[1]

(a) $f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(z-\mu)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

(b) $P(-1 \le z \le 1)$ = st.norm.cdf(1, 0, 1) − st.norm.cdf(−1, 0, 1) = 0.6827

(c) x = st.norm.ppf(0.975, 0, 1) = 1.96

(d) $f_Q(q) = \frac{1}{\sqrt{2}\,\Gamma(\frac{1}{2})} q^{\frac{1}{2}-1} \cdot e^{-\frac{q}{2}} = \frac{1}{\sqrt{2\pi q}} e^{-\frac{q}{2}}$

$$f_Q(q) = \begin{cases} \frac{1}{\sqrt{2\pi q}} e^{-\frac{q}{2}}, & q > 0 \\ 0, & q \le 0 \end{cases}$$

(e) $E[Q] = 1$

(f) $std[Q] = \sqrt{2 \cdot 1} = \sqrt{2}$

(g) $P(Q \le 1)$ = st.gamma.cdf(x=1, α=0.5, scale=2) = 0.6827

[2] (a) $f_T(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t \le 0 \end{cases}$ $(\alpha = 1, \beta = 1)$

(b) $E[T] = \beta = 1$

(c) $std[T] = \sqrt{\beta^2} = \beta = 1$

(d) $P(T > 1)$ = st.gamma.sf(x=1, α=1, scale=1) = 0.3679#

(e) $\alpha = 3, \beta = 1$

$f_{T_3}(t) = \frac{1}{1^3\Gamma(3)} t^{3-1} e^{-\frac{t}{1}} = \frac{1}{2} t^2 e^{-t}$

$$\therefore f_{T_3}(t) = \begin{cases} \frac{1}{2} t^2 e^{-t}, & t > 0 \\ 0, & t \le 0 \end{cases}$$

(f) $E[T_3] = \alpha\beta = 3$

(g) $std[T_3] = \sqrt{\alpha\beta^2} = \sqrt{3}$

(h) $P(T_3 > 3)$ = st.gamma.sf(x=3, α=3, scale=1) = 0.4232

(i) $P(T_3 > 7)$ = st.gamma.sf(x=7, α=3, scale=1) = 0.0296

假設該產品平均壽命為1年，即β=1，在此條件下算出使用3個

產品超過7年以上的機率為2.90%，機會相當小，因此可接受此假設