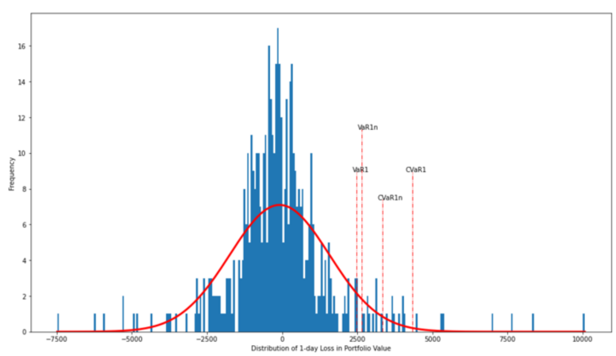
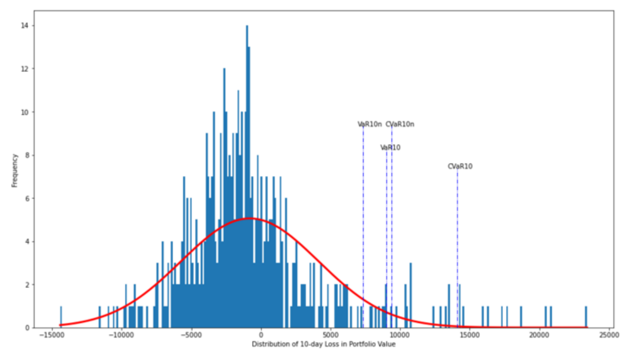
Question 1

Part 1: The 1-day and 10-day VaR and CVaR for both historical scenarios and normal model are shown as below.

```
Historical 1-day VaR 95.0% = $2477.25,   Historical 1-day CVaR 95.0% = $4326.98
    Normal 1-day VaR 95.0% = $2646.49,       Normal 1-day CVaR 95.0% = $3339.94
Historical 10-day VaR 95.0% = $9023.03,   Historical 10-day CVaR 95.0% = $14099.34
    Normal 10-day VaR 95.0% = $7331.15,       Normal 10-day CVaR 95.0% = $9396.60
```

The histograms of the distribution of 1-day (left) and 10-day (right) losses VaR and CVaR in portfolio value are shown as below.





Part 2: Equation *VaR (10-day) = 10 × VaR (one-day)* and *CVaR (10 day) = 10 × CVaR (one day)* are wrong according to the print-out below.

```
VaR for 10-day loss: 9023.029310000004
VaR for 1-day loss:  2477.252192000003
CVaR for 10-day loss: 14099.341740868671
CVaR for 1-day loss:  4326.981496960318
```

The inequality is due to the assumption that stock prices make the so-called random walk, mathematically Wiener Process, but popularly better known as Brownian Motion. Each particular increment of this random walk has variance that is proportional to the time over which the price was moving. Therefore, the standard deviation of stock returns tends to increase with the square root of time. By testing the Var and CVaR, we can show that 10-day Var and CVaR are approximately equal to $\sqrt{10}$ of 1-day Var and CVaR shown in the equations as below.

$$VaR\ (10\text{-}day) \approx \sqrt{10} \times VaR\ (one\text{-}day)$$

$$CVaR\ (10\text{-}day) \approx \sqrt{10} \times CVaR\ (one\text{-}day)$$

```
VaR for 1-day loss x sqrt(10): 7833.759265364759
CVaR for 1-day loss x sqrt(10): 13683.116923799545
```

By using historical scenarios, the sum of 1-day 95% VaR for 100 MSFT, 200 AAPL and 500 IBM stocks does not equal to 1-day 95% VaR for the portfolio. If we use normal model to calculate VaR, the difference decreases from 456.3 to 438.6, which is slightly better.

```
False
Sum of VaR of individual stocks: 2933.5838380000023
VaR of portfolio: 2477.252192000003
```

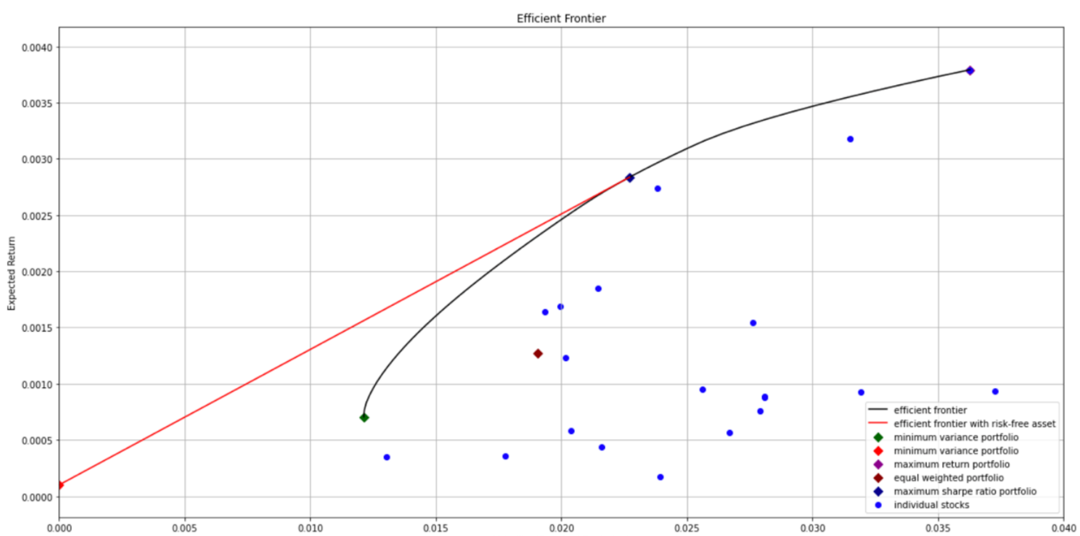
Historical Scenarios

```
False
Sum of VaR of individual stocks: 3085.1433788349605
VaR of portfolio: 2646.4918309374666
```

Normal Model

Question 2

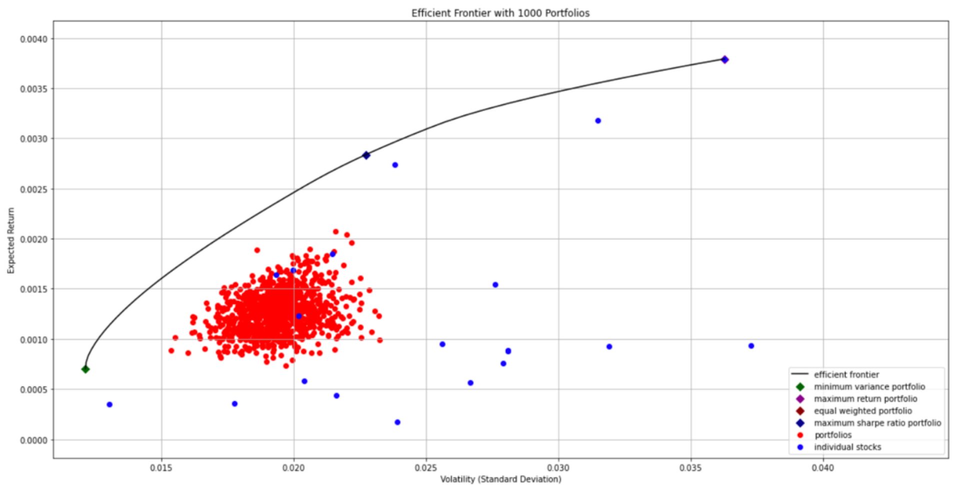
Part 1: The efficient frontier without risk-free asset is shown as a black arc while the efficient frontier with risk-free asset is shown as a red line as below. For the efficient frontier without risk-free asset, the minimum variance portfolio under no-short-sell constraint is plotted as a green square at the left end of the arc while the maximum return portfolio is plotted as a purple square at the right end of the arc. For the efficient frontier with risk-free asset, the minimum variance portfolio with risk-free asset is plotted as a red square on the y-axis and the maximum Sharpe ratio portfolio is plotted as a dark blue square at the end of tangent line. The blue dots fall below the efficient frontier represent the volatility and return of individual stocks.



Part 2: I randomly generated 1000 numbers for 20 stocks from exponential distribution, which resulted in a matrix 1000 x 20. Then I calculated the weights by dividing each number to the sum of the row to make sure the sum of each row is 1. Exponential distribution can produce the portfolio weights that close to uniform distribution and result in a more spread out distribution on mean-variance plot. The formula is shown as following.

$$y_1, \ldots y_n \sim \text{Exp}(1) \text{ i.i.d.}$$

$$x_i = y_i / (\sum_{j=1}^{n} y_j)$$



Question 3

Part 1: The factor loadings for each of the twenty assets in 2019, 2020, and over the two years are shown in the **appendix**. The fitted loadings for three historic periods are shown as below. It can be shown that the R^2 for 20 stocks in 2020 are significantly higher than those in 2019. Since R-Squared is a statistical measure of fit that indicates how much variation of a dependent variable is explained by the independent variable(s) in a regression model, thus the OLS regression model can fit the data in 2020 better than in 2019. Lower value of R^2 in 2019 indicates that the returns on the security is more random, or may be explained by unobserved factors other than the market premium, SMB, and HML. Therefore, the stock market in 2019 is more unpredictable than that in 2020 due to other influential factors.

	MSFT	F	JPM	GOOG	HPQ	C	HOG	VZ	AAPL	IBM	T	CSCO	BAC	INTC	AMD	SNE	NVDA	AMZN	MS	BK
2019	0.733782	0.322942	0.737760	0.468008	0.27425	0.762388	0.489308	0.081453	0.544906	0.445746	0.186319	0.438211	0.720136	0.374959	0.395763	0.297078	0.505823	0.574535	0.692871	0.414714
2020	0.880915	0.646845	0.887034	0.752131	0.60674	0.870422	0.562520	0.547254	0.763396	0.723068	0.734615	0.658981	0.898552	0.576326	0.530567	0.560851	0.788277	0.591290	0.842146	0.754778
2019-2020	0.836645	0.571292	0.854627	0.650872	0.51071	0.846797	0.545291	0.380165	0.693281	0.648105	0.606403	0.585631	0.858923	0.527248	0.442763	0.434783	0.668146	0.555235	0.799401	0.677046

Since the three-factor model predicts the expected return based on market premium, size premium, and value premium, those factors may not capture the effect caused by COVID-19. Therefore, the return estimated by the model may be overestimated because COVID-19 affected many industries, and the cash flow and profit of companies are greatly undermined.

Part 2: I chose 100 scenarios and 504 steps because 100 scenarios is enough to compute the mean value of VaR and 504 is the maximum trading day for 2019-2020.

By using historical scenarios, the sum of 1-day 95% VaR for 100 MSFT, 200 AAPL and 500 IBM stocks does not equal to 1-day 95% VaR for the portfolio. If we use normal model to calculate VaR, the difference decreases from 141 to 7, which is slightly better.

```
Historical Monte Carlo Simulation 1-day 100 MSFT VaR 95.0% = $ 68.71
Historical Monte Carlo Simulation 1-day 200 AAPL VaR 95.0% = $100.51
Historical Monte Carlo Simulation 1-day 500 IBM VaR 95.0% = $381.87
Historical Monte Carlo Simulation 1-day Sum of Individual Stocks VaR 95.0% = $551.09
Historical Monte Carlo Simulation 1-day Portfolio VaR 95.0% = $409.18

-141.90600688271417
```

```
Normal Monte Carlo Simulation 1-day 100 MSFT VaR 95.0% = $629.94
Normal Monte Carlo Simulation 1-day 200 AAPL VaR 95.0% = $642.58
Normal Monte Carlo Simulation 1-day 500 IBM VaR 95.0% = $2626.11
Normal Monte Carlo Simulation 1-day Sum of Individual Stocks VaR 95.0% = $3898.63
Normal Monte Carlo Simulation 1-day Portfolio VaR 95.0% = $3906.46

7.836799858504037
```

The basis of a Monte Carlo simulation is that the probability of varying outcomes cannot be determined because of random variable interference. Therefore, a Monte Carlo simulation focuses on constantly repeating random samples to achieve certain results. Therefore, Monte Carlo simulation heavily rely on the number of times to generate random samples.

The VaR computed by Monte Carlo simulation from historical scenarios is significantly smaller compared to the VaR in question 1, while the VaRn computed by Monte Carlo simulation from normal model is significantly larger comparing to the VaRn in question 1. It may be due to the sampling error from simulating from 100 scenarios or model error of Monte Carlo simulation. If we simulate more scenarios like 10000, the value of VaR computed from Monte Carlo simulation may be smaller due to less sampling error. Also, since we are using three factor model to predict returns, the factors other than market excess, SMB, and HML could not be considered. Moreover, there are correlations between three factor and there’s non-linear relationship in the model while the model in question 1 assume linearity.

Appendix

2019 Factor Loadings

	constant	mkt_excess	SMB	HML
MSFT	-0.008115	0.012480	-0.004860	-0.005243
F	-0.008120	0.010187	0.005505	0.005376
JPM	-0.007749	0.011368	-0.003083	0.009953
GOOG	-0.008755	0.011987	-0.000276	-0.004431
HPQ	-0.009278	0.012463	0.001341	0.002378
C	-0.007809	0.015404	-0.001497	0.010721
HOG	-0.008710	0.014426	0.004634	0.010641
VZ	-0.008317	0.003557	-0.002750	-0.000280
AAPL	-0.007416	0.014750	-0.001725	-0.002657
IBM	-0.008668	0.010820	-0.002796	0.001895
T	-0.007512	0.005742	-0.000151	0.003071
CSCO	-0.009088	0.012441	-0.001421	-0.000288
BAC	-0.007716	0.013013	0.000295	0.012635
INTC	-0.008633	0.013244	-0.004030	0.001250
AMD	-0.007039	0.024222	0.003027	-0.008222
SNE	-0.008094	0.010976	0.002059	-0.001116
NVDA	-0.008027	0.021301	0.004262	-0.000924
AMZN	-0.009064	0.012192	0.000365	-0.005457
MS	-0.008336	0.013966	-0.001379	0.010509
BK	-0.008683	0.009838	0.000308	0.008839

2020 Factor Loadings

	constant	mkt_excess	SMB	HML
MSFT	-0.001585	0.012518	-0.002611	-0.005145
F	-0.000704	0.008952	0.001251	0.010016
JPM	-0.000619	0.010522	-0.002925	0.011202
GOOG	-0.001481	0.010161	-0.001256	-0.002577
HPQ	-0.001037	0.010593	0.007312	0.003554
C	-0.001043	0.012899	-0.000314	0.013294
HOG	-0.000712	0.013514	0.003191	0.008999
VZ	-0.001610	0.005099	-0.002851	0.001182
AAPL	-0.000369	0.012392	-0.004116	-0.004339
IBM	-0.001723	0.009247	-0.000530	0.003232
T	-0.002102	0.007325	-0.003868	0.005033
CSCO	-0.002137	0.009939	-0.003330	0.000240
BAC	-0.000754	0.011506	-0.003540	0.012039
INTC	-0.002892	0.012094	-0.003062	-0.001615
AMD	-0.000397	0.013447	-0.000670	-0.007493
SNE	-0.000679	0.007721	-0.000523	-0.001504
NVDA	-0.000623	0.015457	0.002397	-0.009109
AMZN	-0.000662	0.008549	-0.001927	-0.006813
MS	0.000283	0.012767	-0.002809	0.008049
BK	-0.001318	0.009330	-0.002744	0.008290

2019-2020 Factor Loadings

	constant	mkt_excess	SMB	HML
MSFT	-0.004818	0.012503	-0.002833	-0.005240
F	-0.004436	0.009341	0.002752	0.009034
JPM	-0.004158	0.010692	-0.002539	0.010818
GOOG	-0.005081	0.010496	-0.000499	-0.003147
HPQ	-0.005003	0.010821	0.006537	0.003249
C	-0.004329	0.013317	0.000015	0.012657
HOG	-0.004725	0.013625	0.003922	0.008913
VZ	-0.005008	0.004959	-0.002654	0.000951
AAPL	-0.003882	0.012705	-0.003073	-0.004534
IBM	-0.005112	0.009488	-0.000553	0.002850
T	-0.004912	0.007241	-0.002908	0.004665
CSCO	-0.005579	0.010334	-0.002339	-0.000231
BAC	-0.004268	0.011759	-0.002236	0.011728
INTC	-0.005732	0.012159	-0.002975	-0.001440
AMD	-0.003480	0.014977	0.001523	-0.008650
SNE	-0.004348	0.008198	0.000694	-0.001945
NVDA	-0.004237	0.016024	0.003543	-0.008725
AMZN	-0.004813	0.009046	-0.000714	-0.007173
MS	-0.004037	0.012892	-0.002043	0.008031
BK	-0.005048	0.009441	-0.001695	0.008061