

Part 1a

$$y(0) = 1, y'(0) = -1$$

$$y'' - 2xy' + x^2y = 0 \quad n \leq 4$$
$$x_0 = 0 \quad x = 0$$

$$y'' = 2xy' - x^2y = 2(0)(-1) - (0)^2(1) = 0$$

$$y''' = 2xy''' + 2y' - x^2y' - 2xy$$
$$= 2(0)(0) + 2(-1) - (0)^2(0) - 2(0)(1) = -2$$

$$y'''' = 2xy''' - x^2y'' + 2y'' - 2xy' - 2y - 2xy'$$
$$2(0)(-2) - (0)^2(0) + 2(0) - 2(0)(-1) - 2(1) - 2(0)(-1)$$
$$= -2$$

$$y(x) = \frac{f(0)}{0!} + \frac{f'(0)}{1!}(x-x_0) + \frac{f''(0)}{2!}(x-x_0)^2 + \frac{f'''(0)}{3!}(x-x_0)^3$$

$$+ \frac{f''''(0)}{4!}(x-x_0)^4$$

$$y(x) = 1 + -1x + 0x^2 - \frac{1}{3}x^3 - \frac{1}{12}x^4$$

$$y(3.5) = 1 - 3.5 + 0 - \frac{1}{3}(3.5)^3 - \frac{1}{12}(3.5)^4$$
$$= -29.2969$$

Part 4 b

$$x=3 ; \quad y'' - (x-2)y' + 2y = 0 ; \quad y(3) = 6 \quad y'(3) = 1$$

$$y'' = (x-2)y' - 2y$$

$$y''(3) = (3-2)(1) - 2(6) \Rightarrow 1 - 12 = -11$$

$$y(x) = f(3) + \frac{f'(3)}{1!}(x-x_0) + \frac{f''(3)}{2!}(x-x_0)^2$$

$$y(8) = 6 + 1(x-3) + -\frac{11}{2}(x-3)^2$$