Physics of Life Data Epidemiology

Lect 15: temporal networks

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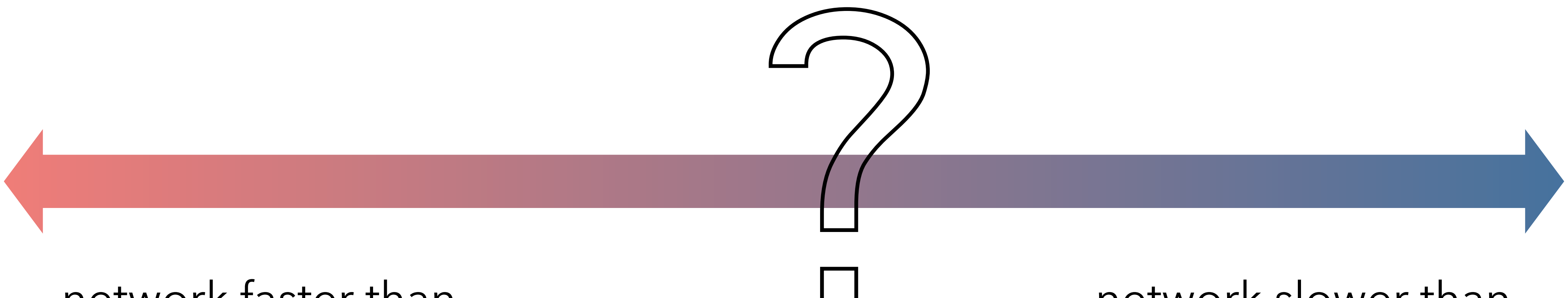
importance of contact dynamics for an epidemic

heterogeneous mean-field approach

Individual-based mean-field approach



network faster than disease

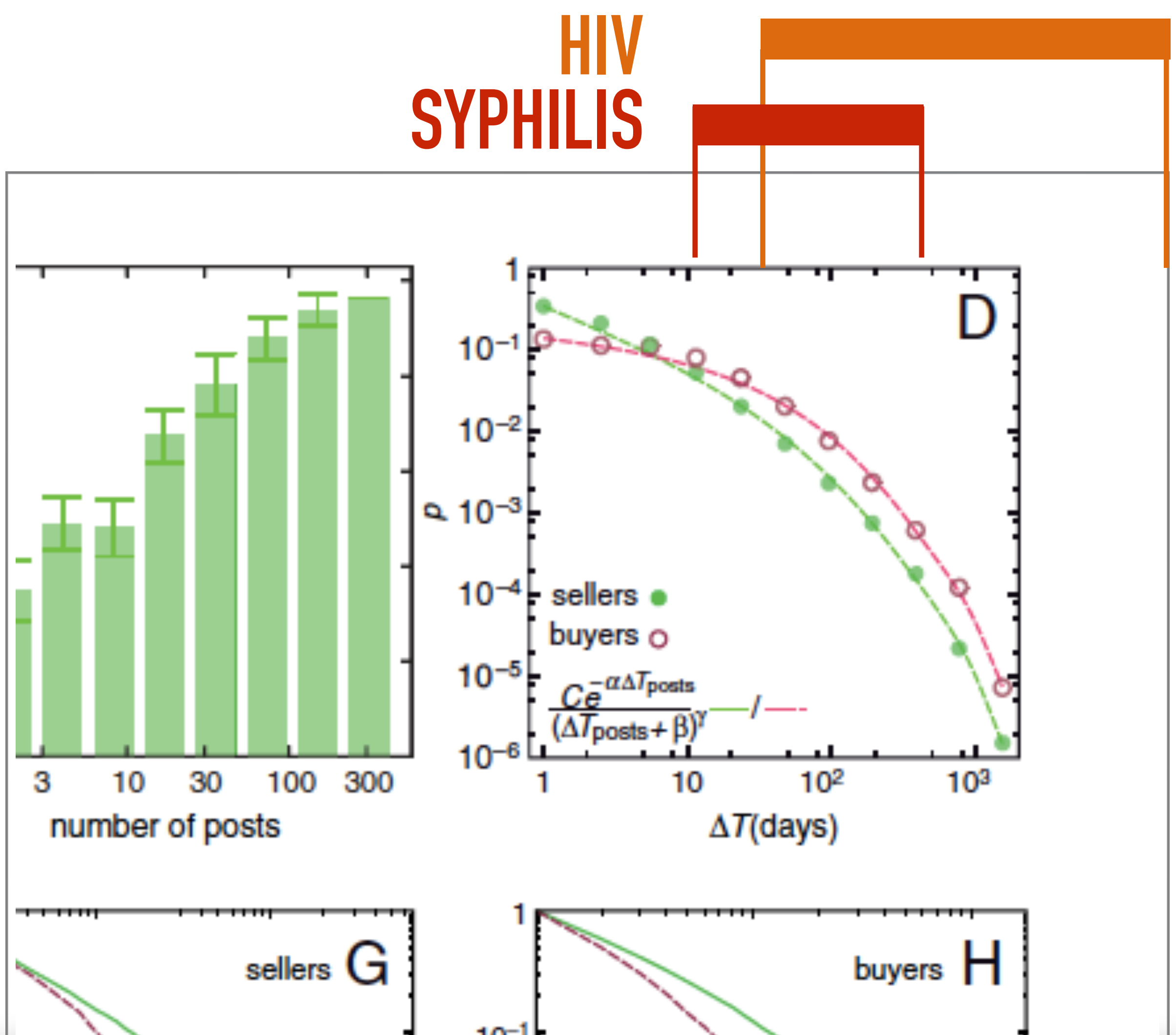
network slower than disease

average infectious duration μ^{-1}

average inter contact time τ

importance of contact dynamics for an epidemic

time scale separation not applicable in many cases



internet mediated prostitution

[LEC. Rocha, et al, PNAS 2009]

approaches to temporal network epidemiology

Botom-up: generative models

activity driven model, and its extensions

Top-down approaches: Randomised Reference Models

compare the epidemics on real data with the outcome in suitable null models

approaches to temporal network epidemiology

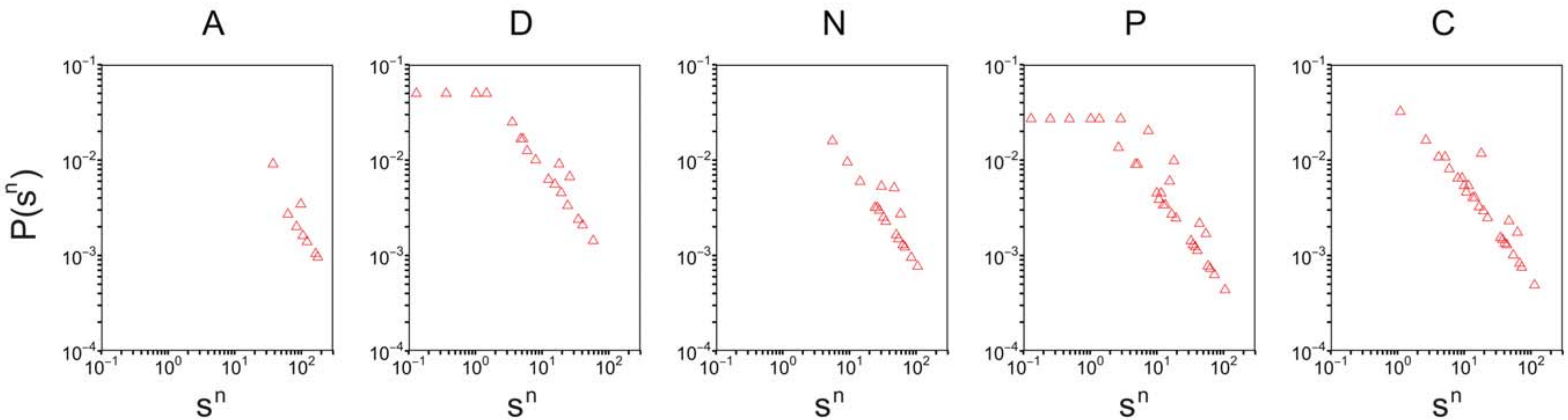
Botom-up: generative models

activity driven model, and its extensions

Top-down approaches

what the degree of static networks hides

probability density function of number of contacts



number of contacts

[Isella et al PLOS ONE 2011]

Modelling heterogeneous activation frequency: **Activity driven model**

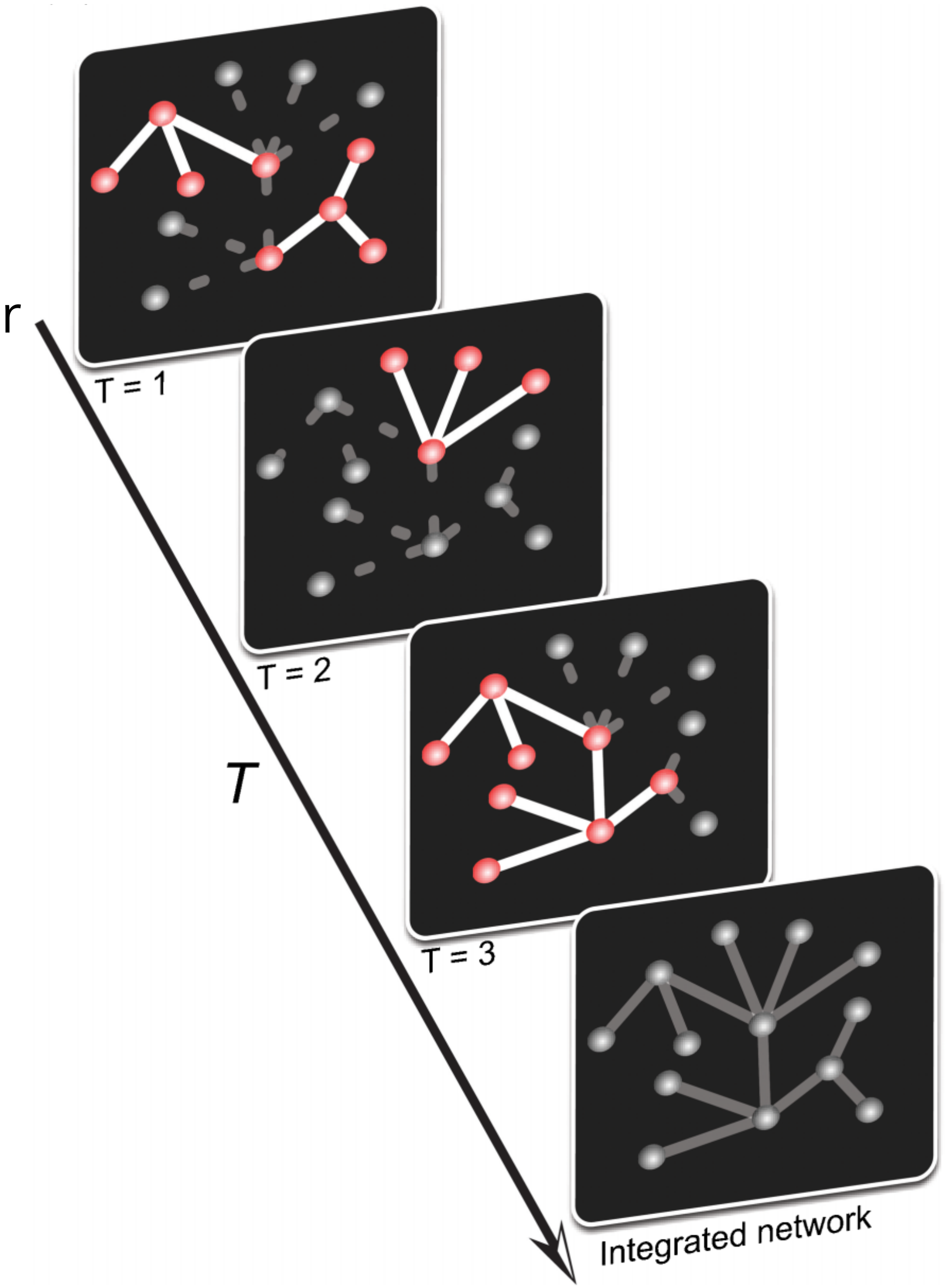
[Perra et al, Sci Rep 2012]

activity driven model

[Perra et al, Sci Rep 2012]

Model ingredients:

- discrete time (time step Δt)
- N: number of nodes
- x_i: activity potential, $\epsilon \leq x_i \leq 1$. This is e.g. number of activation of i during Δt normalised over the total number of activation.
- $F(x)$: distribution of activity potential
- $a_i = \eta x_i$: activation rate, with η rescaling factor chosen to tune the average number of active nodes per unit time in the system $\tilde{N} = \eta \langle x \rangle N$
- m: number of connections made at each activation



activity driven model

[Perra et al, Sci Rep 2012]

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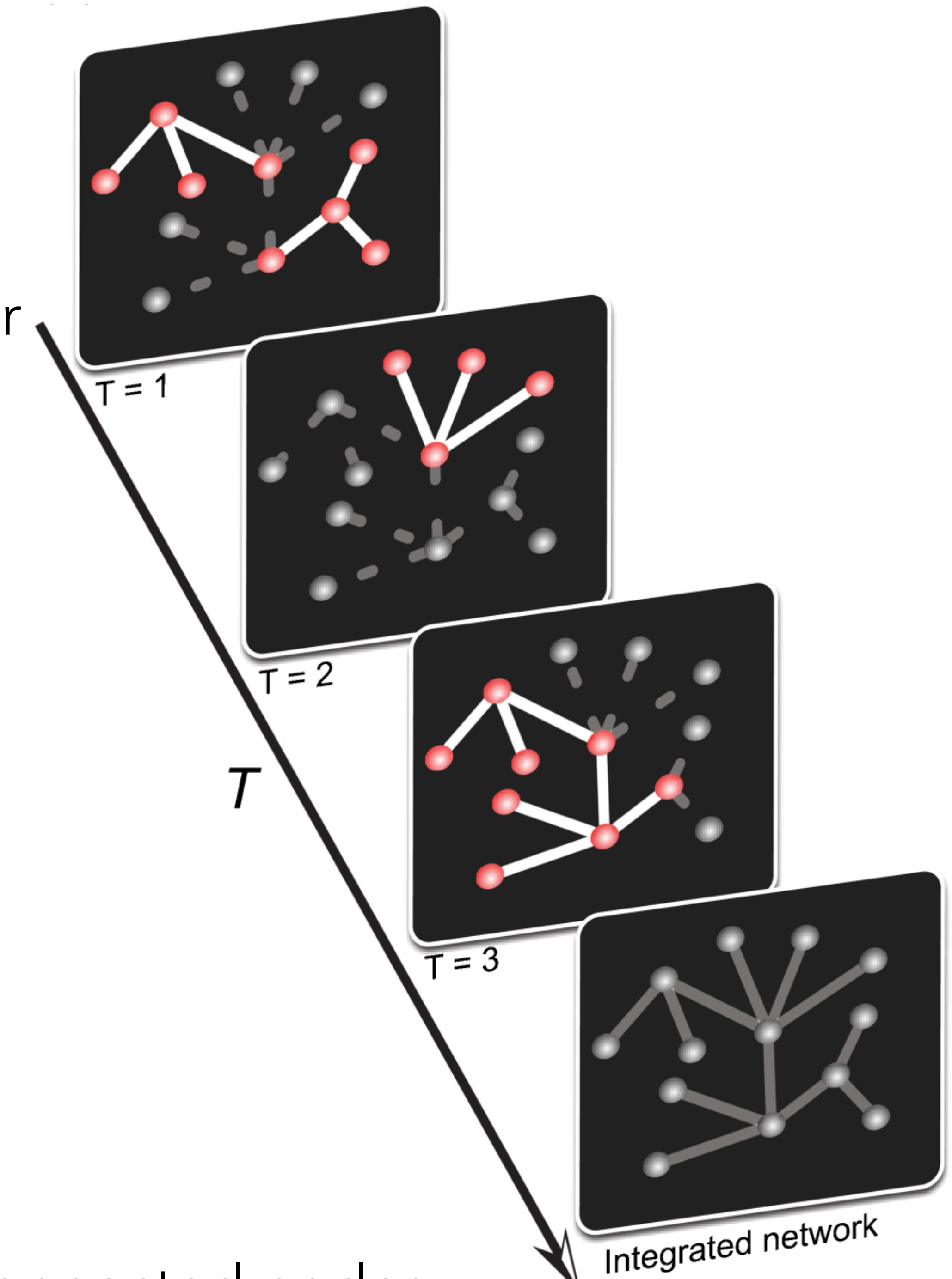

Model algorithm:

- each time step t: the network G_t starts with N disconnected nodes
- node i activates with prob $a_i \Delta t$ and makes m links with other randomly selected nodes. Non-active nodes can still receive connections
- At the next time step $t + \Delta t$ all the edges are deleted → All links last $\tau_i = \Delta t$

activity driven model

[Perra et al, Sci Rep 2012]

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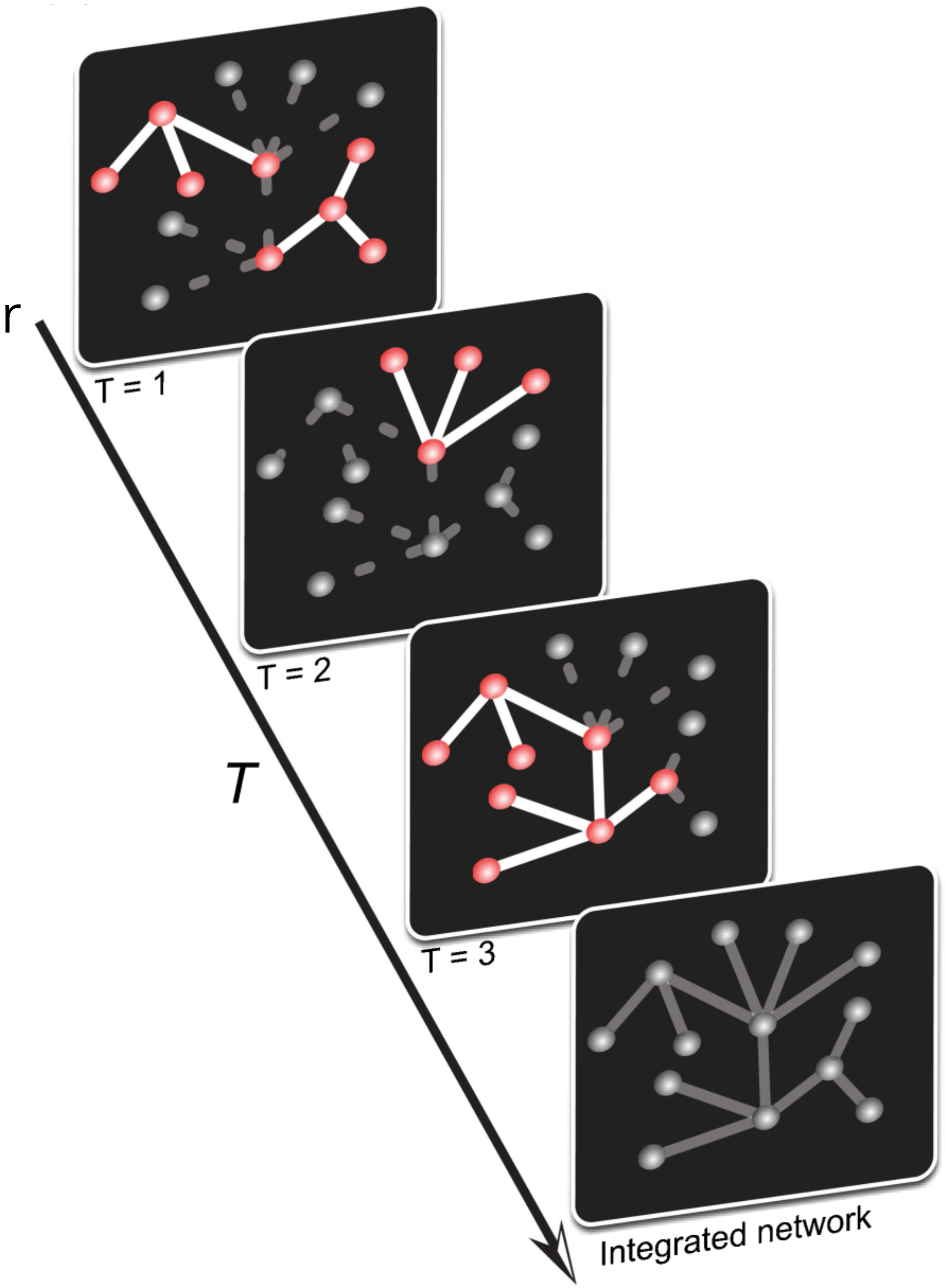


At each time steps:

- number of edges $E_t = m\,\eta \langle x \rangle N$
- average degree $\langle k \rangle_t = \dfrac{2E_t}{N} = 2\,m\,\eta \langle x \rangle$
- The network is homogeneous!

activity driven model

[Perra et al, Sci Rep 2012]

Integrated network over a time window T

- degree of a node i in the aggregated network $k_T(i) = k_T^{\text{out}}(i) + k_T^{\text{in}}(i)$
- $k_T^{\text{out}}(i)$: i makes Ta_im links. How many different nodes does it connect to? (I don't count repeated links with the same node). Urn problem: # of different ball extracted from a urn with N balls after Ta_im extractions
  - prob each ball is extracted $p = 1 - \left[1 - \dfrac{1}{N}\right]^{Ta_im}$
  - prob of extracting d balls is a Binomial $P(d) = \dbinom{N}{d} p^d (1-p)^{N-d}$
  - average # balls $k_T^{\text{out}} = Np = N\left[1 - e^{-Ta_im/N}\right]$, if $N \to \infty$ and $\dfrac{T}{N} \to 0$

activity driven model

[Perra et al, Sci Rep 2012]

Integrated network over a time window T

- degree of a node i in the aggregated network $k_T(i) = k_T^{\text{out}}(i) + k_T^{\text{in}}(i)$
- $k_T^{\text{in}}(i)$: nodes that make connections with i among whose were not target by i (already counted in $k_T^{\text{out}}(i)$)
  - prob a node was not target by i : $\left[1 - \frac{1}{N}\right]^{Ta_i m} = e^{-Ta_i m/N}$
  - average number of links coming form these nodes $T\langle a \rangle mN$
  - They connect to i with probability $\frac{1}{N}$
  - $k_T^{\text{in}}(i) = T\langle a \rangle m e^{-Ta_i m/N}$

activity driven model

[Perra et al, Sci Rep 2012]

Integrated network over a time window T

- degree of a node i in the aggregated network $k_T(i) = k_T^{\text{out}}(i) + k_T^{\text{in}}(i)$

$$k_T(i) = N\left[1 - e^{-Ta_i m/N}\right] + mT\langle a\rangle e^{-Ta_i m/N} \simeq N\left[1 - e^{-Ta_i m/N}\right] = N\left[1 - e^{-T\eta x_i m/N}\right]$$

if $N \to \infty$ and $\dfrac{T}{N} \to 0$

we write the activity potential as a function of the degree $x(k) = -\dfrac{N}{\eta m T}\ln\left(1 - \dfrac{k}{N}\right)$

$$P_T(k)dk \sim F(x)dx \qquad P_T(k) \sim F\left[x(k)\right]\frac{dx(k)}{dk} = \frac{1}{Tm\eta}\frac{1}{1-\frac{k}{N}}F\left[-\frac{N}{\eta m T}\ln\left(1-\frac{k}{N}\right)\right]$$

$\dfrac{k}{N} \to 0$, valid if $T \to 0$

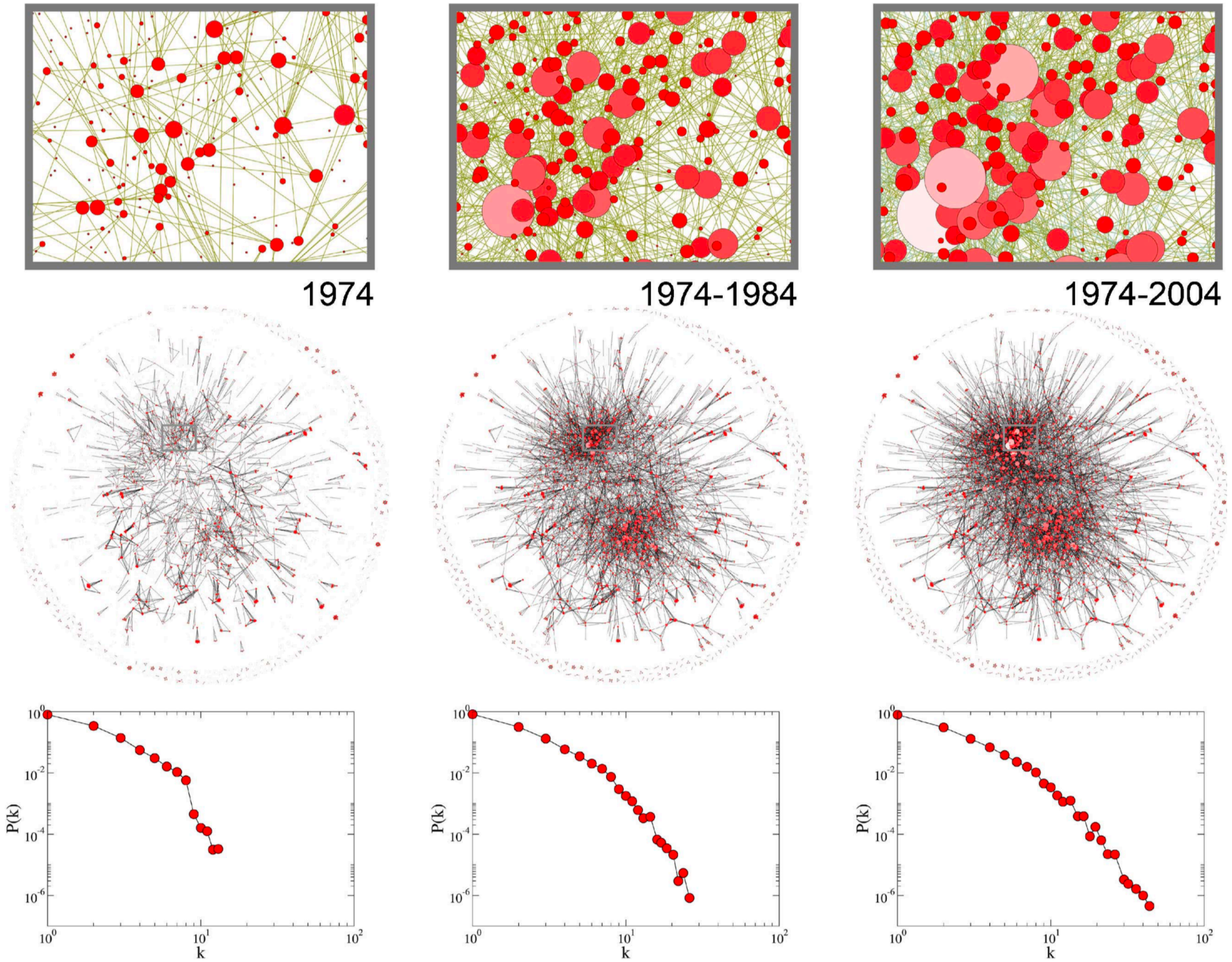
$$P_T(k) \sim \frac{1}{Tm\eta}F\left[\frac{k}{Tm\eta}\right]$$

activity driven model

[Perra et al, Sci Rep 2012]

$$P_T(k) \sim \frac{1}{Tm\eta} F\left[\frac{k}{Tm\eta}\right]$$

heterogenous topology in the aggregated network, over a window T, result from a heterogeneous activity potential



activity driven model

Effect of network dynamics on epidemic spreading

- activity block approximation
- SIR dynamics
- probability of transmission per contact λ
- for simplicity let's assume $m = 1$

$$I_a^{t+\Delta t} = -\mu \Delta t I_a^t + I_a^t + \lambda (N_a^t - I_a^t) a \Delta t \int da' \frac{I_{a'}^t}{N} + \lambda (N_a^t - I_a^t) \boxed{\int da' \frac{I_{a'}^t a' \Delta t}{N}}$$

$$\int da I_a^{t+\Delta t} = I^{t+\Delta t} = I^t - \mu \Delta t I^t + \lambda \langle a \rangle I^t \Delta t + \lambda \theta^t \Delta t$$

$$\theta^t = \int da' I_{a'}^t a'$$

$$\theta^{t+\Delta t} = \theta^t - \mu \theta^t \Delta t + \lambda \langle a^2 \rangle I^t \Delta t + \lambda \langle a \rangle \theta^t \Delta t$$

$$\begin{aligned} \partial_t I &= -\mu I + \lambda \langle a \rangle I + \lambda \theta, \\ \partial_t \theta &= -\mu \theta + \lambda \langle a^2 \rangle I + \lambda \langle a \rangle \theta \end{aligned}$$

[Perra et al, Sci Rep 2012]

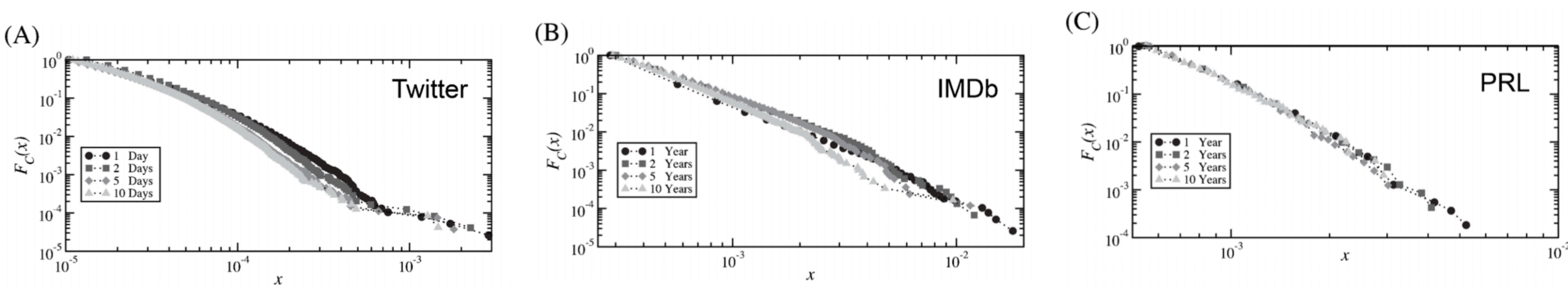
activity driven model

$$\partial_t I = -\mu I + \lambda\langle a\rangle I + \lambda\theta,$$
$$\partial_t \theta = -\mu\theta + \lambda\langle a^2\rangle I + \lambda\langle a\rangle\theta$$

$$J = \begin{pmatrix} -\mu + \lambda\langle a\rangle & \lambda \\ \lambda\langle a^2\rangle & -\mu + \lambda\langle a\rangle \end{pmatrix} \qquad \Lambda_{(1,2)} = \lambda\langle a\rangle - \mu \pm \lambda\sqrt{\langle a^2\rangle}$$

$$\boxed{\frac{\lambda}{\mu} > \frac{1}{\langle a\rangle + \sqrt{\langle a^2\rangle}} + \mathcal{O}(\frac{1}{N})}$$

heterogeneities in the activation rate lower the epidemic threshold

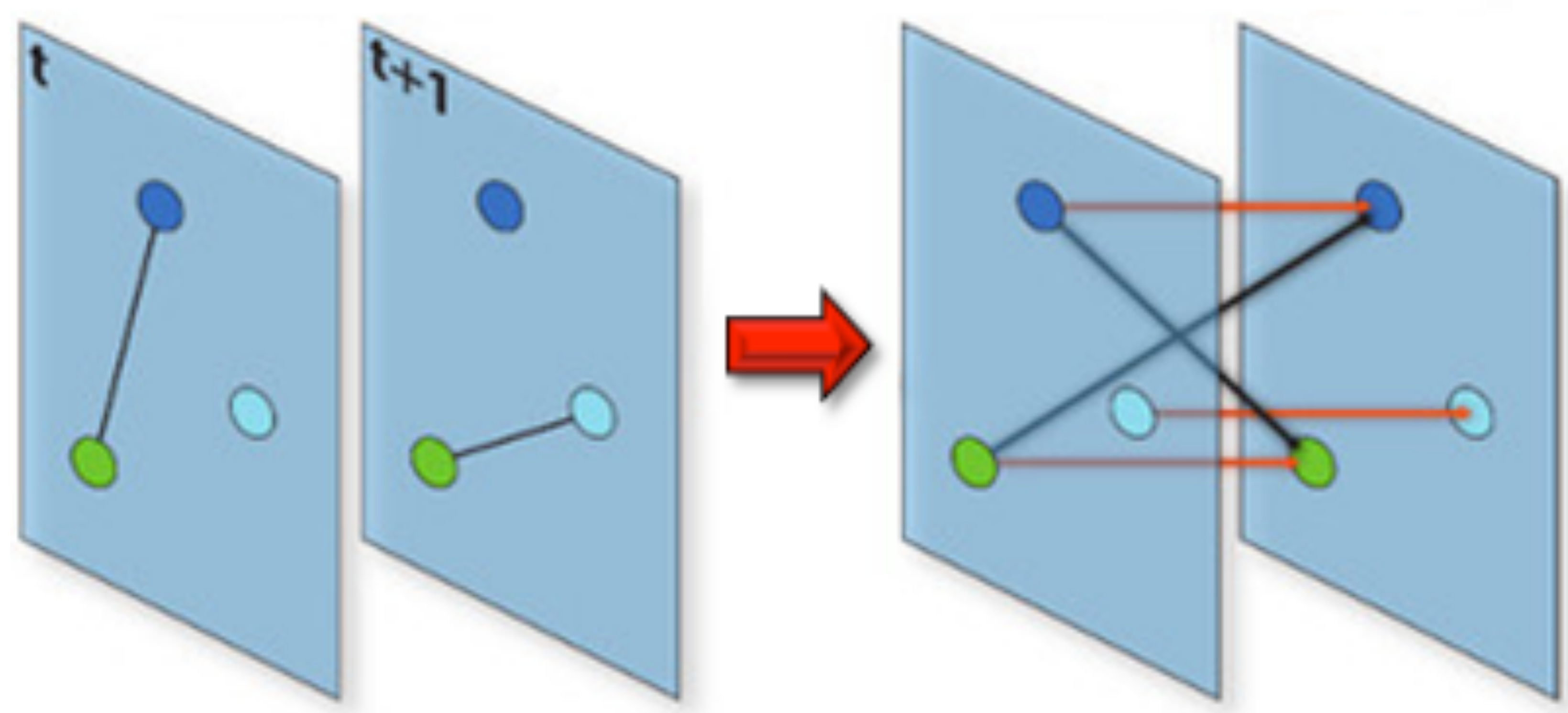


[Perra et al, Sci Rep 2012]

activity driven model

- A model that captures a realistic property of human behaviour (face-to-face, sexual contacts, phone call, email, tweets)
- human have heterogeneous activity rate
- the contact network at a certain instant of time is sparse, with homogeneous degree
- the aggregated network over a certain window is well connected with heterogeneous degree
- pattern of activation unfolds at the same time scale of the spreading process
- calculation possible in the activity-block approximation (same scheme as the degree block approximation)
- contact heterogeneities lower the epidemic threshold

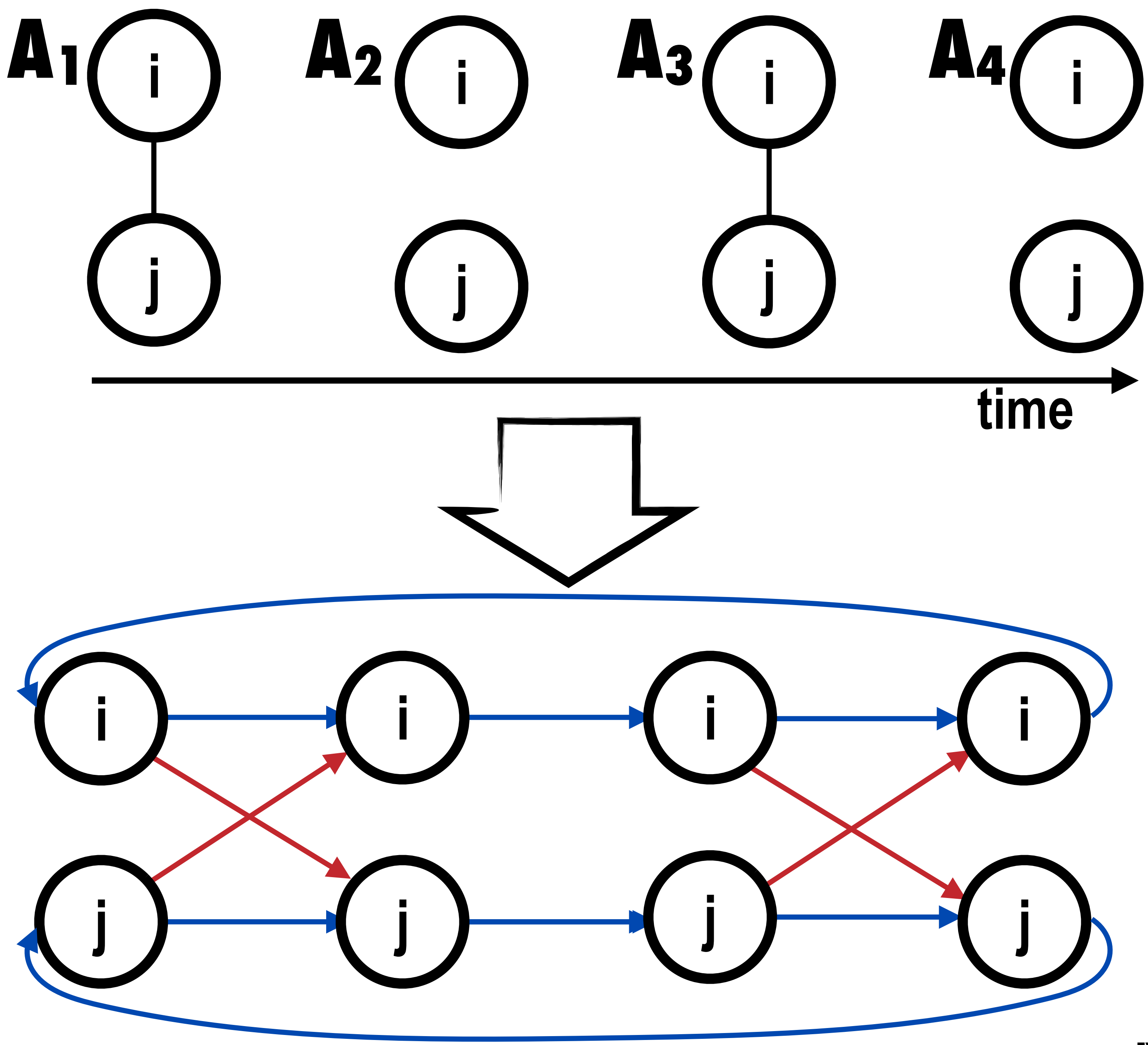
individual-based mean field approach



multilayer formalism + individual-based mean field approach = **Infectious propagator approach**

[multilayer: De Domenico et al PRX 2013, Valdano et al PRX 2015]

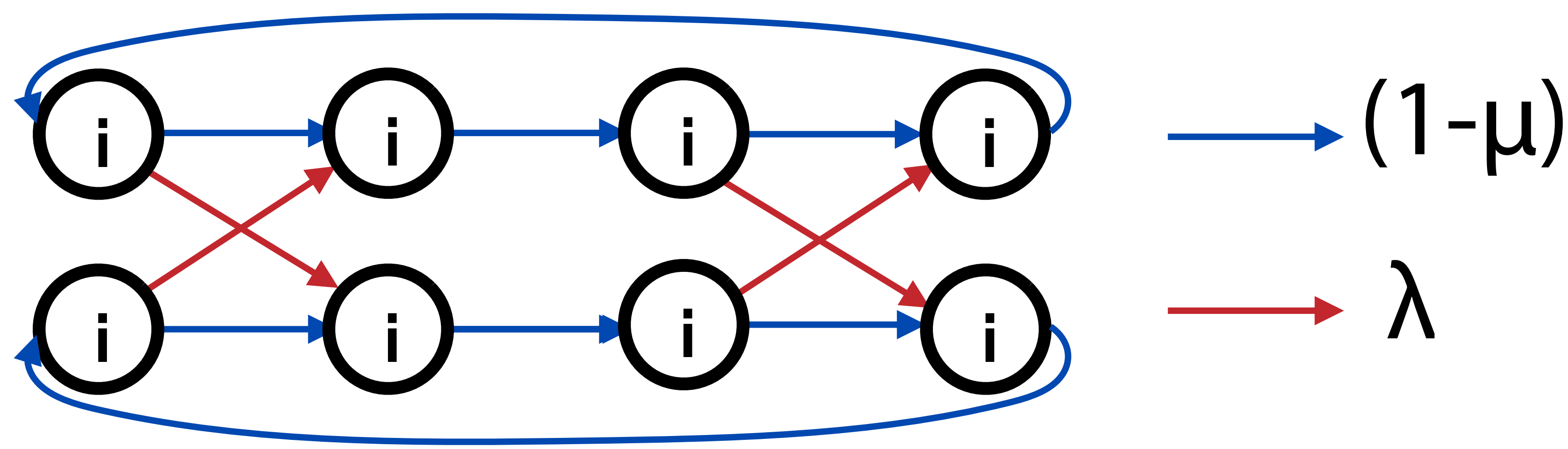
infectious propagator approach



[Valdano et al PRX 2015]

infectious propagator approach

$S \xrightarrow{\lambda} I \xrightarrow{\mu} S$



NT x NT supra-adjacency matrix

$$M = \begin{pmatrix} 0 & 1-\mu+\lambda A_1 & 0 & \cdots & 0 \\ 0 & 0 & 1-\mu+\lambda A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-\mu+\lambda A_{T-1} \\ 1-\mu+\lambda A_T & 0 & 0 & \cdots & 0 \end{pmatrix}$$

N x N blocks

[Valdano et al PRX 2015]

infectious propagator approach

threshold on temporal network $\Leftrightarrow$ threshold on STATIC network

epi threshold

$$\rho[\mathbf{M}(\lambda_{thr},\mu)] = 1$$

dim = NT

[Powell, arXiv, 2011]

dim = N

$$\rho[\mathbf{P}(\lambda_{thr},\mu)] = 1$$

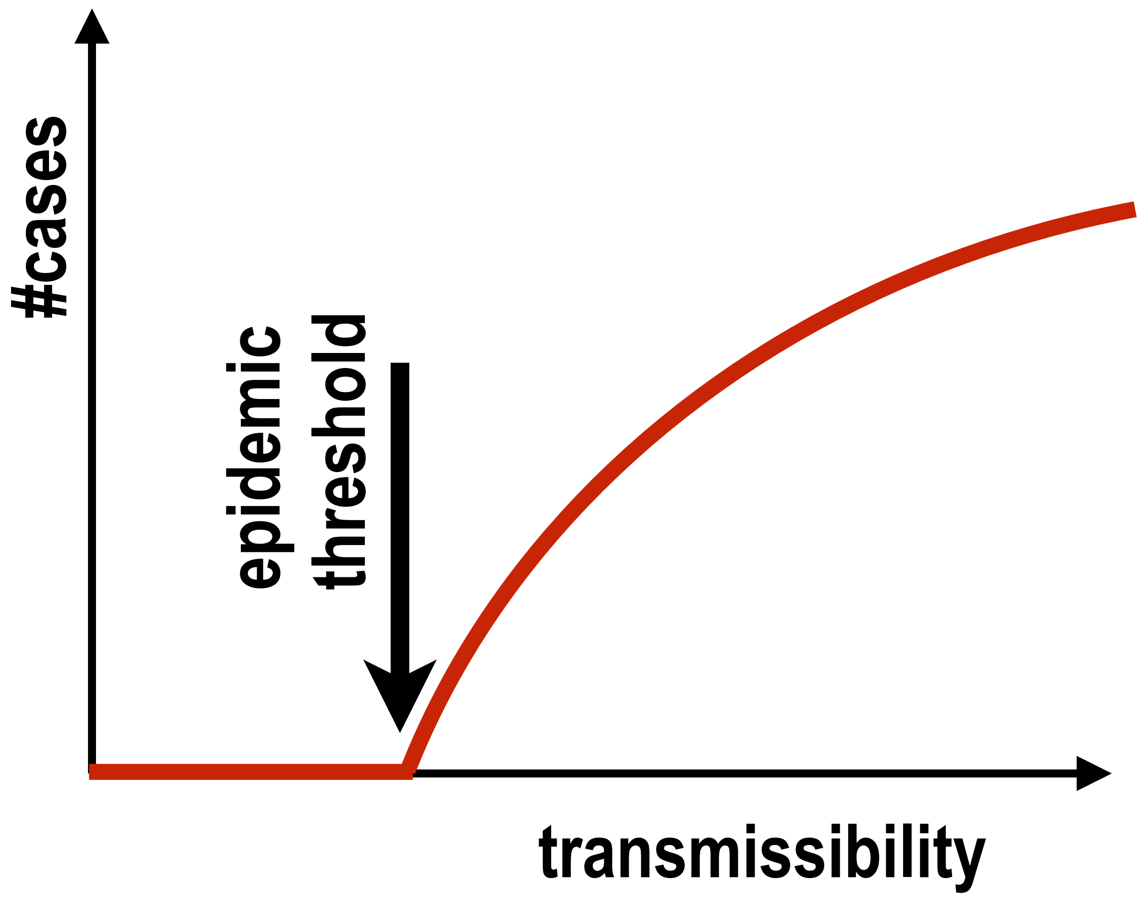
$$\mathbf{P} = (1-\mu + \lambda \mathbf{A_1})(1-\mu + \lambda \mathbf{A_2}) \cdots (1-\mu + \lambda \mathbf{A_T})$$

[Lentz et al, PRL 2013]

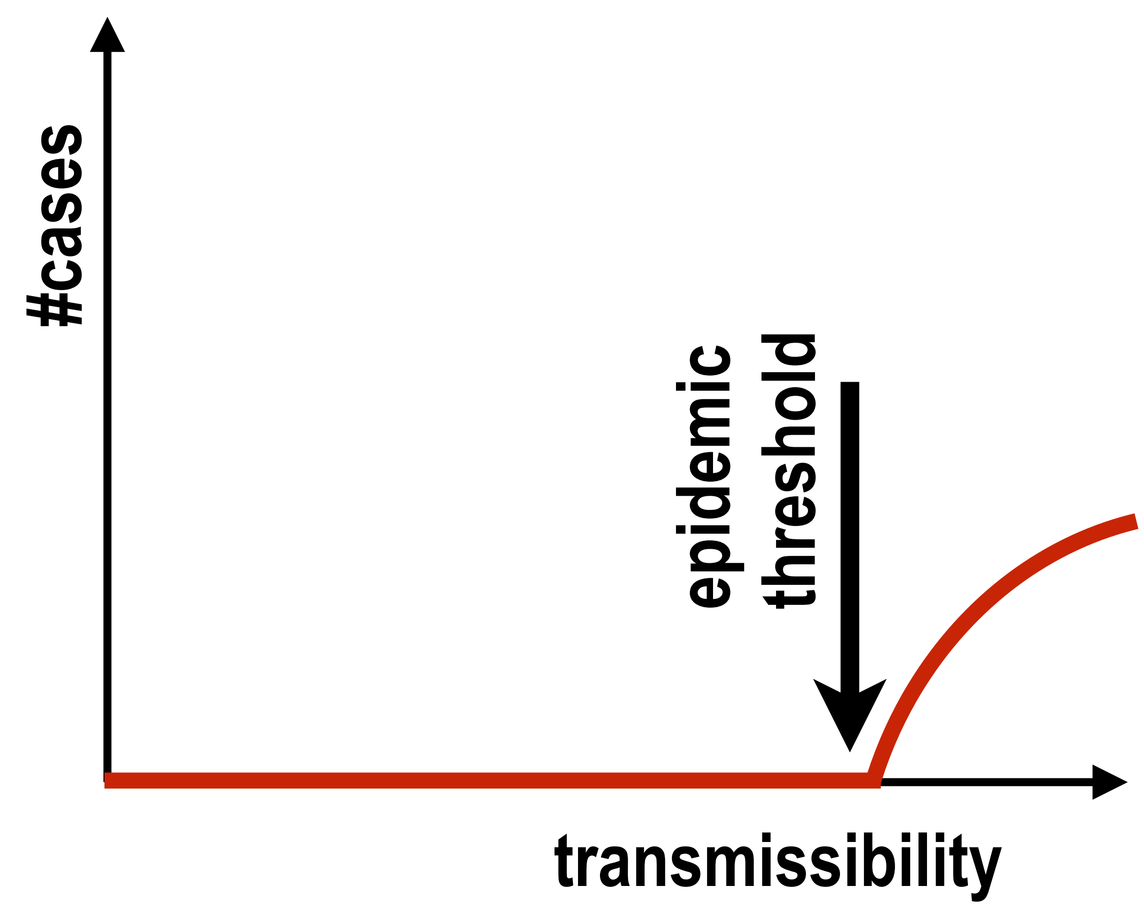
infection propagator

[Valdano et al PRX 2015]

reduce system's vulnerability



vaccinate
personal protective measures
hygien
alter individuals contact

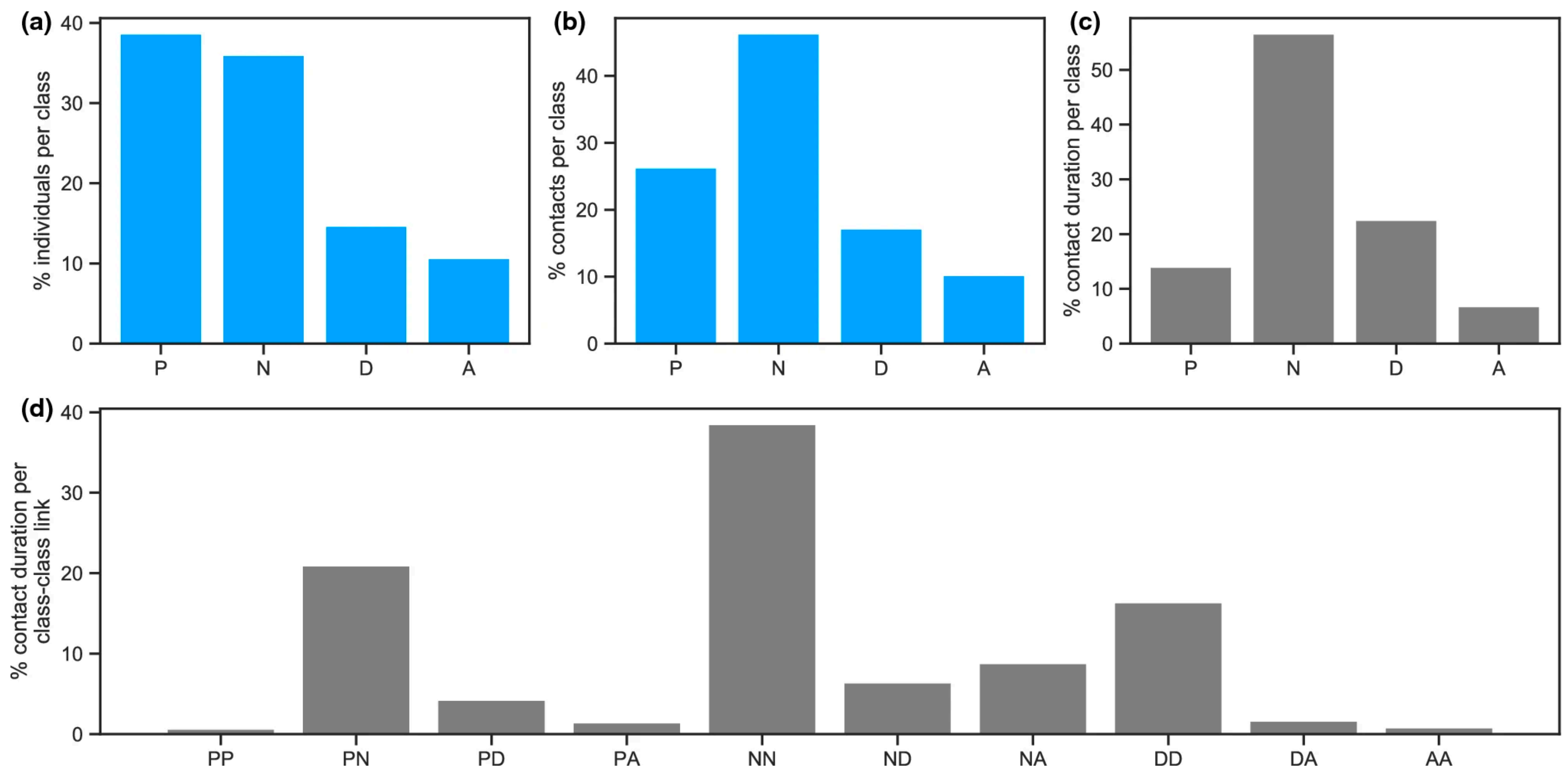


spread of nosocomial diseases

Sociopatterns.org

[Vanhems et al PLoS ONE 2013; Valdano et al Sci Rep 2021]

- geriatric ward
- 75 participants
- **27 nurses (N), 11 doctors (D), 8 administrative staff (A), 29 patients (P)**
- duration 4 days and 4 nights



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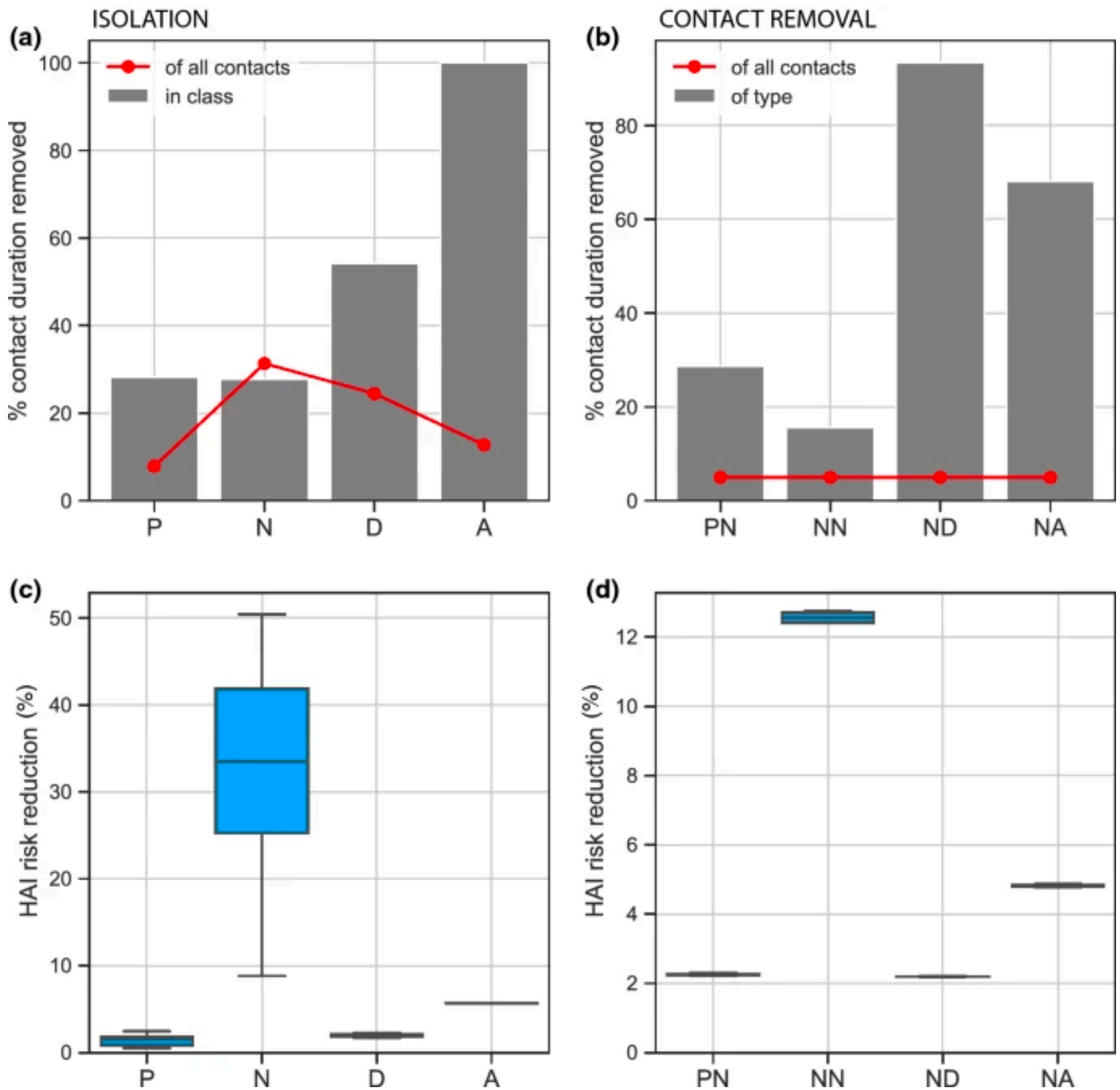

[Valdano et al Sci Rep 2021]