Part 2 $y'' + P(x)\,y' + yQ(x) = 0$

$x=0;\quad (x^2+4)y'' + y = x$

$P(x) = 0 \qquad Q(x) = \frac{1}{x^2+4} = \frac{1}{0+4} = \frac{1}{4}$

When x equal 0 (x=0) it give P(x)=0 and Q(x) = 1/x²+4 hence they are analytic everywhere. This makes them ordinary points.

$y = [a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + \dots + a_n x^n] \cdot 1$

$y' = [a_1 + 2x a_2 + 3x^2 a_3 + 4x^3 a_4 + 5x^4 a_5 + 6x^5 a_6 + 7x^6 a_7 + 8x^7 a_8 + \dots + n a_n x^{n} + n(n-1) a_n x^n] \cdot 0$

$x^2+4 \qquad y'' = [2a_2 + 6x a_3 + 12x^2 a_4 + 20x^3 a_5 + 30x^4 a_6 + 42x^5 a_7 + 56x^6 a_8 + \dots + n(n-1) a_n x^{n-2} + (n+2)(n+1) a_n x^n] \cdot 1$

$x^n\,[4(n+2)(n+1)a_{n+2} + n(n-1)a_n + a_n] = 0$

$4(n+2)(n+1)a_{n+2} + n(n-1)a_n + a_n = 0$

$\frac{4(n+2)(n+1)a_{n+2}}{4(n+2)(n+1)} = \frac{-n(n-1)a_n + a_n}{4(n+2)(n+1)}$

$\boxed{a_{n+2} = -\frac{n^2-n+1}{4(n+2)(n+1)}\,a_n}$ Recurrence formula

$n=0$: $a_2 = -\frac{0^2 - 0 + 1}{4(0+2)(0+1)} a_0 = -\frac{1}{8} a_0$

$n=1$: $a_3 = -\frac{1^2 - 1 + 1}{4(1+2)(1+1)} a_1 = -\frac{1}{24} a_1$

$n=2$: $a_4 = -\frac{2^2 - 2 + 1}{4(2+2)(2+1)} a_2 = -\frac{3}{48} a_2 = -\frac{3}{48} \cdot -\frac{1}{8} a_0 = \frac{1}{128} a_0$

$n=3$: $a_5 = \frac{-3^2 - 3 + 1}{4(2+3)(3+1)} a_3 = -\frac{7}{80} a_3 = -\frac{7}{80} \cdot -\frac{1}{24} a_1 = \frac{7}{1920} a_1$

$n=4$: $a_6 = -\frac{4^2 - 4 + 1}{4(2+4)(4+1)} a_4 = -\frac{13}{120} a_4 = \frac{-13}{120} \cdot \frac{1}{128} a_0 = -\frac{13 a_0}{15360}$

$n=5$: $a_7 = -\frac{5^2 - 5 + 1}{4(2+5)(5+1)} a_5 = -\frac{21}{168} a_5 = \frac{-21}{168} \cdot \frac{7}{1920} a_1 = -\frac{147 a_1}{322560}$

$n=6$: $a_8 = -\frac{6^2 - 6 + 1}{4(2+6)(6+1)} a_6 = -\frac{31}{224} a_6 = -\frac{31}{224} \cdot \frac{-13}{15360} a_0 = \frac{403}{3440640} a_0$

$n=7$: $a_9 = -\frac{7^2 - 7 + 1}{4(2+7)(7+1)} a_7 = -\frac{43}{288} a_7 = -\frac{43}{288} \cdot \frac{-147}{322560} a_1 = \frac{6321}{92897280} a_1$

$n=8$: $a_{10} = -\frac{8^2 - 8 + 1}{4(2+8)(8+1)} = -\frac{57}{360} a_8 = \frac{-57}{360} \cdot \frac{403}{3440640} a_0 =$

$= \frac{-22971}{1238630400} a_0$

Part 2 Conti General Solution

$$y = a_0 + a_1 x - \frac{1}{8}a_0 x^2 - \frac{1}{24}a_1 x^3 + \frac{1}{128}a_0 x^4 + \frac{7}{1920}a_1 x^5 - \frac{13}{15360}a_0 x^6$$

$$-\frac{147}{322560}a_1 x^7 + \frac{403}{3440640}a_0 x^8 + \frac{6321}{92897280}a_1 x^9 - \frac{22971}{1238630400}a_0 x^{10}$$

$$y = a_0\left(1 - \frac{1}{8}x^2 + \frac{1}{128}x^4 - \frac{13}{15360}x^6 + \frac{403}{3440640}x^8 - \frac{22971}{1238630400}x^{10}\right)$$

$$+ a_1\left(x - \frac{1}{24}x^3 + \frac{7}{1920}x^5 - \frac{147}{322560}x^7 + \frac{6321}{92897280}x^9\right)$$

$$y = a_0 y_1(x) + a_1 y_2(x)$$