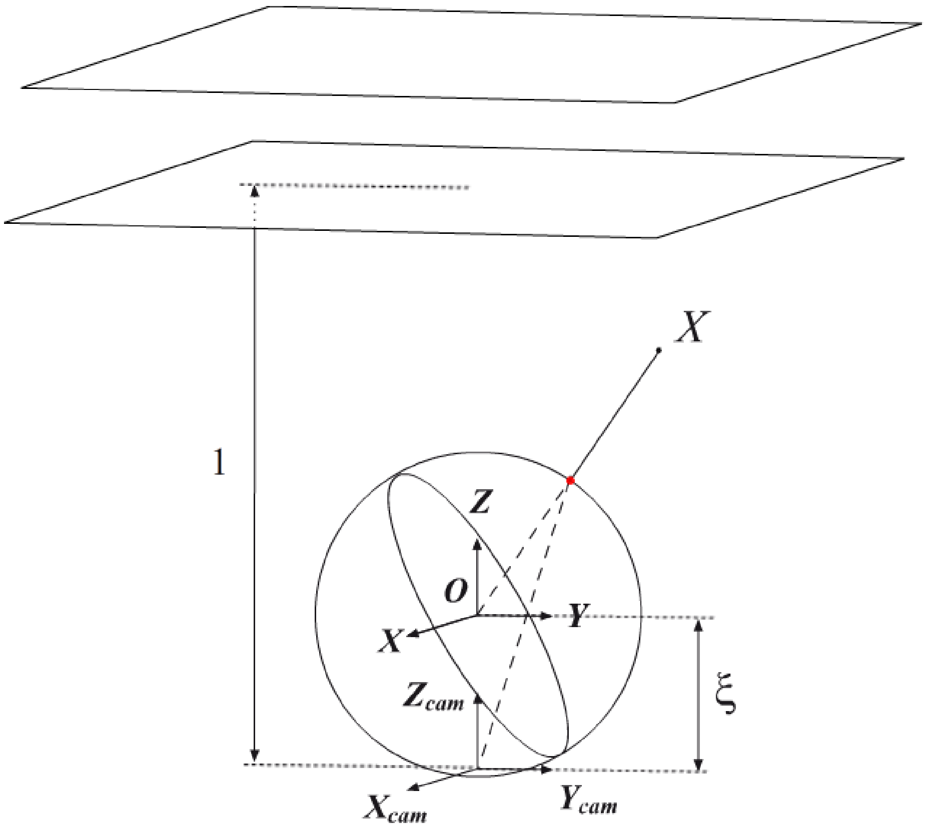


Abstract model projecting on
sphere and then projecting on
a plane (no reflections at all)



GEOMETRICALLY
EQUIVALENT

Sphere projection model for catadioptric systems



$$\mathbf{X} \approx \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T, \quad \mathbf{X} \in P^3$$

$$\boxed{\hbar(\mathbf{x}) = \begin{pmatrix} X \\ Y \\ Z + \xi\sqrt{X^2 + Y^2 + Z^2} \end{pmatrix}}$$

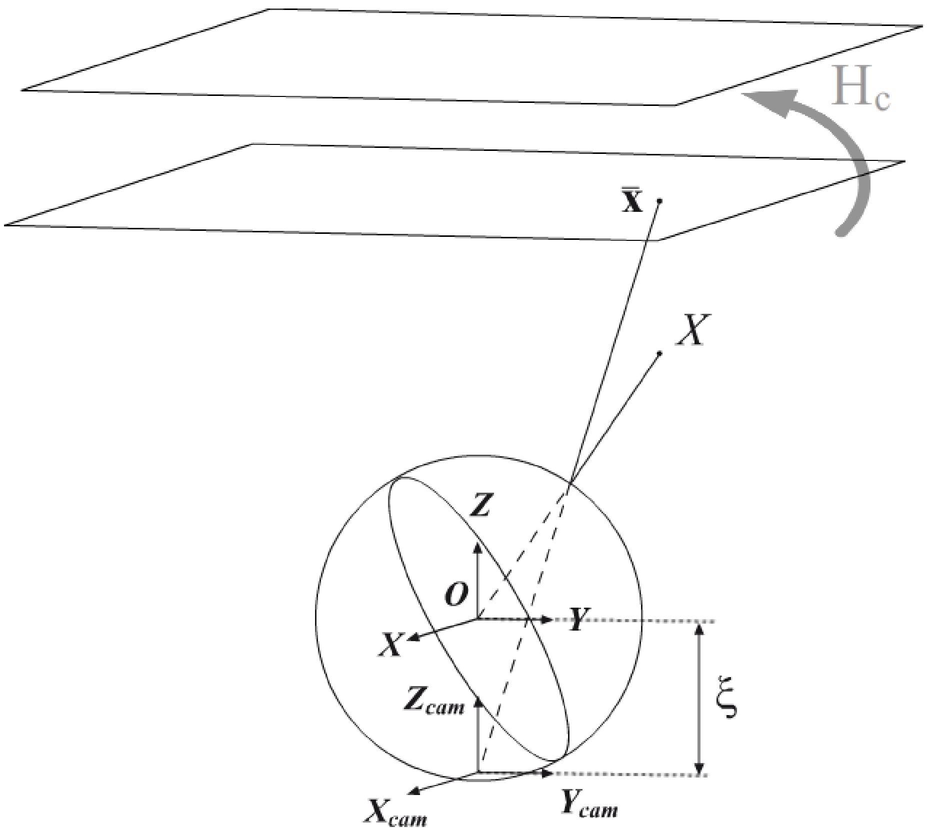
$$\overline{\mathbf{x}} = \hbar(\mathbf{x})$$

$$H_c = \underbrace{\begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{K_c} \underbrace{\begin{pmatrix} \psi - \xi & 0 & 0 \\ 0 & -\psi + \xi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{M_c}$$

$$\mathbf{u} = H_c \overline{\mathbf{x}}$$

$\xi = 0$ perspective $\quad 0 < \xi < 1$, hyperbolic mirror $\quad \xi = 1$ parabolic mirror

Sphere projection model for catadioptric systems



$$\mathbf{X} \approx \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T, \quad \mathbf{X} \in P^3$$

$$\hbar(\mathbf{x}) = \begin{pmatrix} X \\ Y \\ Z + \xi\sqrt{X^2+Y^2+Z^2} \end{pmatrix}$$

$$\bar{\mathbf{x}} = \hbar(\mathbf{x})$$

$$\boxed{H_c = \underbrace{\begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{K_c} \underbrace{\begin{pmatrix} \psi-\xi & 0 & 0 \\ 0 & -\psi+\xi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{M_c}}$$

$$\mathbf{u} = H_c \bar{\mathbf{x}}$$

$\xi = 0$ perspective $\quad 0 < \xi < 1$, hyperbolic mirror $\quad \xi = 1$ parabolic mirror

Sphere unprojection model for catadioptric systems

- Analytical bijective unprojection function

$$\mathbf{x} = \hbar\left(\bar{\mathbf{x}}\right)$$

$$\hbar^{-1}(\mathbf{x}) = \begin{pmatrix} \frac{z\xi + \sqrt{z^2 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+z^2} x \\ \frac{z\xi + \sqrt{z^2 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+z^2} y \\ \frac{z\xi + \sqrt{z^2 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+z^2} z - \xi \end{pmatrix}$$

$$\bar{\mathbf{x}} = H_c^{-1}\mathbf{u}$$

also radially symmetric!

$$r = \frac{f(\psi - \xi)\tan\theta}{1 + \xi\sqrt{\tan^2\theta + 1}}$$

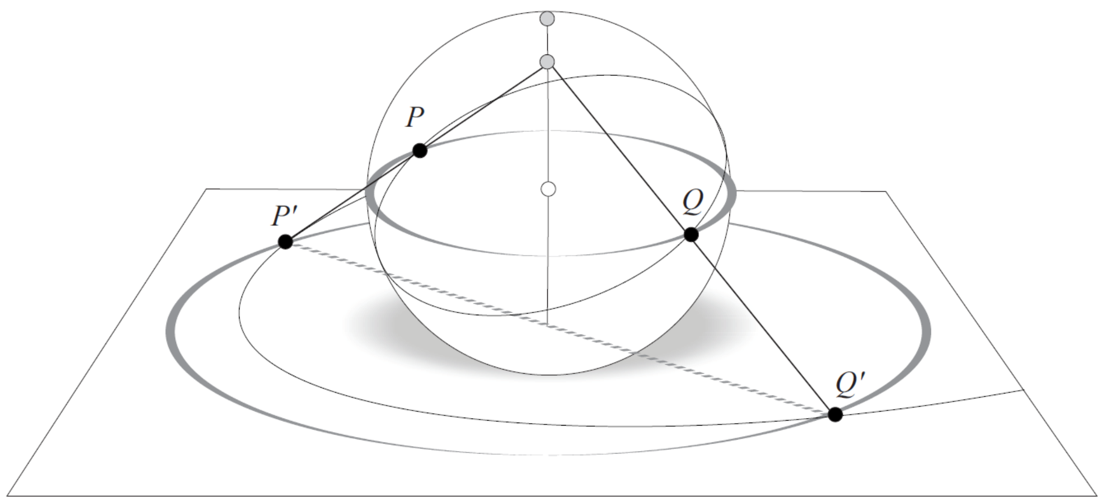
	ξ	ψ
Parabolic	1	$1+2p$
Hyperbolic	$\frac{d}{\sqrt{d^2+4p^2}}$	$\frac{d+2p}{\sqrt{d^2+4p^2}}$

d: distance between foci

4p: latus rectum

Paracatadioptric camera system

- Equivalent to stereographic projection



$$\hbar(\mathbf{x}) = \begin{pmatrix} X \\ Y \\ Z + \xi\sqrt{X^2 + Y^2 + Z^2} \end{pmatrix}$$

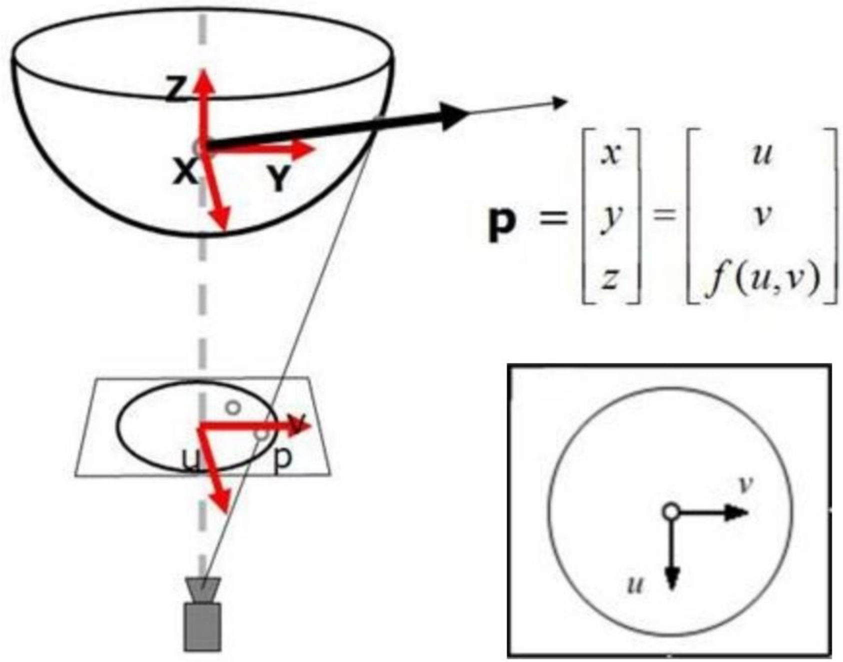
$$H_c = \underbrace{\begin{pmatrix} \alpha_x & 0 & u_0 \\ 0 & \alpha_y & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{K_c} \underbrace{\begin{pmatrix} 2p & 0 & 0 \\ 0 & -2p & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{M_c}$$

Empirical central models: Scarammuza

- Scaramuzza undistortion model: Taylor expansion

$$\mathbf{p} = \begin{pmatrix} u \\ v \\ f(u,v) \end{pmatrix} \in P^2$$

$$f(\rho) = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + \dots$$



Empirical central models: Kannala-Brandt projection model

- Rotational Symmetry

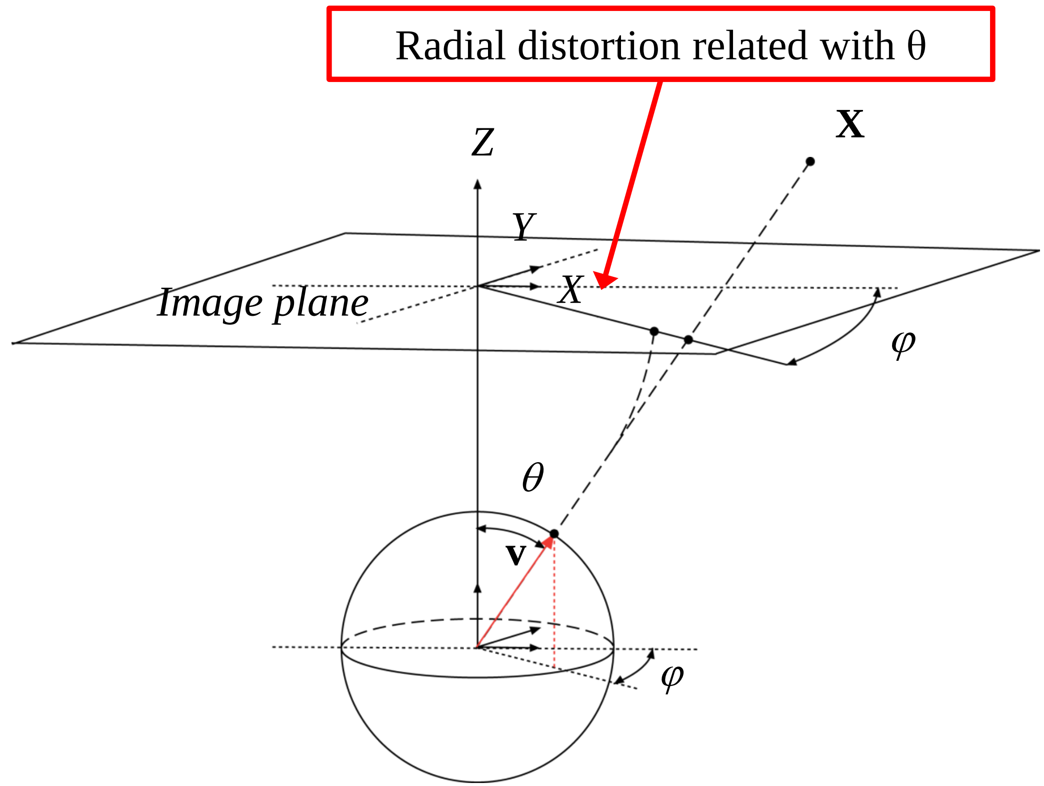
$$\mathbf{X} \approx \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T, \quad \mathbf{X} \in P^3$$

$$R = \sqrt{X^2 + Y^2}$$

$$\theta = \arctan 2\left(R, Z\right)$$

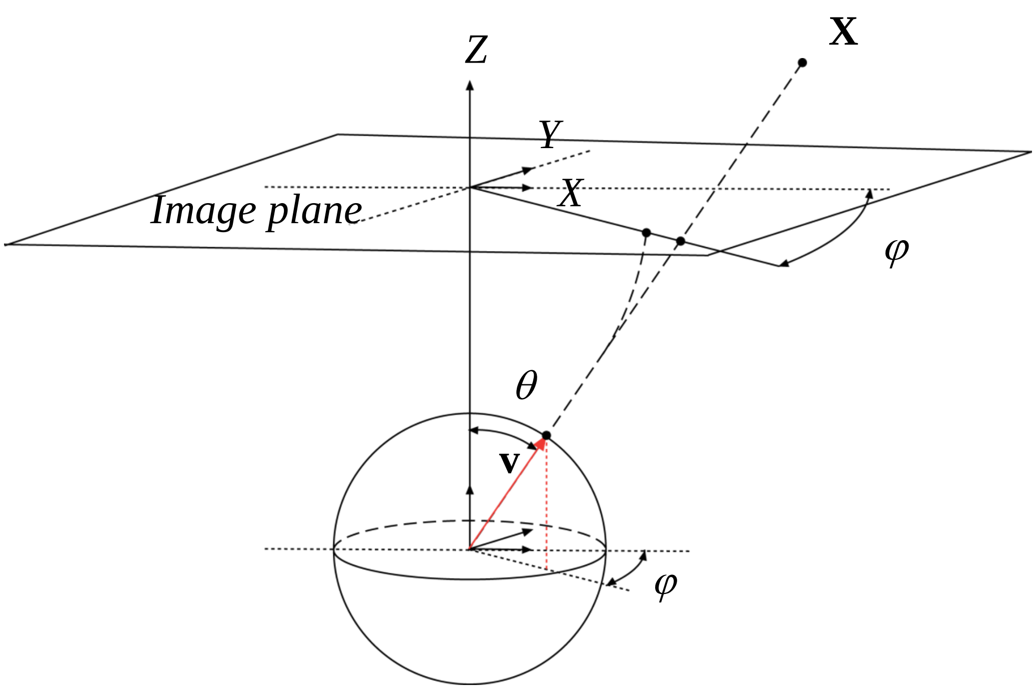
$$d\left(\theta\right) = \theta + k_1\theta^3 + k_2\theta^5 + k_3\theta^7 + k_4\theta^9$$

$$\mathbf{u} = \begin{pmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d\left(\theta\right)\cos\varphi \\ d\left(\theta\right)\sin\varphi \\ 1 \end{pmatrix}$$



Empirical central models: Kannala-Brandt unprojection model

- Obtaining theta means solving a 9th degree polynomial.
- In practice there is just 1 real solution and LUTs can be used.



$$d(\theta) = \theta + k_1\theta^3 + k_2\theta^5 + k_3\theta^7 + k_4\theta^9$$

$$\mathbf{x}_c \sim \mathbf{K}_c^{-1}\mathbf{u}$$

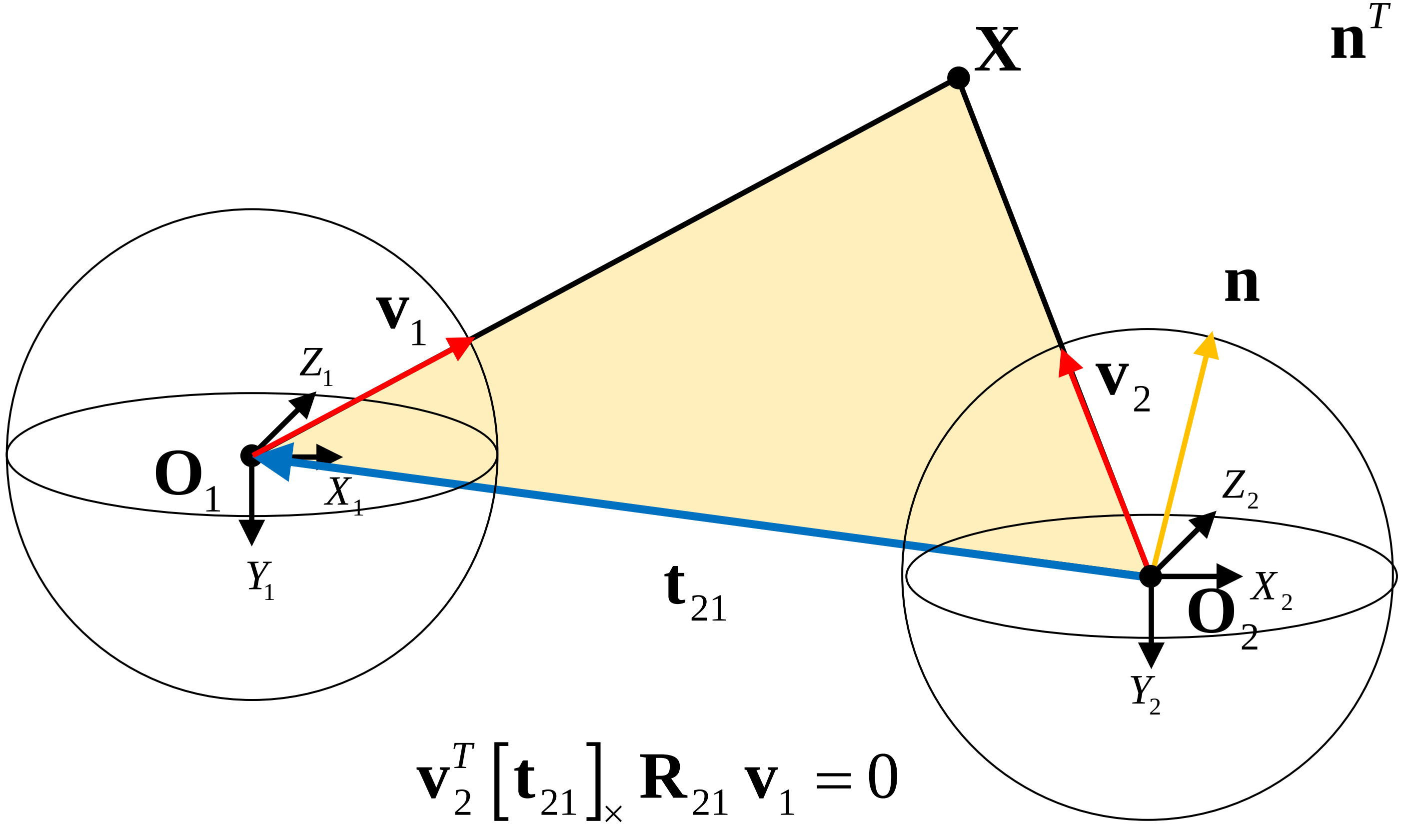
$$d(\theta) = r = \sqrt{\frac{x_c^2 + y_c^2}{z_c^2}}$$

$$\varphi = \arctan 2(y_c, x_c)$$

$$\theta = d^{-1}(r)$$

$$\mathbf{v} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix} \in S^2$$

3D Geometry from rays: Essential matrix



$$\mathbf{n}^T = \mathbf{v}_2^T [\mathbf{t}_{21}]_\times$$

$$\underbrace{\mathbf{v}_2^T [\mathbf{t}_{21}]_\times \mathbf{R}_{21}}_{\mathbf{E}_{21}} \mathbf{v}_1 = 0$$

Epipolar lines in omnidirectional projections

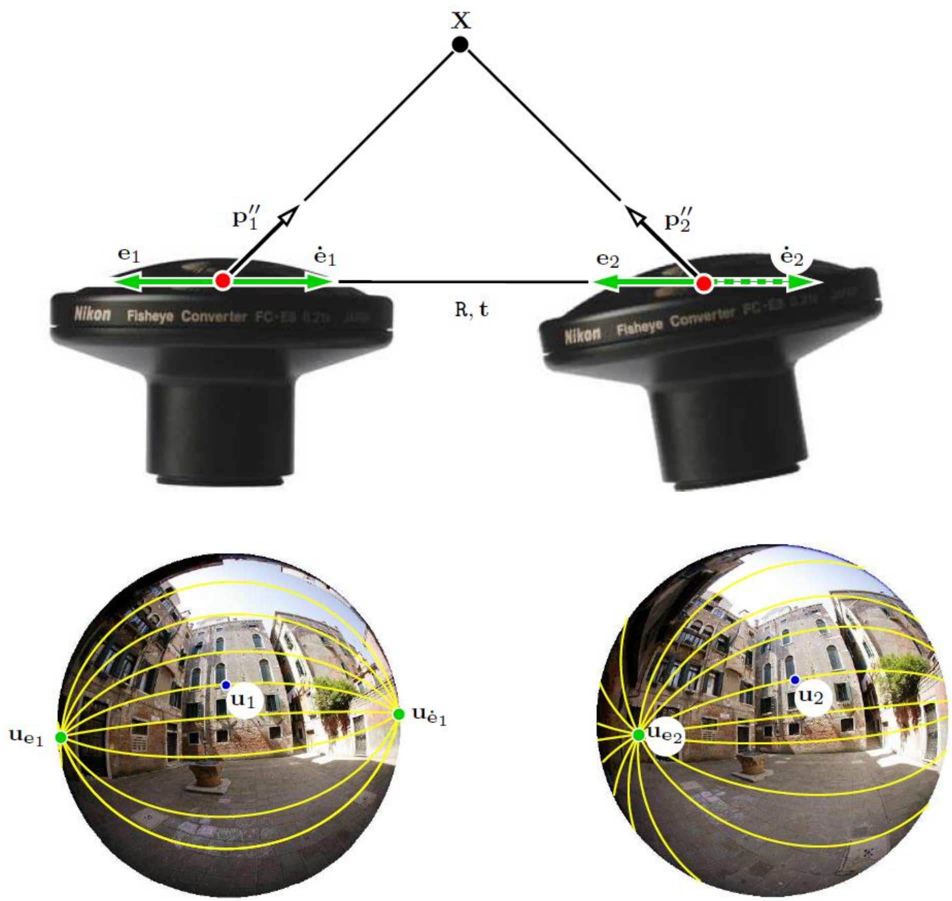
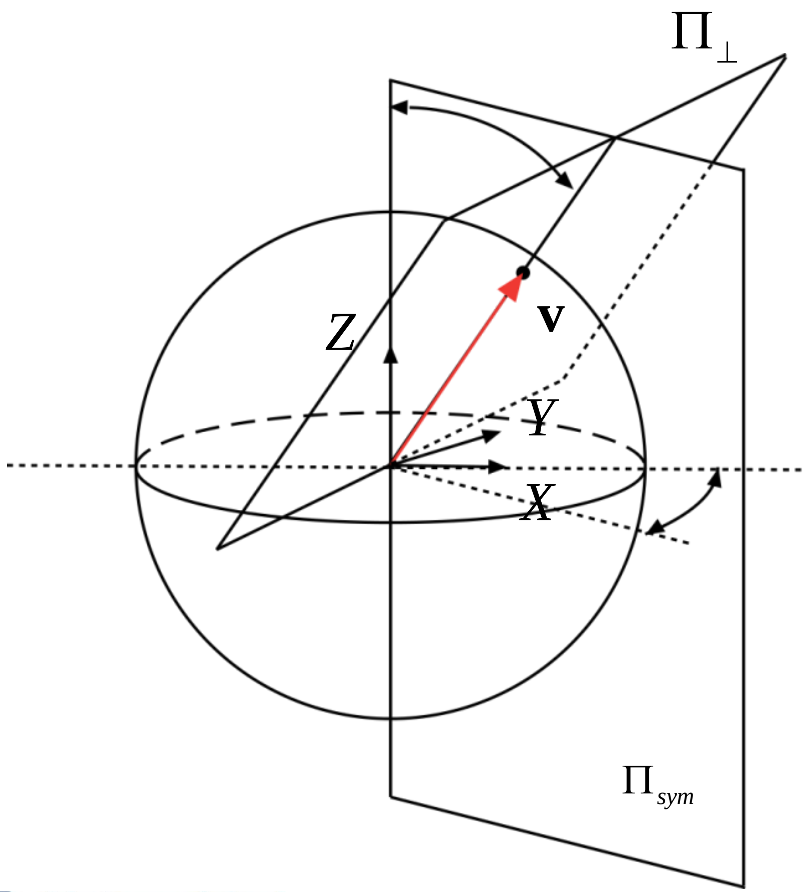


Figure courtesy of Micusik. Two-View Geometry of Omnidirectional Cameras PhD Thesis, Branislav Micusık, 2004

Triangulation using planes

- Defining a ray with two planes

$$\Pi_{sym} = \begin{pmatrix} -v_y \\ vx \\ 0 \\ 0 \end{pmatrix} \qquad \Pi_{\perp} = \begin{pmatrix} -v_z v_x \\ -v_z v_y \\ v_x^2 + v_y^2 \\ 0 \end{pmatrix}$$

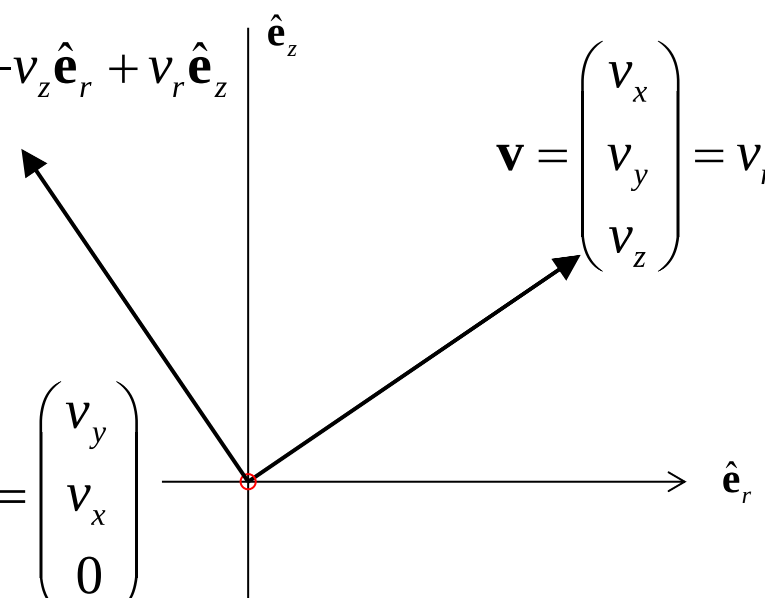


$$if\ \mathbf{v} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T \Rightarrow \Pi_{sym} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^T, \Pi_{\perp} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}^T$$

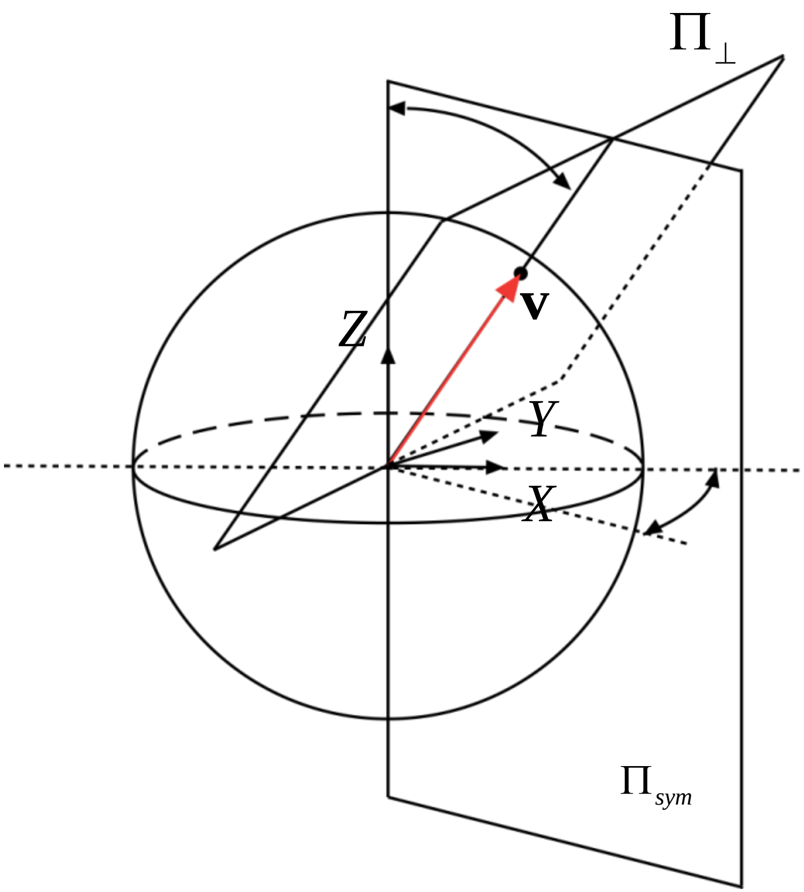
$$\mathbf{n}_{\perp} = -v_z \hat{\mathbf{e}}_r + v_r \hat{\mathbf{e}}_z$$

$$\mathbf{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = v_r \hat{\mathbf{e}}_r + v_z \hat{\mathbf{e}}_z$$

$$n_{sym} = \begin{pmatrix} v_y \\ v_x \\ 0 \end{pmatrix}$$



Triangulation using planes



$$\boldsymbol{\Pi}_{sym1} = \begin{pmatrix} -v_y \\ v_x \\ 0 \\ 0 \end{pmatrix} \qquad \boldsymbol{\Pi}_{\perp 1} = \begin{pmatrix} -v_z v_x \\ -v_z v_y \\ v_x^2 + v_y^2 \\ 0 \end{pmatrix} \qquad \mathbf{X}_1 = {}^1\mathbf{T}_2 \mathbf{X}_2$$

$$\left\{\Pi_{sym1}\right\}_2 = {}^1\mathbf{T}_2{}^T \Pi_{sym1} \qquad \left\{\Pi_{\perp 1}\right\}_2 = {}^1\mathbf{T}_2{}^T \Pi_{\perp 1}$$

$$\mathbf{AX} = \mathbf{0} \quad \text{such that} \quad \mathbf{A} = \begin{bmatrix} \left\{\Pi_{sym1}\right\}_2^T \\ \left\{\Pi_{\perp 1}\right\}_2^T \\ \Pi_{sym2}^T \\ \Pi_{\perp 2}^T \end{bmatrix}$$

Triangulation using planes

$$\mathrm{A}\mathbf{X} = \mathbf{0} \quad \text{such that} \quad \mathbf{A} = \begin{bmatrix} \left\{\Pi_{sym1}\right\}_2^T \\ \left\{\Pi_{\perp 1}\right\}_2^T \\ \Pi_{sym2}^T \\ \Pi_{\perp 2}^T \end{bmatrix}$$

$\mathrm{rank}(\mathbf{A}) \cong 3$ otherwise the points does not fulfill the epipolar constraint

$$\left[\mathbf{U}, \mathbf{S}, \mathbf{V}\right] = \mathrm{svd}\left(\mathbf{A}\right)$$

$$\mathbf{X} = \mathbf{V}_4 \qquad \mathbf{V} = \left[\mathbf{V}_1 \quad \mathbf{V}_2 \quad \mathbf{V}_3 \quad \mathbf{V}_4\right]$$

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