Introduction To Numerical Methods of Engineering Analysis
University of Florida
Mechanical and Aerospace Engineering
HW9 Angel Lopez Pol

Problem 1 :

The following pictures display my computations for the integral of the given function using four different integration methods

12/3/2022

Problem 1

integrate $f(x) = x^3 + x^2 + x + 1$ from 0 to 10

a) analytically

$$\int_0^{10} x^3 + x^2 + x + 1 \, dx = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \Big|_0^{10}$$

$$= \frac{10000}{4} + \frac{1000}{3} + \frac{100}{2} + 10$$

$$= 2500 + 333.\bar{3} + 50 + 10$$

$$= 2893.333...$$

b) trapezoidal rule

$$\int f(x)dx = (b-a)\frac{f(a)+f(b)}{2} = 10\frac{(1111)}{2} = 5555$$

c) 1/8 Rule

$$I = (b-a)\frac{f(x_0) + 4f(x_1) + f(x_2)}{6} = 10\left(\frac{f(0) + 4f(5) + f(10)}{6}\right)$$

$$= 10 \cdot \frac{4(156) + 1111}{6}$$

$$= 2891.666$$

d) 3/8ths Rule

$$I = (b-a)\frac{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)}{8}$$

$$= 10\left(\frac{f(0) + 3f(3.\bar{3}) + 3f(6.\bar{6}) + f(10)}{8}\right)$$

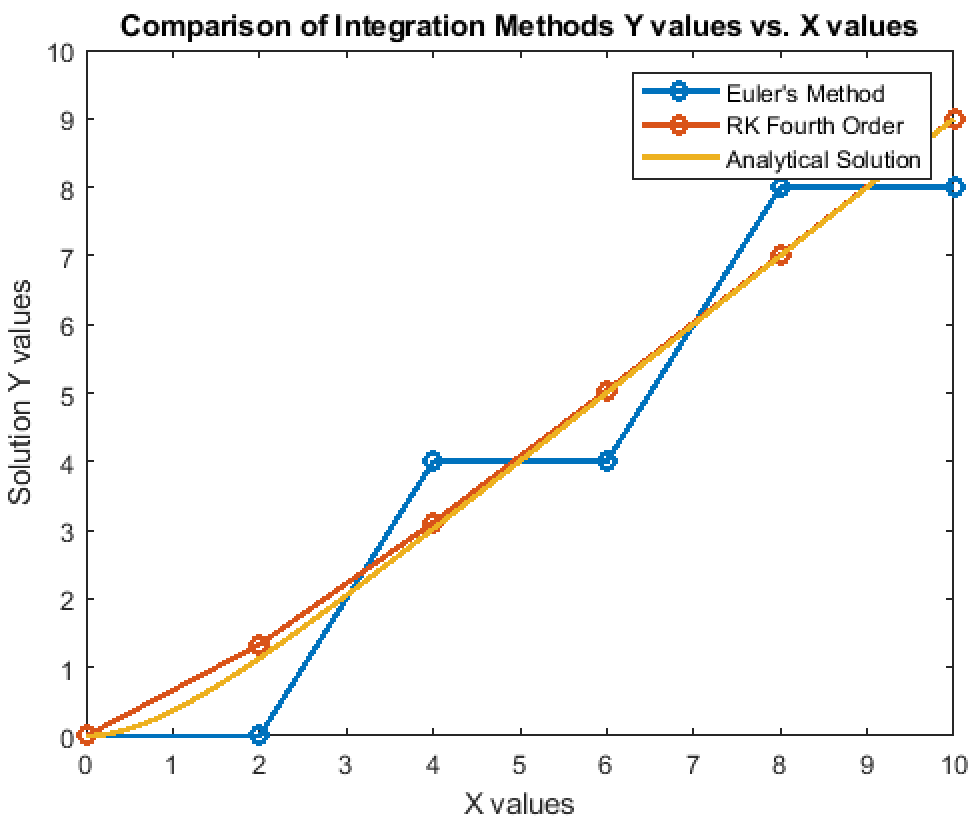
$$= 10\left(\frac{3 \cdot 52.481 + 3 \cdot 348.407 + 1111}{8}\right)$$

$$\approx 2892.08$$

We can see that the analytical solution is the best, with 3/8ths, 1/8ths, and regular trapezoidal following behind. This makes sense theoretically for the same reason that Riemann Sums get more accurate as step-size decreases: The more data points we have for a specific function in a given interval, the better we can model the area under that curve using geometric shapes like rectangles and trapezoids. Trapezoidal uses the least amount of data points, while 1/8ths uses more. The 3/8ths method uses even more data points in the given interval, and finally the analytical method provides us with the result from using an infinite number of data points in that given interval, by leveraging an infinitesimal step-size (dx).

Problem 2 :

The code used to perform these methods and plot this data can be found in the hw9_p2.m file



As we can see, there is a massive difference in terms of accuracy between Euler's Method and Classical Fourth Order RK method performed on the same ODE. The crazy thing is that we could most likely generate a similar plot by making the step size in Euler's Method smaller/having more data points for the function.

The way that the Fourth Order RK method works, to my understanding, is that it takes the space between two data points and divides it into n spots to calculate the slope at, where n is the order of the RK method. It then calculates the slopes at these n points between the two data points, and uses them to generate a curve. In summary, the RK Method of 4th order calculates more meaningful data per data point than Euler (4 slopes between 2 data points, versus just 1 slope between two data points), which ends up giving us more bang-for-our-buck in situations where we can only gather a limited number of data points.