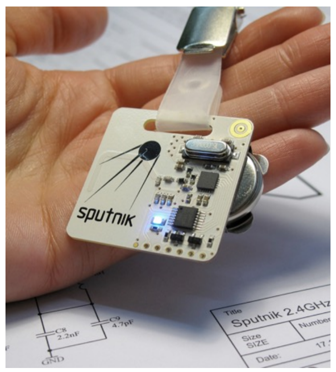
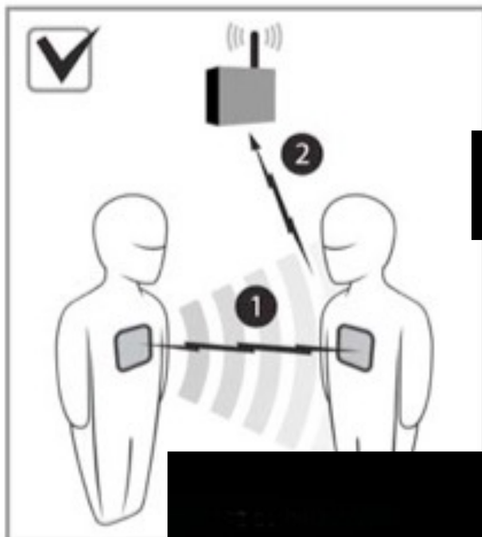
Physics of Life Data Epidemiology

Lect 13: temporal networks

Chiara Poletto
mail: chiara.poletto@unipd.it
web: chiara-poletto.github.io
bsky: @chpoletto.bsky.social

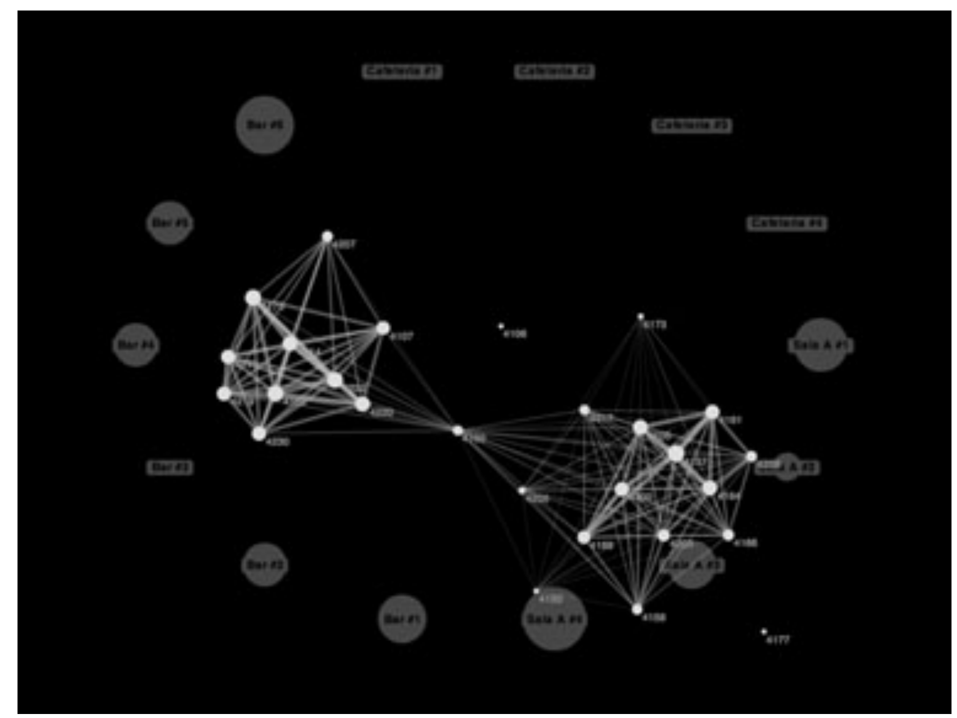
high resolution network data





face-to-face contacts

RFID technology



schools - workplaces - hospitals - museums - conferences -households - rural Africa

[**Sociopatterns.org**]

high resolution network data

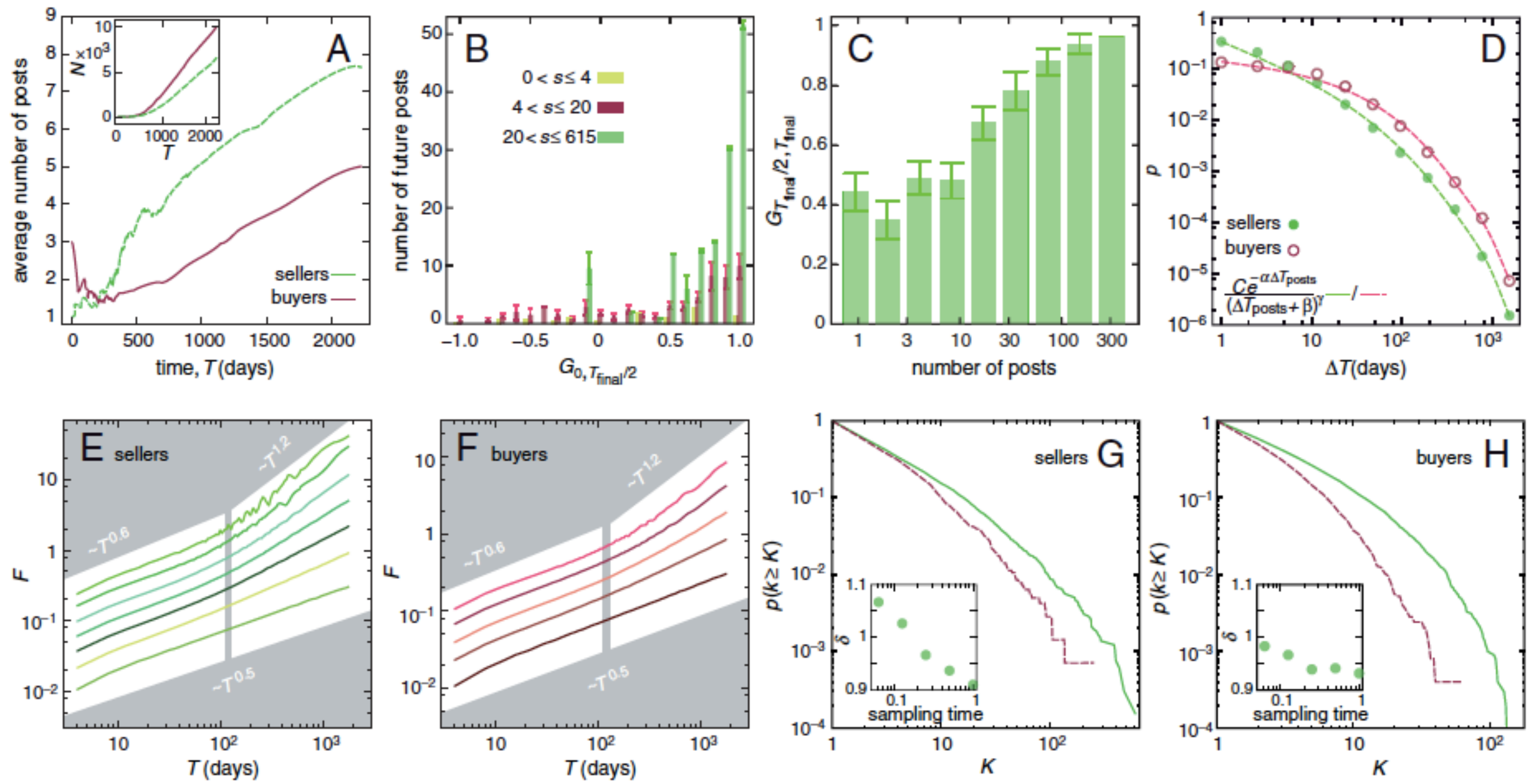


Fig. 1. Statistics of the dynamics of the community. (*A*) Time evolution of the average number of posts by sex buyers and about sex buyers. The *Inset* shows the growth in the number of sex sellers and sex buyers in the data. (*B*) The number of new posts according to the previous average grade at $T_{\mathrm{final}}/2 = 1{,}116$ days for three different activity levels, or total number of posts, s. The R^2-values of these data are 0.19 ($0 < s \leq 4$), 0.29 ($4 < s \leq 20$), and 0.33 ($20 < s$). (*C*) The average future grade of sellers as a function of their number of contacts at half of the total sampling time (the data is logarithmically binned along the abscissa). (*D*) Shows the distribution of the time elapsed between two posts T_{posts} for buyers and sellers. Many posts were written during the same day, respectively, $p(T_{\mathrm{posts}} = 0) = 0.495$ and $p(T_{\mathrm{posts}} = 0) = 0.246$. The distributions are well fitted by $p(T_{\mathrm{posts}}) = C\exp(-\alpha T_{\mathrm{posts}}) = (T_{\mathrm{posts}} + \beta)^{\gamma}$, with: $C = 2.9 \pm 0.5$ days$^{\gamma}$, $\alpha = 0.0023 \pm 0.0001$ days^{-1}, $\beta = 3.1 \pm 0.4$ days, and $\gamma = 1.49 \pm 0.04$ (for sellers); and $C = 12 \pm 8$ days$^{\gamma}$, $\alpha = 0.0021 \pm 0.0002$ days^{-1}, $\beta = 18 \pm 4$ days, and $\gamma = 1.5 \pm 0.1$ (for buyers). (*E*) and (*F*) shows statistics the DFA fluctuation function as a function of the time-scale ΔT for sellers and buyers, resp. The different curves correspond to different activity levels—from bottom to top they represent less than 3, 3–7, 8–20, 21–54, 55–148, 149–403, and more than 403 posts (about sellers or from buyers) resp. *Black Lines* are inserted for reference. $T^{1/2}$ corresponds to uncorrelated interaction. (*G*) and (*H*) show degree distributions for sex sellers (*G*) and buyers (*H*) cumulative degree distributions for the full sampling time (*Solid Line*) and a yearlong window (starting one year after the full dataset; *Dashed Line*) for sex sellers and -buyers, resp. The *Insets* show the exponent of preferential attachment (Eq. 1).

internet mediated prostitution

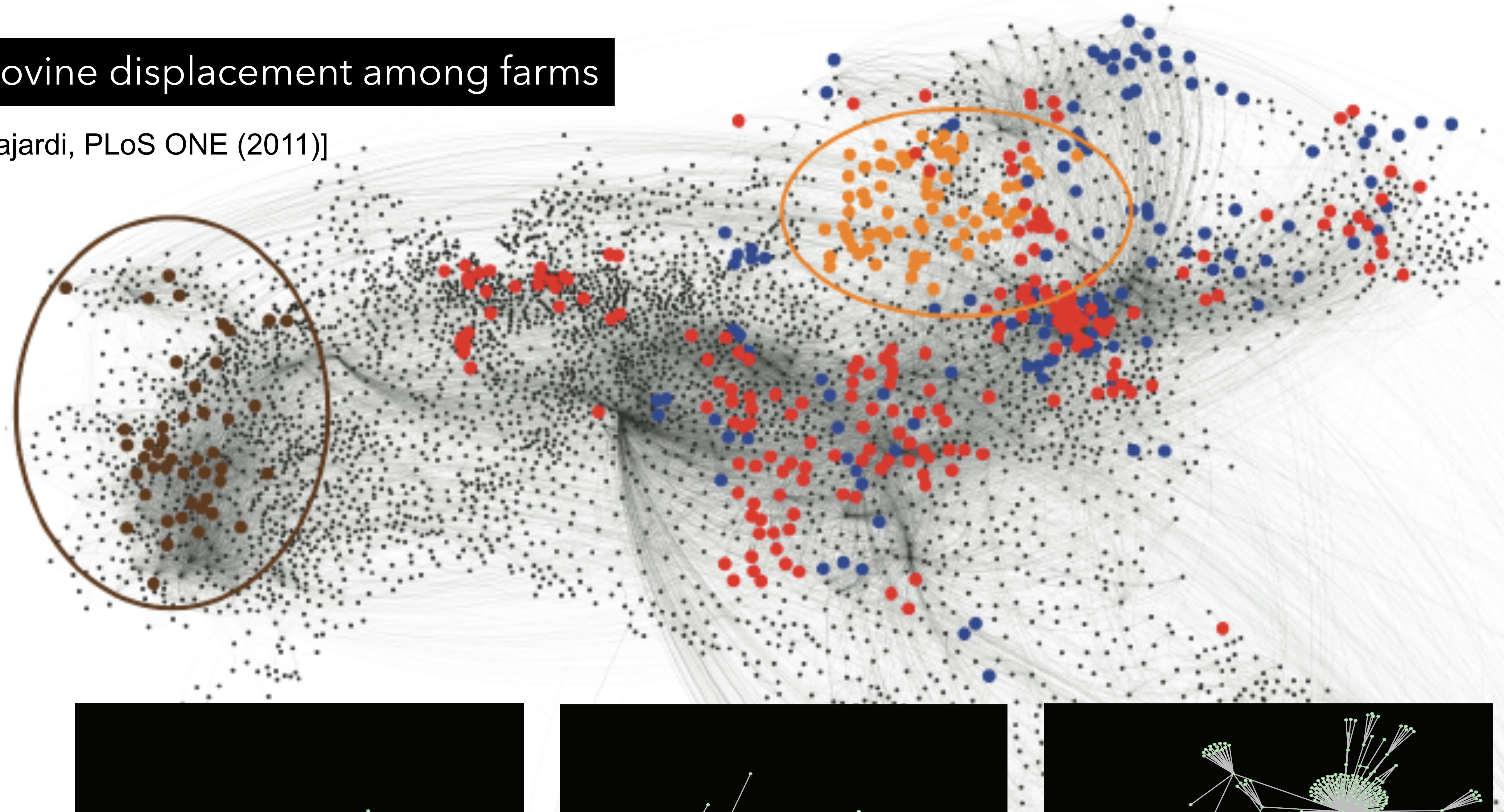
sexual contacts between 6,624 escorts and 10,106 sex buyers extracted from an online community

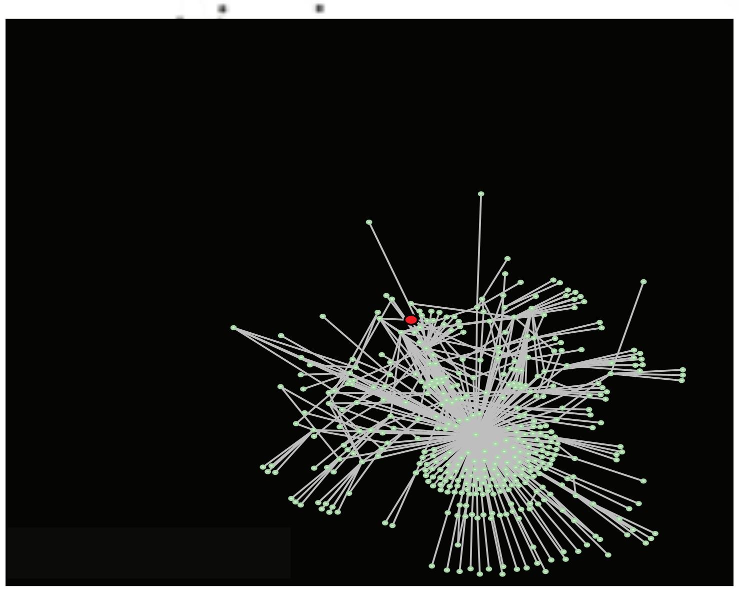
[LEC. Rocha, et al, PNAS 2009]

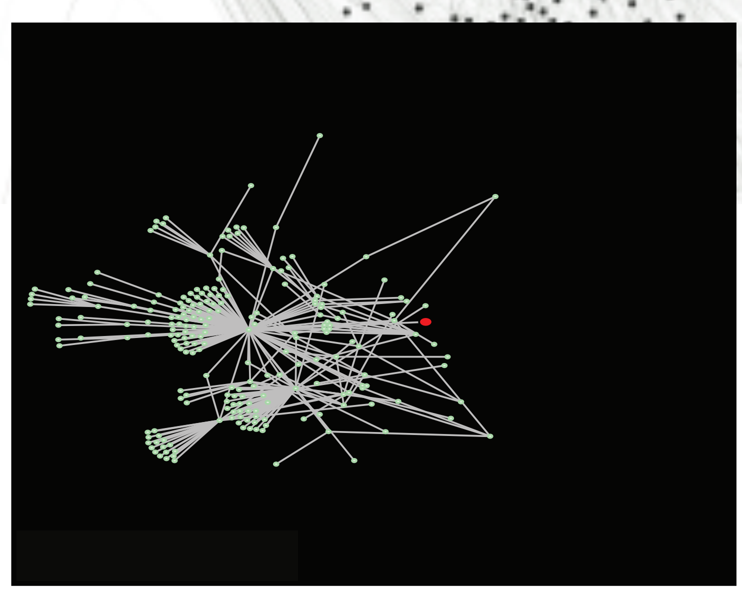
high resolution network data

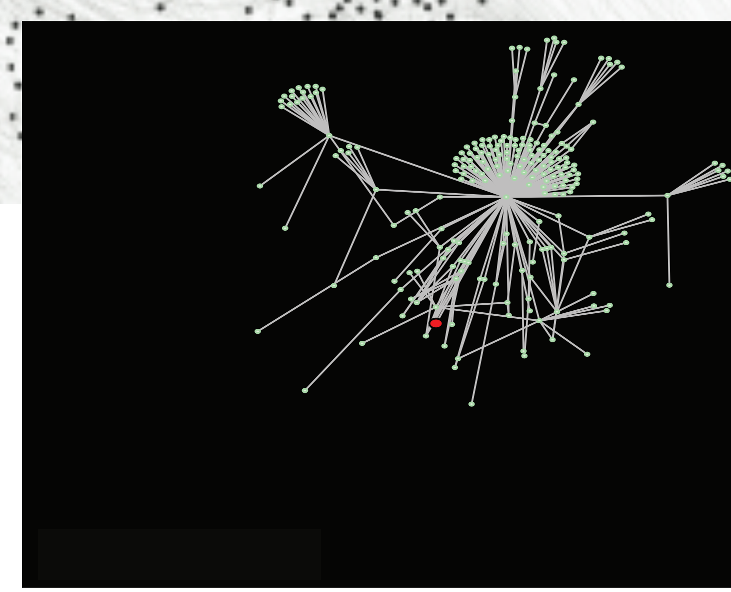
bovine displacement among farms

[Bajardi, PLoS ONE (2011)]

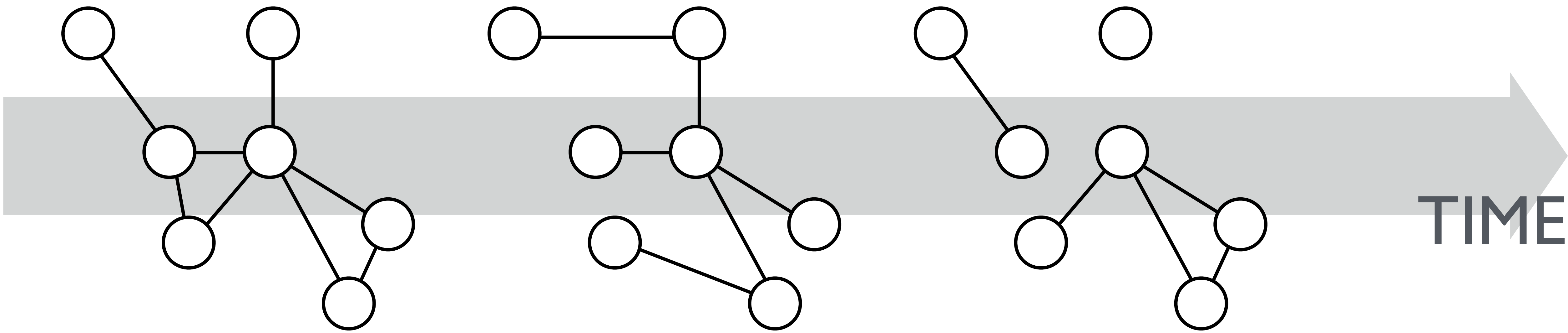






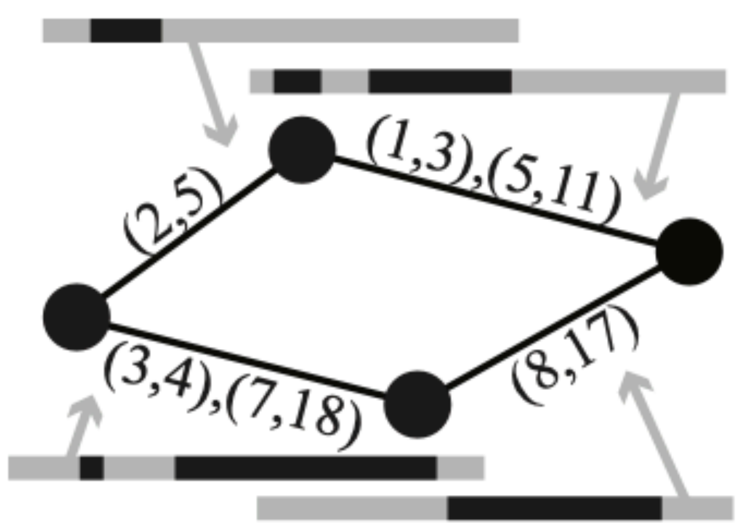


temporal dimension of networks



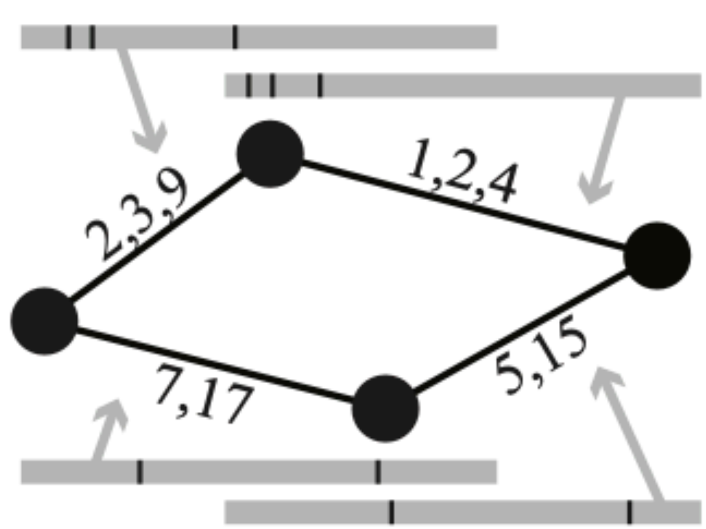
[Holme, Saramaki Phys. Rep. (2012)]

temporal network: definition



A temporal network $G^T = (\mathbf{V}, \mathbf{E}, \mathbf{T})$ is composed by

- a set nodes $\mathbf{V} = \{1, \ldots, N\}$
- a set of edges $\mathbf{E} = \{(i, j), \ldots\}$
- a set of **activation timelines** $T = \{T_{(i,j)}, \ldots\}$, with $T_{(i,j)} = \{(t_1, t_1'), (t_2, t_2'), \ldots\}$ for each $(i, j) \in \mathbf{E}$



If we neglect the contact duration

A temporal network $G^T = (\mathbf{V}, \mathbf{E})$ is composed by

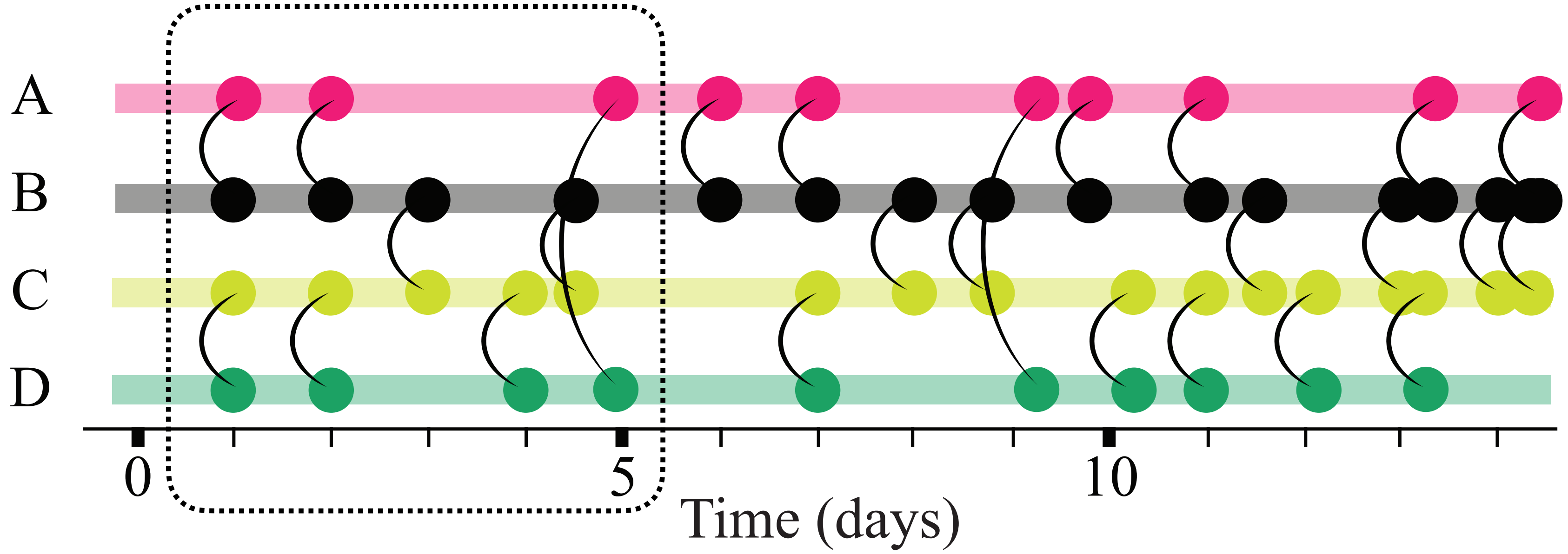
- a set of nodes $\mathbf{V} = \{1, \ldots, N\}$
- a set of edges $\mathbf{E} = \{(i, j, t), \ldots\}$, where edges are triplets with i and $j \in \mathbf{V}$ and t instant of time

[Holme, Saramaki Phys. Rep. (2012)]

temporal network: representation

[Holme, Saramaki Phys. Rep. (2012)]

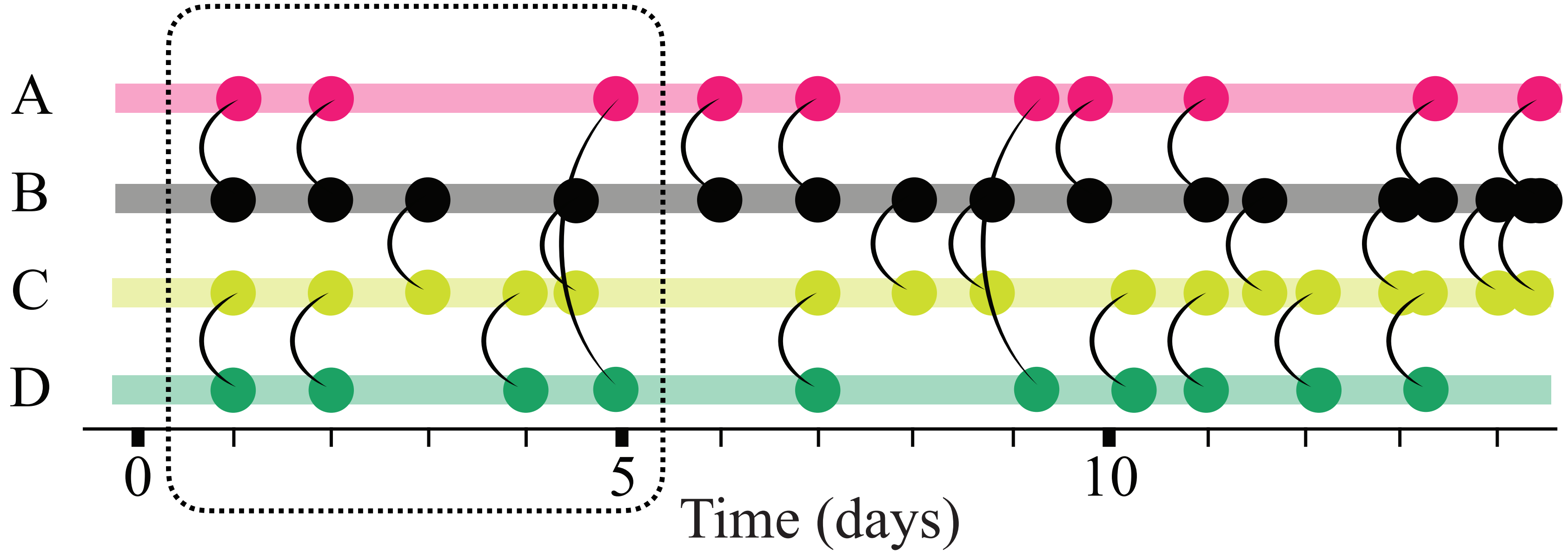
sequence of links: continuous time



temporal network: representation

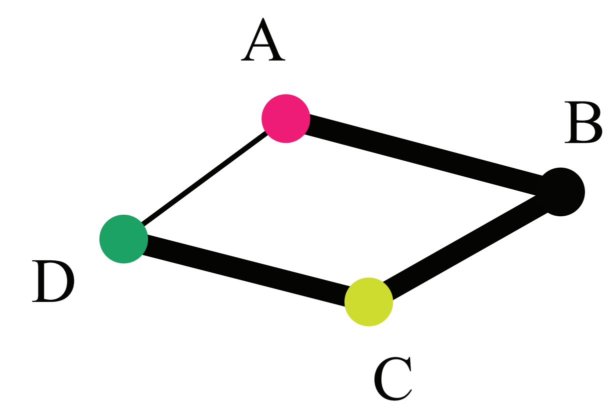
[Holme, Saramaki Phys. Rep. (2012)]

sequence of links: continuous time



weighted

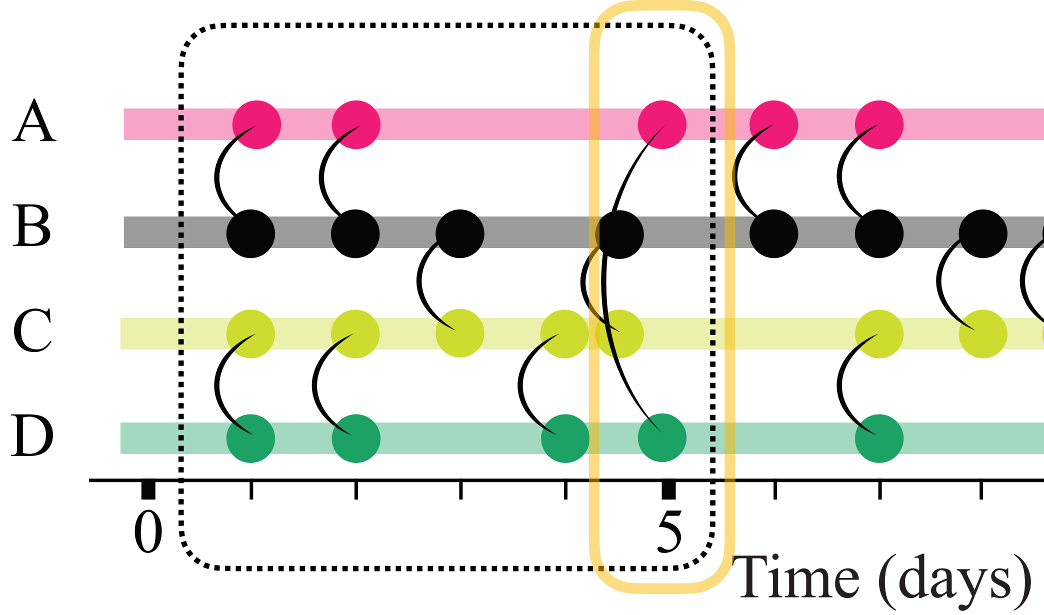
aggregated: static



temporal network: representation

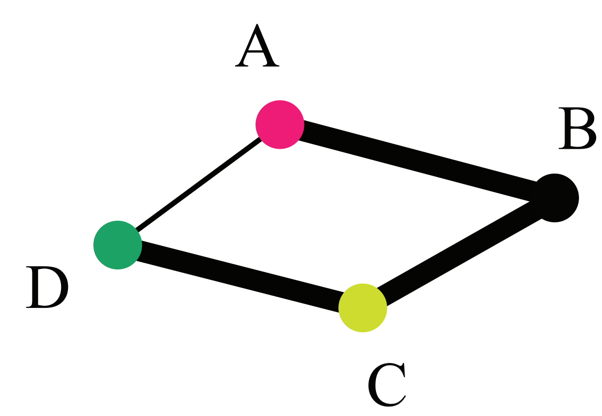
[Holme, Saramaki Phys. Rep. (2012)]

sequence of links: continuous time

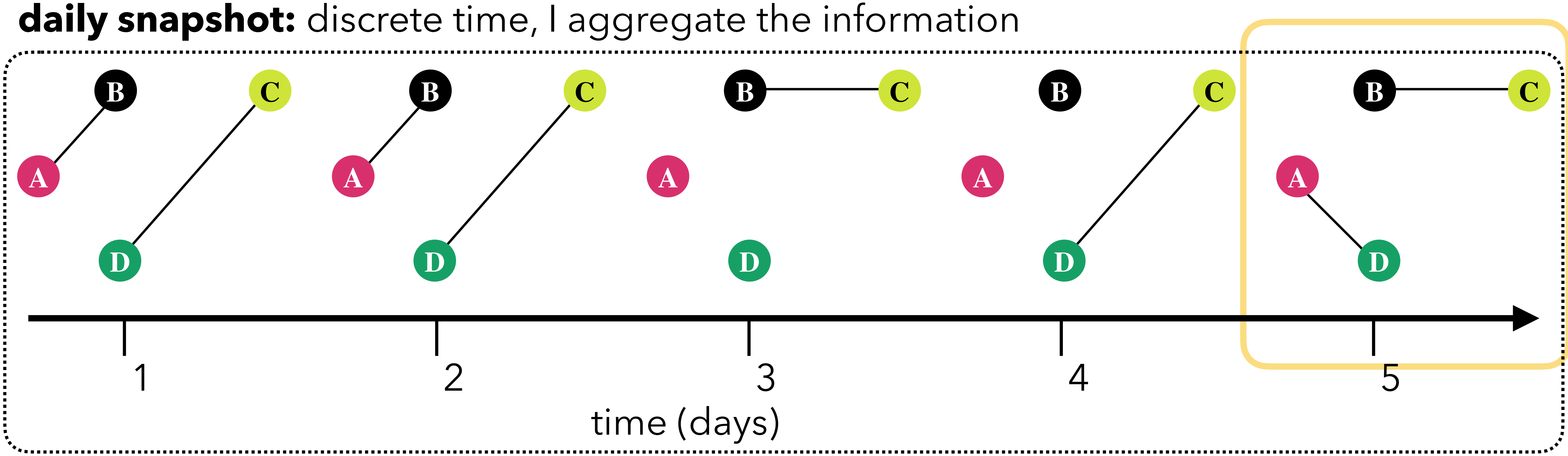


weighted

aggregated: static



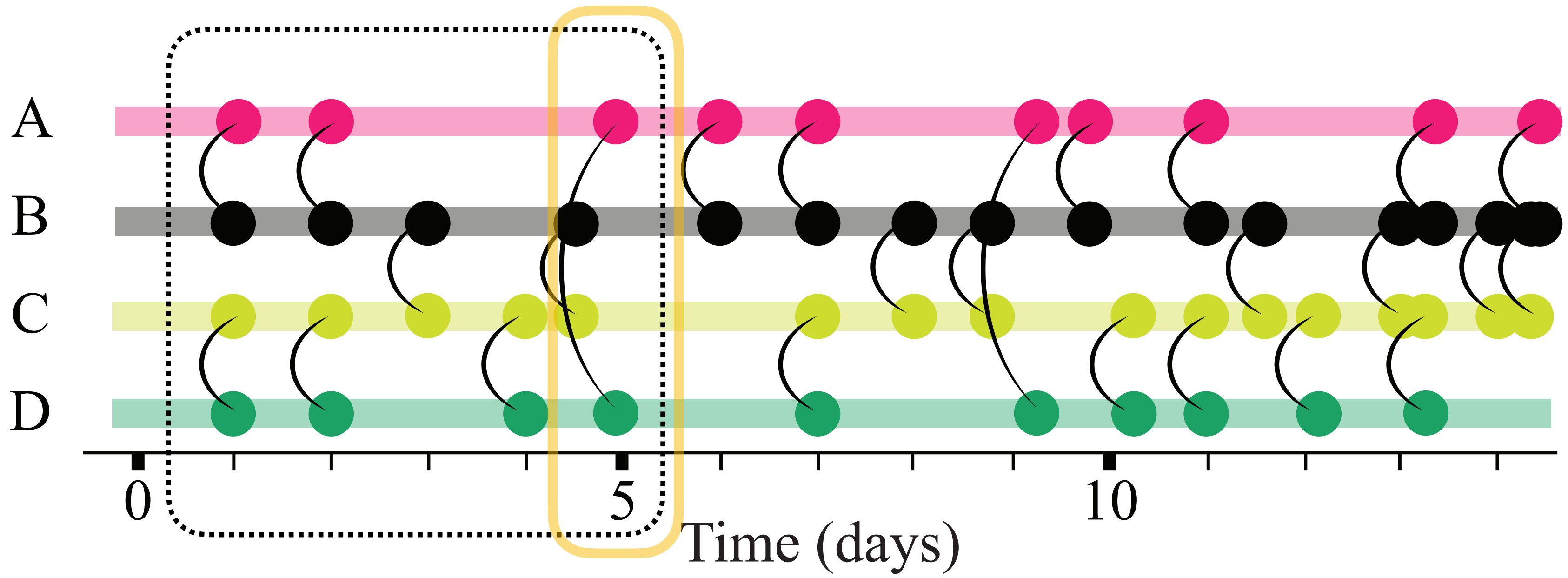
daily snapshot: discrete time, I aggregate the information



temporal network: representation

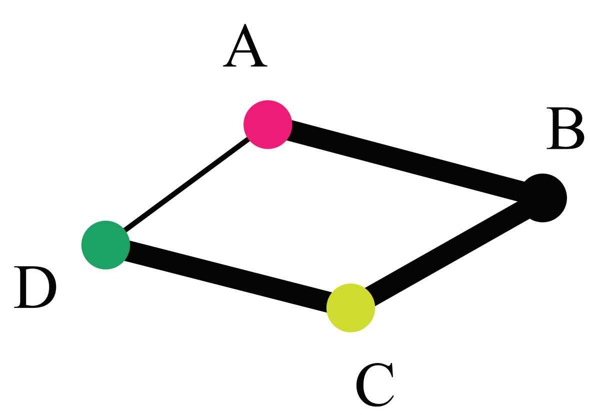
[Holme, Saramaki Phys. Rep. (2012)]

sequence of links: continuous time

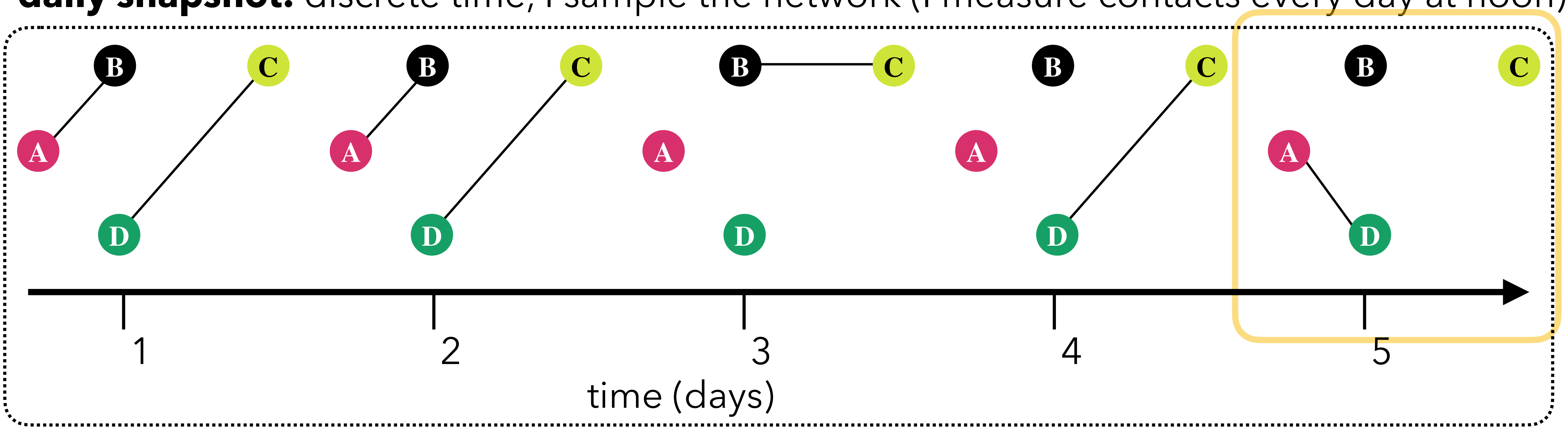


weighted

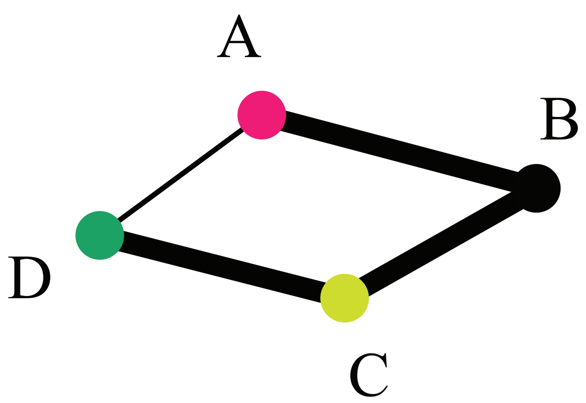
aggregated: static



daily snapshot: discrete time, I sample the network (I measure contacts every day at noon)

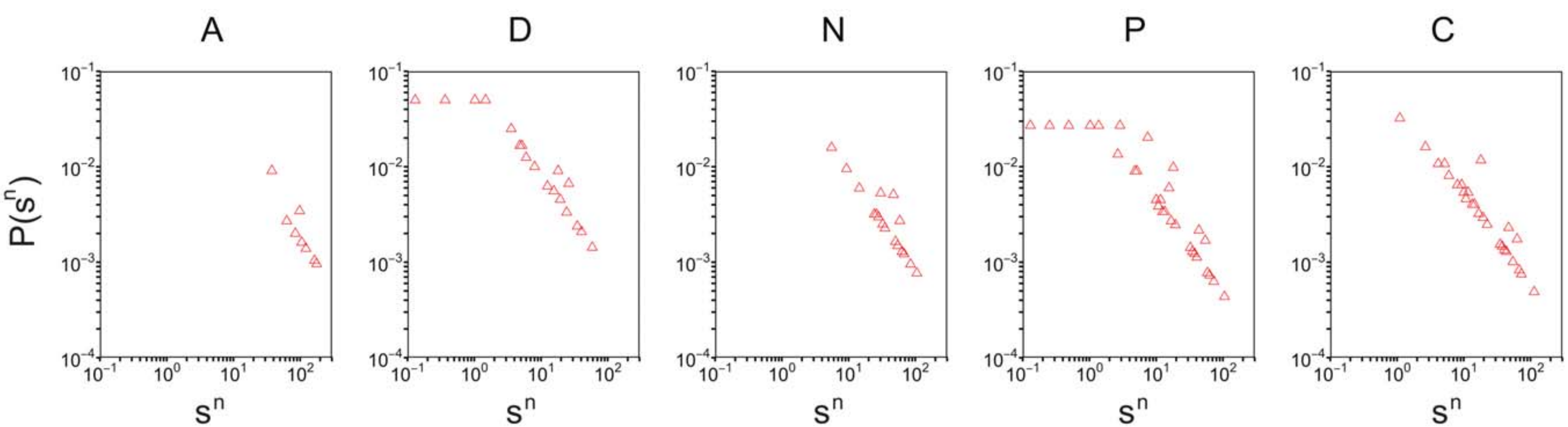


what the degree of static networks hides



Cumulative number of contacts results from both activation frequency and number of contacts made at each activation

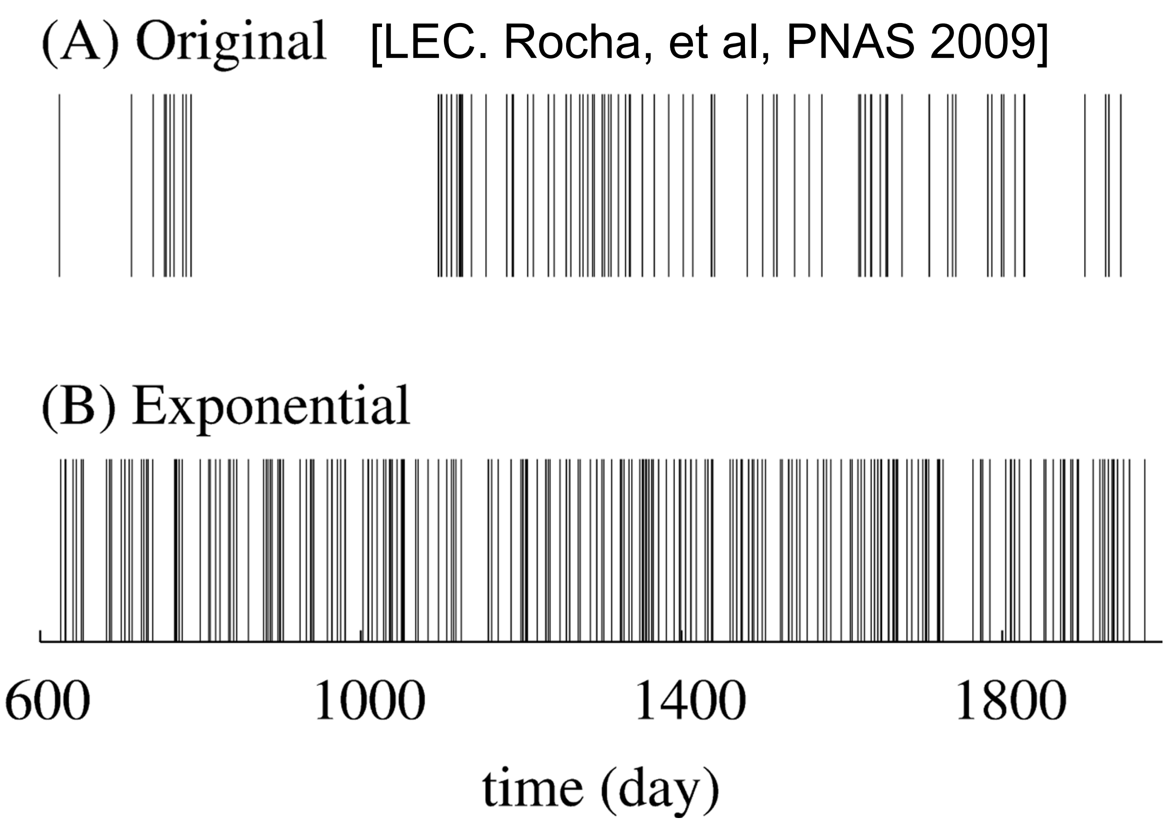
probability density function of number of contacts



number of contacts

[Isella et al PLOS ONE 2011]

Timeline of activation



more realistic model: $P_E(\tau) = A\tau^{-\alpha}e^{-\tau/\tau_E}$

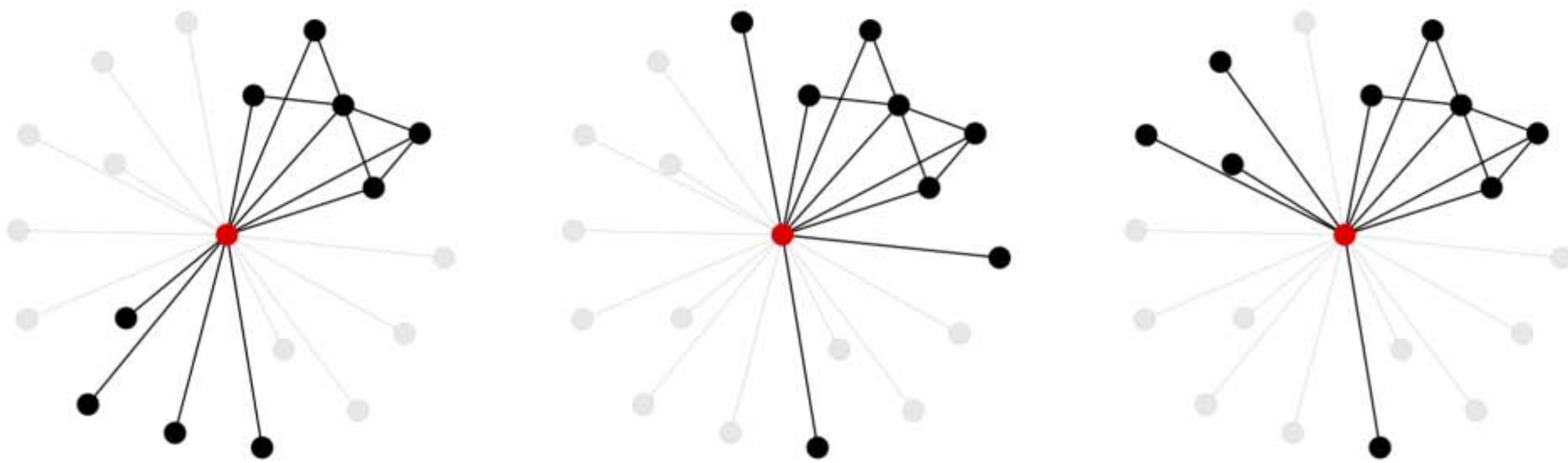
Poisson model: $P_P(\tau) = \dfrac{e^{-\tau/\langle\tau\rangle}}{\langle\tau\rangle}$

inter-contact time: time from two consecutive activations

human behaviour is bursty

burstiness: broader-than-expected distributions of inter-contact times

Temporal correlations



[Miritello, et al, Sci Rep 2013]

$k_{i,t}$= degree of i in the network aggregated over the interval $[t-\delta, t]$

$s_{i,t}$= weighted degree of i in the network aggregated over the interval $[t-\delta, t]$

social strategy: $\gamma_{i,t} = \dfrac{k_{i,t}}{s_{i,t}}$

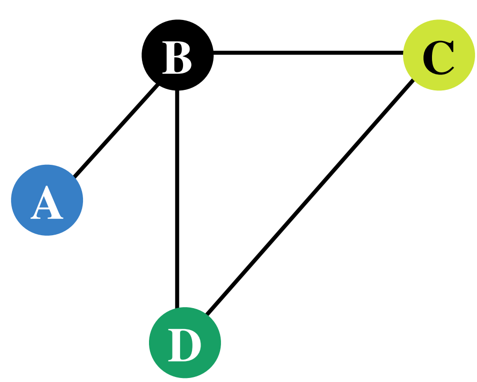
$\gamma \to 0$: memory-driven behavior (a node tends to make contacts always with the same nodes)

$\gamma \to 1$: memoryless behavior (a node shows a more socially exploratory behavior)

Network reachability

network reachability:

i is reachable from j if it exists a path from i to j

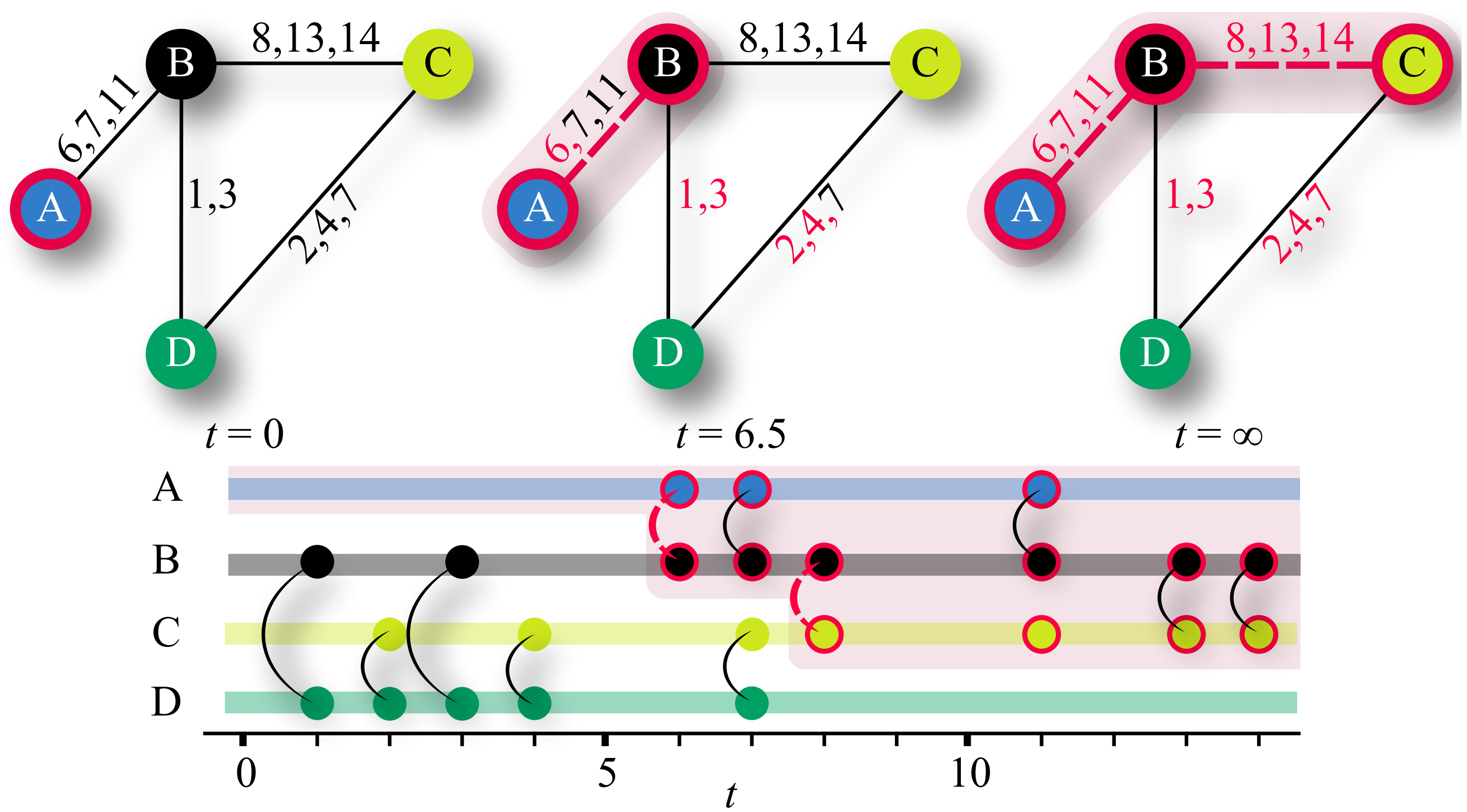


in an undirected static network every node is reachable from every node in its connected component

Network reachability

network reachability:

i is reachable from *j* if it exists a path from *i* to *j*

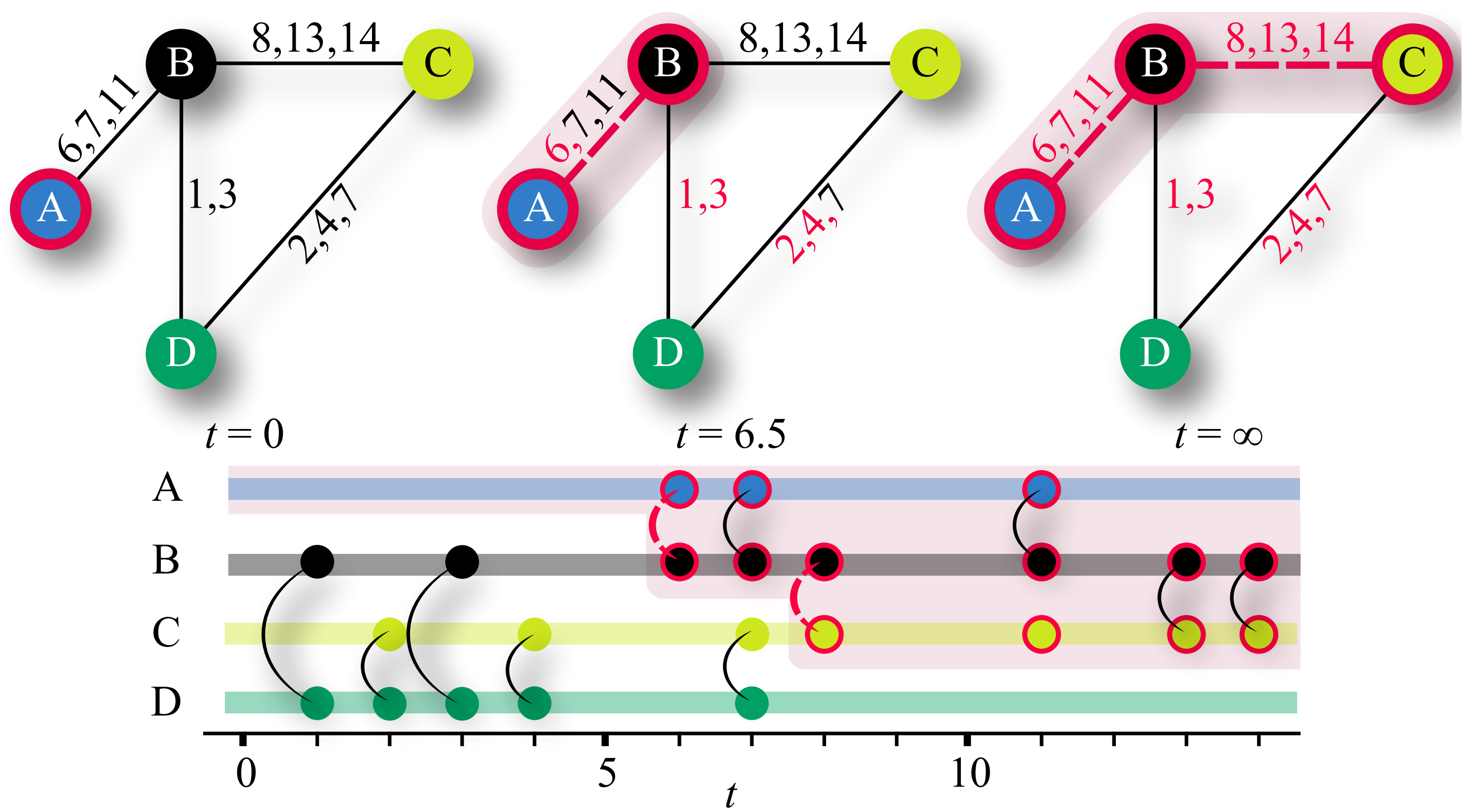


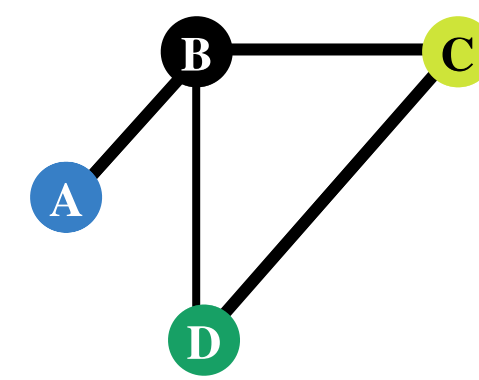
in a undirected temporal network, *j* is reachable from *i* only if there exists a **time respecting path** from *i* to *j*, i.e. a sequence of contacts that connect *i* and *j* with each contact in the path coming after the one before it in time

Network reachability

network reachability:

i is reachable from j if it exists a path from i to j



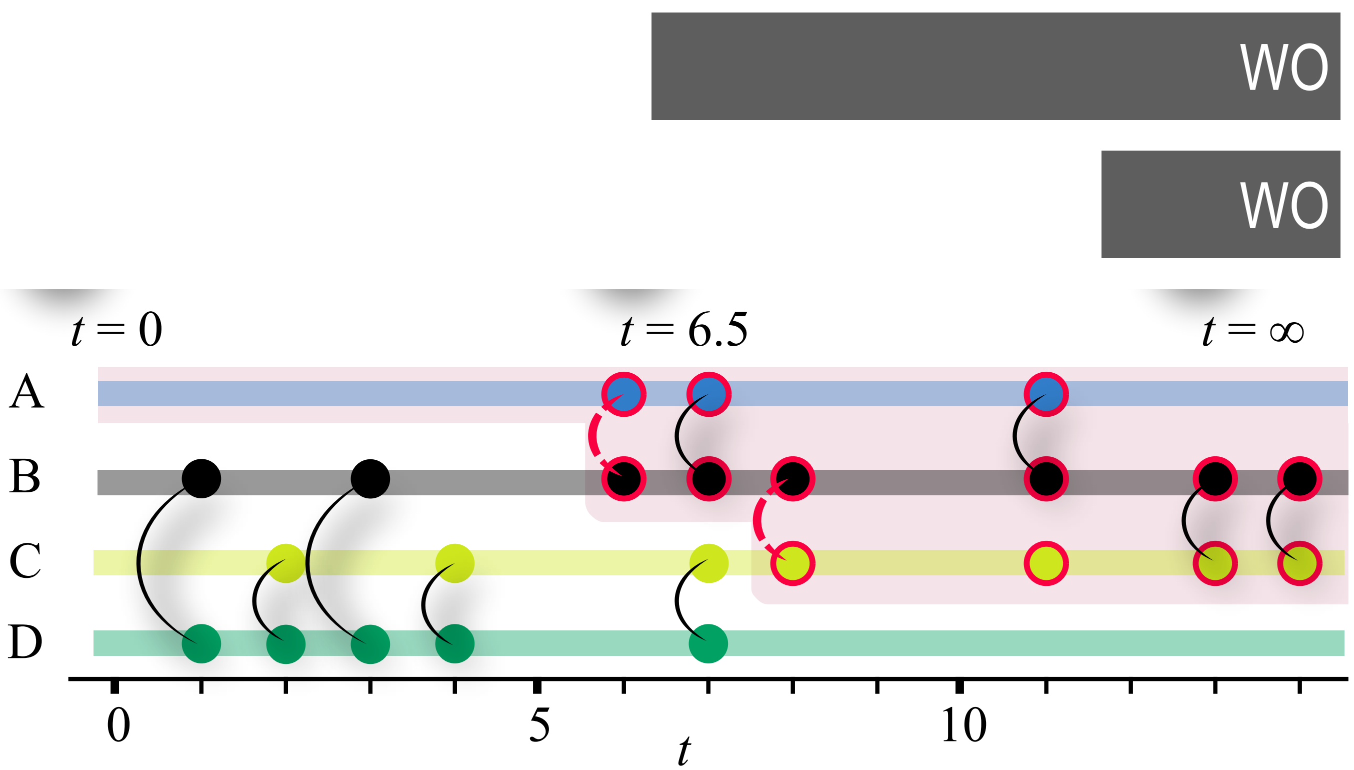


In the weighted aggregated network I lose a lot of information!

in a undirected temporal network, j is reachable from i only if there exists a **time respecting path** from i to j, i.e. a sequence of contacts that connect i and j with each contact in the path coming after the one before it in time

Network reachability

The existence of a time respecting path depends on the window $[t, T]$ of observation



For $t = 6.5$ there is a path from A to C

For $t = 11.5$ there is no path from A to C

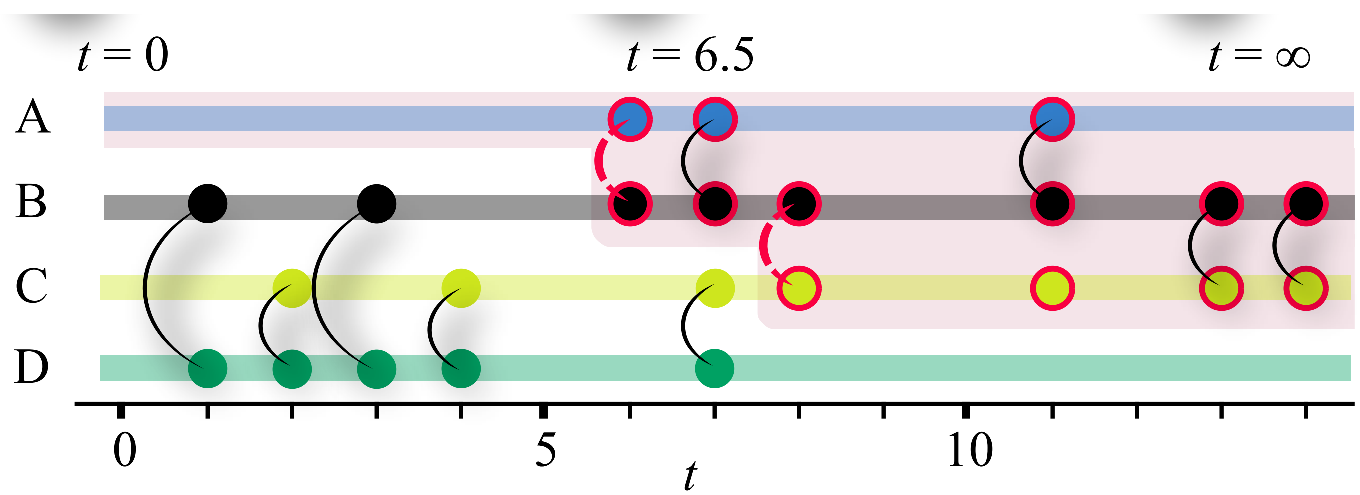
Network reachability

In the window $[t, T]$ a path exist from i to j. Is i able to infect j?

WO

For $\mu^{-1} = 3$ days YES

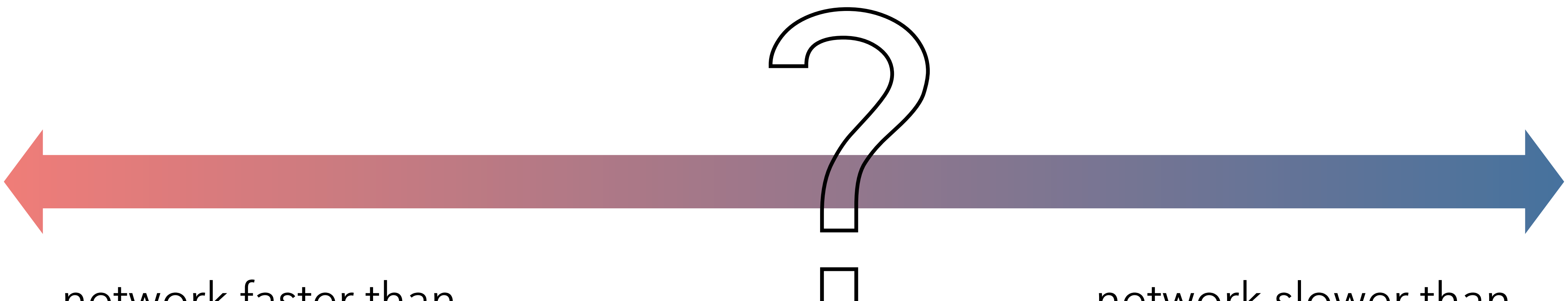
For $\mu^{-1} = 1$ days NO



importance of contact dynamics for an epidemic

heterogeneous mean-field approach

Individual-based mean-field approach



network faster than disease

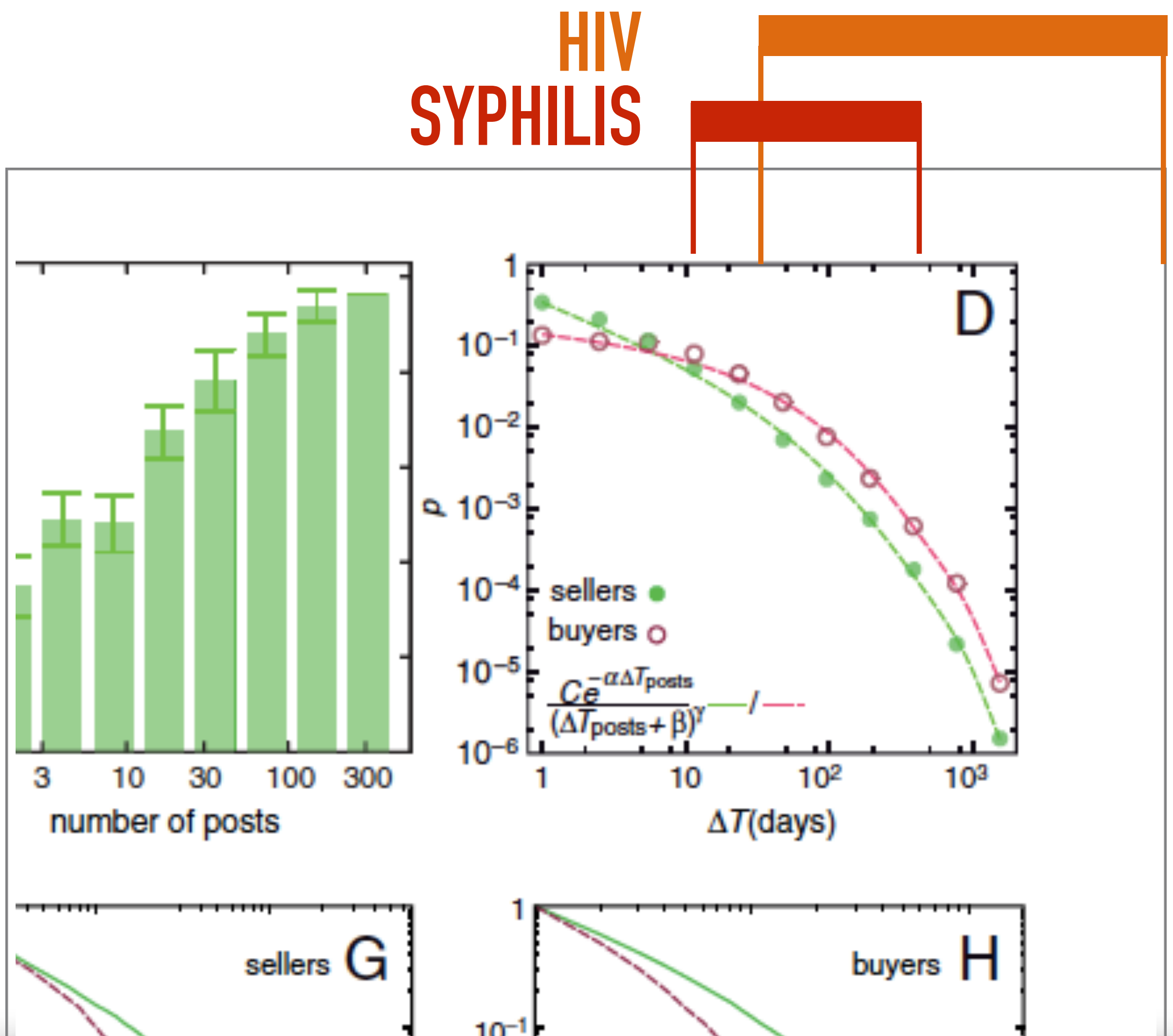
network slower than disease

average infectious duration μ^{-1}

average inter contact time τ

importance of contact dynamics for an epidemic

time scale separation not applicable in many cases



internet mediated prostitution

[LEC. Rocha, et al, PNAS 2009]

approaches to temporal network epidemiology

Botom-up: generative models

activity driven model, and its extensions

Top-down approaches: Randomised Reference Models

compare the epidemics on real data with the outcome in suitable null models

approaches to temporal network epidemiology

Botom-up: generative models

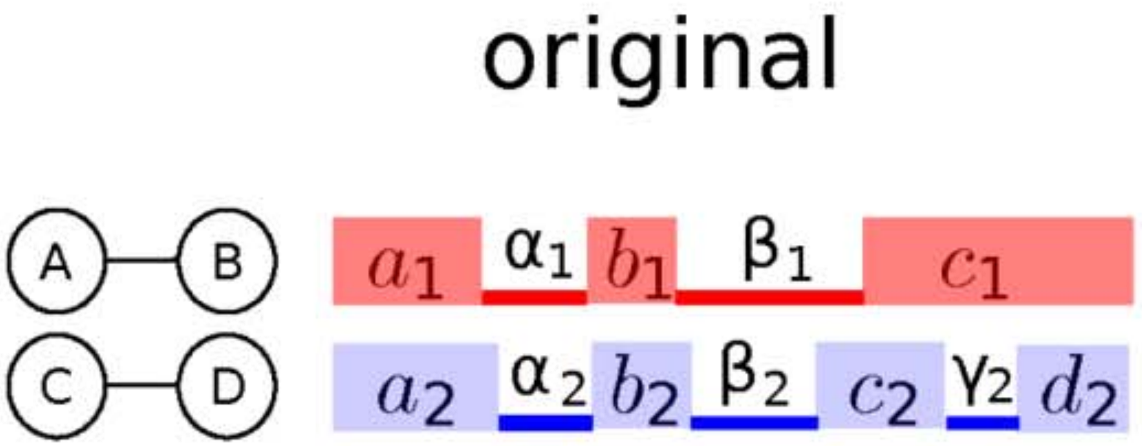
activity driven model, and its extensions

Top-down approaches: Randomised Reference Models

compare the epidemics on real data with the outcome in suitable null models

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link



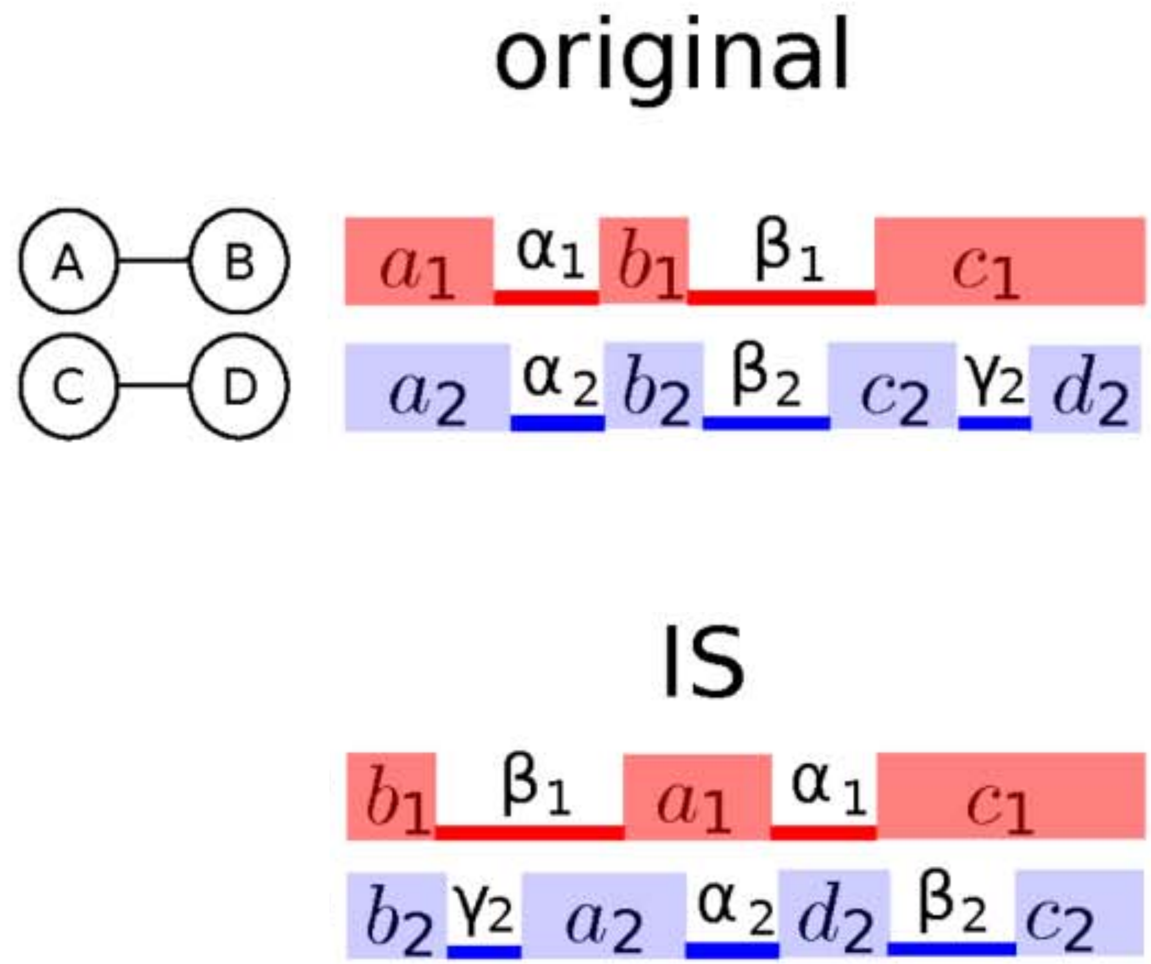
RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$

[Gauvin et al Sci Rep 2013]

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS							



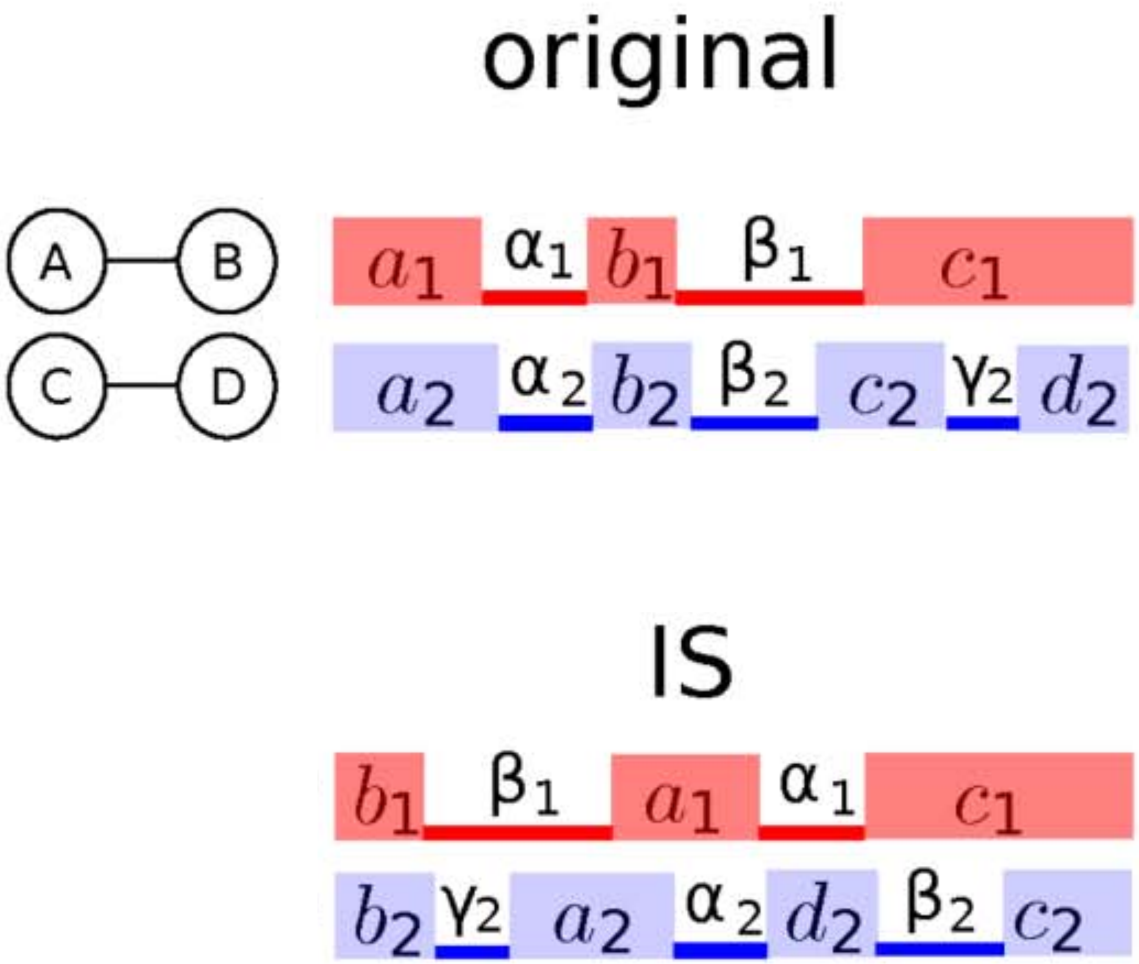
interval shuffling (IS): the sequences of contact and inter-contact durations are reshuffled for each link separately

[Gauvin et al Sci Rep 2013]

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V



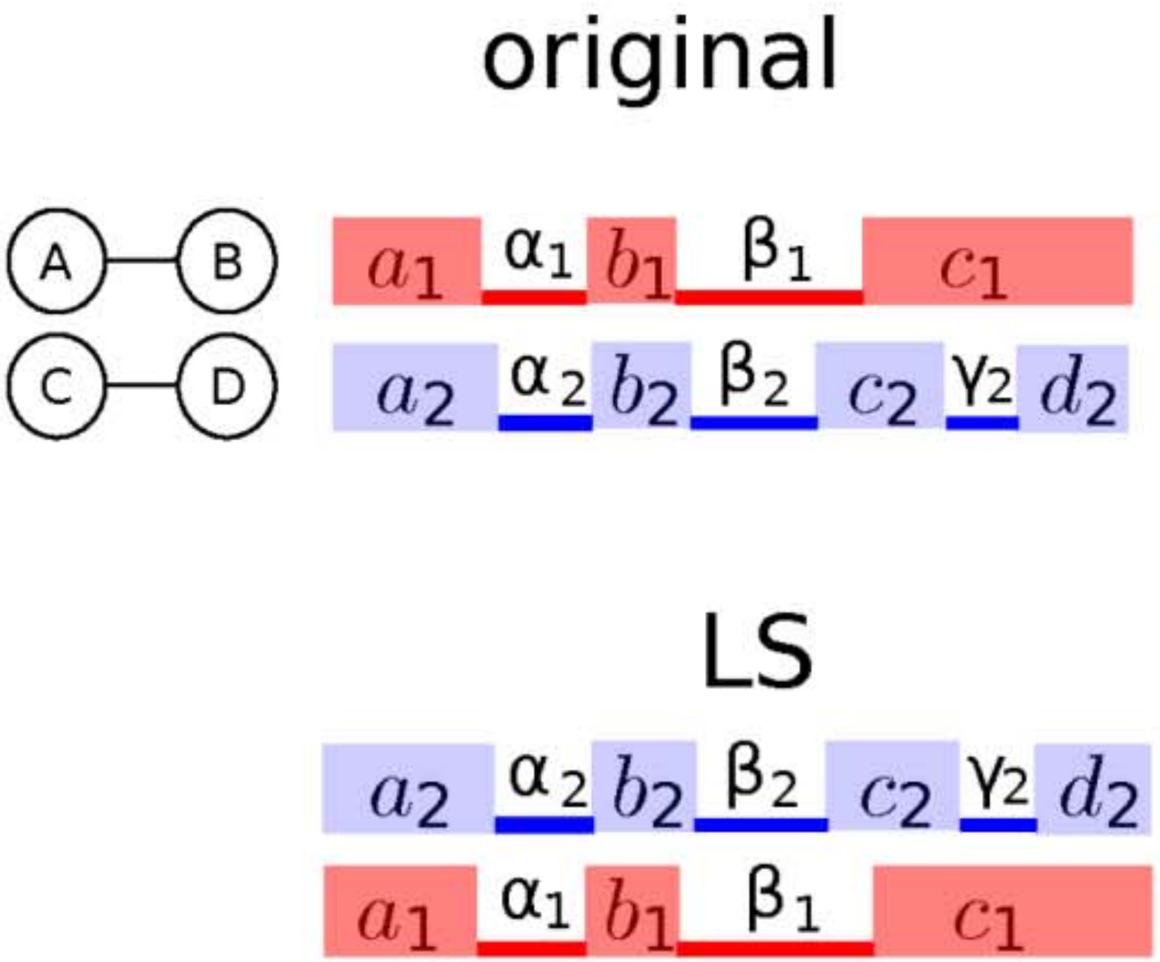
interval shuffling (IS): the sequences of contact and inter-contact durations are reshuffled for each link separately

[Gauvin et al Sci Rep 2013]

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V
LS							



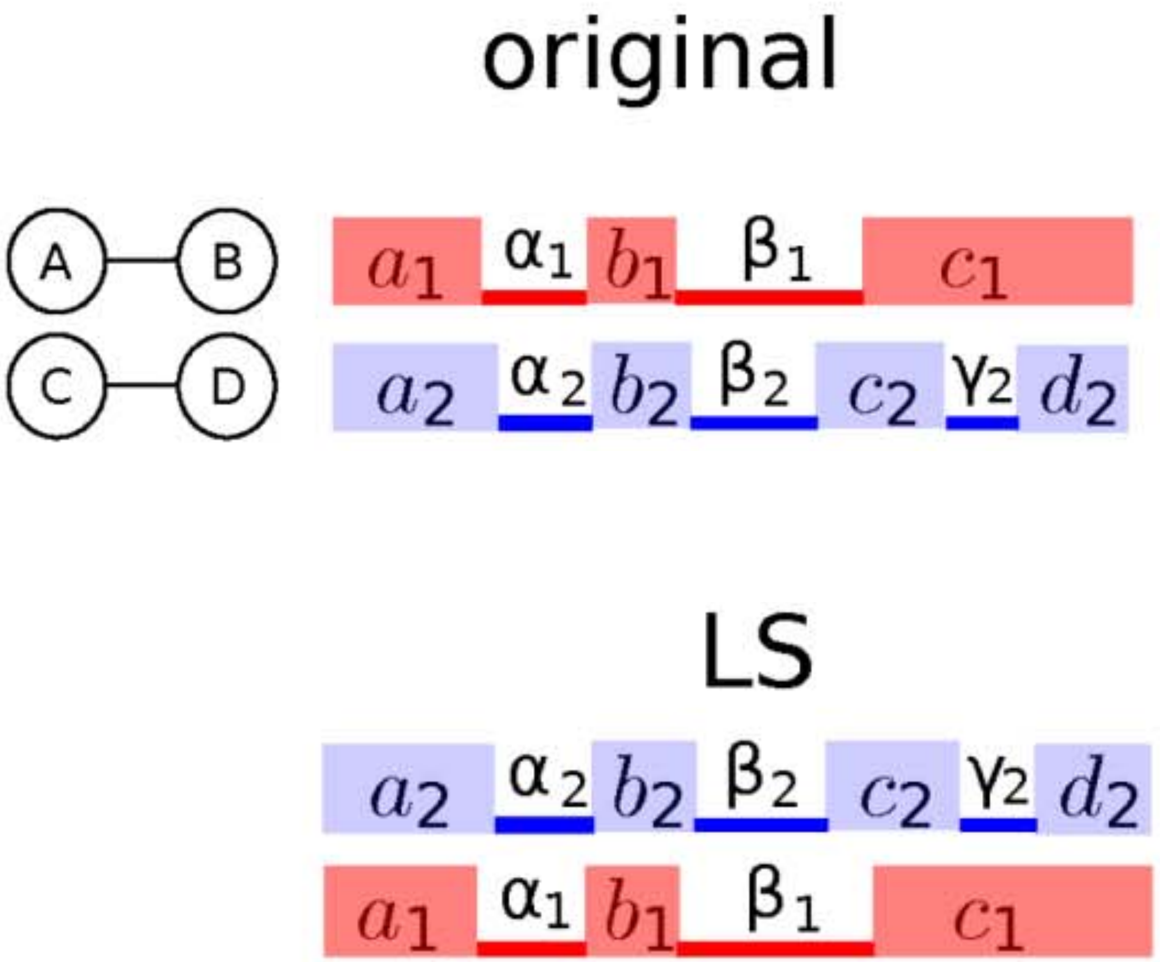
link shuffling (LS): the unaltered sequences of events are swapped between link pairs

[Gauvin et al Sci Rep 2013]

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V
LS	V	X	V	X	V	X	V



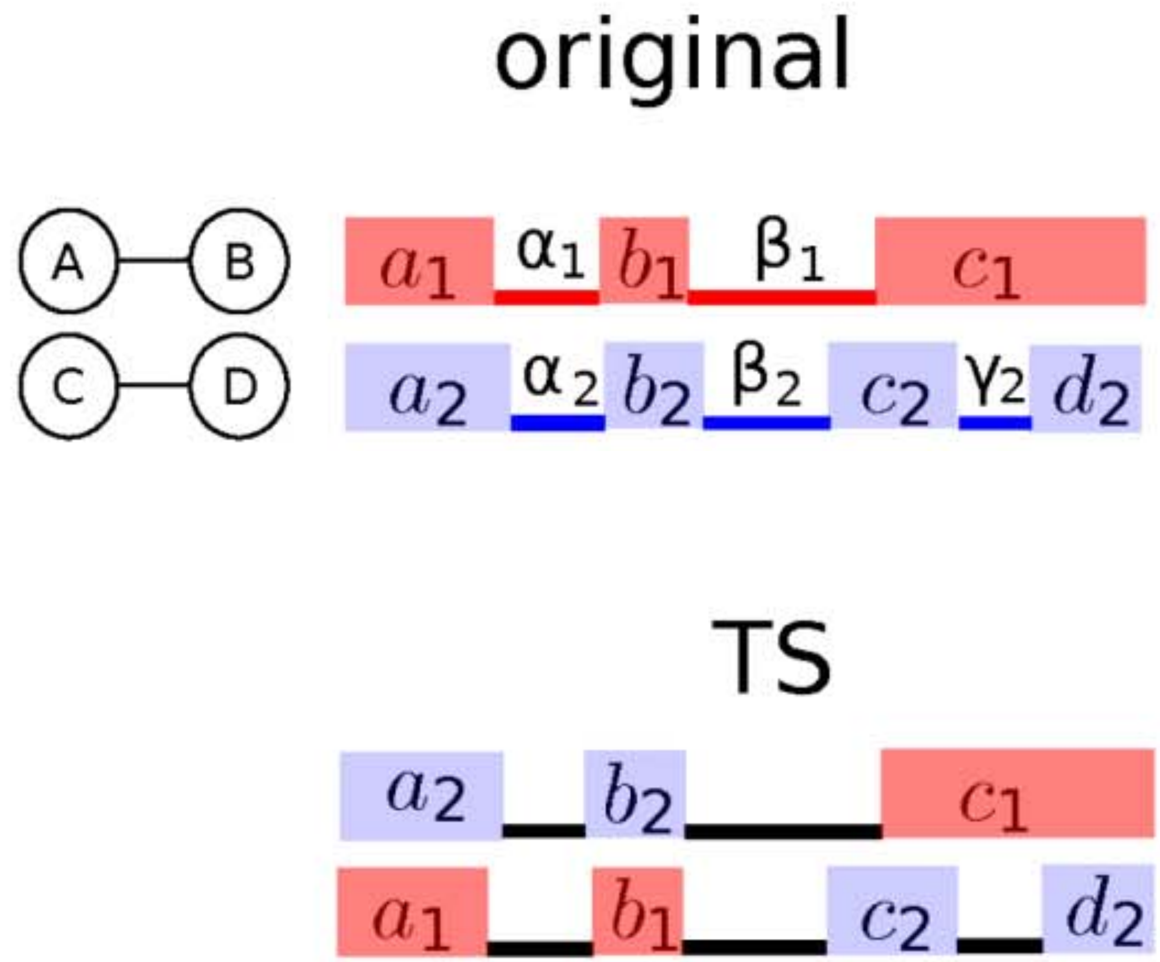
link shuffling (LS): the unaltered sequences of events are swapped between link pairs

[Gauvin et al Sci Rep 2013]

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V
LS	V	X	V	X	V	X	V
TS							

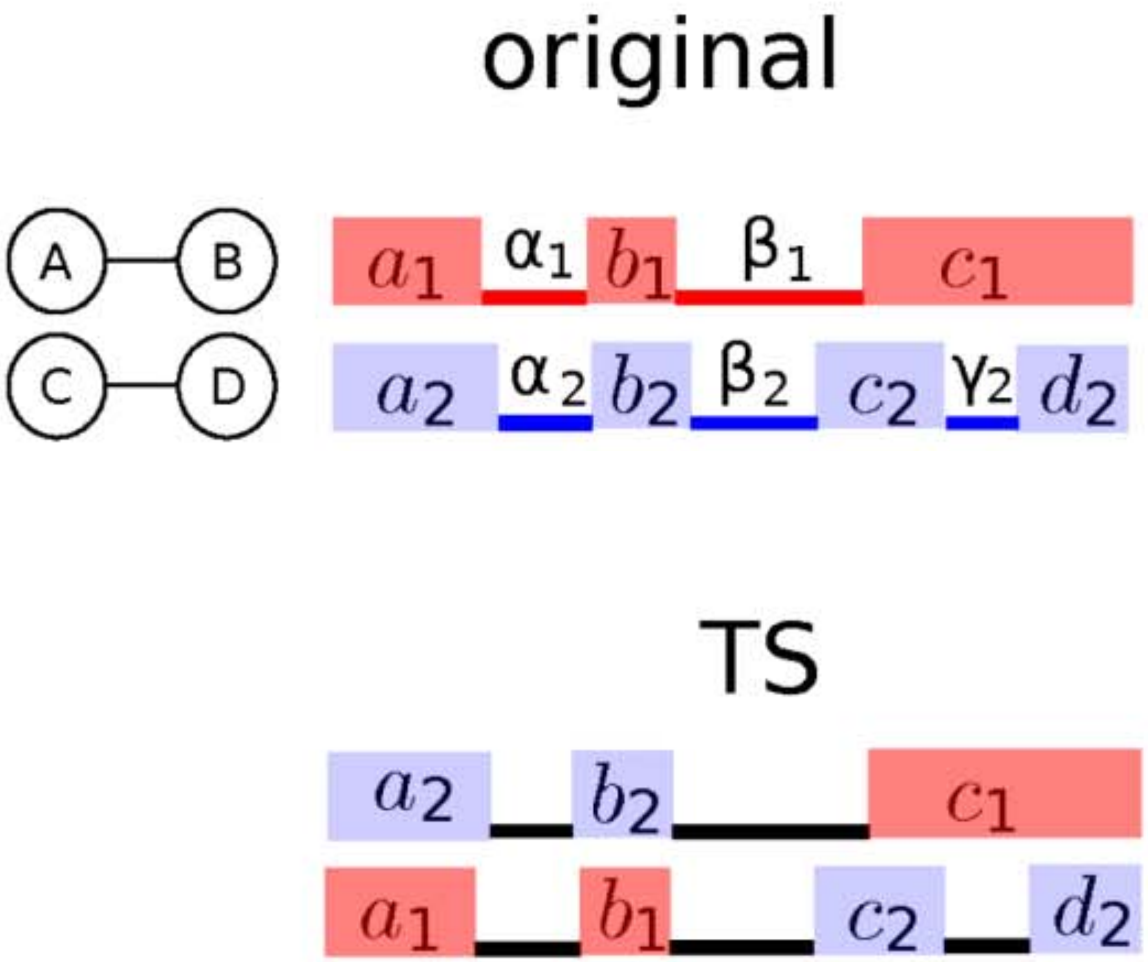


global time shuffling (TS): build a global list of the contact durations. For each link, generate a synthetic activity timeline by sampling with replacement the global list according to the original number of contacts for that link

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V
LS	V	X	V	X	V	X	V
TS	V	X	X	X	X	V	V

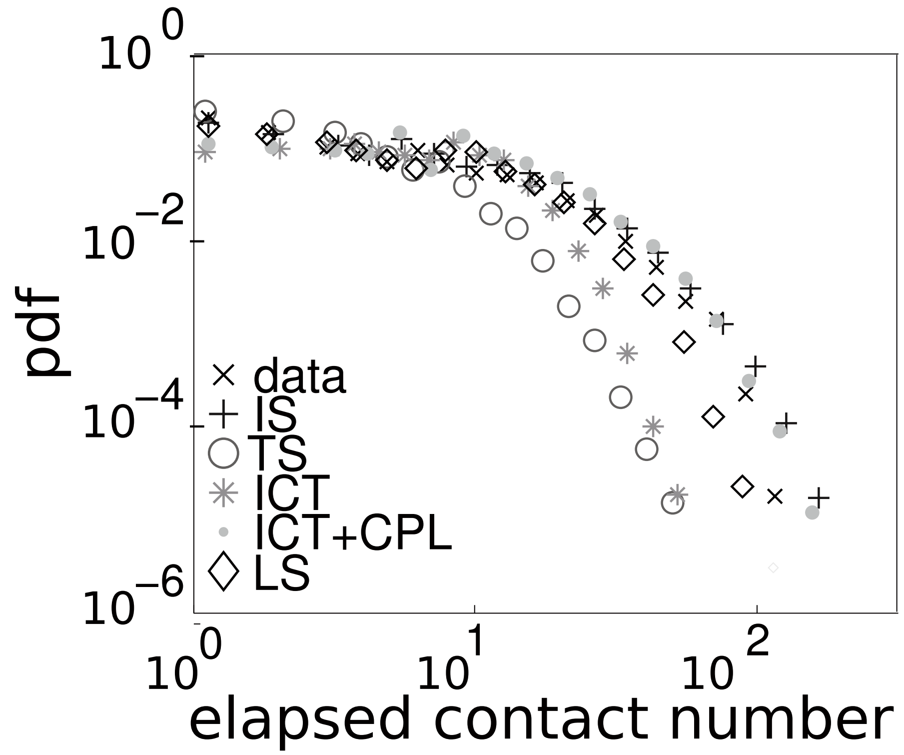


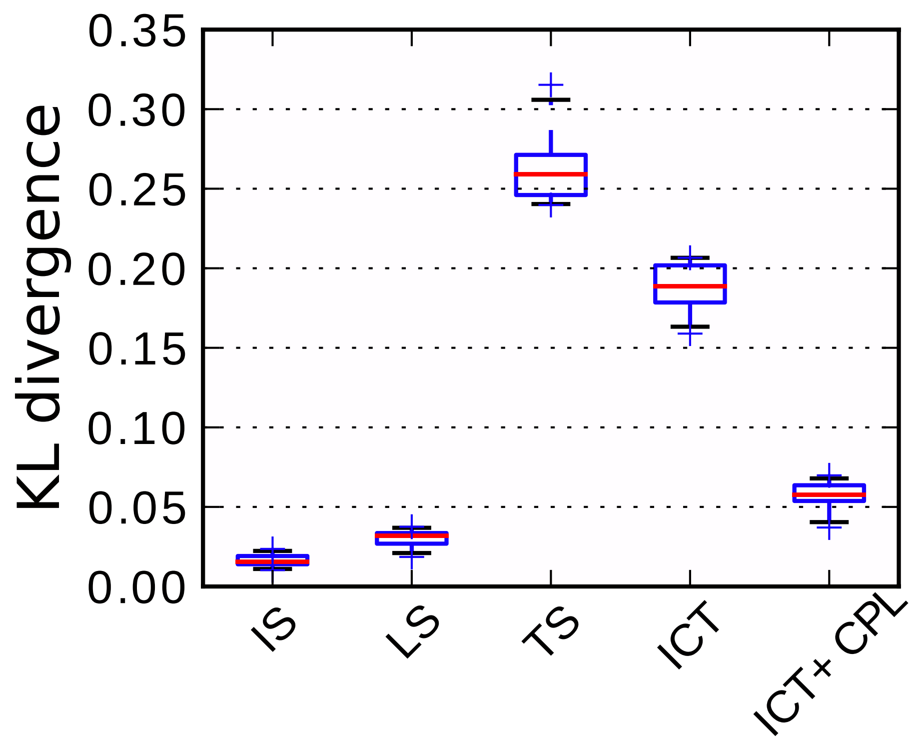
global time shuffling (TS): build a global list of the contact durations. For each link, generate a synthetic activity timeline by sampling with replacement the global list according to the original number of contacts for that link

Randomised reference models (RRM)

- $P(\tau)$: inter-contact time distribution
- ω_{AB}: cumulated contact durations of an arbitrary link
- $P(\omega)$: distribution of the cumulated contacts duration
- n_{AB}: number of contacts per link of an arbitrary link
- $P(n)$: distribution of the number of contacts per link

RRM	Topology	Causality	$P(\tau)$	ω_{AB}	$P(\omega)$	n_{AB}	$P(n)$
IS	V	X	V	V	V	V	V
LS	V	X	V	X	V	X	V
TS	V	X	X	X	X	V	V





[Gauvin et al Sci Rep 2013]