Laboratory Session 05 : May 13, 2025

Exercises due : June 1, 2025

Exercise 1: Sampling Gaussian Mixture

- Let $g(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\sigma})$ be the un-normalized distribution of weighted mixture of two Gaussian distributions

$$g(\boldsymbol{\theta}|\boldsymbol{x}) = \sum_{i=1}^{2} \phi_i \mathcal{N}(\boldsymbol{\mu_i}, \boldsymbol{\sigma_i})$$

  where

  - $\phi_0 = 0.6$ and $\phi_1 = 0.4$
  - $\boldsymbol{\mu} = \{-3, 3\}$ and $\boldsymbol{\sigma} = \{1, 1\}$

- Draw a Markov Chain from the posterior distribution using a Metropolis-Hastings algorithm, usings a Norm $(0, 1)$ as random-walk candidate density
- Plot the sampled distribution
- Evaluate the *mean* and the *variance*
- Analyze the chain with the `CODA` package and plot the chain autocorrelation
- Try different burn-in cycles and thinning, plot the related posterior distribution and the chain autocorrelation function. What are the best parameters ?

Exercise 2: Gaussian model

- Let

$$\vec{x} = \{13.427, 8.588, 10.908, 11.582, 11.011, 9.735, 13.779, 9.763\}$$

  be $n = 8$ observed data from a Gaussian distribution with unknown mean m and variance s^2
- Assuming a uniform prior distributions for the parameters,

  - `m` $\sim$ `dunif(-10,30)`
  - `s` $\sim$ `dnorm(0, 50)`

  build a simple JAGS model and run a Markov Chain Monte Carlo to obtain the posterior distribution of the mean and variance.
- Compute the posterior distribution for `m/s`

Exercise 3 : Power Law fitting

- Using `R`, generate 100 synthetic data from power law distribution

$$p(x \mid \alpha, x_{\min}) = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}} \right)^{-\alpha}, \quad x \geq x_{\min}$$

  with fixed $x_{\min} = 10$ and power law exponent $\alpha = 2.5$

- Assuming $x_{\min}$ fixed and α unknown, build JAGS models using the priors
  - `alpha` $\sim$ `dunif(1.01, 10)`
  - `alpha` $\sim$ `dnorm(2.5, 100)`
- Run MCMC to get the posterior distribution. Evaluate the mean, the most probable value, the credibility interval at 95% level.
- Compare the Bayesian estimate for α with the one achieved in frequentist setting.