Application of System of Linear Equations to Traffic Flow for a Network of Four One-Way Streets in Kumasi, Ghana

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Abstract

We used a system of linear equations to determine the number of vehicles that should be allowed to route a four one-way streets in Kumasi, in order to keep traffic flowing. The systems of equations used in the model were solved analytically using the method of Gauss-Jordan elimination. Our work shows that if 155vph, 276vph, and 240vph are allowed to route intersection A and B, A and D and D and C of the

model respectively, traffic congestion in the area of Kumasi discussed in the model would be minimized. We recommend to the stakeholders of Kumasi to provide exclusive lanes for public transport, ensure the use of traffic regulations and traffic engineers to control the traffic, use innovative ideas to reduce traffic impacts on public transport, provide traffic lights at the four intersections under discussion in this research and adjust them in the direction of the results of this research.

Keywords: Mathematical model, traffic congestion, Traffic flow, traffic volume

1. Introduction

In mathematics and civil engineering, traffic flow is the study of interactions between vehicles, drivers, and infrastructure (including highways, signage, and traffic control devices), with the aim of understanding and developing an optimal road network with efficient movement of traffic and minimal traffic congestion problems [1].

Mathematical theory of traffic flow and traffic equilibrium analysis was first introduced by Frank Knite in 1920's, and was refined into Wardrop's first and second principles of equilibrium. Current traffic models use a mixture of empirical and theoretical techniques. These models are then developed into traffic forecast, to take account of proposed local or major changes, such as increased vehicle use, changes in land use or changes in mode of transport and to identify areas of congestion where the network needs to be adjusted [1].

Traffic congestion has a number of negative effects on humanity. These include wasting time of motorists and passengers which therefore reduce regional economic health; delays, which may result in late arrival for employment, meetings and education, resulting in loss of businesses, disciplinary action or other personal losses. Blocked traffic may interfere with the passage of emergency vehicles travelling to their destinations where they are urgently needed; wasted fuel, increasing air pollution and carbon dioxide emissions owing to increasing idling, acceleration and breaking of vehicles; wear and tear on vehicles as a result of idling in traffic and frequent accelerating and breaking, leading to more frequent repairs and replacement of car parts; stressed and frustrated motorists, encouraging road rage and reduced health of motorists[2].

In United States of America, the Texas Transportation Institute estimated that, in 2000, the 75 largest metropolitan areas experienced 3.6 billion vehicle-hours of delay, resulting in 5.7 billion U.S. gallons (21.6 billion liters) in wasted fuel and $67.5 billion in lost productivity, or about 0.7% of the nation's GDP [3].

In the United Kingdom, the inevitability of congestion in some urban road networks has been officially recognized since the Department for Transport set down policies based on the reported Traffic in Towns in 1963. The Department for Transport sees growing congestion as one of the most serious transport

problems facing the UK [3]. Eddington (2006) published a UK government-sponsored report into the future of Britain's transport infrastructure. The Eddington Transport Study set out the case for action to improve road and rail networks, as a "crucial enabler of sustained productivity and competitiveness". Eddington has estimated that congestion may cost the economy of England £22 bn a year in lost time by 2025. He warned that roads were in serious danger of becoming so congested that the economy would suffer [4].

In China, the August 2010 China National Highway 110 traffic jam in Hebei province, is considered the world's worst traffic jam ever, as traffic congestion stretched more than 100 kilometres (62 miles) from August 14 to 26, including at least 11 days of total gridlock [5, 6, 7]. The New York Times has called this event the "Great Chinese Gridlock of 2010 [8].

In Ghana, Transport experts have attributed the depressing vehicular congestion to an increase in vehicular population on the city's already inadequate roads. There are an estimated 1.2 million vehicles in Ghana, 60 per cent in Accra alone, and with a total road network of 1,632 kilometres of 1,310 kilometres are tarred. Accra's roads appear woefully inadequate as heavy traffic congestion characterizes travelling on most roads within the national capital [9].
Traffic congestion on Kumasi roads is becoming an ever-present nightmare for road users, negatively affecting productivity and the environment. The city of Kumasi has spread out over the past years. Being the second largest city in Ghana, it is experiencing rapid urbanisation and accelerated population growth and an exploded traffic on its roads. Nowhere is this more evident than in the Central Business District (CBD) and on other arterial roads. One result of this phenomenon is the severe traffic congestion as witnessed on the Lake Road, 24th February Road and the Sunyani Road which results in loss of working time, affecting productivity, higher vehicle running cost and negative environmental impact. Information at the Driver and Vehicular Licensing Authority (DVLA) indicate that the number of vehicles imported into Kumasi keep increasing year after year [10].

The current study therefore aims at applying a system of linear equations to traffic flow for a network of four one-way streets in Kumasi, Ghana. It also aims to determine the number of vehicles that should be allowed to route the four one-way streets under study in the model in order to reduce traffic congestion in Kumasi.

2. Mathematical Model

A system of linear equations was used to analyze the flow of traffic for a network of four one-way streets in Kumasi, Ghana. The pioneering work done by Gareth Williams on Traffic flow [11] has led to greater understanding of this research. The variables $x, y, z,$ and w represent the flow of the traffic between the four

intersections in the network. The data was obtained by counting the number of vehicles that travelled around the four one-way streets between the hours of 6am to 10pm, and 2pm to 6pm during the mid-week peak traffic hours. The arrows in the diagram indicate the direction of flow of traffic in and out of the network that is measured in terms of number of vehicles per hour (vph). The diagram in Figure 1 below describes the four one-way streets in Kumasi under study in the model:
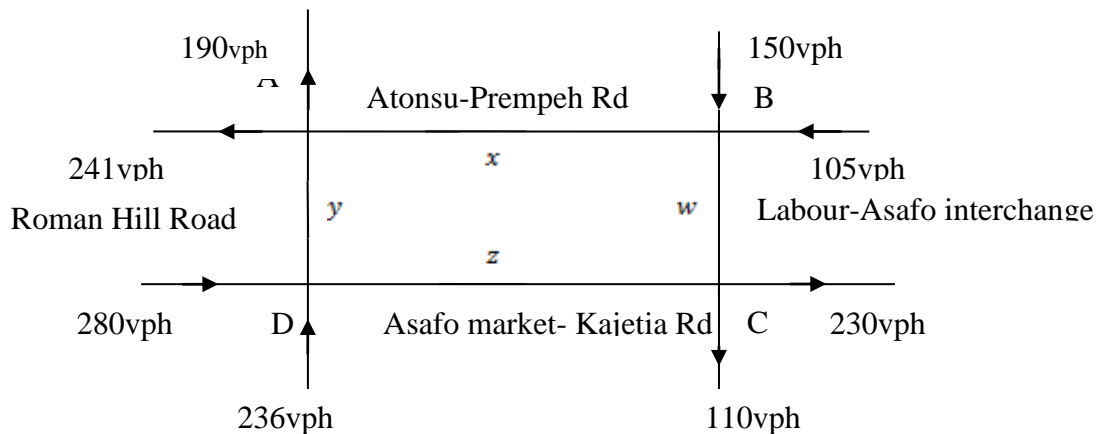

Figure 1: Diagram of the four one-way streets, in Kumasi

Model Assumptions

The following assumptions were made in order to ensure the smooth flow of the traffic;

    i)   Vehicles entering each intersection should always be equal to the number of vehicles leaving the intersection.

    ii)   The streets must all be one-way with the arrows indicating the direction of traffic flow.

The system of equations for the model was formulated as follows:

At intersection A: Traffic in $= x + y$, traffic out $= 241 + 190$, thus, $x + y$

$= 431$.

At intersection B: Traffic in $= 150 + 105$, traffic out $= x + w$, thus $x + w$

$= 255$

At intersection C: Traffic in $= z + w$, traffic out $= 230 + 110$, thus, $z + w = 340$

Intersection D: Traffic in $= 280 + 236$, Traffic out $= y + z$, thus, $y + z = 516$

The constraints were written as a system of linear equations as follows:

$x + y = 431$

$x + w = 255$

$z + w = 340$

$y + z = 516$

We then used the Gauss-Jordan elimination method to solve the system of equations. The augmented matrix and reduced row-echelon form of the above system are as follows:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 1 & 0 & 0 & 1 & 255 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 1 & 1 & 0 & 516 \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 0 & 1 & 255 \\ 0 & 1 & 0 & -1 & 176 \\ 0 & 0 & 1 & 1 & 340 \\ & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system of equations that corresponds to this reduced row-echelon form is;

$x+w = 255$

$y - w = 176$

$z + w = 340$

Expressing each leading variable in terms of the remaining variable, we had

$x = -w + 255$

$y = w + 176$

$z = -w + 340$

If we take a construction limit on Labour-Asafo Interchange Rd (w) to be 100vph, then the values of $x, y,$ and z will be;

x = -100 + 255 = 155vph

y = 100 + 176 = 276vph

z = -100 + 340 = 240vph

Discussion of Results

The system of the modeling equations has many solutions, and therefore many traffic flows are possible. A driver has a certain amount of choice at the intersection, due to the nature of the model. Considering the stretch DC, it is desirable to have small traffic flow z as possible along this stretch of road. The flows can therefore be controlled along the various branches by the use of traffic lights. According to the model, the third equation in the system shows that z will be a minimum when w is as large as possible, as long as it does not exceeds 340. The largest value w can be assumed without causing negative values of x, or y is 255. Thus the smallest value of w is -255+340 or 85. Any road work on Asafo interchange to Roman hill down should allow for traffic volume of at least 85vph. Therefore, to keep the traffic flowing 240vph must be routed between D and C, 155vph between A and B and 276vph between the intersections A and $D.$

Conclusion

We have established that traffic congestion at the four one-way street linking Labour-Prempeh Rd, Roman Hill Rd, Asafo-Roman Rd and Asafo interchange-Labour Rd can be minimized if any road work on Asafo interchange to Roman hill down should allow for traffic volume of at least 85vph. Therefore, to keep the traffic flowing, 240 vehicles per hour must be routed between D and C, 155vph between A and B, and 276vph between the intersections A and D respectively

Recommendations

In order to reduce the impact of traffic congestion and ensure the free flow of traffic in Kumasi, we made the following recommendations to the stakeholders of Kumasi:

- provide exclusive lanes for public transport
- use regulations and traffic engineers to control the traffic
- ensure the use of innovative ideas to reduce traffic impacts on public transport.
- provide traffic lights at the four intersections under discussion and adjusted in the direction of the results of this research.

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