ROB521 Aid Sheet

Kinematics: $\underline{C}_{12} = \underline{C}_{21}^T = \underline{C}_{21}^{-1}$ $\quad \underline{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}$ $\quad \underline{C}_2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$ $\quad \underline{C}_3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ glo ref fra

$\underline{C}_{21} = \cos\varphi \, \underline{1} + (1-\cos\varphi)\, \underline{a}\,\underline{a}^T - \sin\varphi \, \underline{a}^\times$ $\qquad \underline{p}_v^{p_j v} = \underline{T}_{vi}\, \underline{p}_i^{p_j i} \Leftrightarrow \underline{p}_v^{p_j v} = \underline{C}_{vi}(\underline{r}_i^{p_j i} - \underline{r}_i^{vi})$

2D: $\underline{r}_i^{vi} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ $\quad \underline{C}_{vi} = \underline{C}_3$ $\quad \underline{q} = [x \; y \; \theta]^T$

$\underset{\rightarrow}{r}^{\bullet} = \underset{\rightarrow}{r}^{\circ} + \underset{\rightarrow}{\omega}_{21} \times \underset{\rightarrow}{r}$ (frame 2 w.r.t. frame 1) $\qquad \underset{\rightarrow}{r}^{\bullet\bullet} = \underset{\rightarrow}{r}^{\circ\circ} + 2\underset{\rightarrow}{\omega}_{21} \times \underset{\rightarrow}{r}^{\circ} + \underset{\rightarrow}{\omega}_{21}^{\circ} \times \underset{\rightarrow}{r} + \underset{\rightarrow}{\omega}_{21} \times (\underset{\rightarrow}{\omega}_{21} \times \underset{\rightarrow}{r})$

acc. coriolis acc. angular acc. centripetal acc.

$\dot{\underline{C}}_{21} = -\underline{\omega}_2^{21\times}\underline{C}_{21} \qquad \underline{\omega}_2^{21\times} = -\dot{\underline{C}}_{21}\underline{C}_{21}^T$

translation vector: $\underline{r}_i^{vi}$ dest → orig, → frame

Wheel Model: (in $\underset{\rightarrow}{\mathcal{F}}_w$) $\underline{v}_w^{wi} = \underline{v}_w^{vi} + \underline{\omega}_w^{vi\times}\underline{r}_w^{wv}$ $\qquad$ pose rate (inertial frame) $\dot{\underline{q}} = [\dot{x} \; \dot{y} \; \dot{\theta}]^T$

$\begin{bmatrix} \underline{c}_{long,r}^T \\ \underline{c}_{long,l}^T \\ \underline{c}_{lat,r}^T \\ \underline{c}_{lat,l}^T \end{bmatrix} \dot{\underline{\xi}} = \begin{bmatrix} \dot{\varphi}_r r \\ \dot{\varphi}_l r \\ 0 \\ 0 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_r r \\ \dot{\varphi}_l r \\ 0 \\ 0 \end{bmatrix}$ $\qquad$ pose rate (vehicle frame) $\dot{\underline{\xi}} = [u \; v \; \omega]^T = \underline{C}_3(\theta)\dot{\underline{q}}$

long. rolling w/o sliding [
lat. no sideways sliding [-

Differential-Drive Kinematics $\qquad$ inverse: $\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = \frac{1}{r}\begin{bmatrix} 1 & b \\ 1 & -b \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix}$ $\qquad$ for

Swedish

long $\; [\cos(\alpha+\beta+\gamma) \quad \sin(\alpha+\beta+\gamma) \quad d\sin(\beta+\gamma)]\,\dot{\underline{\xi}} = \dot{\varphi}_r r\cos\gamma + \dot{\varphi}_{swe} r_{swe}$

lat $\; [-\sin(\alpha+\beta+\gamma) \quad \cos(\alpha+\beta+\gamma) \quad d\cos(\beta+\gamma)]\,\dot{\underline{\xi}} = -\dot{\varphi}_r \sin\gamma$

$[v \; u \; \omega]^T = (\underline{A}^T\underline{A})^{-1}\underline{A}^T [\dot{\varphi}_1 r \;\; \dot{\varphi}_2 r \;\; \dot{\varphi}$

Vehicle Model: $\underset{\text{unicycle}}{\dot{\underline{q}}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}}_{\underline{G}(q)} \begin{bmatrix} r/2 & r/2 \\ r/2b & -r/2b \end{bmatrix}\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$ $\qquad$ curvature: $k = \frac{\omega}{v} = \frac{1}{b}\frac{\dot{\varphi}_r - \dot{\varphi}_l}{\dot{\varphi}_r + \dot{\varphi}_l}$ $\quad R = \frac{1}{|k|}$

$\overbrace{[\sin\theta \quad -\cos\theta \quad 0}^{\underline{H}(q)^T}$ non-holonomic constraint

Wheel Odometry: $\dot{\underline{x}} = \underline{A}(\underline{x})\underline{x} + \underline{B}(\underline{x})\underline{u}$ $\qquad \dot{x} = u + \overset{\text{noise}}{w}$ $\qquad x(Kh) = x(0) + h\sum_{k=0}^{K-1} u(kh) + h\sum_{k=0}^{K-1} w(kh)$ $\qquad \mu(Kh) = \mathbb{E}[x$

quantization noise $\; w(t) \sim \mathcal{U}(-\frac{\pi}{2^N}, \frac{\pi}{2^N})$, N-bit $\qquad \underline{q}(t+h) = \underline{q}(t) + \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} r/2 & r/2 \\ r/2b & -r/2b \end{bmatrix} h \begin{bmatrix} \dot{\varphi}_r(t) \\ \dot{\varphi}_l(t) \end{bmatrix}$ $\qquad h\begin{bmatrix} \dot{\varphi}_r(t) \\ \dot{\varphi}_l(t) \end{bmatrix} \approx \begin{bmatrix} \Delta\varphi_r(t) \\ \Delta\varphi_l(t) \end{bmatrix}$

$\underline{\mu}(t+h) = \underline{\mu}(t) + \begin{bmatrix} \cos\bar{\theta}(t) & 0 \\ \sin\bar{\theta}(t) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} r/2 & r/2 \\ r/2b & -r/2b \end{bmatrix}\begin{bmatrix} \Delta\varphi_r(t) \\ \Delta\varphi_l(t) \end{bmatrix}$ $\qquad \underline{\Sigma}(t+h) = \underline{A}(t)\underline{\Sigma}(t)\underline{A}(t)^T + \underline{B}(t)\underline{Q}\underline{B}(t)^T \Rightarrow$ uncertainty grows w/o bound $\qquad$ Cal

Calibrate radius: $r = 2x_{true} / \sum_k(\Delta\varphi_r(kh) + \Delta\varphi_l(kh))$ $\qquad$ calibrate separation: $b = r\sum_k(\Delta\varphi_r(kh) - \Delta\varphi_l(kh)) / 2N\cdot 2\pi$

Path-tracking Control: