

SparseMatrixSoluter v1.2 Test Report

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2025.10.6

I Sparse Matrix

i Construction and Print

Try Matrix Construction:

```
SparseMatrix A(3, 3);
A.setValue(0, 0, 1.0);
A.setValue(0, 2, 2.0);
A.setValue(1, 1, 3.0);
A.setValue(2, 2, 5.0);
A.setValue(2, 0, 4.0);
```

Print A both in Dense and Sparse form. Output is correct.

```
Matrix A(Concentrate):
1      0      2
0      3      0
4      0      5

Matrix A(Sparse):
row: 3, col: 3, elements: 5
values: 1 2 3 4 5
col_indices: 0 2 1 0 2
row_ptr: 0 2 3 5
```

Figure 1: Print

ii Transpose

Try transposing the Matrix A(defined above):

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```

Transpose A(Dense):
1      0      4
0      3      0
2      0      5

Transpose A(Sparse):
row: 3, col: 3, elements: 5
values: 1 4 3 2 5
col_indices: 0 2 1 0 2
row_ptr: 0 2 3 5

```

Figure 2: Transpose

Output is correct.

iii Cut

Try cutting Matrix into diagonal part, upper triangle part and lower triangle part.

```

Origin Matrix
1      2      3
4      5      6
7      8      9

Diagonal Part
1      0      0
0      5      0
0      0      9

Upper triangle Part
0      2      3
0      0      6
0      0      0

Down triangle Part
0      0      0
4      0      0
7      8      0

```

Figure 3: Cut

Operation is correct.

iv Add and Multyply

Try defining Matrix B, and testing + operator and * operator.

```

Matrix B(Dense):
5      6      0
0      7      8
9      0      0

Matrix A+B(Dense):
6      6      2
0      10     8
13     0      5

Matrix A*B(Dense):
23     6      0
0      21     24
65     24     0

```

Figure 4: Add and Multiply

Operation is correct.

II Matrix Solver

I have write two method for solving matrix:

Gauss-Sider Iteration is suitable for most cases,especially when the diagonal element is dominant.

Conjunction Gradient is based on Krylov subspace. It can only be used when the matrix is positive defined and symmetric.

i Correction Test

I test a 6*6 Matrix when the RHS is 12.0, 20.0, 30.0, 60.0, 20.0, 30.0, for each equation respectively:

```

Matrix A(Dense):
4      1      0      0      0      0
1      5      2      0      0      0
0      2      6      4      0      0
0      0      4      8      5      0
0      0      0      5      9      2
0      0      0      0      2      7

Gauss-Seidel finished.
Gauss-Seidel iter: 70
Gauss-Seidel residual: 8.8329e-07
Gauss-Seidel solution: 1.07108 7.71567 -9.82471 18.3792 -9.547 7.01343
Conjugate gradient converged in 6 iterations. Residual: 2.27268e-13
Conjugate Gradient solution: 1.07108 7.71567 -9.82471 18.3792 -9.547 7.01343

```

Figure 5: Correction Test

And the result is correct for each method.

Noticing that Conjunction Gradient's Iteration is obviously less than Gauss-Sider, and it's residual is much less than the other one. Conjunction Gradient is recommanded for positive symmetric case.

ii Time Test

I test a 100000*100000 Matrix when the RHS is periodic. There are three elements in each row:

```

Huge Sparse Matrix, Scale:100000.
Now Testing...
Now Starting chrono
Now transposing...
Start...
Gauss-Seidel finished.
Gauss-Seidel iter: 9
Gauss-Seidel residual: 6.56674e-07
First Solution: -2.14367

Execution time:
- 39 milliseconds
- 39627 microseconds
- 39627700 nanoseconds

```

Figure 6: Time Test

Only takes 39ms for both transposing and Gauss-Sider Iteration.

III PDE Solver

i Dirichlet BC in Rectangle area

I solve Poisson equation as:

$$\nabla^2 u = 0$$

with Boundary Condition:

$$u(x, y) = x^2 - y^2 \quad (x = 0 \mid x = 1 \mid y = 0 \mid y = 1)$$

I make grid with interval x,y is both 0.01, meaning that there are 100*100 nodes.

The problem is solved rapidly:

```
Gauss-Seidel finished.  
Gauss-Seidel iter: 2884  
Gauss-Seidel residual: 9.9891e-05  
Successfully wrote 10000 doubles to vector_data.bin  
Press ENTER to finish
```

Figure 7: Solve Poisson EQ

ii Visualize

In this part I choose Python language to Plot the result. Mention that the solution of the equation is obvious $u(x, y) = x^2 - y^2$, the computing result is obviously right.

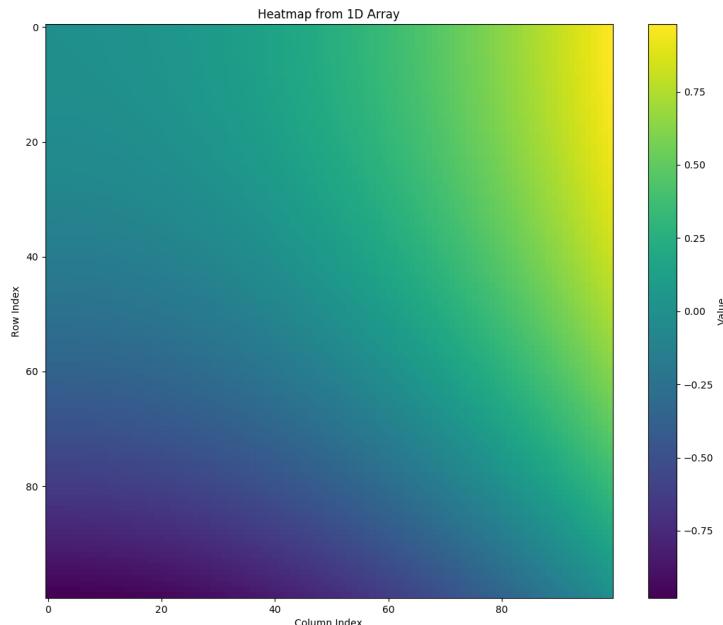


Figure 8: Visualize the Solution