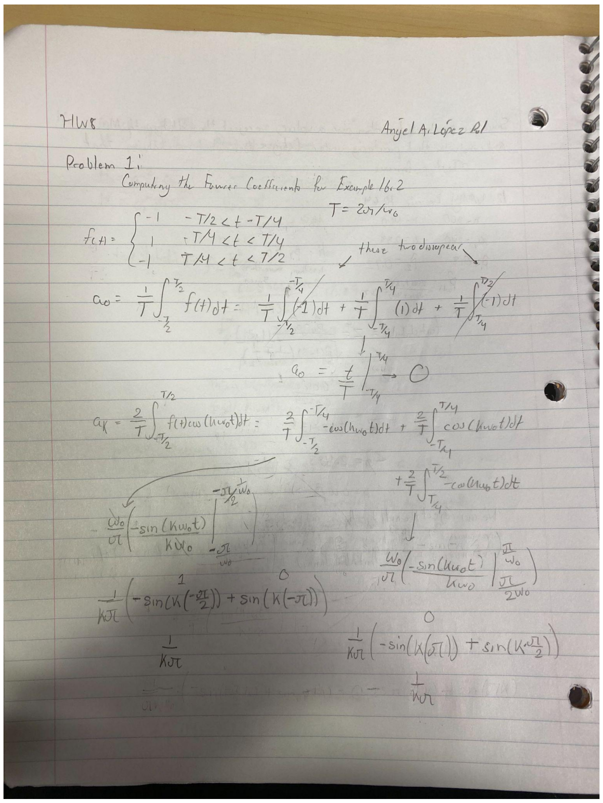
Introduction To Numerical Methods of Engineering Analysis
University of Florida
Mechanical and Aerospace Engineering

Problem 1 :

The following pictures display my computations of the Fourier Coefficients for the square wave in Example 16.2 in the book..



HW8 Angel A. López Pol

Problem 1:

Computing the Fourier Coefficients for Example 16.2

$T = 2\pi/\omega_0$

$f(t) = \begin{cases} -1 & -T/2 < t < -T/4 \\ 1 & -T/4 < t < T/4 \\ -1 & T/4 < t < T/2 \end{cases}$

these two disappear

$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt = \frac{1}{T}\int_{-T/2}^{-T/4}(-1)dt + \frac{1}{T}\int_{-T/4}^{T/4}(1)dt + \frac{1}{T}\int_{T/4}^{T/2}(-1)dt$

$a_0 = \frac{t}{T}\Big|_{-T/4}^{T/4} \to 0$

$a_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(k\omega_0 t)dt = \frac{2}{T}\int_{-T/2}^{-T/4} -\cos(k\omega_0 t)dt + \frac{2}{T}\int_{-T/4}^{T/4}\cos(k\omega_0 t)dt + \frac{2}{T}\int_{T/4}^{T/2} -\cos(k\omega_0 t)dt$

$\frac{\omega_0}{\pi}\left(\frac{-\sin(k\omega_0 t)}{k\omega_0}\right)\Big|_{-\pi/\omega_0}^{-\pi/2\omega_0}$

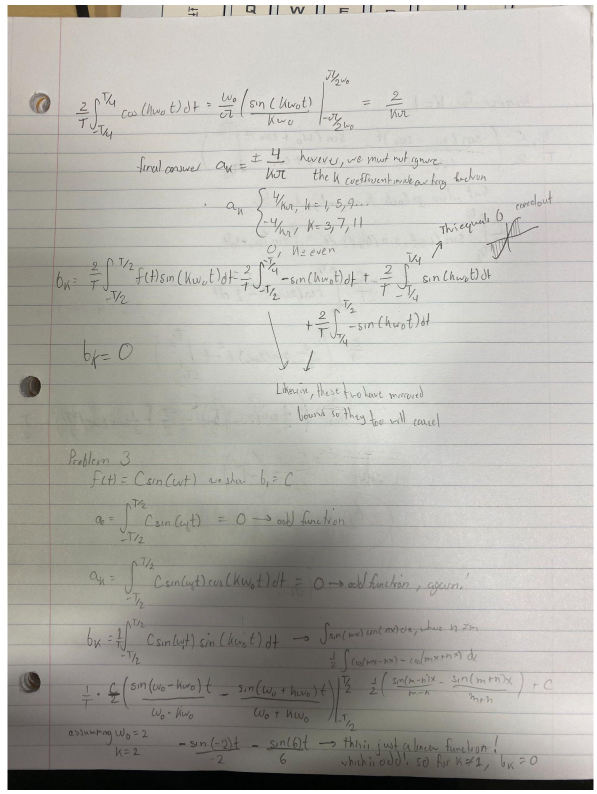
$\frac{1}{k\pi}\left(-\sin(k(-\frac{\pi}{2})) + \sin(k(-\pi))\right)$

$\frac{1}{k\pi}$

$\frac{\omega_0}{\pi}\left(\frac{-\sin(k\omega_0 t)}{k\omega_0}\Big|_{\pi/2\omega_0}^{\pi/\omega_0}\right)$

$\frac{1}{k\pi}\left(-\sin(k(\pi)) + \sin(k\frac{\pi}{2})\right)$

$\frac{1}{k\pi}$



$\frac{2}{T}\int_{-T/4}^{T/4}\cos(k\omega_0 t)dt = \frac{\omega_0}{\pi}\left(\frac{\sin(k\omega_0 t)}{k\omega_0}\right)\Big|_{-\pi/2\omega_0}^{\pi/2\omega_0} = \frac{2}{k\pi}$

final answer $a_k = \pm\frac{4}{k\pi}$ however, we must not ignore the k coefficient inside our trig function

$a_k \begin{cases} 4/k\pi, & k = 1, 5, 9 \ldots \\ -4/k\pi, & k = 3, 7, 11 \\ 0, & k = \text{even} \end{cases}$

This equals 0 / cancel out

$b_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(k\omega_0 t)dt = \frac{2}{T}\int_{-T/2}^{-T/4} -\sin(k\omega_0 t)dt + \frac{2}{T}\int_{-T/4}^{T/4}\sin(k\omega_0 t)dt + \frac{2}{T}\int_{T/4}^{T/2} -\sin(k\omega_0 t)dt$

$b_k = 0$

Likewise, these two have mirrored bounds so they too will cancel

Problem 3

$f(t) = C\sin(\omega t)$ we show $b_1 = C$

$a_0 = \int_{-T/2}^{T/2} C\sin(\omega t) = 0 \to$ odd function

$a_k = \int_{-T/2}^{T/2} C\sin(\omega t)\cos(k\omega_0 t)dt = 0 \to$ odd function, again!

$b_k = \frac{1}{T}\int_{-T/2}^{T/2} C\sin(\omega t)\sin(k\omega_0 t)dt \to \int \sin(mx)\sin(nx)dx$, where $n \neq m$

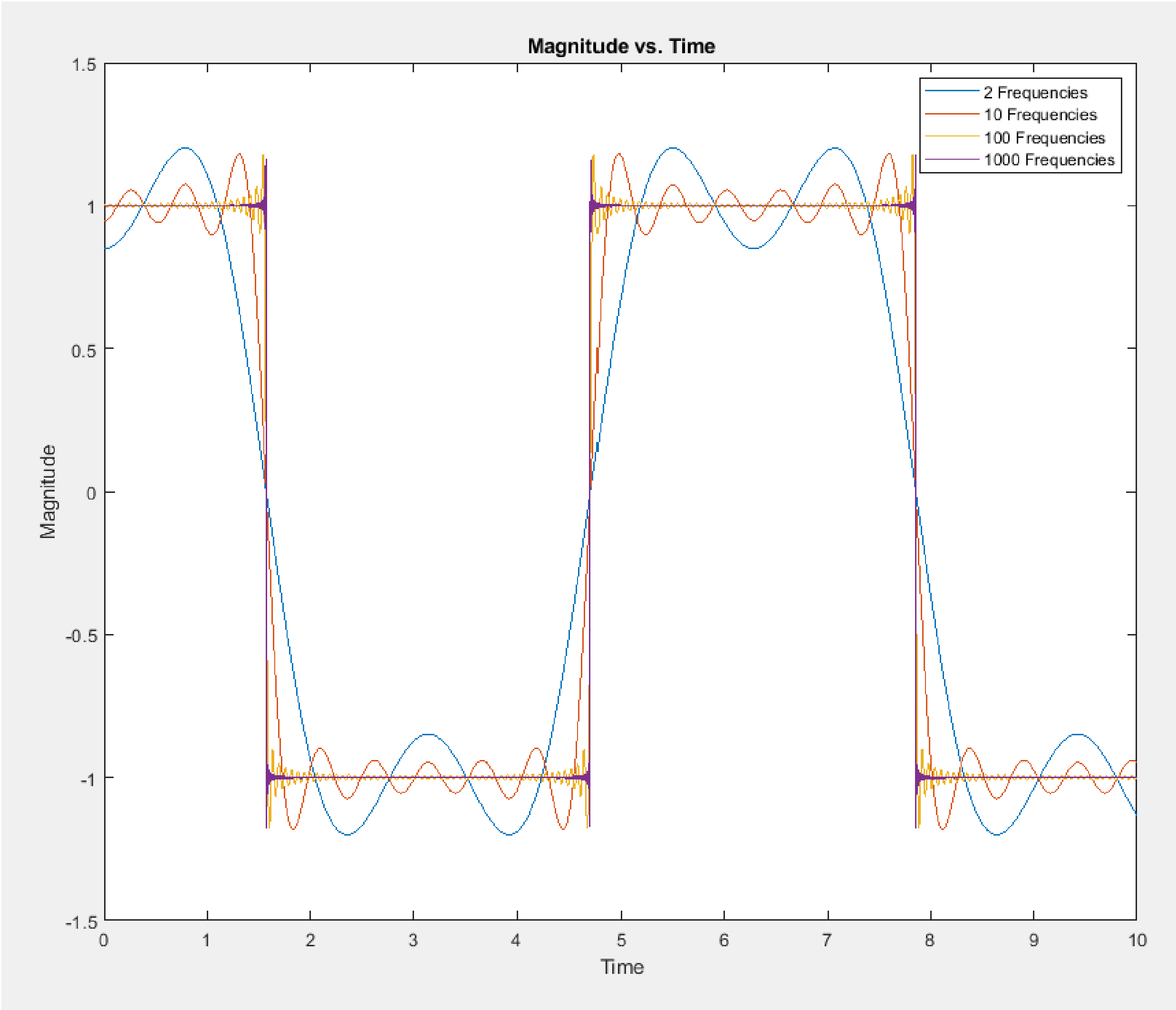
$\frac{1}{2}\int(\cos(mx-nx) - \cos(mx+nx))dx$

$\frac{1}{T}\cdot\frac{C}{2}\left(\frac{\sin((\omega_0 - k\omega_0)t)}{\omega_0 - k\omega_0} - \frac{\sin((\omega_0 + k\omega_0)t)}{\omega_0 + k\omega_0}\right)\Big|_{-T/2}^{T/2}$ $\quad \frac{1}{2}\left(\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n}\right) + C$

assuming $\omega_0 = 2$, $k = 2$: $-\frac{\sin(-2)t}{-2} - \frac{\sin(6)t}{6} \to$ this is just a linear function! which is odd! so for $k \neq 1$, $b_k = 0$

Problem 2 :

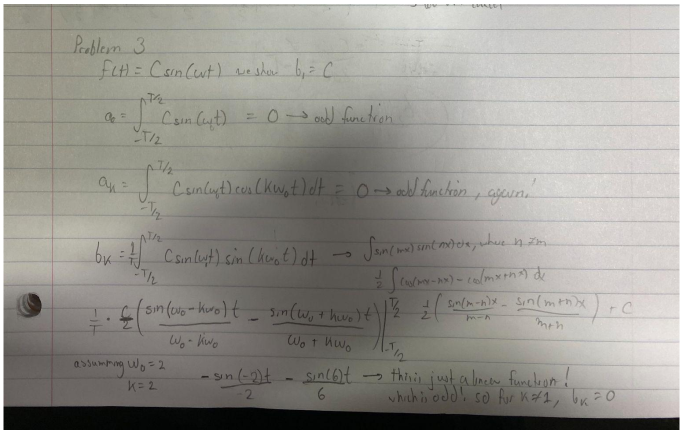
The code for this discussion can be found in the square_fourier.m file.



We notice that as the amount of frequencies we use to approximate the square wave increases, the more "square" our wave gets. This is because the more frequencies we are allowed to use to model these square waves, the more frequencies we have to "dampen" the waviness of the function. Basically we are using destructive and constructive interference in a way that creates these square waves, and the higher the variety of frequencies we can use, the more precise our destructive and constructive interference will be.

Problem 3:

The following Images display my work for for showing that the only nonzero Fourier Coefficient of the given function is b1 = C:



Problem 3

$f(t) = C\sin(\omega t)$ we show $b_1 = C$

$$a_0 = \int_{-T/2}^{T/2} C\sin(\omega_0 t) = 0 \rightarrow \text{odd function}$$

$$a_k = \int_{-T/2}^{T/2} C\sin(\omega_0 t)\cos(k\omega_0 t)\,dt = 0 \rightarrow \text{odd function, again.}$$

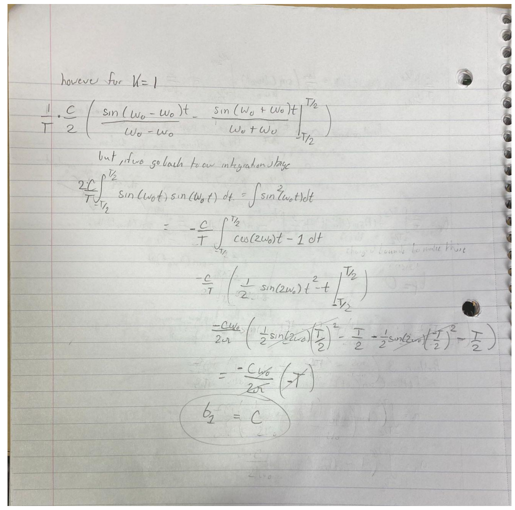
$$b_k = \frac{1}{T}\int_{-T/2}^{T/2} C\sin(\omega_0 t)\sin(k\omega_0 t)\,dt \rightarrow \int \sin(mx)\sin(nx)\,dx, \text{ where } n \neq m$$

$$\frac{1}{2}\int \cos(mx-nx) - \cos(mx+nx)\,dx$$

$$\frac{1}{T}\cdot\frac{C}{2}\left(\frac{\sin(\omega_0 - k\omega_0)t}{\omega_0 - k\omega_0} - \frac{\sin(\omega_0 + k\omega_0)t}{\omega_0 + k\omega_0}\right)\Bigg|_{-T/2}^{T/2} \quad \frac{1}{2}\left(\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n}\right) + C$$

assuming $\omega_0 = 2$, $k = 2$

$$-\frac{\sin(-2)t}{-2} - \frac{\sin(6)t}{6} \rightarrow \text{this is just a linear function! which is odd! so for } k \neq 1,\ b_k = 0$$



however for $k = 1$

$$\frac{1}{T}\cdot\frac{C}{2}\left(\frac{\sin(\omega_0 - \omega_0)t}{\omega_0 - \omega_0} - \frac{\sin(\omega_0 + \omega_0)t}{\omega_0 + \omega_0}\Bigg|_{-T/2}^{T/2}\right)$$

but, if we go back to our integration stage

$$\frac{2C}{T}\int_{-T/2}^{T/2} \sin(\omega_0 t)\sin(\omega_0 t)\,dt = \int \sin^2(\omega_0 t)\,dt$$

$$= -\frac{C}{T}\int_{-T/2}^{T/2} \cos(2\omega_0)t - 1\,dt$$

$$-\frac{C}{T}\left(\frac{1}{2}\sin(2\omega_0)t^2 - t\Bigg|_{-T/2}^{T/2}\right)$$

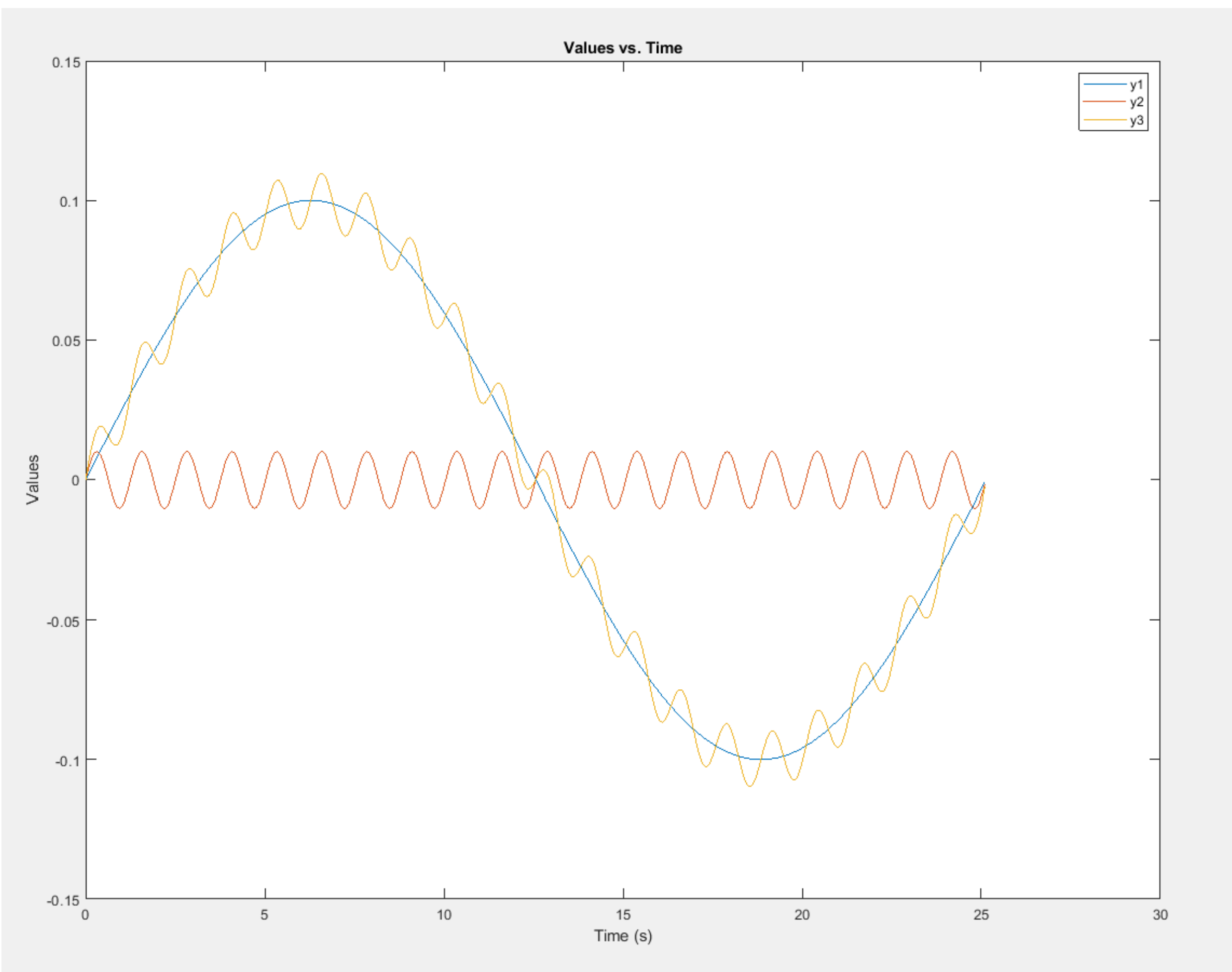
$$\frac{-C\omega_0}{2\pi}\left(\frac{1}{2}\cancel{\sin(2\omega_0)}\left(\frac{T}{2}\right)^2 - \frac{T}{2} - \frac{1}{2}\cancel{\sin(2\omega_0)}\left(\frac{-T}{2}\right)^2 - \frac{T}{2}\right)$$

$$= \frac{-C\omega_0}{2\pi}(-T)$$

$$b_1 = C$$

Problem 4:

Below are my graphs for y1, y2, and y3 put together. We can see that y3 is a combination of y1 and y2

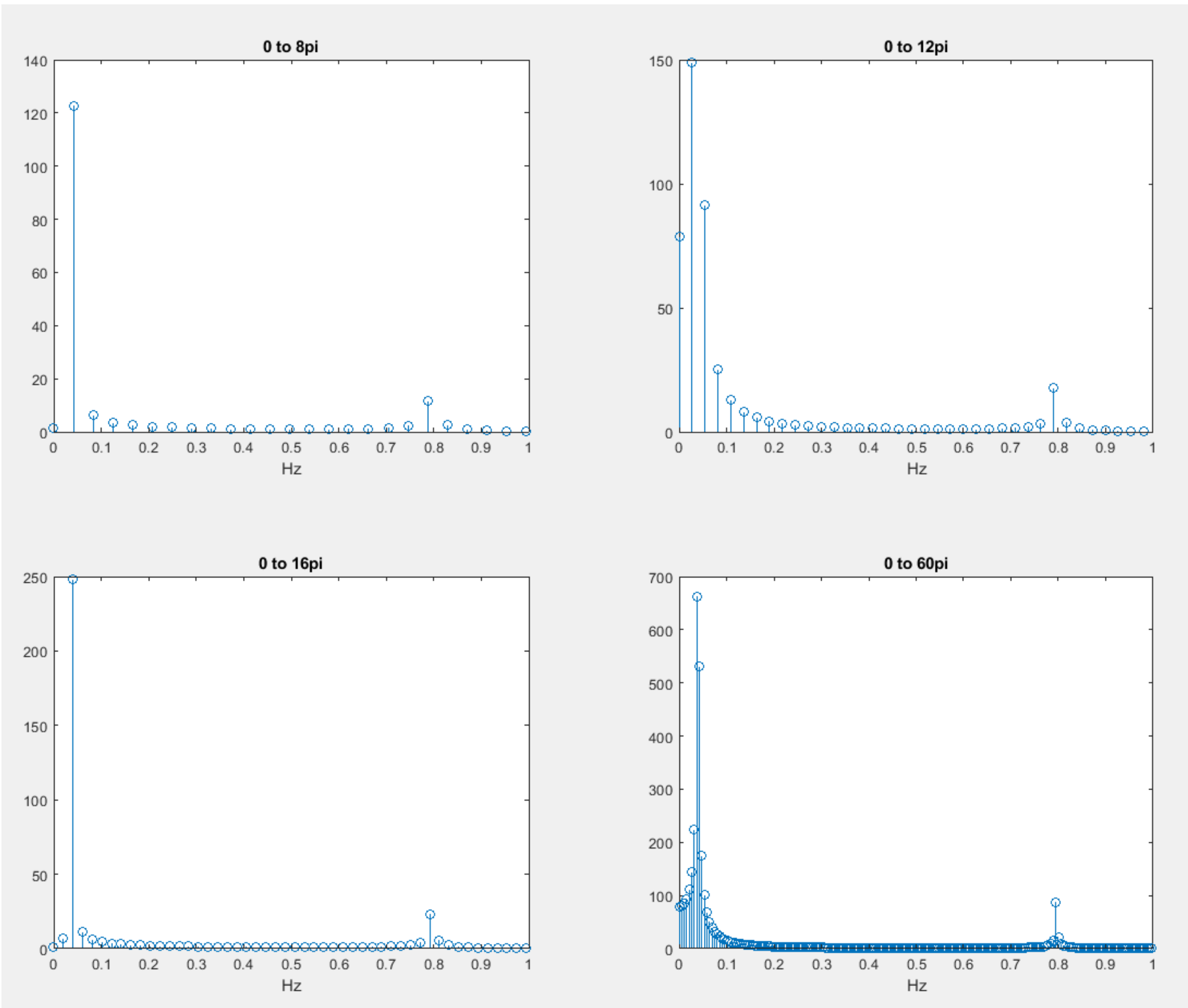


Problem 5:

The code that computes the Discrete Fourier Transform for a given f[n] can be found in the compute_dft.m function file.

Problem 6:

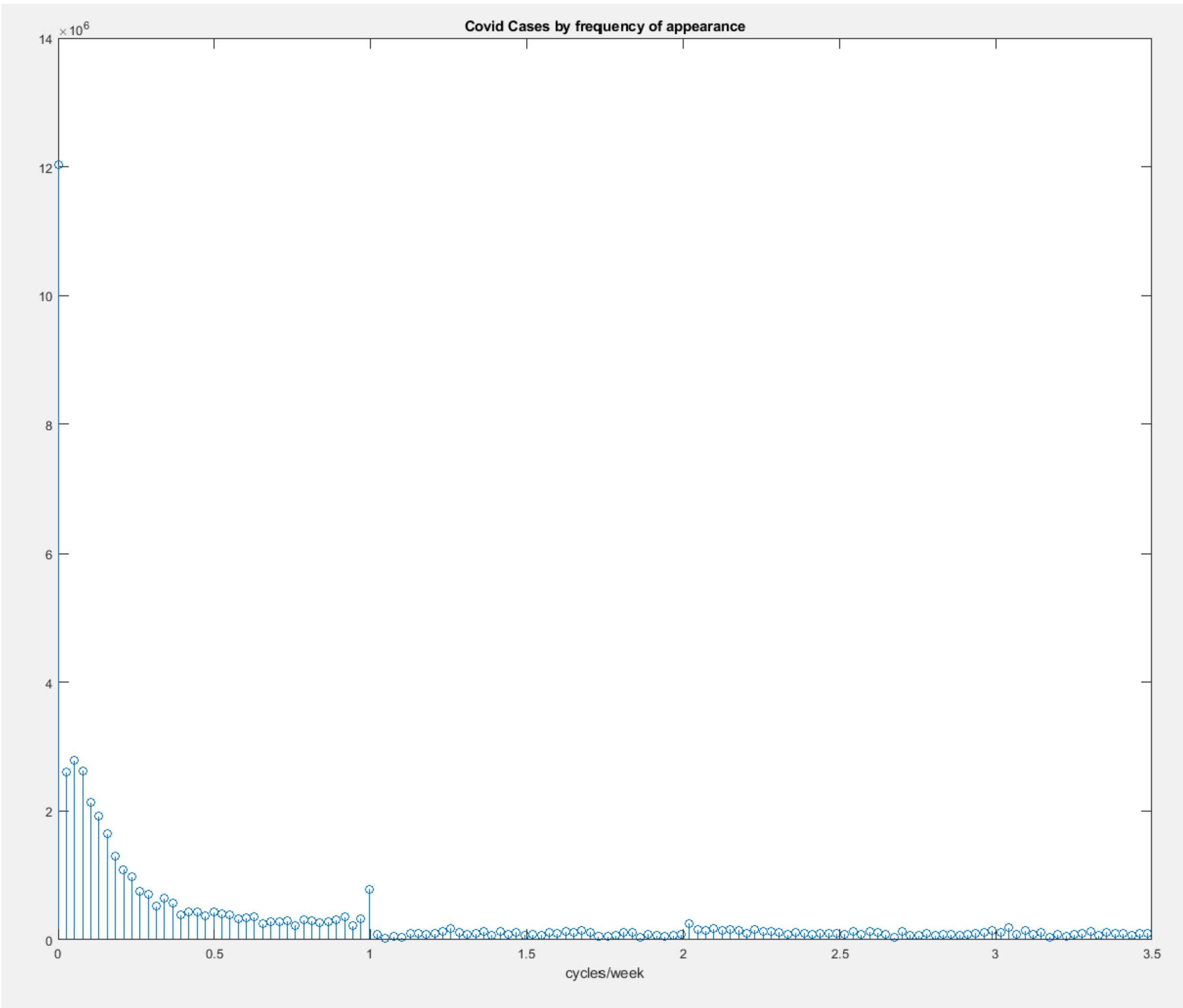
Below are my graphs from problem 6. The code used to obtain them can be found on hw8_p6.m



Some of the properties these graphs share are high magnitudes near the same frequencies. This makes sense because these stems are supposed to model the FFT's guess regarding what frequencies of sin dominate in the data points that we provided. We can recall that our function had two sine functions with frequencies of 0.25 rad/s and 5 rad/s respectively. We can see that the FFT's most likely guesses (those with the highest magnitude) are around those two values (slightly different due to round-off error and period).

Problem 7:

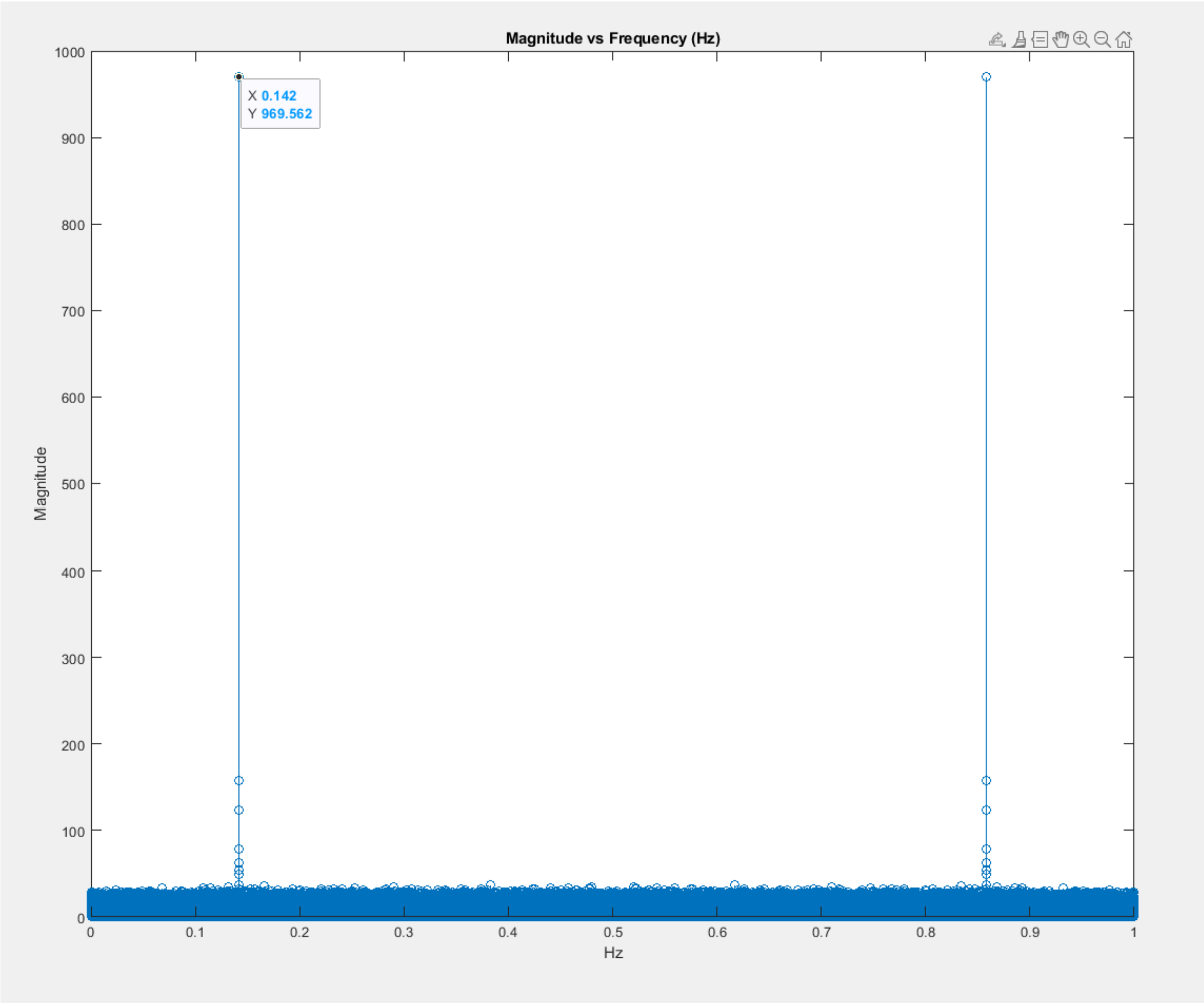
The following is a plot that portrays covid cases by frequency of appearance. A good way of interpreting this plot is that it portrays how covid case reports fluctuate in terms of frequency. For instance, they might not fluctuate much day-by-day, but week-by-week the amount of reported cases fluctuates hugely.



We have small sign waves everywhere else but at 1 cycle / week we have a spike of cases. This is because covid cases are reported on a weekly schedule meaning that the brunt of the reported cases (that may have surfaced at any point during that given week) are reported or put into data with a frequency of 1 cycle per week, or once a week. The first initial spike at 0 corresponds to the k = 0th value. We know that k = 0 denotes the mean value of the function- suggesting that what we are seeing at 0 cycles/week is literally the mean value of all covid cases. What this means is that at this time we have 12 million confirmed covid cases.

Problem 8

The following plot shows my obtained result after performing a Fourier Transform on the data found in hw8_p8.mat



We see that one of the signals (not considering the other due to mirroring being the likely culprit behind its existence), namely the signal with a frequency of 0.142 Hz has a massive frequency content, about 31 times higher than the rest of the frequency contents. This suggests that this signal could be a sign of extraterrestrial life trying to communicate with us.