

Life Data Epidemiology

Lect 19: epidemics in space 2

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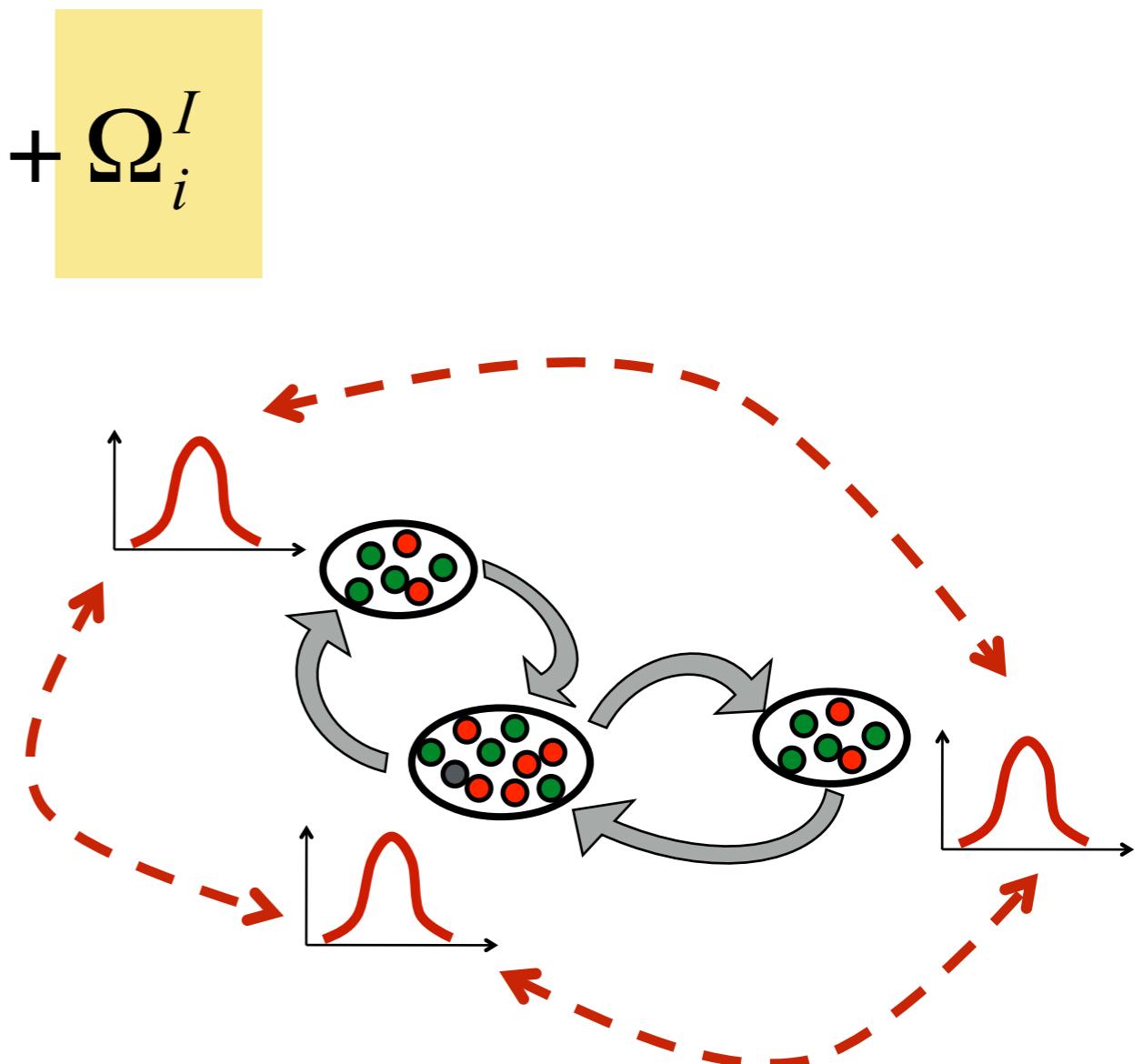
SIR metapopulation model: markovian mobility

$$\frac{dS_i}{dt} = -\beta \frac{I_i(t)S_i(t)}{N_i} + \Omega_i^S$$

$$\frac{dI_i}{dt} = \beta \frac{I_i(t)S_i(t)}{N_i} - \mu I_i(t) + \Omega_i^I$$

$$\frac{dR_i}{dt} = \mu I_i(t) + \Omega_i^R$$

$$\Omega_i^X = \sum_j \frac{w_{ji}}{N_j} X_j - \frac{w_{ij}}{N_i} X_i$$

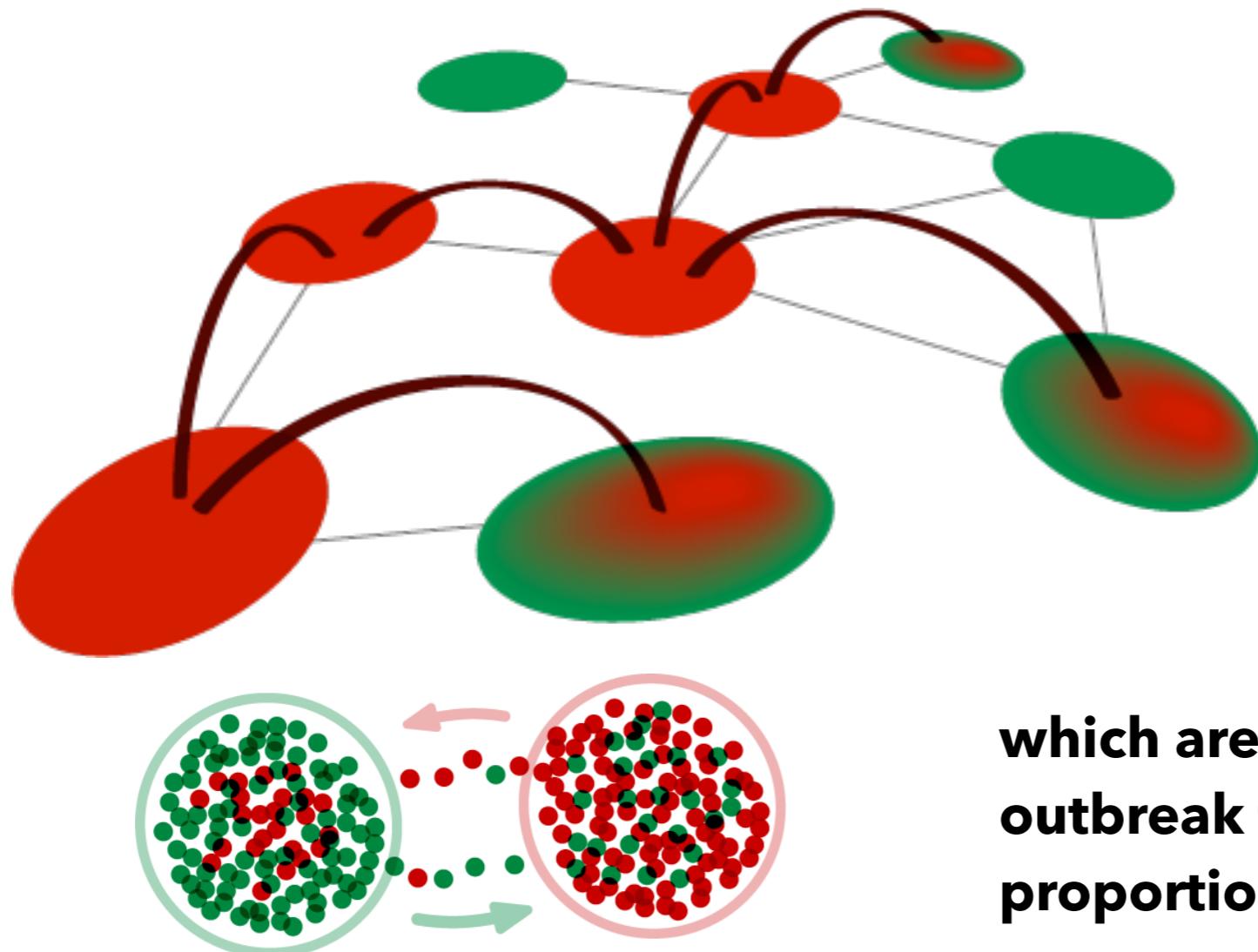


SIR metapopulation model: markovian mobility

What can I do with that?

- analytical understanding
 - spatial propagation & predictability
 - **global invasion threshold**
- application for epidemic assessment
- computer simulations

global invasion threshold



which are the conditions for a local outbreak to spread at global proportion?

global invasion threshold

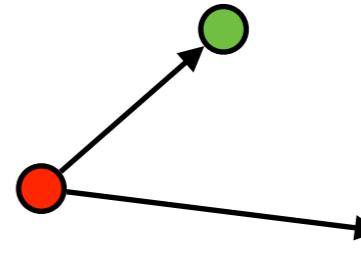
coarse graining

following the spread from one
subpopulation to another

mapping the spreading dynamics among
subpopulation into the spreading on a
network

individual
level

$$R_0$$



virus extinction

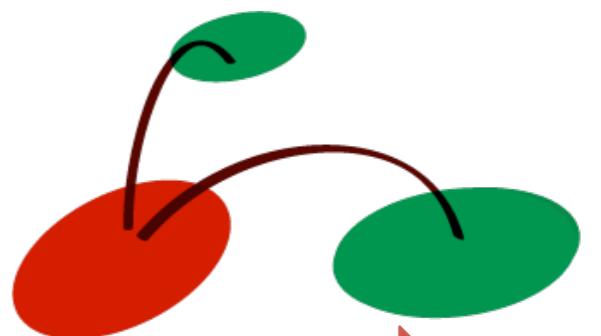
epidemic

$$1$$

$$R_0$$

subpopulation
level

$$R_*$$



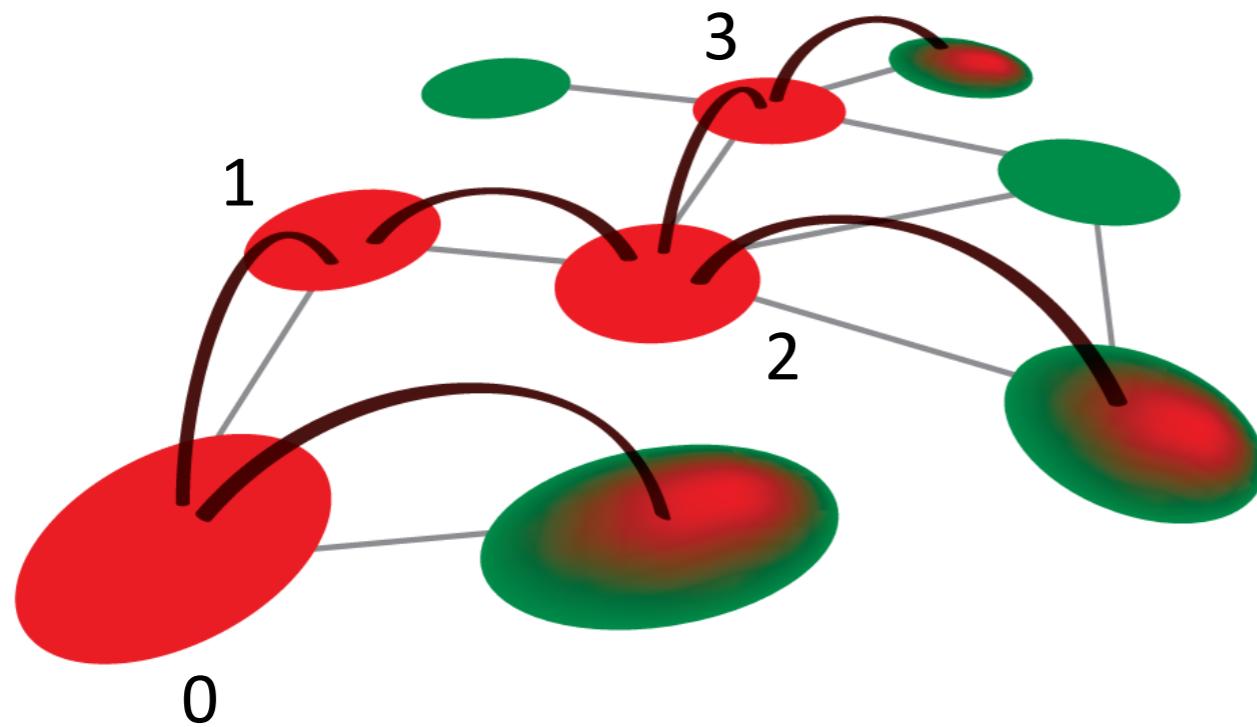
containment

spatial invasion

$$1$$

$$R_*$$

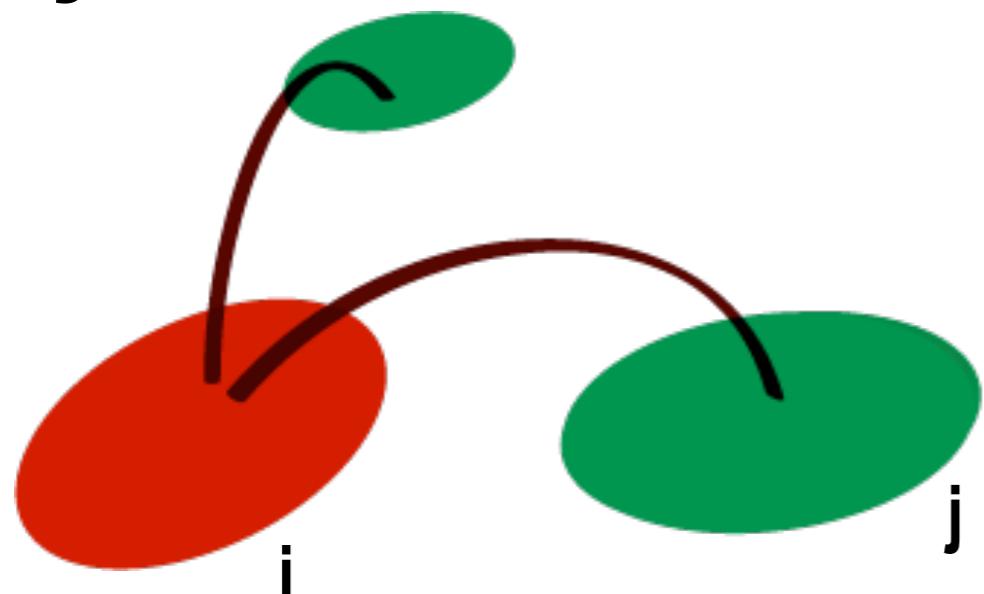
global invasion threshold



- Invasion dynamics at the subpopulation level
- branching process approximation

D^n : diseased subpopulations at generation n

invasion threshold: homogeneous systems



w_0 travellers along each link

$\langle k \rangle$ # connection of each subpopulation

N population of each subpopulation

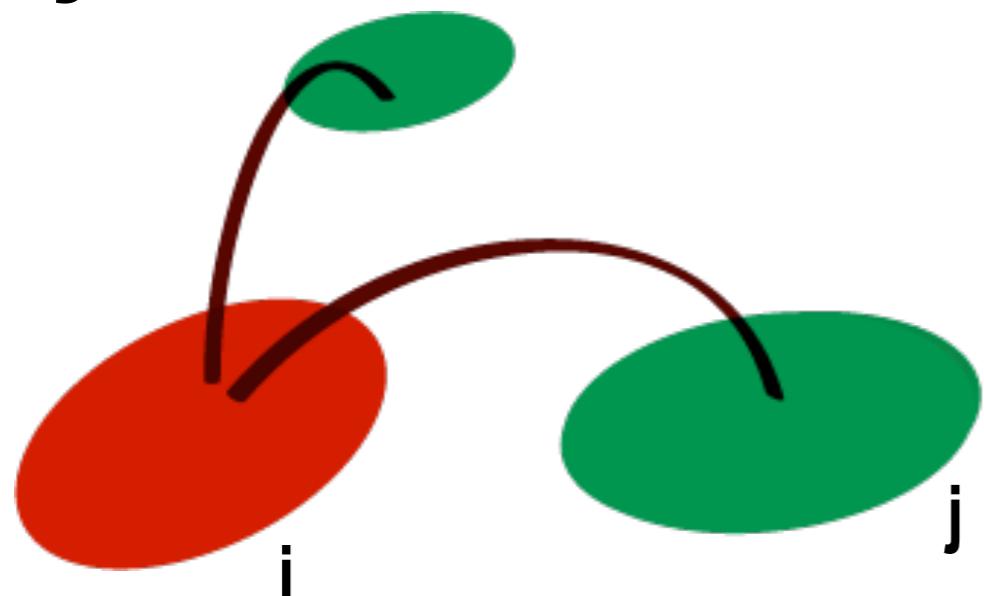
α epidemic attack rate

total # infectious individuals sent from *i* to *j* during the local outbreak

$$\lambda_{ij} = \frac{w_0}{N} \frac{\alpha N}{\mu}$$

probability of early extinction $p_{\text{ext}} = \left(\frac{1}{R_0}\right)^{\lambda_{ij}}$

invasion threshold: homogeneous systems



w_0 travellers along each link
 $\langle k \rangle$ # connection of each subpopulation
 N population of each subpopulation
 α epidemic attack rate

probability of early extinction $p_{\text{ext}} = \left(\frac{1}{R_0}\right)^{\lambda_{ij}}$

$$D^n = (\langle k \rangle - 1) (1 - P_{\text{ext}}) \left(1 - \sum_{m=0}^{n-1} \frac{D^m}{V}\right) D^{n-1}$$

invasion threshold: homogeneous systems

$$D^n = (\langle k \rangle - 1) (1 - P_{\text{ext}}) \left(1 - \sum_{m=0}^{n-1} \frac{D^n}{V} \right) D^{n-1}$$

$$R_* = (\langle k \rangle - 1) (1 - P_{\text{ext}})$$

$$1 - P_{\text{ext}} = 1 - \left(\frac{1}{R_0} \right)^{\lambda_{ij}} \simeq \lambda_{ij}(R_0 - 1) = \frac{\alpha w_0}{\mu}(R_0 - 1)$$

$$R_* = (\langle k \rangle - 1) \frac{\alpha w_0}{\mu} (R_0 - 1)$$

invasion threshold: homogeneous systems

$$R_* = (\langle k \rangle - 1) \frac{\alpha w_0}{\mu} (R_0 - 1)$$

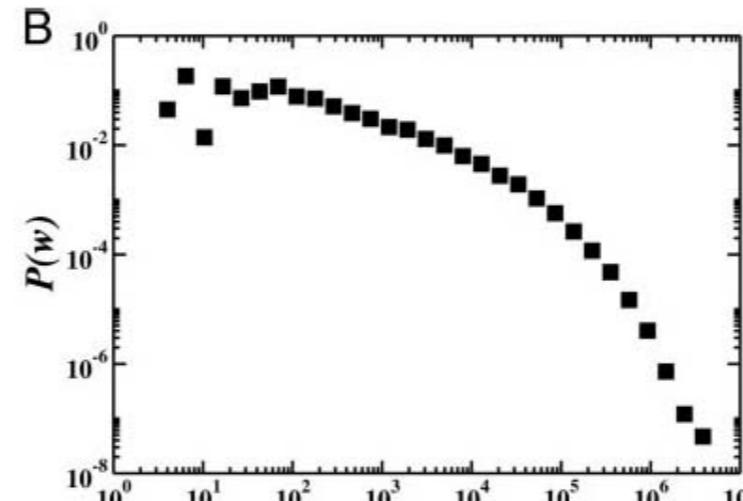
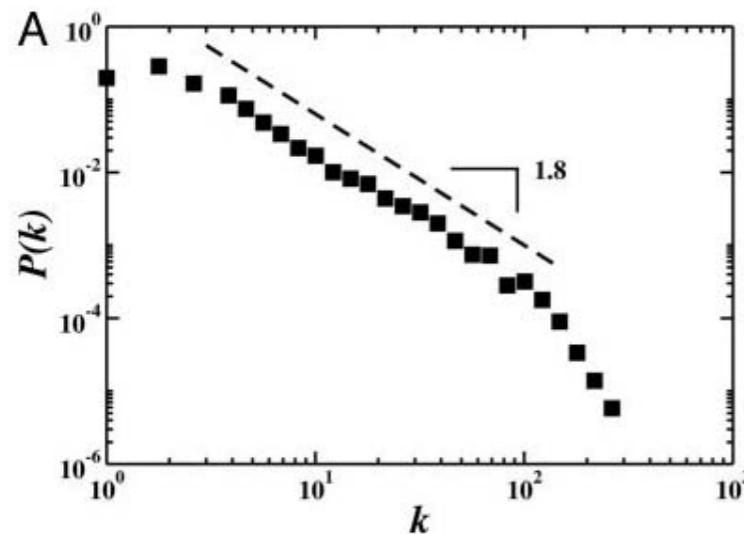
invasion potential growing function of :

- R_0
- overall traffic rescaling
- average number of connections
- infectious duration

invasion threshold: heterogeneous systems

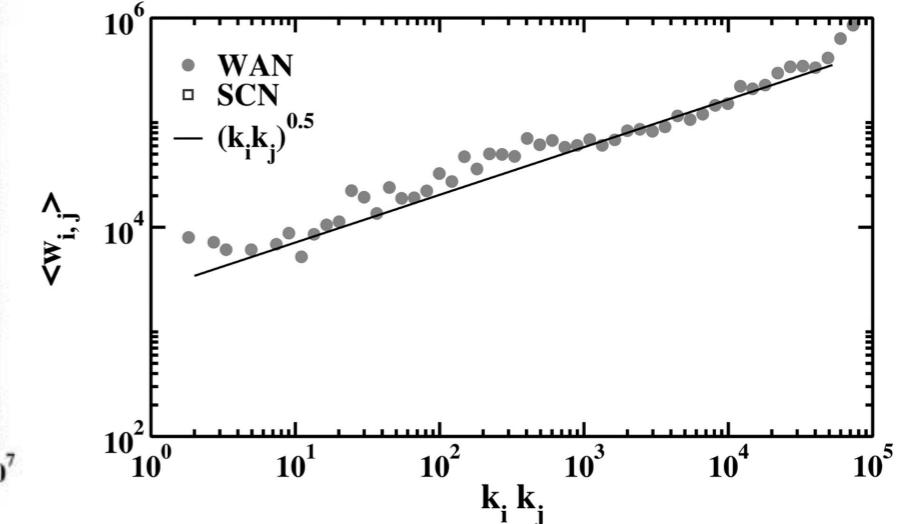
Real systems are highly heterogeneous. E.g.: air transportation network

Bad news :(



- number of connections and travellers along the connections is heterogeneous
- average quantities are not good representative of the properties of patches
- homogenous approximation is bad

Good news :)



- scaling relations: approximate laws that make possible calculations **and justify the degree-block description**

invasion threshold: heterogeneous systems

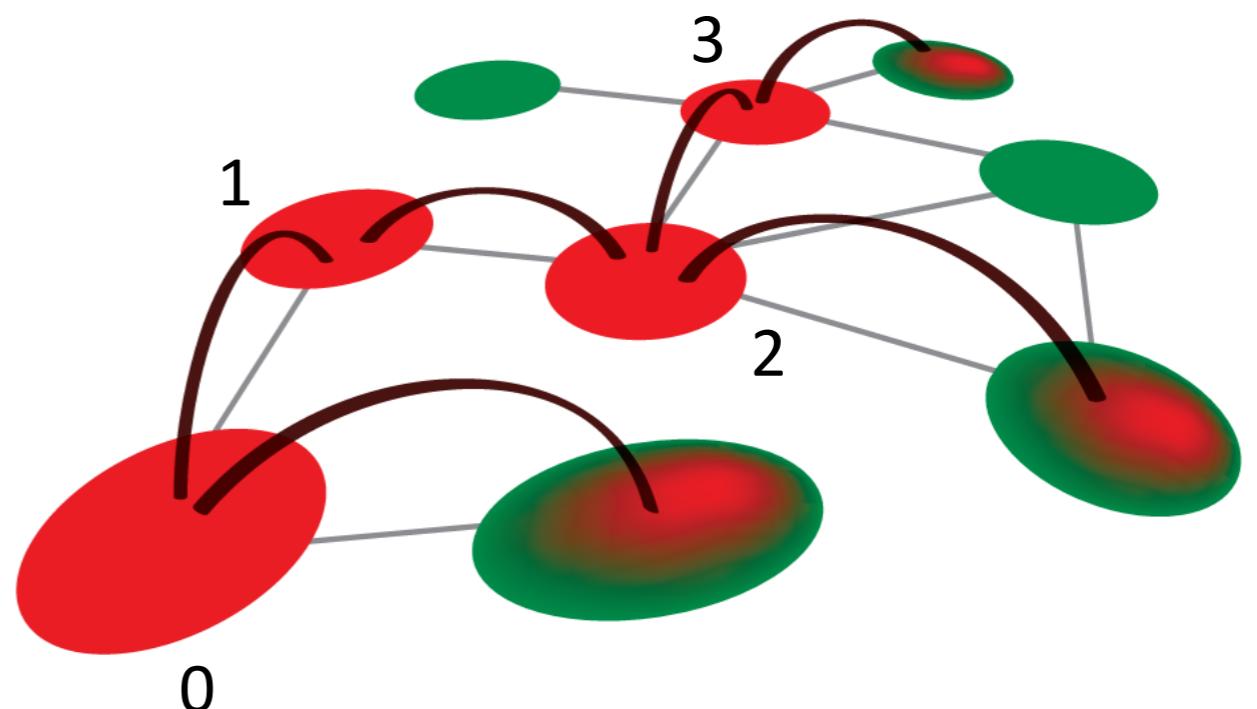
Degree-block description:

we group patches according to their degree and consider patches within the same degree-class homogeneous

$$N_k = N_0 k^\phi$$

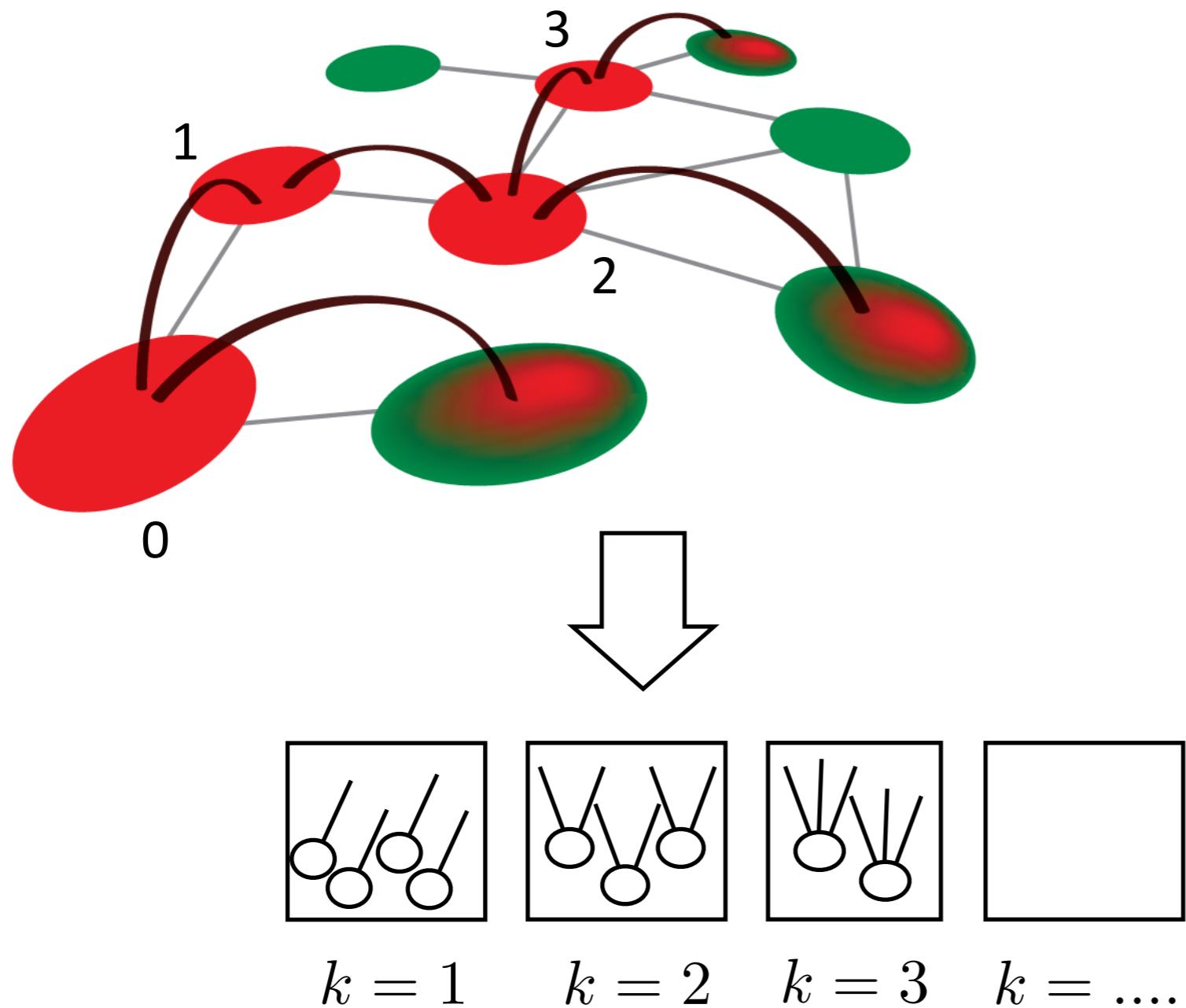
$$w_{kk'} = w_0 (k k')^\theta$$

$$p_{kk'} = \frac{w_0}{N_0} \frac{(k k')^\theta}{k^\phi}$$



D_k^n : diseased subpopulations at generation n , with k mobility connections

heterogeneous mean field approach



$$P(k)$$

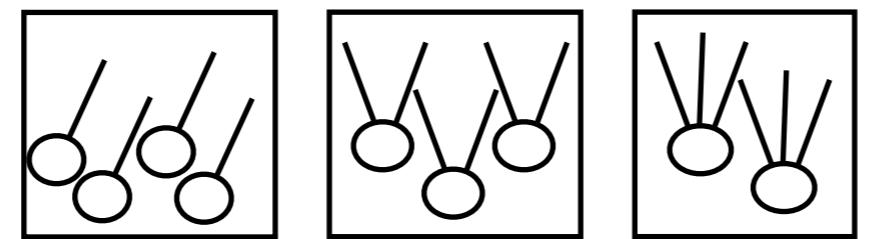
Degree distribution

$$N_k = N_0 k^\phi$$

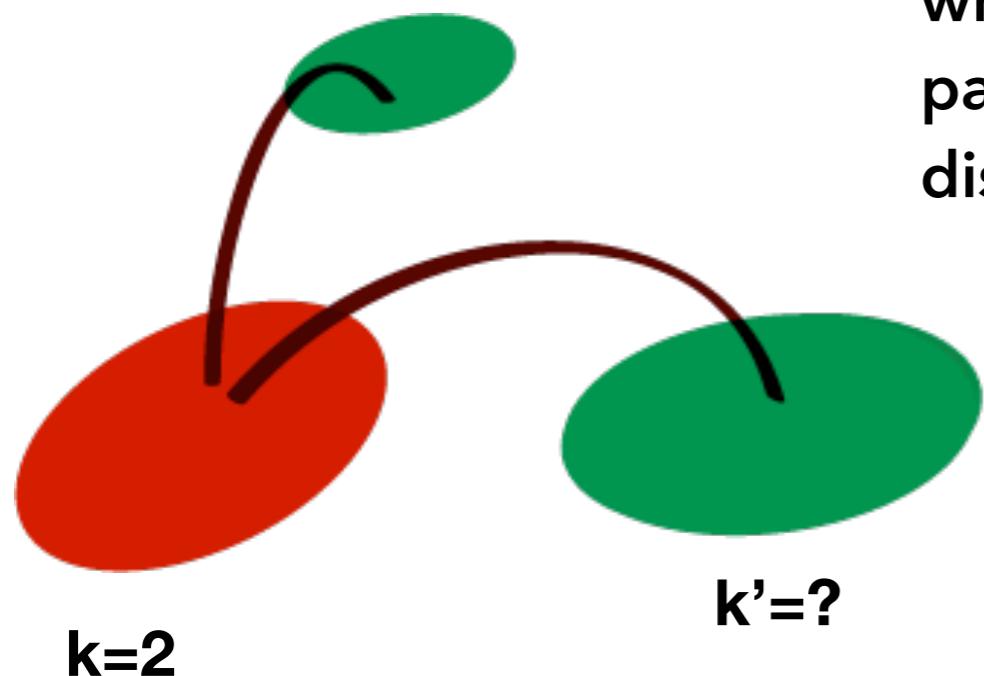
$$w_{kk'} = w_0 (k k')^\theta$$

$$p_{kk'} = \frac{w_0}{N_0} \frac{(k k')^\theta}{k^\phi}$$

heterogeneous mean field approach

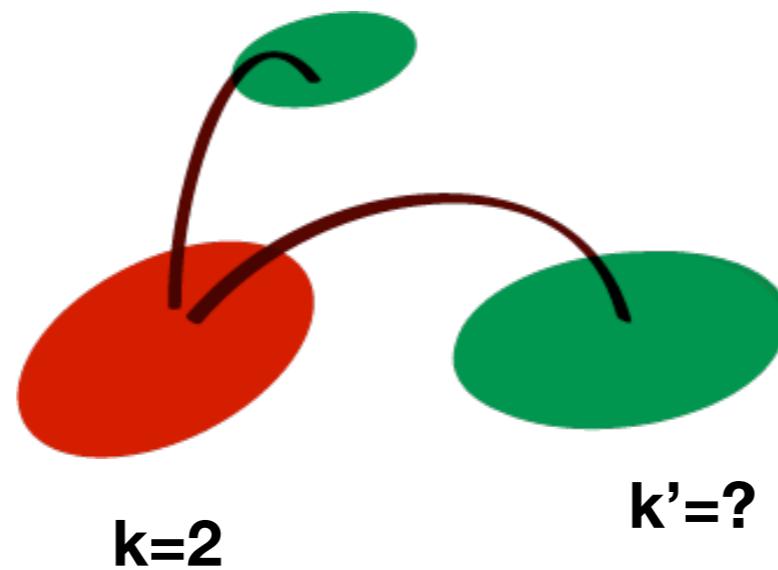


$k = 1 \quad k = 2 \quad k = 3 \quad k = \dots$



what is the probability that an infected
patch with degree k is connected to a
disease-free patch of degree k' ?

heterogeneous mean field approach



number of mobility
connections
through which the
seeding may
potentially occur:

$$k - 1$$

x

probability that
contact has
degree k' :

$$P(k'|k)$$

x

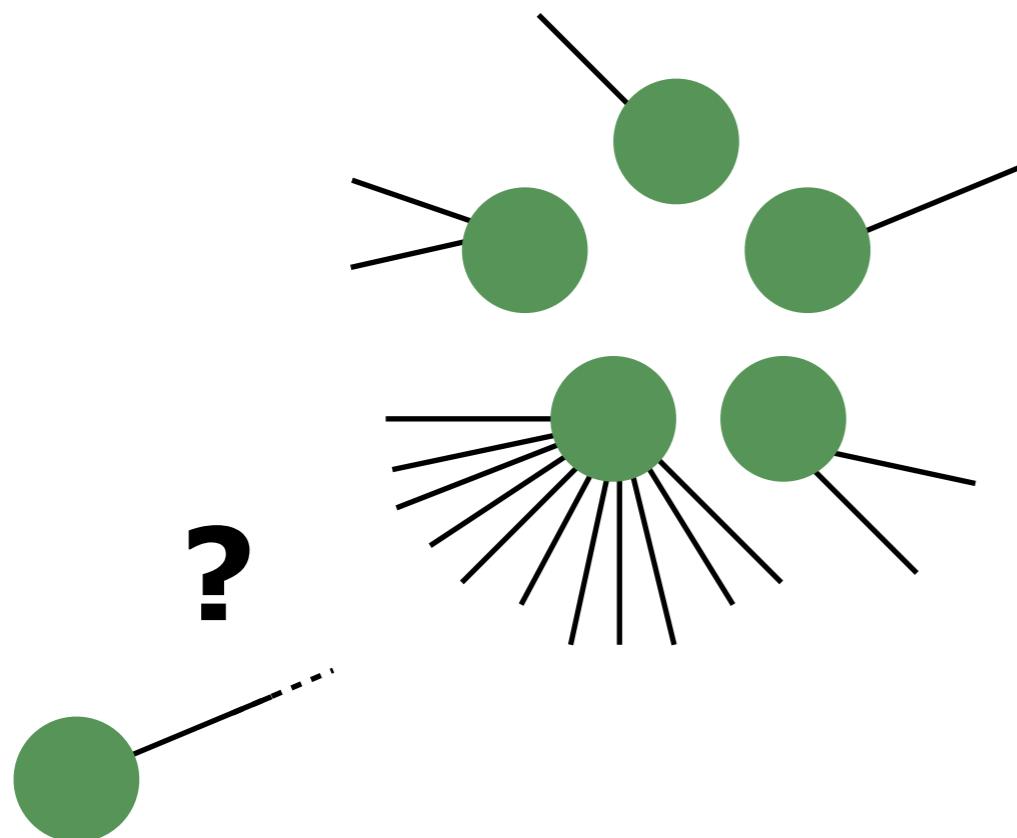
number of disease-free
patches within the k' -
class:

$$\left(1 - \sum_{m=0}^{n-1} \frac{D_{k'}^m}{V_{k'}} \right)$$

heterogeneous mean field approach

$$P(k'|k) = k' \frac{P(k')}{\langle k \rangle}$$

If I make a connection at random I will do it more likely with a node that is well connected (more stubs)



Friendship paradox:
my friends have more friends than me!

invasion threshold: heterogeneous systems

$$D^n = (\langle k \rangle - 1) (1 - P_{\text{ext}}) \left(1 - \sum_{m=0}^{n-1} \frac{D^m}{V} \right) D^{n-1}$$

$$D^n \rightarrow D_k^n$$

$$D_k^n = \sum_{k'} D_{k'}^{n-1} (k' - 1) P(k | k') \left(1 - \sum_{m=0}^{n-1} \frac{D_k^m}{V_k} \right) (1 - P_{\text{ext}}(\lambda_{k'k}))$$

invasion threshold: heterogeneous systems

$$D_k^n = \sum_{k'} D_{k'}^{n-1} \quad (k'-1)P(k|k') \quad \left(1 - \sum_{m=0}^{n-1} \frac{D_k^m}{V_k} \right) \quad (1 - P_{\text{ext}}(\lambda_{k'k}))$$

$P(k|k') = k \frac{P(k)}{\langle k \rangle}$

$$1 - P_{\text{ext}}(\lambda_{k'k}) = 1 - \left(\frac{1}{R_0} \right)^{\lambda_{k'k}} \simeq \lambda_{k'k}(R_0 - 1)$$

$\lambda_{ij} = \frac{w_0 \alpha}{\mu}$

homogenous case

$$\lambda_{k'k} = \frac{w_0 (kk')^\theta \alpha}{N_0 k^\phi} - (N_0 k^\phi)$$

invasion threshold: heterogeneous systems

$$D_k^n = \sum_{k'} D_{k'}^{n-1} (k' - 1) P(k | k') \left(1 - \sum_{m=0}^{n-1} \frac{D_k^m}{V_k} \right) (1 - P_{ext}(\lambda_{k'k}))$$

$$D_k^n = (R_0 - 1) \frac{\alpha w_0}{\mu} \frac{k^{1+\theta} P(k)}{\langle k \rangle} \boxed{\sum_{k'} D_{k'}^{n-1} (k' - 1) k'^\theta} \Theta^{n-1}$$

$$\Theta^n = (R_0 - 1) \frac{\alpha w_0}{\mu} \frac{\langle k^{2+2\theta} \rangle - \langle k^{1+2\theta} \rangle}{\langle k \rangle} \Theta^{n-1}$$

$$R_* = (R_0 - 1) \frac{\alpha w_0}{\mu} \frac{\langle k^{2+2\theta} \rangle - \langle k^{1+2\theta} \rangle}{\langle k \rangle} > 1$$

invasion threshold: heterogeneous system

$$R_* = (R_0 - 1) \frac{\alpha w_0}{\mu} \frac{\langle k^{2+2\theta} \rangle - \langle k^{1+2\theta} \rangle}{\langle k \rangle} > 1$$

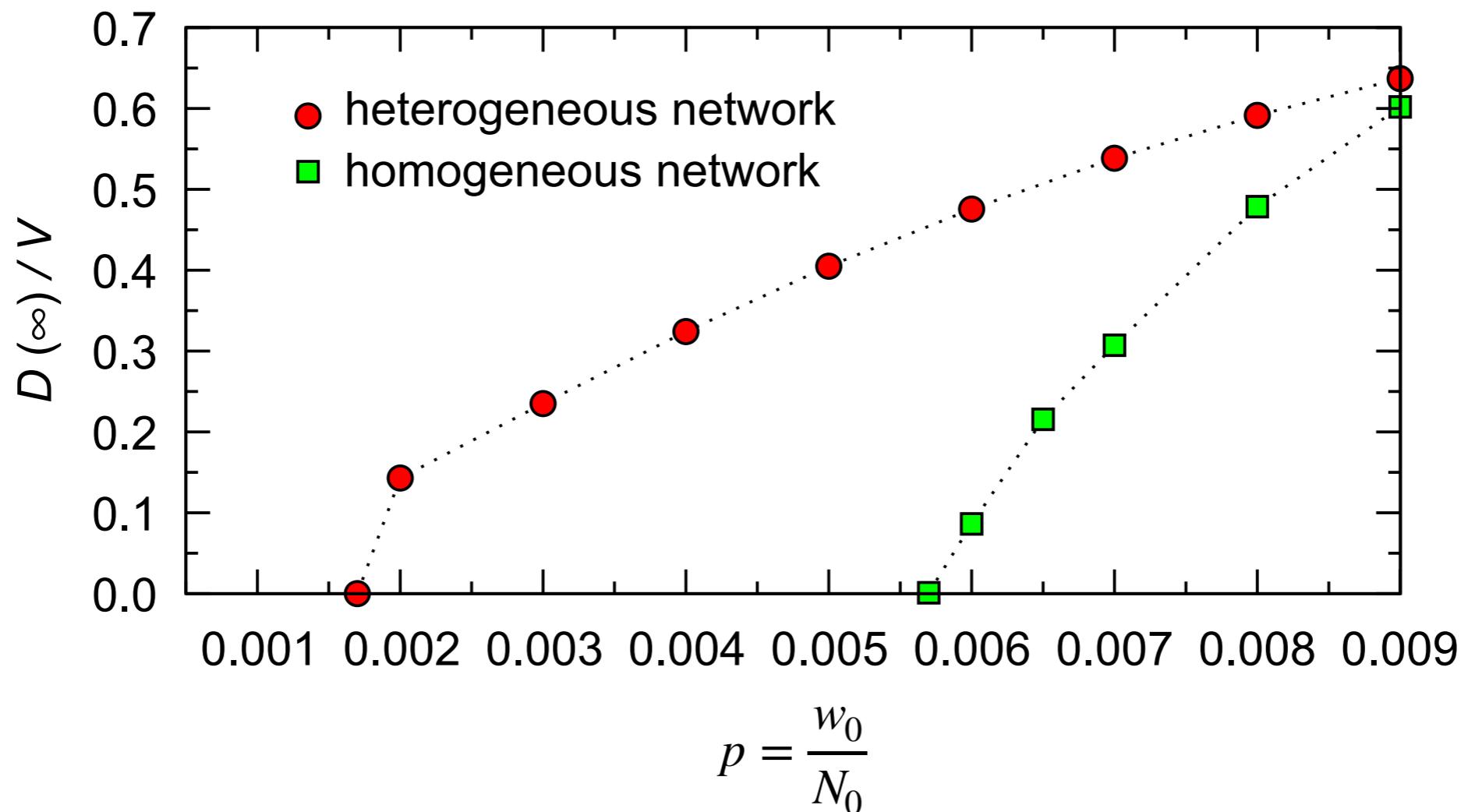
invasion potential growing function of :

- R_0
- overall traffic rescaling
- average number of connections
- infectious duration
- moments of the degree distribution and its fluctuations

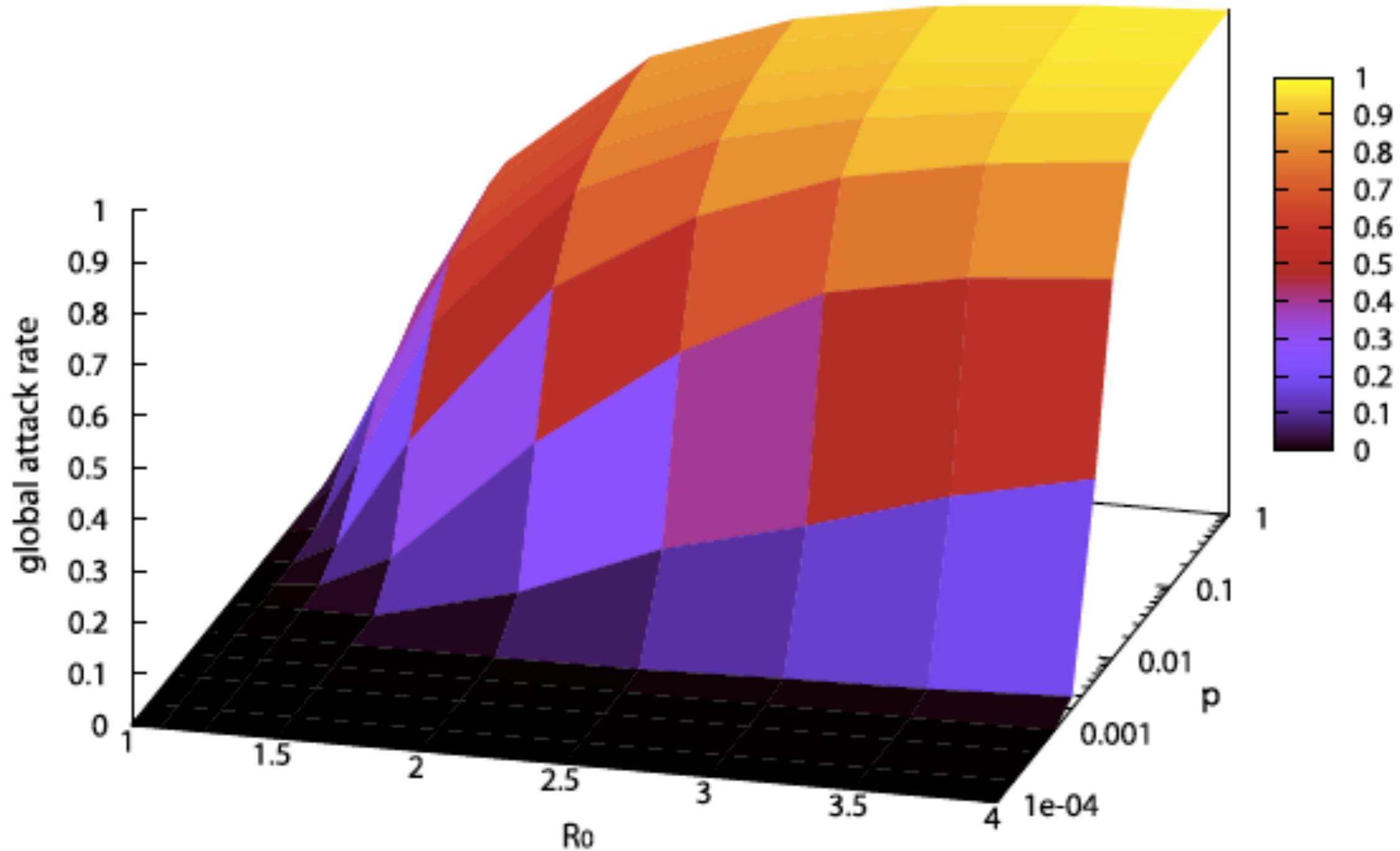
$$\langle k^{2+2\theta} \rangle - \langle k^{1+2\theta} \rangle \simeq 7 \cdot 10^4$$

$$\langle k \rangle \simeq 10$$

invasion threshold: heterogeneous system



invasion threshold: heterogeneous system



[Colizza & Vespignani, PRL 2007, JTB 2008]

SIR metapopulation model: markovian mobility

What can I do with that?

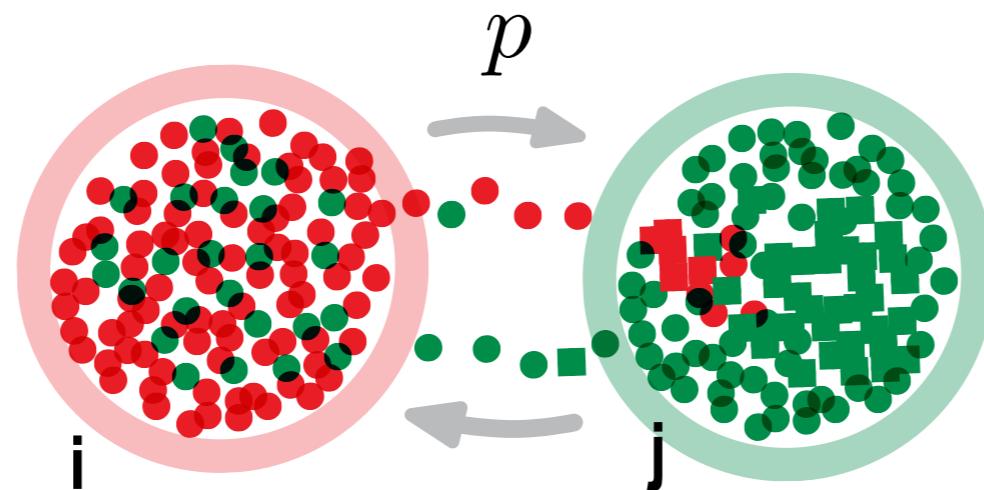
- analytical understanding
 - **spatial propagation & predictability**
 - global invasion threshold
- application for epidemic assessment
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spatial propagation

Dynamics of spatial spread above the epidemic threshold

An epidemic starts in a given city i how does it spread to j, h , etc.?

seeding time (or arrival time), t_{seeding} : time of arrival of the first case in patch j



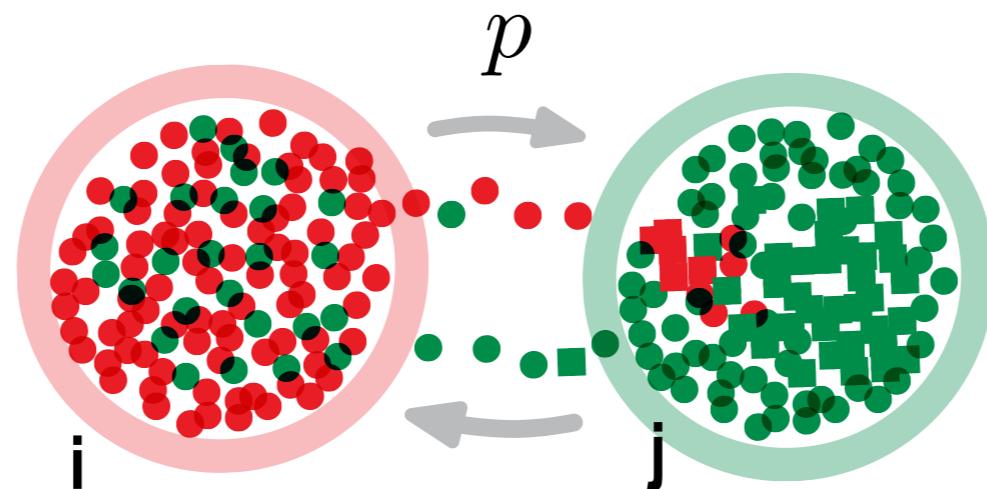
p traveling probability from i to j

$I(t)$ infectious in i

probability that an infectious arrives in j at time t :

$$\left[1 - (1 - p)^{I(t) dt} \right]$$

spatial propagation



probability that an infectious arrives in *j* at time *t*: $\left[1 - (1 - p)^{I(t \ dt)} \right]$

probability that the **first** infectious arrives in *j* at time *t*:

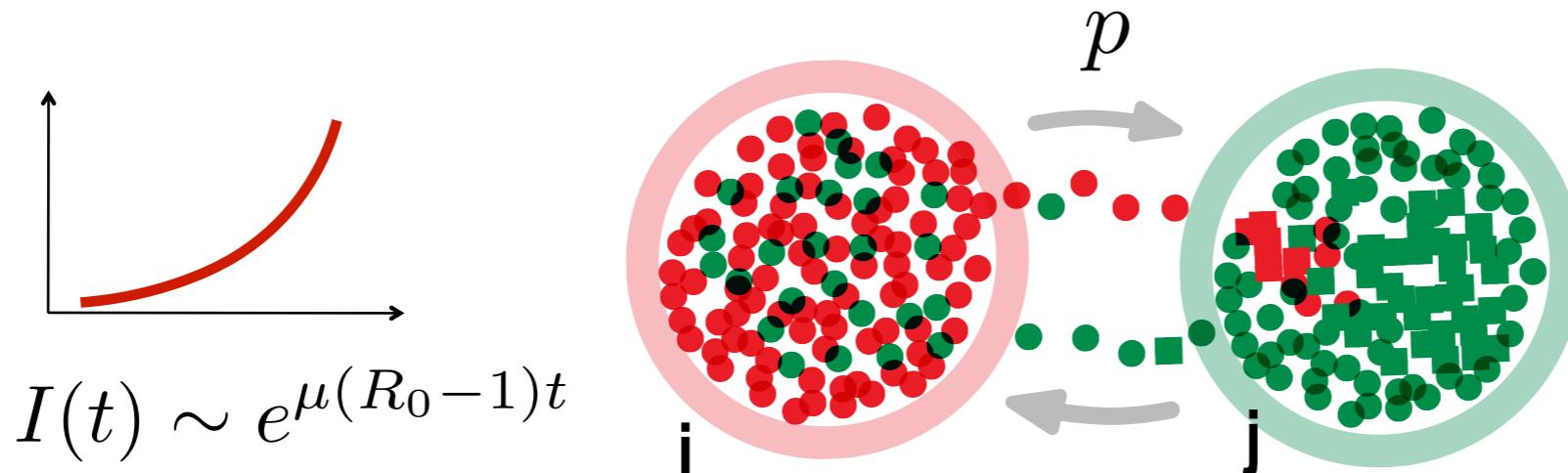
$$P(t_{\text{seeding}} = t \ dt) = \prod_{s=1}^{t-1} (1 - p)^{I(s \ dt)} \times \left[1 - (1 - p)^{I(t \ dt)} \right]$$

\downarrow $p \rightarrow 0$

$$P(t_{\text{seeding}} = t) = p I(t) e^{-p \int_0^t I(s) ds}$$

[Gautreau et al JTB 2008]

spatial propagation



$$P(t_{\text{seeding}} = t) = p I(t) e^{-p \int_0^t I(s) ds}$$

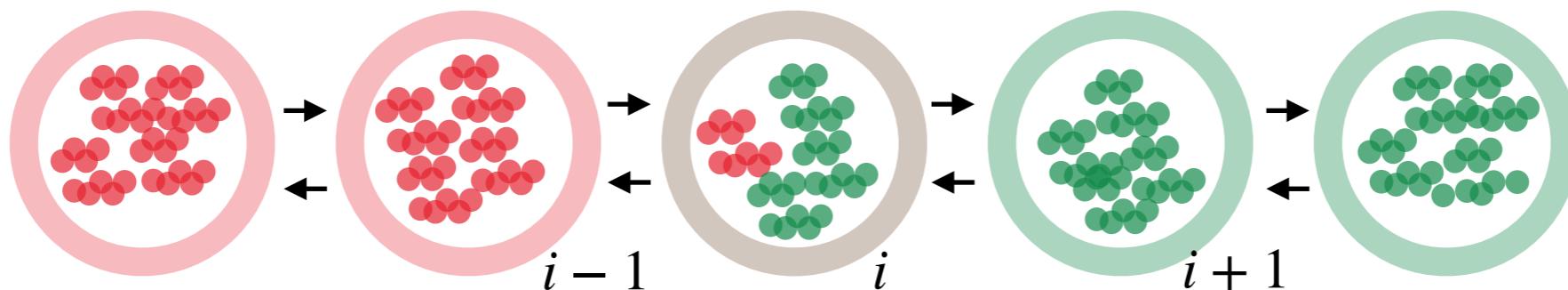
$$a = \mu(R_0 - 1)$$

$$P(t_{\text{seeding}} = t) = p e^{at} e^{-p/a e^{at}}$$

Gumbel distribution

$$\langle t_{\text{seeding}} \rangle \simeq -\frac{1}{a} \ln \left(\frac{p}{a} \right)$$

spatial propagation



chain of identical cities (population N traveling weight p): what is the time of importation in city $i + 1$?

$$\langle t_{\text{seeding},i} \rangle - \langle t_{\text{seeding},i-1} \rangle = \Delta_i$$

$$\langle t_{\text{seeding},n} \rangle = \sum_{i=1}^n \Delta_n$$

$\{\Delta_i\}$ are correlated and not identically distributed.

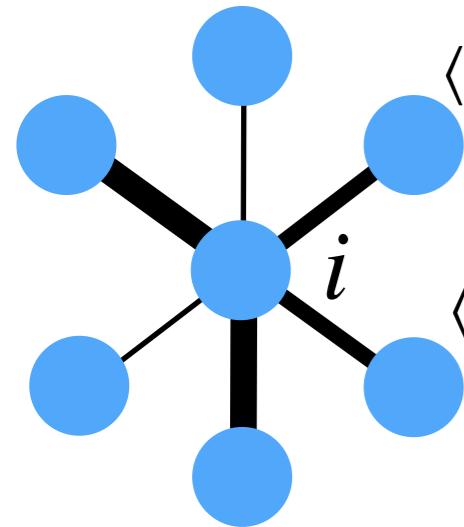
Arrival time in city $i + 1$ depends on the incidence dynamics in city i that is combination of introductions from $i - 1$ and infection transmission within i .

Still, $\Delta_n \simeq \langle \Delta \rangle = \langle t_{\text{seeding},1} \rangle$ is a valid first order approximation

$$t_{\text{seeding},i+1} \simeq (i+1)\Delta = -(i+1)\frac{1}{a} \ln \left(\frac{p}{a} \right)$$

[Gautreau et al JTB 2008]

spatial propagation: spreading pathways



$$\langle t_{\text{seeding}, j} \rangle \simeq -\frac{1}{a} \ln \left(\frac{p_{ij}}{a} \right)$$

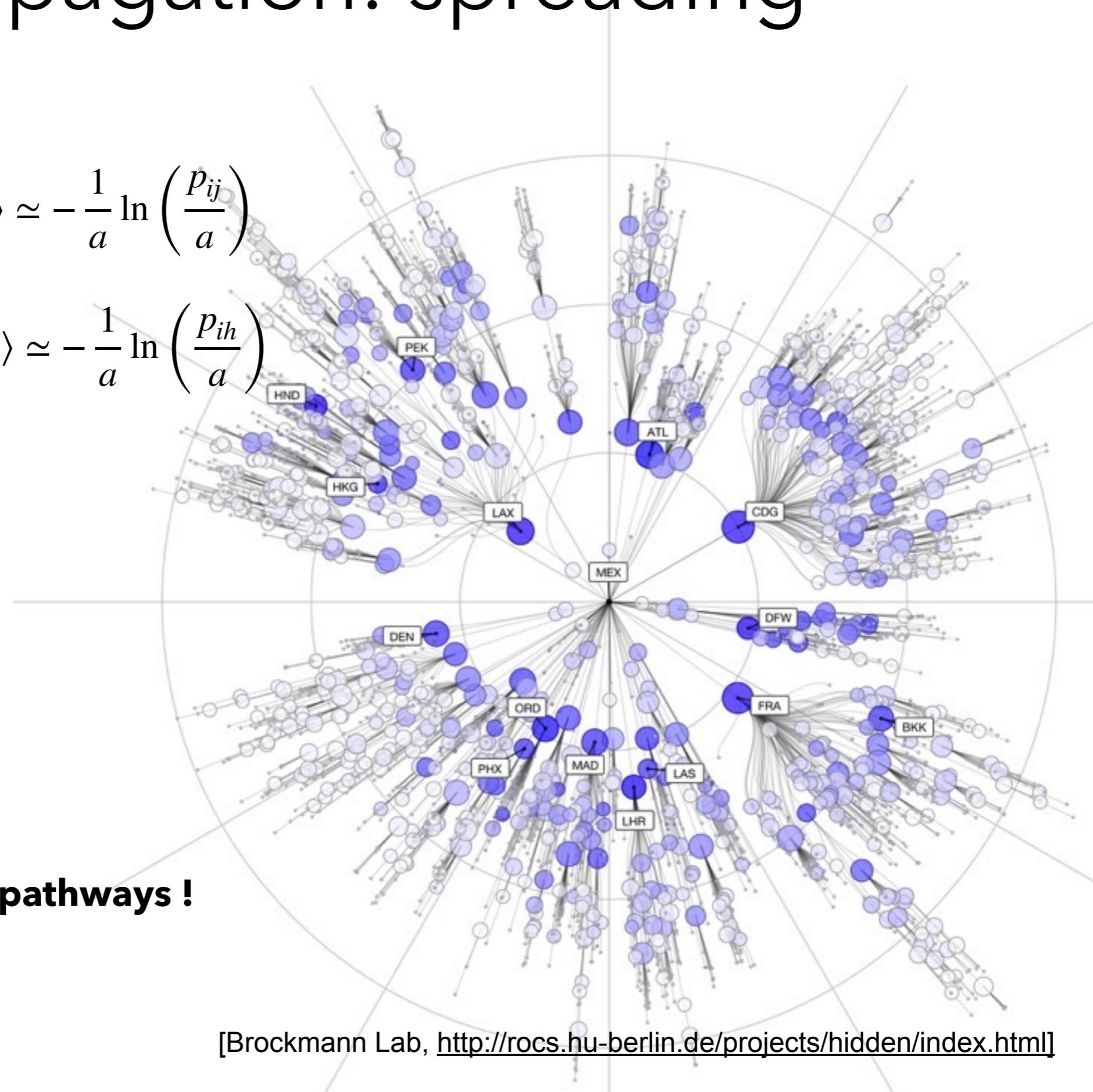
$$\langle t_{\text{seeding}, h} \rangle \simeq -\frac{1}{a} \ln \left(\frac{p_{ih}}{a} \right)$$

effective distance
between i and j
 $-\ln(p_{ij})$

[Brockmann, Helbing, Science
2013]

good news: existence of pathways !
risk assessment analysis, ...

[Colizza, et al PNAS (2006)]

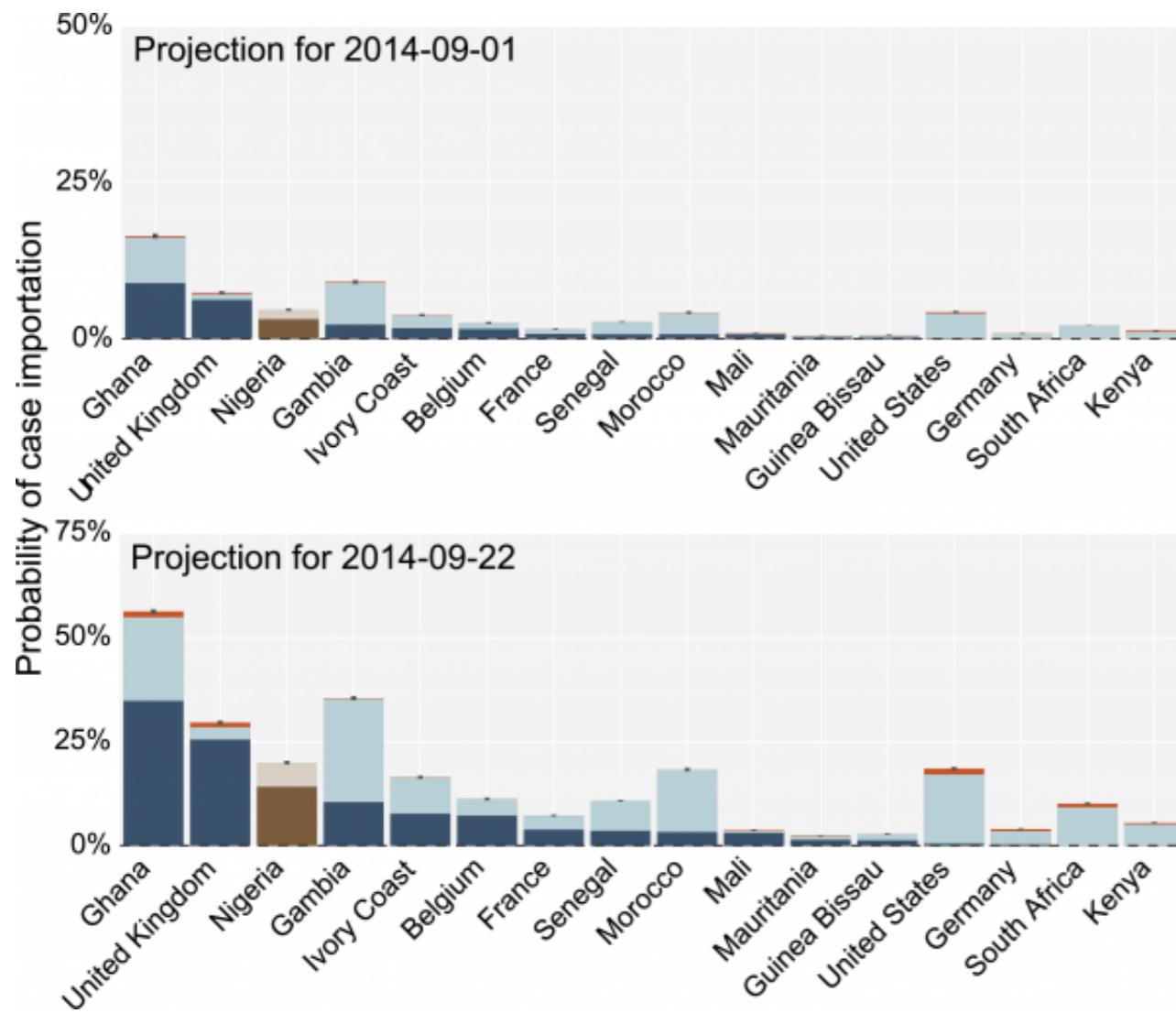


[Brockmann Lab, <http://rocs.hu-berlin.de/projects/hidden/index.html>]

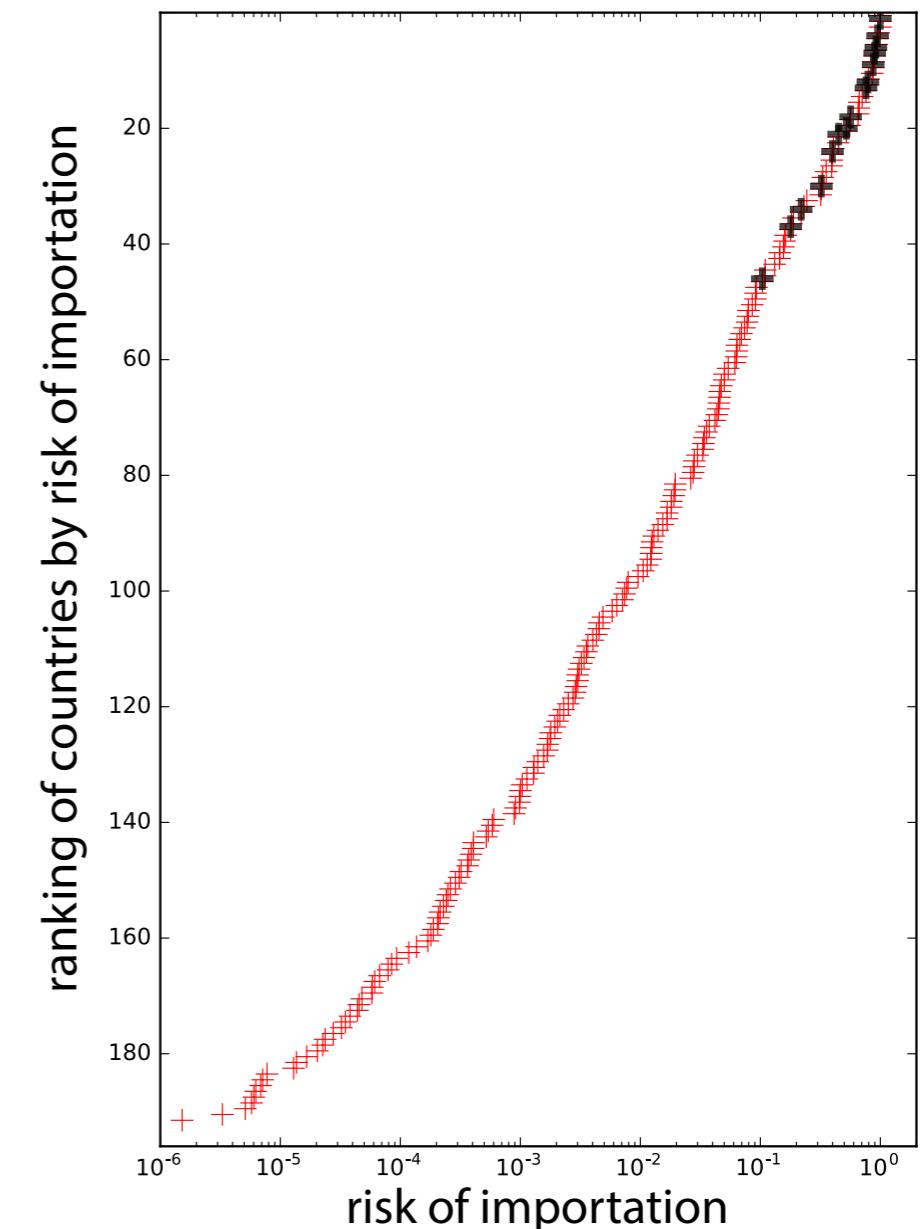
spatial propagation: spreading pathways

Air-travel quantifies the risk of importation

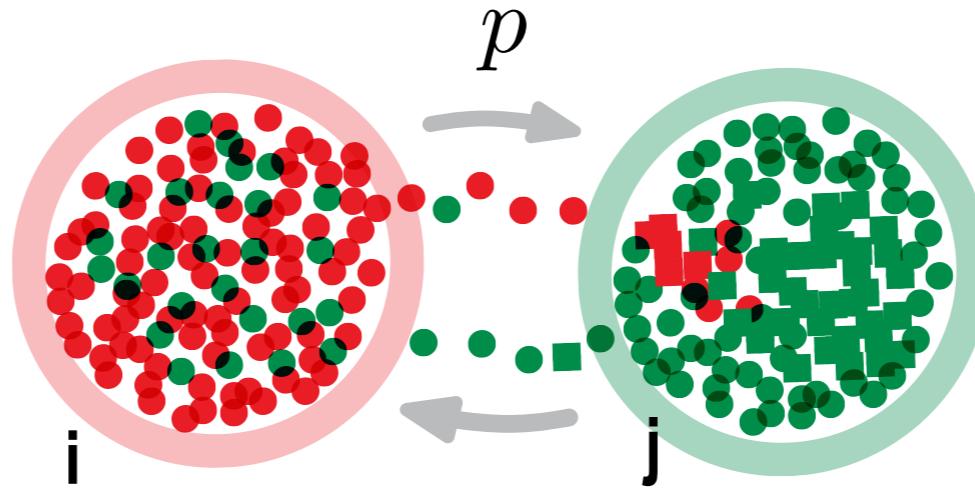
Western-Africa Ebola outbreak 2014



MERS 2012-2015



spatial propagation: travel restrictions



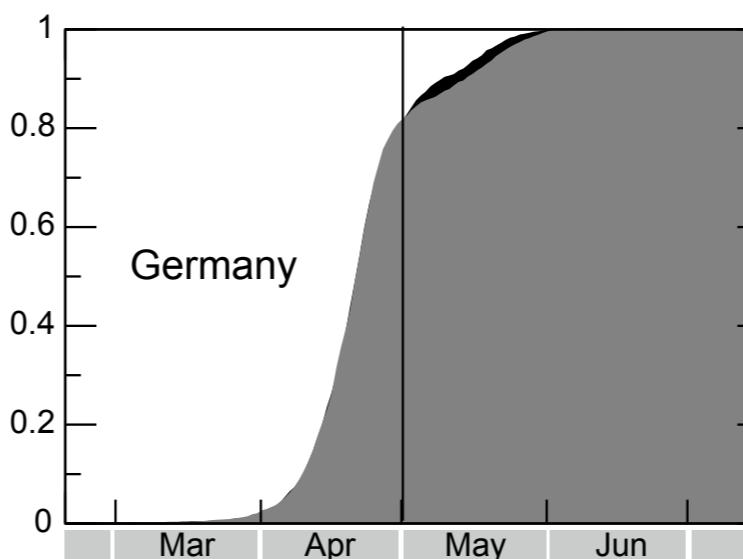
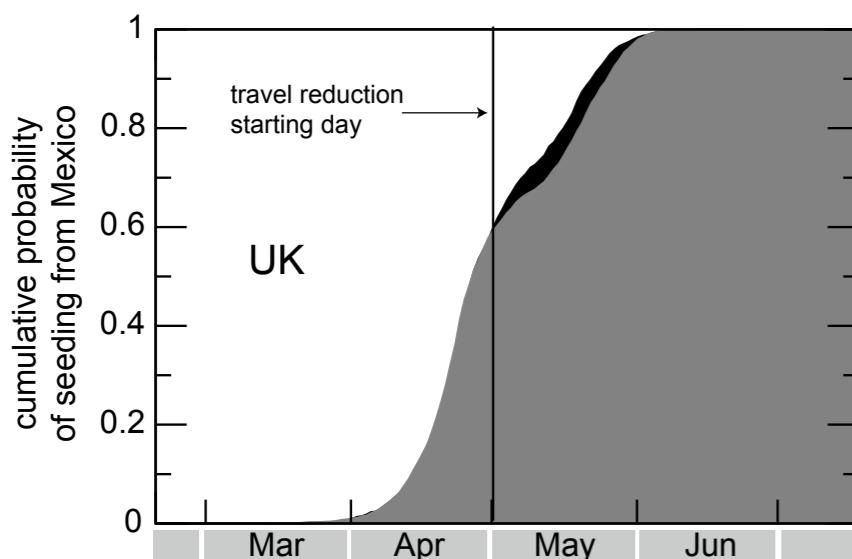
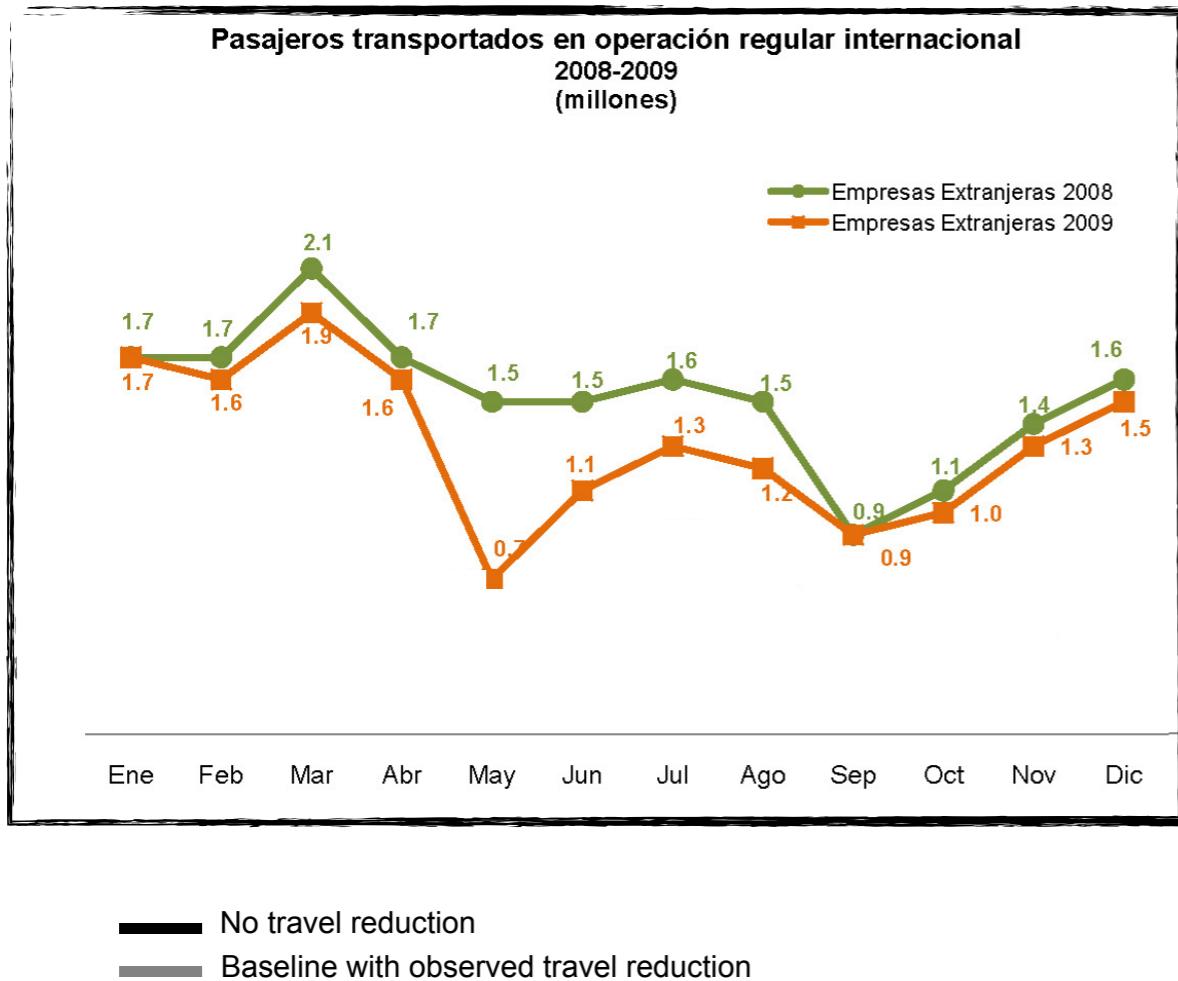
I reduce the traffic with the epidemic origin: is it effecting in containing or delaying the propagation?

I rescale the traveling probability of a factor ω

$$\langle t_{\text{seeding, T.R.}} \rangle \simeq \frac{1}{a} \ln(\omega p a)$$

$$\begin{aligned}\langle t_{\text{seeding, T.R.}} \rangle - \langle t_{\text{seeding}} \rangle &\simeq -\frac{1}{a} \ln\left(\frac{\omega p}{a}\right) + \frac{1}{a} \ln\left(\frac{p}{a}\right) \\ &= -\frac{1}{a} \ln(\omega) - \frac{1}{a} \ln\left(\frac{p}{a}\right) + \frac{1}{a} \ln\left(\frac{p}{a}\right) \\ &= -\frac{1}{a} \ln(\omega)\end{aligned}$$

spatial propagation: travel restrictions



H1N1 pandemic 2009

- drop of 40% in the air-travel to/from Mexico
- simulations with a global spreading model for influenza show negligible delay

[Bajardi et al, PLoS ONE 2011]

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COVID-19 early warning



- "urgent notice on the treatment of pneumonia of unknown cause" by Wuhan Municipal Health Committee. Picked up by ProMED-mail
- Epidemic situation in Jan 17:
 - 41 cases + 1 deaths in Wuhan
 - 2 cases in Thailand, 1 case in Japan (among travellers)

a large number of importations compared to local cases

[The New York Times,
<https://www.nytimes.com/interactive/2020/03/22/world/coronavirus-spread.html>]

COVID-19 early warning



When WHO gives an international alert, surveillance is heightened in foreign countries. Information of international cases more reliable. I use this information to infer the epidemic situation in the country of origin

[The New York Times,
<https://www.nytimes.com/interactive/2020/03/22/world/coronavirus-spread.html>]

COVID-19 early warning

$$I_{\text{trav}} = I_{\text{Wuhan}} p_{\text{travel}}$$

$$p_{\text{travel}} = p_{\text{daily}} T_d$$

- daily probability of travelling out of Wuhan $p_{\text{daily}} = \frac{w}{N}$
- time to detect a case, T_d , incubation period + time to hospitalisation
- Wuhan airport, passenger per day, $w= 3301$
- Wuhan airport, catchment population, $N = 19$ millions
- incubation period 5-6 days; time to hospitalisation 4-5 days (from SARS)
- estimated cases 1,723 (95% CI: 427 - 4,471)**