

國立臺北科技大學 資訊與財金管理系碩士班
財務管理與資訊應用 第三組



Investment Decision Rules

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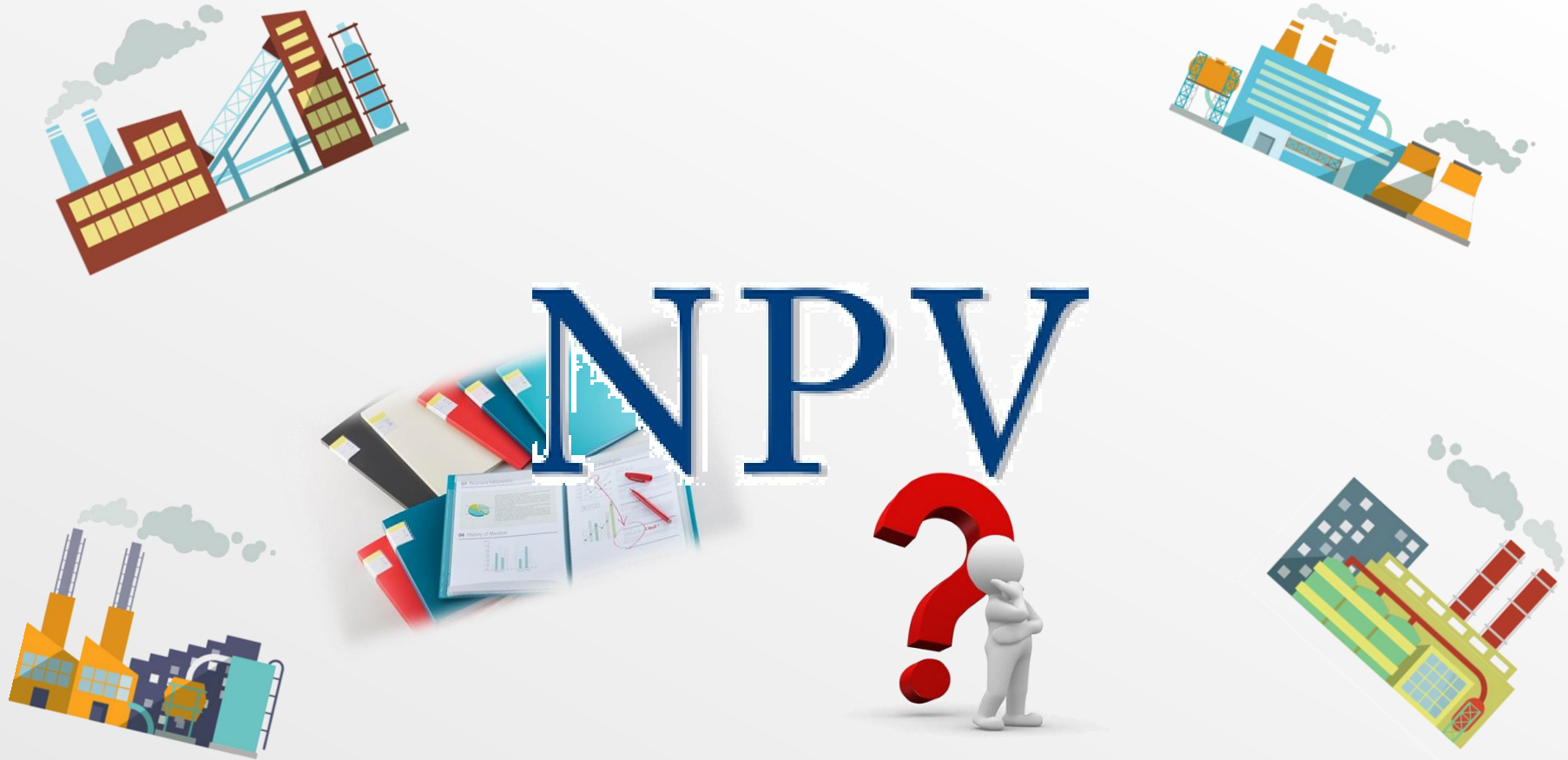
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7.1 NPV and Stand-Alone Projects

NPV and Stand-Alone Projects



NPV

淨現值（Net Present Value）：

一項投資所產生的未來現金流的「折現值」，
與投資成本之間的差值。

Applying the NPV Rule



成本：2.5億美元

Example

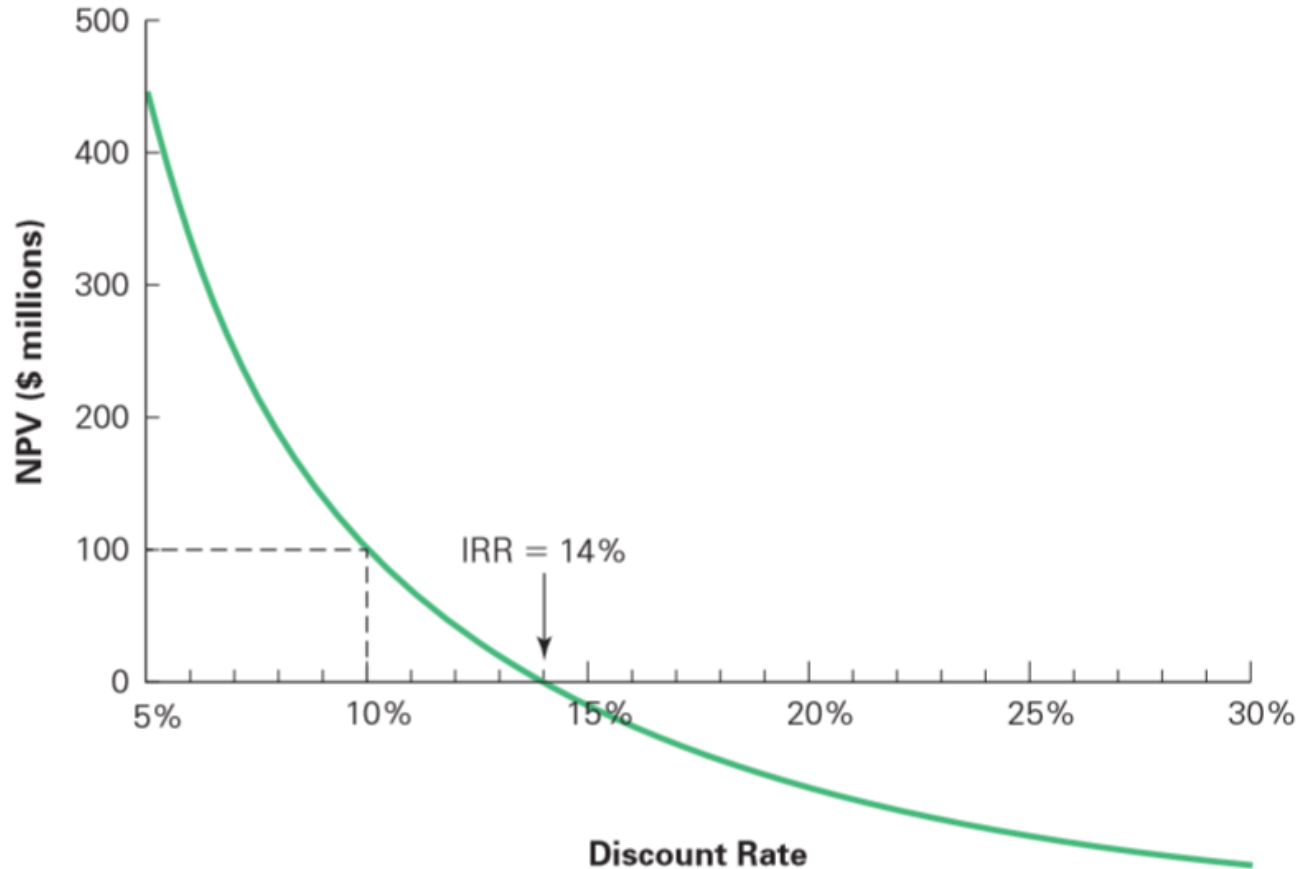


$$\text{NPV} = -250 + \frac{35}{r}$$

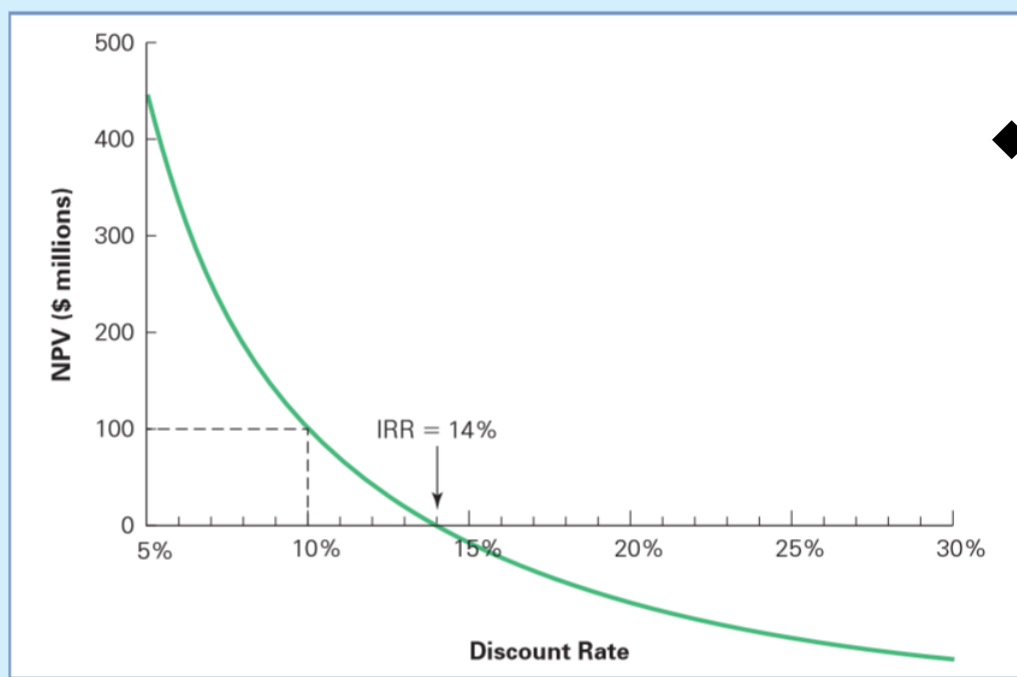
- ◆ The financial managers responsible for this project estimate a cost of capital of 10% per year.

The NPV Profile and IRR

internal rate of return (IRR)



The NPV Profile and IRR



◆ In general, the difference between the cost of capital and the IRR is the maximum estimation error in the cost of capital that can exist without altering the original decision.

- 因為資本成本都是用預估的方式，假設估算15 %、18 %、20 % 等等，只要超過14%的內部收益率，則淨現值將為負。
- 資本成本和IRR之間的差異是在不變原始決策下所容忍的最大估計誤差。

- Although the NPV rule is the most accurate and reliable decision rule, **in practice a wide variety of tools are applied, often in tandem with the NPV rule.**
- **In a 2001 study, 75% of the firms John Graham and Campbell Harvey surveyed used the NPV rule for making investment decisions.**
- However, because you may encounter these techniques in the business world, you should know what they are, how they are used, and how they compare to NPV.



7.2 The Internal Rate of Return Rule

The Internal Rate of Return(内部報酬率法[IRR])

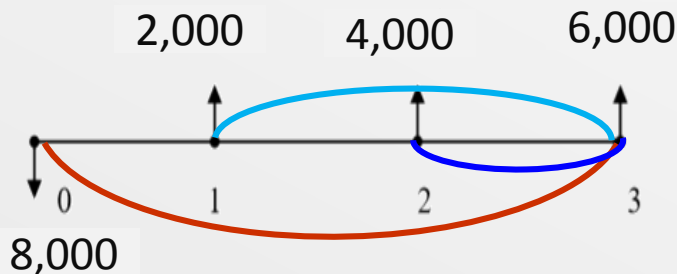
- Internal rate of return (IRR), defined as the interest rate that sets the **net present value** (NPV) of the cash flows **equal to zero**.
- In other words, this investment is economically a **break-even** proposition when the NPV is zero because value **is neither created nor destroyed**.
- One interpretation of the internal rate of return(IRR) is the **average return earned** by taking on the investment opportunity.

Example

方案1: 假設期初小明拿了8,000元去存台幣定存，存3年，定存利率為 r ，三年後所獲得的本利和為： $8,000 \times (1 + r)^3$

方案2: 小明除了期初投資定存外，還把第一年的獲利2,000元、第二年的獲利4,000元與第三年的獲利6,000，進行為期3年的再投資，假設再投資率為 r ，其所獲得的報酬：
為： $2,000 \times (1 + r)^2 + 4,000 \times (1 + r)^1 + 6,000$

而假設當 $8,000 \times (1 + r)^3 = 2,000 \times (1 + r)^2 + 4,000 \times (1 + r)^1 + 6,000$ 時，所算出來的 r ，亦等同於內部報酬率(IRR)，所以IRR也被視為平均報酬



- IRR Investment Rule:

Take any investment opportunity where the IRR exceeds the opportunity cost of capital.

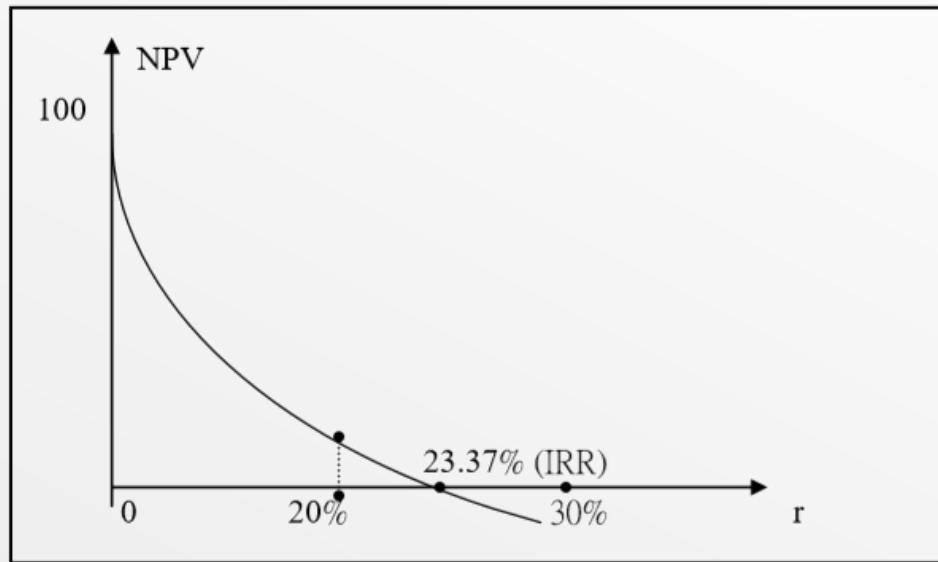
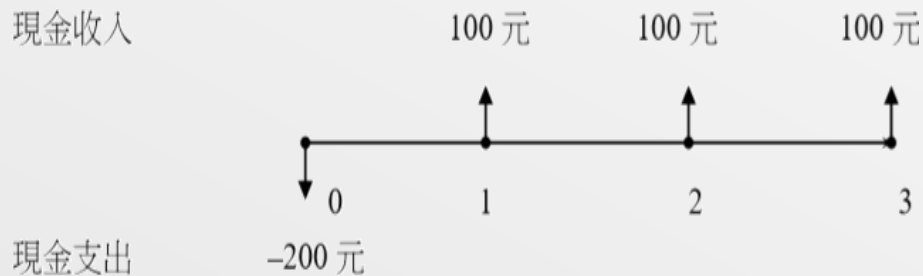
Turn down any opportunity whose IRR is less than the opportunity cost of capital.

→若一個投資計畫的IRR大於所要求的最低報酬率，則可進行。
反之，則不可進行投資。

- The IRR investment rule will give the correct answer (that is, the same answer as the NPV rule) in many—but not all—situations.
- In fact, the IRR rule is only guaranteed to work for a **stand-alone project** if all of the project's **negative cash flows precede its positive cash flows**. If this is not the case, the IRR rule can lead to incorrect decisions.

Example

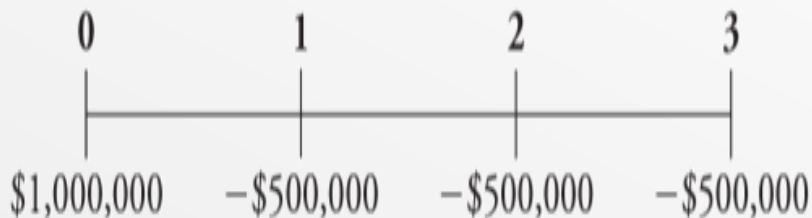
- 假設現在有一個3期的投資計畫，期初需要投入200元，之後每一期會有100元的現金流入，而資金成本為10%，則這個計畫的IRR為：



Pitfall #1: Delayed Investments

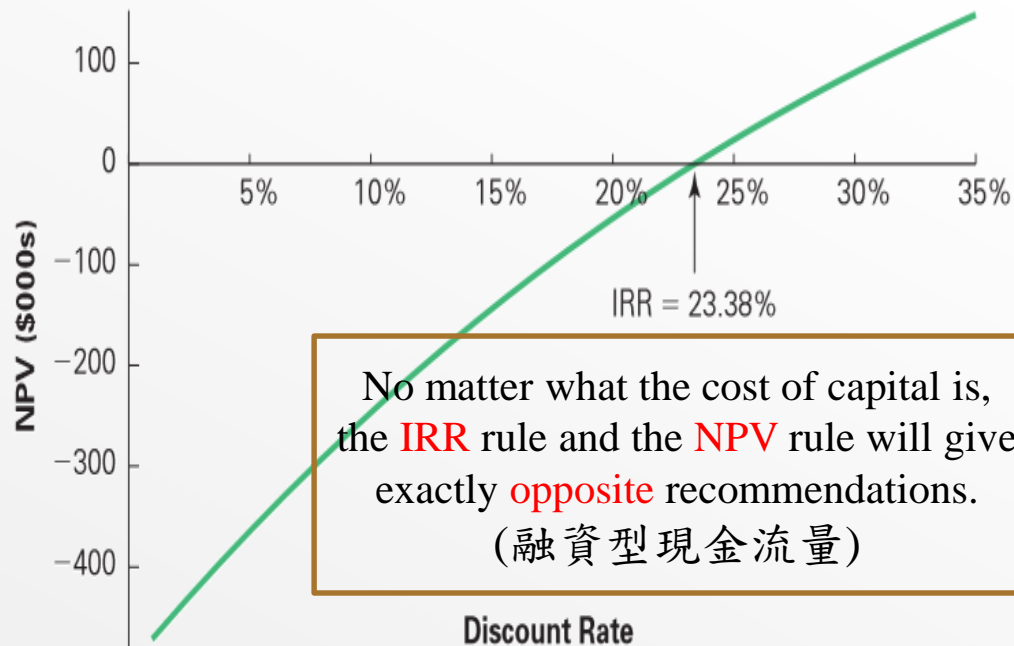
A publisher pay Star \$1 million upfront. He estimates that it will take him three years to write the book. The time that he spends writing will cause him to forgo alternative sources of income amounting to \$500,000 per year. Considering the risk of his alternative income sources and available investment opportunities, Star estimates his opportunity cost of capital to be 10%.

Problems With the IRR-續



$$NPV = 1,000,000 - \frac{500,000}{1+r} - \frac{500,000}{(1+r)^2} - \frac{500,000}{(1+r)^3}$$

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		1,000,000	-500,000	0	
Solve for I		23.38%				=RATE(3,-500000,1000000,0)



→ The 23.38% **IRR** is larger than the 10% opportunity cost of capital. According to the IRR rule, Star should **sign the deal**.

But, At a 10% discount rate, the **NPV** is **negative**, so signing the deal would reduce Star's wealth. He should **not** sign the book deal.

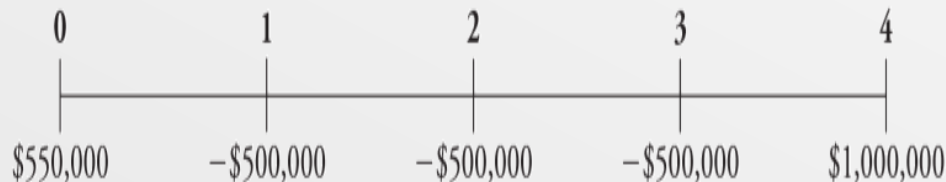
Problems With the IRR-續

Pitfall #2: Multiple IRRs

The publisher offers to give him a royalty payment when the book is published in exchange for taking a smaller upfront payment.

Specifically, Star will receive \$1 million when the book is published and sold four years from now, together with an upfront payment of \$550,000.

Ans:



$$NPV = 550,000 - \frac{500,000}{1+r} - \frac{500,000}{(1+r)^2} - \frac{500,000}{(1+r)^3} + \frac{1,000,000}{(1+r)^4}$$

Ans:



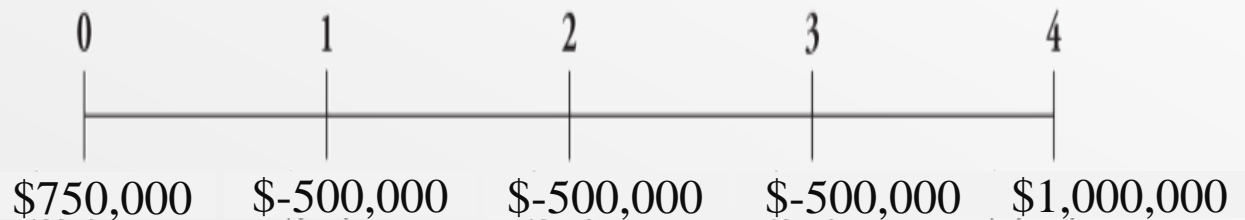
- 雖然IRR在此情況下，無法針對投資計畫進行有效的評估，但唯一可以確認的是當IRR介於7.164%~33.673%時，因為 $NPV < 0$ ，所以要拒絕此投資計畫。
- 當有多個IRR時，大概只能參考NPV是否為正值來做決定。

Problems With the IRR-續

Pitfall #3: Nonexistent IRR

After protracted negotiations, Star is able to get the publisher to increase his initial payment to \$750,000, in addition to his \$1 million royalty payment when the book is published in four years.

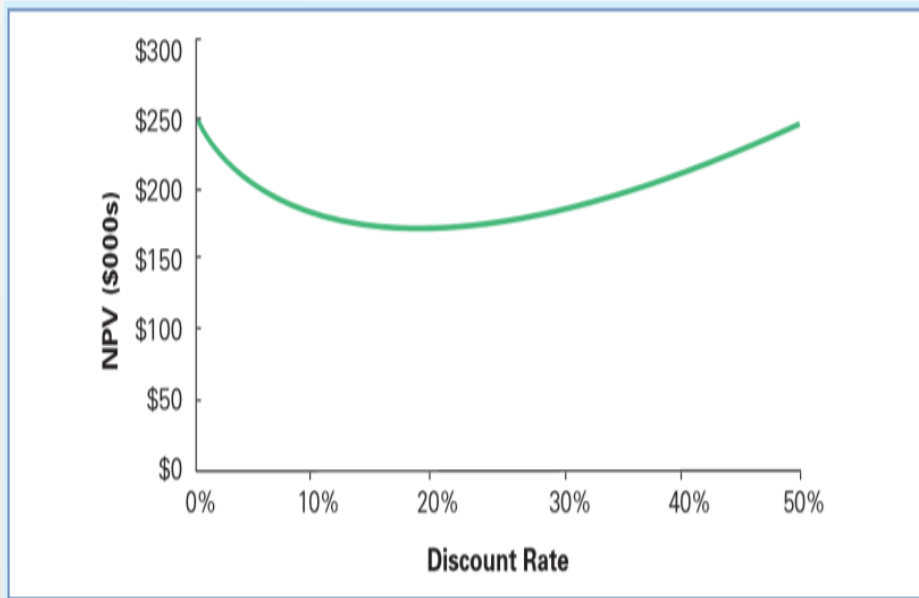
Ans:



$$NPV = 750,000 - \frac{500,000}{1 + IRR} - \frac{500,000}{(1 + IRR)^2} - \frac{500,000}{(1 + IRR)^3} + \frac{1,000,000}{(1 + IRR)^4} = 0$$

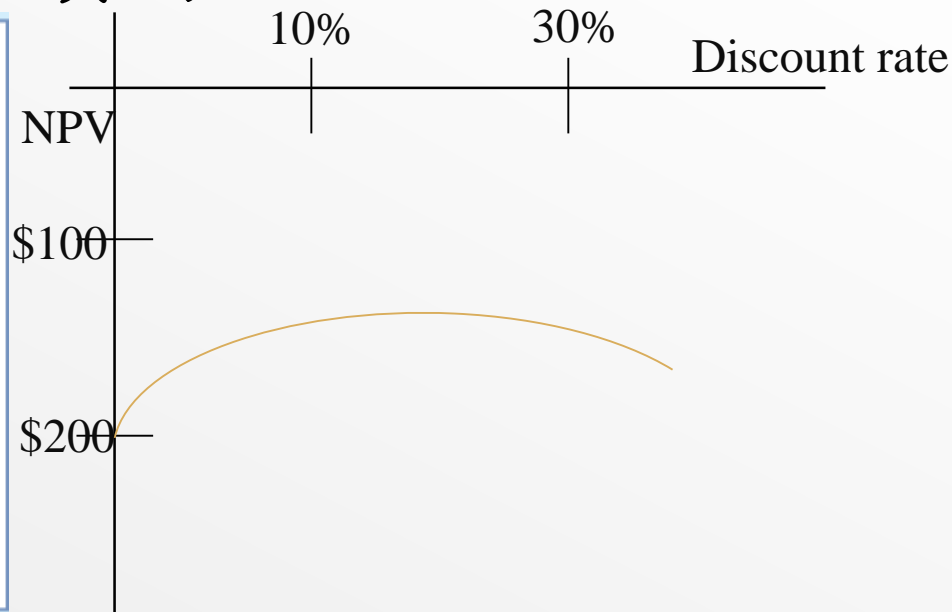
Problems With the IRR-續

Ans:



→ In this case, the NPV is positive for every discount rate, and so there is **no IRR**. Thus, we cannot use the IRR rule.

實務上:



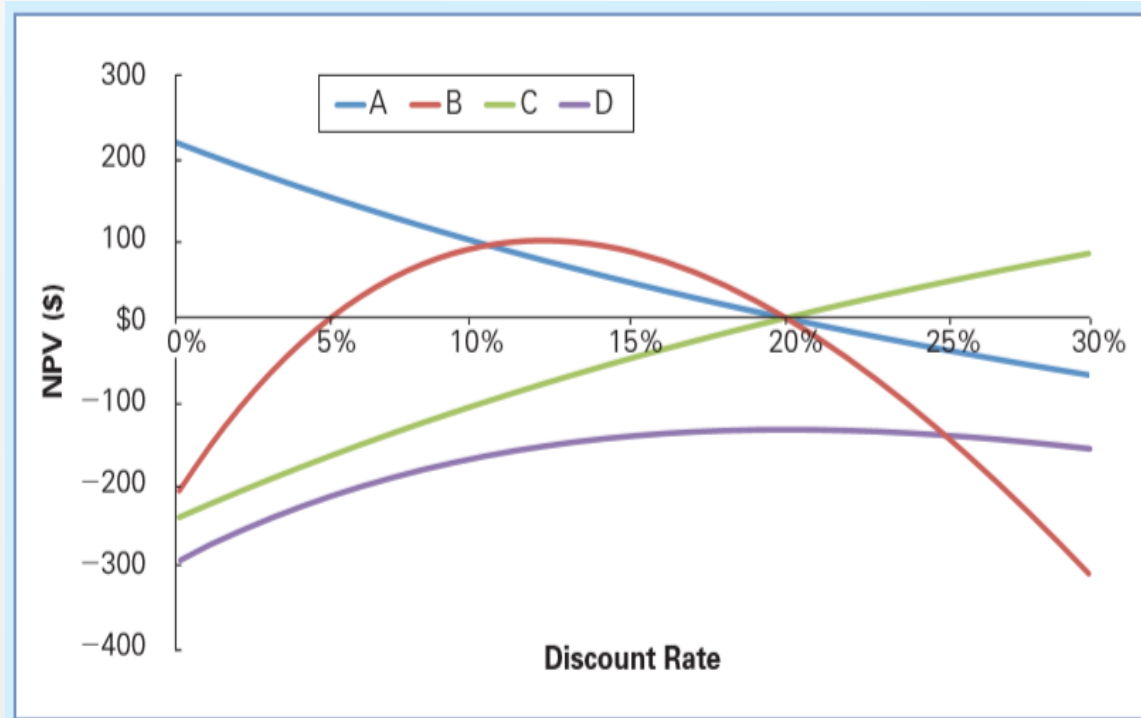
→ 當IRR為無解的時候，大多數的NPV其實都是小於0的。

- Consider projects with the following cash flows:
- Which of these projects have an IRR close to 20%?
 - For which of these projects is the IRR rule valid?

Project	0	1	2
A	-375	-300	900
B	-22,222	50,000	-28,000
C	400	400	-1,056
D	-4,300	10,000	-6,000

Ans:

- projects A, B, and C each have an IRR of approximately 20%, while project D has no IRR.
- The IRR rule is valid only if the project has a positive NPV for every discount rate below the IRR. Thus, the IRR rule is only valid for project A. This project is the only one for which all the negative cash flows precede the positive ones.





7.3 The Payback Rule

The Payback Rule(還本期法)

- Loosely, the pay-back is the length of time it takes to recover our initial investment.
- The payback investment rule states that you should only accept a project if its cash flows pay back its initial investment within a prespecified period.



Example

The Payback Rule

Problem

Assume Fredrick's requires all projects to have a payback period of five years or less. Would the firm undertake the fertilizer project under this rule?

Solution

Recall that the project requires an initial investment of \$250 million, and will generate \$35 million per year. The sum of the cash flows from year 1 to year 5 is $\$35 \times 5 = \175 million, which will not cover the initial investment of \$250 million. In fact, it will not be until year 8 that the initial investment will be paid back ($\$35 \times 8 = \280 million). Because the payback period for this project exceeds five years, Fredrick's will reject the project.

Advantages

- 簡單易行，許多日常決策不值得花費太多成本去做詳細分析。
- 在計算NPV之前，就能先評斷投資計畫的績效，因為還本期法只要資本支出回收後即可開始評估。

Disadvantages

- 未考慮金錢的時間價值，僅單純地加總未來現金流量
- 忽略了公司目標還本期後的現金流量
- 公司所訂的目標還本期法並無理論基礎



Example

Suppose we have somehow decided on an appropriate payback period of two years or less.

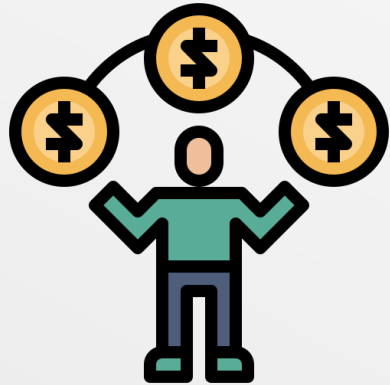
Year	Long	Short
0	-\$250	-\$250
1	100	100
2	100	200
3	100	0
4	100	0
conclusion	The payback on Long is 2.5years.	The payback on Short is 1.75years.
NPV	$-\$250 + (100 * \{[1 - (1/1.15^4)] / .15\}) = \35.50	$-\$250 + \left(\frac{100}{1.15}\right) + \left(\frac{200}{1.15^2}\right) = \$ -\11.81

→ With a cutoff of two years, Short is acceptable and Long is not.



7.4 Choosing Between Projects

Independent



Mutually Exclusive



Mutually Exclusive Projects

➤ NPV Rule

- The highest NPV.

➤ IRR Rule

- IRRs cannot be meaningfully compared.
 - Differences in Scale.
 - Differences in Timing.
 - Differences in Risk.

Review :

$$NPV = -C_0 + \sum_{t=1}^n \frac{C_t}{(1+r)^t}$$

-期初資金 + $\sum_{t=1}^n \frac{\text{第}t\text{期現金流量}}{(1+\text{報酬率})^t}$

Example —

NPV Rule & Mutually Exclusive Investments

Project	Initial Investment	First-Year Cash Flow	Growth Rate	Cost of Capital
Book Store	\$300,000	\$63,000	3.0%	8%
Coffee Shop	\$400,000	\$80,000	3.0%	8%
Music Store	\$400,000	\$104,000	0.0%	8%
Electronics Store	\$400,000	\$100,000	3.0%	11%



Example —

NPV Rule & Mutually Exclusive Investments

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Electronics Store	\$400,000	\$100,000	3.0%	11%

增長型永續年金
(constant growing perpetuity)

$$PV = \frac{\frac{C}{1+i}}{1 - \frac{(1+g)}{(1+i)}} = \frac{C}{i - g}$$

$$NPV(\text{Book Store}) = -300,000 + \frac{63,000}{8\% - 3\%} = \$960,000$$

$$NPV(\text{Coffee Shop}) = -400,000 + \frac{80,000}{8\% - 3\%} = \$1,200,000$$

$$NPV(\text{Music Store}) = -400,000 + \frac{104,000}{8\%} = \$900,000$$

$$NPV(\text{Electronics Store}) = -400,000 + \frac{100,000}{11\% - 3\%} = \$850,000$$

Example —

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IRR Rule & Mutually Exclusive Investments

1 Differences in Scale

1. A 500% return
2. A 20% return

IRR Rule & Mutually Exclusive Investments

1 Differences in Scale

1. A 500% return on \$1
2. A 20% return on \$1 million

IRR Rule & Mutually Exclusive Investments

1 Differences in Scale

1. A 500% return on \$1 = \$5



2. A 20% return on \$1 million = \$200,000

IRR Rule & Mutually Exclusive Investments

1 Differences in Scale

1. A 500% return on \$1 = \$5

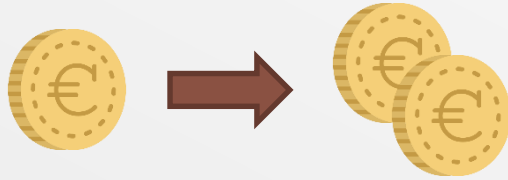


2. A 20% return on \$1 million = \$200,000

IRR Rule & Mutually Exclusive Investments

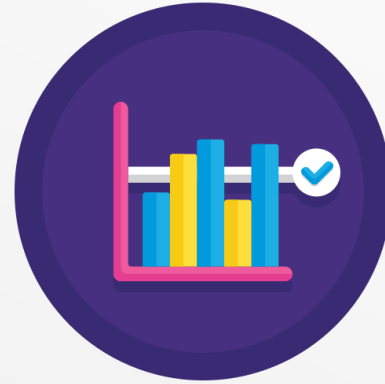
1 Differences in Scale

NPV rule



Law of One Price

IRR rule



the average return

Example —

1 Differences in Scale

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Music Store	\$400,000	\$104,000	0.0%	8%
Electronics Store	\$400,000	\$100,000	3.0%	11%

$$\text{Book Store: } -300,000 + \frac{63,000}{IRR - 3\%} = 0 \Rightarrow IRR = 24\%$$

$$\text{Coffee Shop: } -400,000 + \frac{80,000}{IRR - 3\%} = 0 \Rightarrow IRR = 23\%$$

$$\text{Music Store: } -400,000 + \frac{104,000}{IRR} = 0 \Rightarrow IRR = 26\%$$

$$\text{Electronics Store: } -400,000 + \frac{100,000}{IRR - 3\%} = 0 \Rightarrow IRR = 28\%$$

Example —

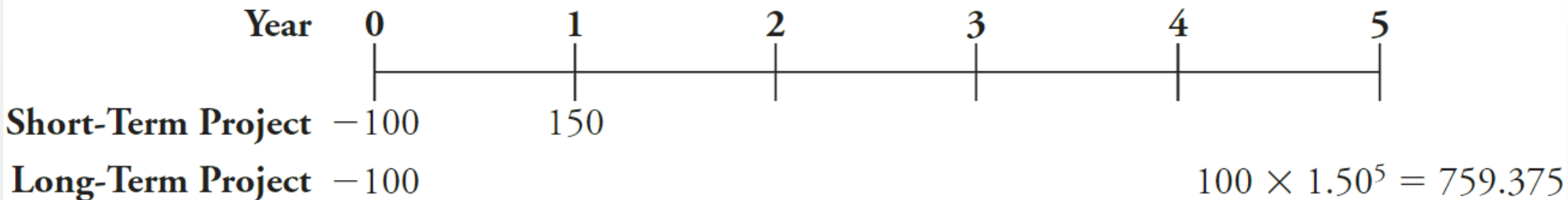
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Project	NPV	IRR
Book Store	\$960,000	24%
Coffee Shop	\$1,200,000	23%

IRR Rule & Mutually Exclusive Investments

2 Differences in Timing



	Short-term project	Long-term project
IRR	50	50
Cost of capital	10%	10%
NPV	$-100 + 150/1.10 = \$36.36$	$-100 + 759.375/1.10^5 = \371.51

10倍多

Example —

2 Differences in Timing

Project	Initial Investment	First-Year Cash Flow	Growth Rate	Cost of Capital
Coffee Shop	\$400,000	\$80,000	3.0%	8%
Music Store	\$400,000	\$104,000	0.0%	8%

Project	NPV	IRR
Coffee Shop	\$1,200,000	23%
Music Store	\$900,000	26%

IRR Rule & Mutually Exclusive Investments

3 Differences in Risk

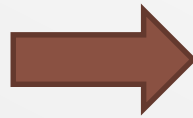
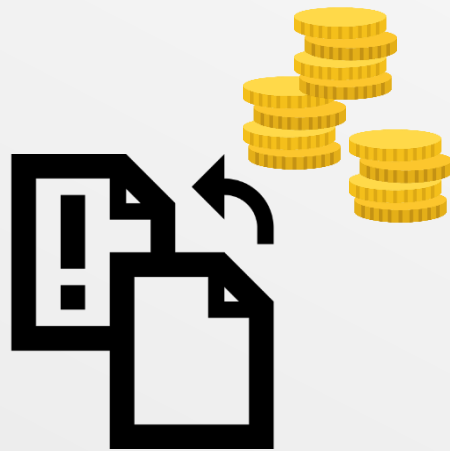
Project	NPV	IRR
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Music Store	\$900,000	26%
Electronics Store	\$850,000	28%

Example —

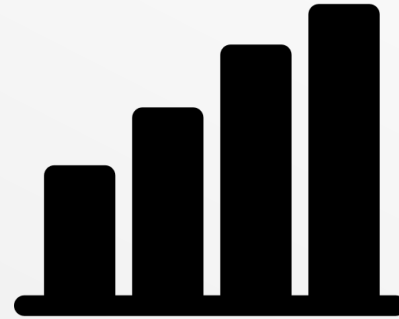
3 Differences in Risk

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Electronics Store	\$400,000	\$100,000	3.0%	11%

Project	NPV	IRR
Book Store	\$960,000	24%
Coffee Shop	\$1,200,000	23%
Music Store	\$900,000	26%
Electronics Store	\$850,000	28%



Incremental IRR



Example —

Using the Incremental IRR to Compare Alternatives



Proposal	0	1	2	3
Minor Overhaul	−10	6	6	6
Major Overhaul	−50	25	25	25

(in millions of dollars)

Problem

1. IRR?
2. Incremental IRR?
3. If the cost of capital for both of these projects is 12%, what should your firm do?

Minor Overhaul

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		-10	6	0	
Solve for Rate		36.3%				=RATE(3,6,-10,0)

IRR = 36.3%

Major Overhaul

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		-50	25	0	
Solve for Rate		23.4%				=RATE(3,25,-50,0)

IRR = 23.4%

$$NPV = -C_0 + \sum_{t=1}^n \frac{C_t}{(1+r)^t}$$

Major Overhaul 取代 Minor Overhaul

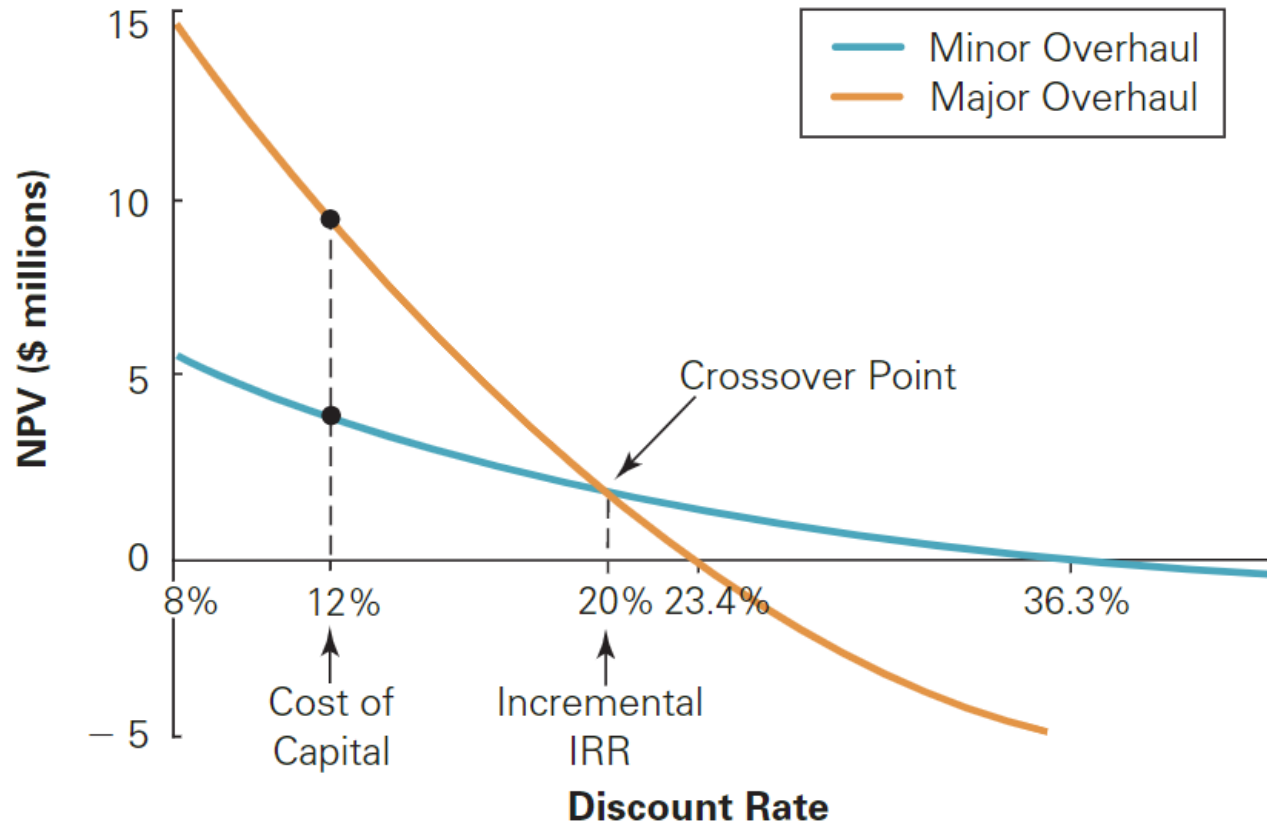
Proposal	0	1	2	3
Major Overhaul	-50	25	25	25
Less: Minor Overhaul	$-(-10)$	-6	-6	-6
Incremental Cash Flow	-40	19	19	19

These cash flows have an IRR of 20.0%:

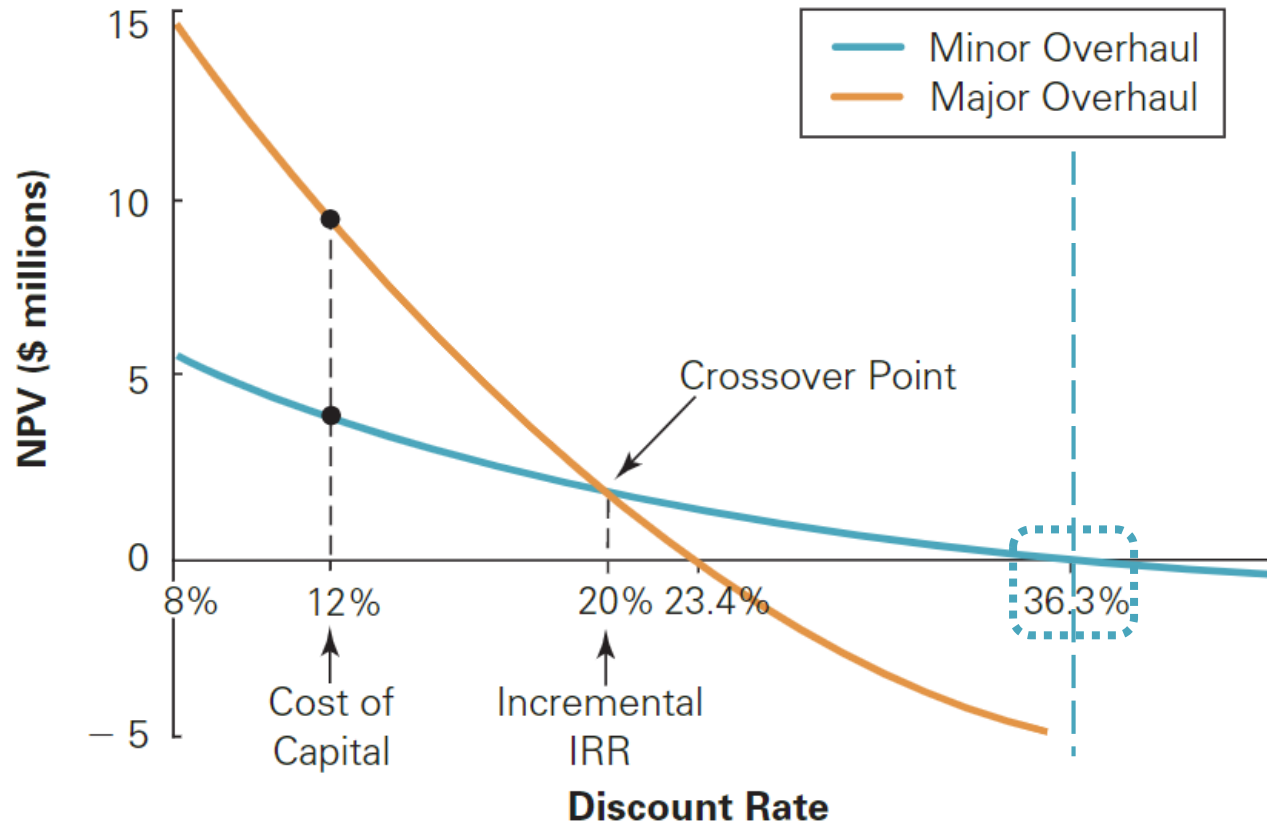
	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		-40	19	0	
Solve for Rate		20.0%				=RATE(3,19,-40,0)

Incremental IRR = 20.0%

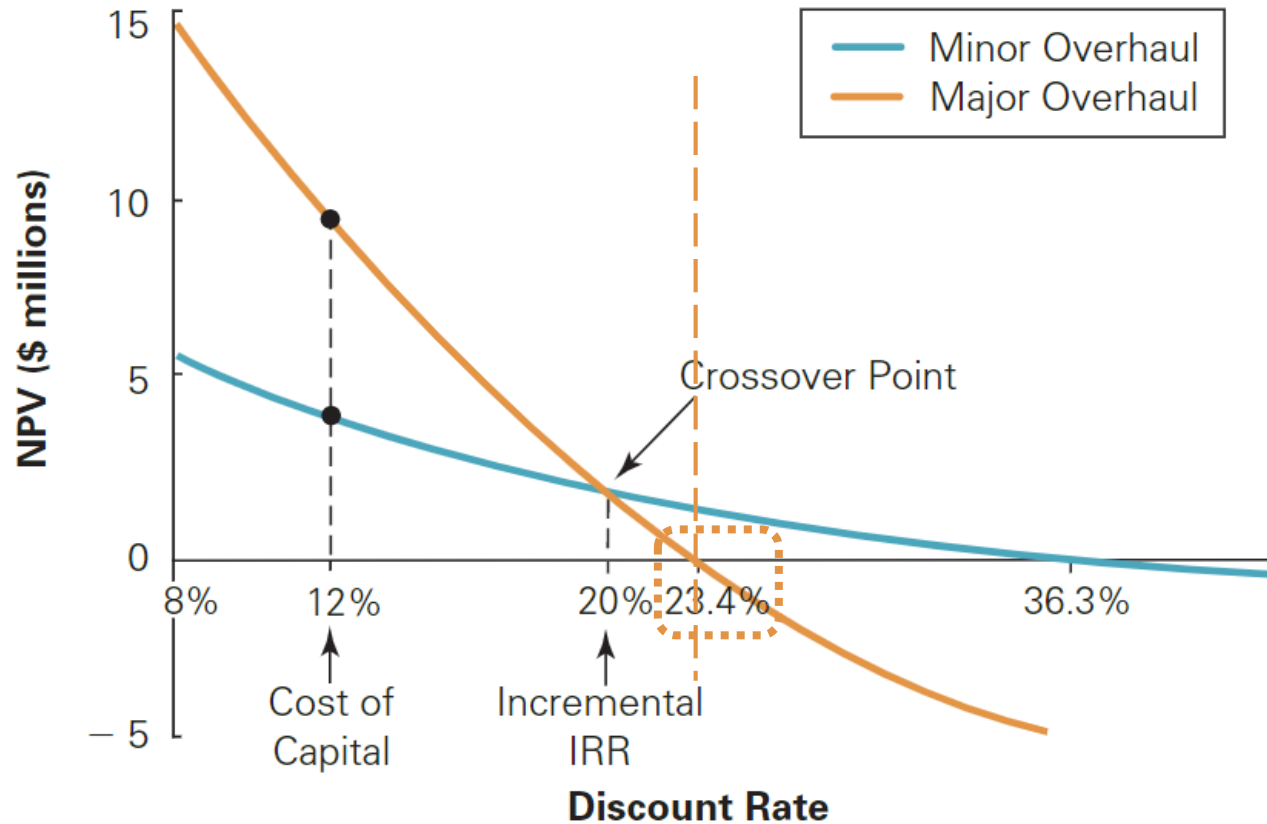
Comparison of Minor and Major Overhauls



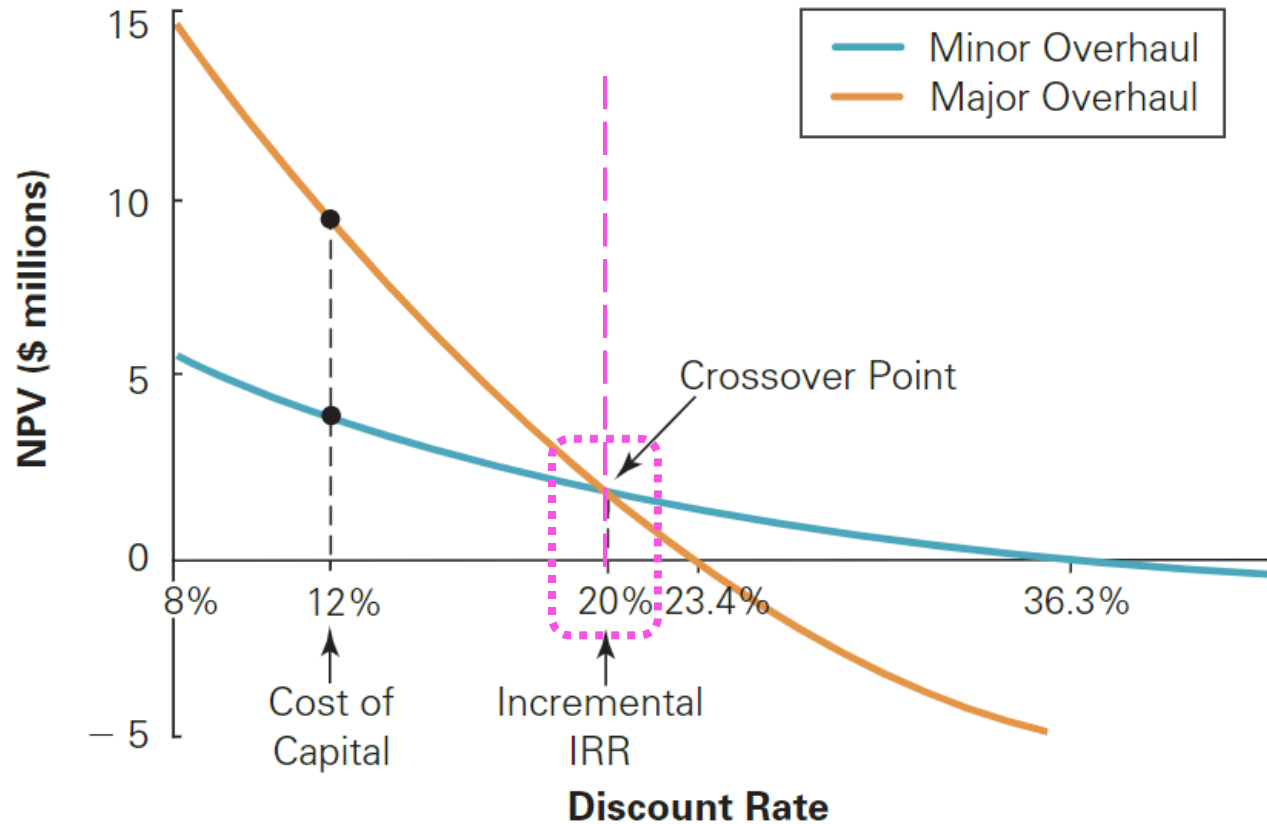
Comparison of Minor and Major Overhauls



Comparison of Minor and Major Overhauls



Comparison of Minor and Major Overhauls



Shortcoming of The Incremental IRR

1. It does not indicate whether either project has a positive NPV on its own.
2. When the individual projects have different costs of capital, it is not obvious what cost of capital the incremental IRR should be compared to.
3. Even if the negative cash flows precede the positive ones for the individual projects, it need not be true for the incremental cash flows. If not, the incremental IRR is difficult to interpret, and may not exist or may not be unique.



NPV Rule

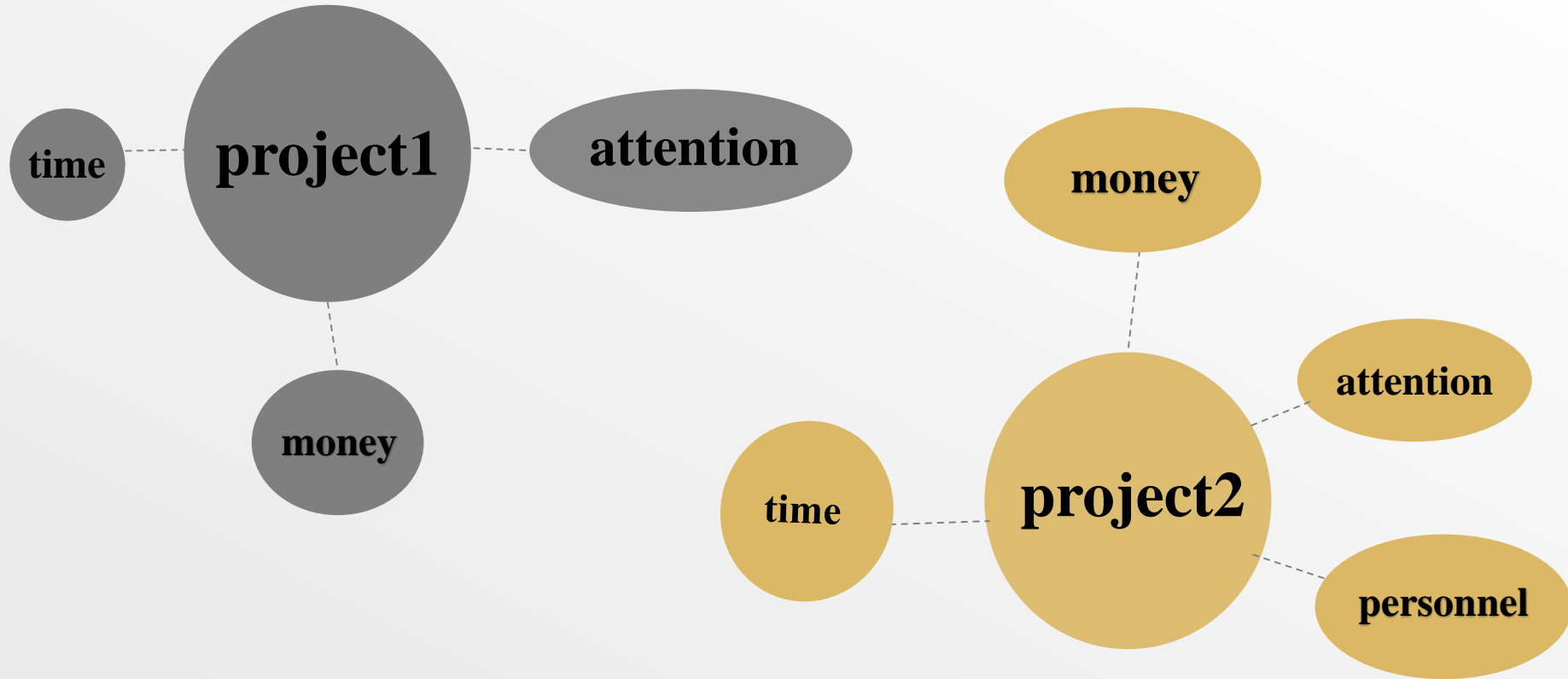
股東財富最大化



7.5 Project Selection with Resource Constraints

項目選擇的約束

Projects with Different Resource Requirements :



Which one do you choose

NPV=110

①

NPV=70

②

NPV=60

③

Which one do you choose

NPV=110

①

投資100元

NPV=70

②

投資50元

投資50元

NPV=60

③

在評估資源要求不同的專案時，根據淨現值對專案進行排名可能不是最佳的。

$$\text{Profitability Index (PI)} = \frac{\text{NPV}}{\text{Resour Consumed}}$$

盈利能力指數衡量的是所創造的價值，
以消耗的資源單位的NPV計算

PI>1 代表項目可行

PI越高該項目越好

Profitability Index

project	NPV	initial investment	PI
1	110	100	1.1
2	70	50	1.4
3	60	50	1.2

當初始投資為稀缺資源的時候：
可得到最佳組合：②+③>①

PI with a Human Resource Constraint

A公司有一個路由器專案，該專案的預期淨現值（NPV）為1770萬美元，並且需要50名軟體工程師。

但是該公司項目很多，但是工程師一共190個，路由器專案必須與以下其他專案競爭這些工程師。

project	NPV (million)	engineers headcount
Router	17.7	50
A	22.7	47
B	8.1	44
C	14.0	40
D	11.5	61
E	20.6	58
F	12.9	32
Total	107.5	332

How should prioritize these project



Project	NPV (\$ millions)	Engineering Headcount (EHC)	Profitability Index (NPV per EHC)	Cumulative EHC Required
Project A	22.7	47	0.483	47
Project F	12.9	32	0.403	79
Project E	20.6	58	0.355	137
Router	17.7	50	0.354	187
Project C	14.0	40	0.350	
Project D	11.5	61	0.189	
Project B	8.1	44	0.184	

最大190
剩餘3人

雖然盈利能力指數易於計算和使用，但為了完全可靠，必須滿足兩個條件：

- 1、根據盈利能力指數排名的專案集耗盡可用資源。
- 2.只有一個相關的資源約束。

Shortcomings of PI

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Project D	11.5	61	0.189	
Project B	8.1	44	0.184	
Project G	0.12	3	0.04	190

Q & A

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