Metro Schedule Optimization for Société de transport de Montréal

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1 Introduction

The Société de transport de Montréal (STM) is a public transport agency that offers transit services such as buses and metro in the city of Montreal, Quebec, Canada. Established in 1861, it has grown a large service map comprising of 4 metro lines, 68 metro stations, and 209 bus routes. According to the Activity Report 2019, the daily ridership of the busiest bus line is 31,209 while the busiest metro station, Berri-UQAM with an interchange station for three lines, has around 34,630 people entering the station on a daily basis [1]. However, these numbers change drastically after COVID-19 hit the city of Montreal. The STM reports that the total number of trips made in 2020 is around 171.7 million, a 54% decrease from 2019 [2].

STM has made some adjustments to react to the sudden decrease in ridership. For example, it adjusts the frequency of train departures of orange and green lines from 3 to 4 minutes respectively, but the blue and yellow lines remain unchanged [3]. Continuing operating the pre-COVID schedule sent STM into a serious financial crisis. Moreover, based on the survey of transit users conducted by Trajectoire Québec, about 25% [4] of users said they would not use transit again, meaning that Montreal is doubling down on public transport during the pandemic 2021. This information made our group wonder whether STM is currently operating on a reasonable schedule.

In this project, we limit our scope on examining the operating schedule of the STM metro system. By optimizing the frequency of departures among different time slots throughout a week, we obtained a more reasonable operating schedule that meets the current transit demand in Montreal. We also examined the yellow, orange, blue line's expansion projects as an extension to our basic model. As the expansion projects will bring more passengers to the metro service, we want to examine how STM can adjust its current metro schedule to meet the new demand.

2 Problem Description and Formulation

2.1 Parameters Description

- VCost_k: Variable cost for line k, including remuneration, goods and services, and etc.
- FCost: Fixed cost per week, including net debt service, special operating budget, and etc.
- Capacity_k: The capacity of a train per line
- $\text{Demand}_{i,j,k}$: The number of passenger for line k at time i and day j
- $\text{WaitTime}_{i,j,k}$: The maximum average waiting time for line k at time i and day j

2.2 Decision Variables

There are four metro lines currently operating in Montreal, which are: orange, green, yellow, and blue line. On average, each line operates from 5 a.m. to midnight the next day during weekdays and Sundays. On Saturdays, each line operates from 5 a.m. to 1 a.m. the next day. Therefore, we separate daily operational hours into 20 timeslots in which each timeslot represents an hour. Moreover, based on distinct demand patterns within a week, we define four types of days that the metro operates. The definitions are listed as below:

Type	**Weekday**
Type 1	Monday – Thursday
Type 2	Friday
Type 3	Saturday
Type 4	Sunday

Since there are 4 metro lines, 20 operational hours/timeslots in a day, and 4 types of days, there are $4 * 4 * 20 = 320$ variables in total. Each variable is defined as the number of train departure times within an hour which can be mathematically expressed as X_{ijk},where i represents an operational time slot of a day, j represents one of the four types of days, and k represents one of the four metro lines.

2.3 Objective Function

The goal of our project is to minimize STM's metro lines' operation costs. Due to simplicity purpose, we will only calculate the operation cost for one week of all four lines. The weekly operational cost is composed of two types of costs: variable cost and fixed cost, and the variable cost varies among different types of metro lines. The objective function is defined as below:

$$\text{Minimize} \sum_i \sum_k 4 * X_{i,1,k} * \text{VCost}_k + \sum_i \sum_{j \neq 1} \sum_k X_{i,1,k} * \text{VCost}_k + \text{FCost}$$

Where i i is operational time slot, j is type of day, k is type of operational line. And VCost is the variable cost for each departure of a train, FCost is the weekly fixed cost for operating the 4 metro lines.

Variable costs are separated into two parts in the above equation because there are 4 days for type 1 day (from Monday to Thursday). Thus, when j is equal to 1, the variable cost needs to be multiplied by 4 to calculate the weekly costs.

2.4 Constraints

2.4.1 Budget Constraint

The objective function is restricted by the weekly budget constraint in which the budget is estimated from STM's planned 2021 annual budget for metro

$$\$335,594,000/52 = \$6,453,730$$

The equation is as follows:

$$\sum_i \sum_k X_{i,1,k} * \text{VCost}_k * 4 + \sum_i \sum_{j \neq 1} \sum_k X_{i,1,k} * \text{VCost}_k + \text{FCost} \leq 6453730$$

2.4.2 Demand/Capacity Constraint

This constraint requires that the total capacity of the trains within hour i ($X_{ijk} * \text{Capacity}_{ijk}$) for day j and line k should be greater than the demand of the passengers. The total capacity is calculated by the capacity of one train times the number of trains operating in hour i.

$$X_{ijk} * \text{Capacity}_{ijk} \geq \text{Demand}_{ijk}$$

2.4.3 Maximum Waiting Time Constraint

The average waiting time for each line is restricted by the maximum waiting time. The maximum waiting time is measured in minutes. Therefore, the waiting time for each train should be represented as $60/X_{ijk}$ which should be smaller than or equal to the maximum waiting time. The equation is as follows:

$$60/X_{ijk} \leq \text{WaitTime}_{ijk}$$

However, since Gurobi package does not allow division expression in model, we re-formulate the equation as follows:

$$X_{ijk} * \text{WaitTime}_{ijk} \geq 60$$

2.4.4 Decision Variable Constraint

Based on our problem formulation, our decision variables have to be integer and non-negative. Hence we need to add the following constrain to our model:

$$X_{ijk} \geq 0 \text{ ,Where } X_{ijk} \in \mathbb{Z}$$

3 Numerical Implementation and Results

3.1 Data Collection

3.1.1 Operation Costs: Variable Costs and Fixed Costs

To estimate the total number of train departure times per line per year, we used the data of frequency of departures[5] from the Budget and Reports[6] of STM website. It gives a range of estimated frequency of train departure based on whether it is peak or non-peak hours, and whether it is weekday or weekend. We used 60 minutes per hour divided by the average frequency of departure per category per line, times the number of hours in each category. Then we aggregated numbers from all three categories. Finally, we multiplied it by 365 to get the annual total number of train departure times per line.

To get the weighted average variable costs per departure per line, we first used the total number of stations for each line to estimate the proportion of their contribution to the total variable costs per year. Then, by using this proportion to multiply with total variable costs per year, we get the weighted average variable costs per line per year. Next, we used the total number of train departure times for each line to estimate the cost of every departure for the metro in each line, we get these numbers by dividing weighted average variable costs per line per year by the total number of train departure times, and we named these number as weighted average variable costs per departure per line.

To get the fixed costs per week, we divide the total fixed cost per year by the number of weeks in one year (approximately 52 weeks).The results are as follows:

Operation Costs			
Total Variable Costs Per Year	**$**	**481,085,000**	
Total Number of Train Departure Times Per Line Per Year		**494,940**	
Green		126,655	
Orange		126,655	
Yellow		127,020	
Blue		114,610	
Total Number of Stations Per Line		**70**	**Weighted Percentage**
Orange		28	40%
Blue		12	17%
Green		27	39%
Yellow		3	4%
Weighted Average Variable Costs Per Line Per Year			
Orange	$	192,434,000	
Blue	$	82,471,714	
Green	$	185,561,357	
Yellow	$	20,617,929	
Weighted Average Variable Costs Per Departure Time Per Line			
Orange	**$**	**1,519**	
Blue	**$**	**651**	
Green	**$**	**1,461**	
Yellow	**$**	**180**	
Total Fixed Costs Per Year	**$**	**97,644,000**	
Fixed Costs Per Week	**$**	**1,877,769**	

3.1.2 Demand Matrix

The Société de transport de Montréal does not provide detailed ridership on each operation day. We formulated our daily demand constraint based on the total annual ridership and the estimated hourly occupancy from STM.

Based on the annual total ridership of metro service in 2020, we distributed the total ridership into four lines by weight. The weight is calculated based on the number of stations each line has. The result is in the following table:

Demand Distribution				
Total Annual Ridership			**118,982,861**	
	Number of Stations	**Weighted Percentage**	**Annual Ridership**	**Weekly Ridership**
Orange	28	40%	47,593,144	915,253
Blue	12	17%	46,403,316	892,371
Green	27	39%	4,759,314	91,525
Yellow	3	4%	20,227,086	388,982

Then, we utilized the estimated occupancy level to estimate the number of demands per hour. The estimated occupancy level has three categories: nearly empty, some seats available, and standing room only. Since STM does not provide relationships among these three occupancy levels, we assume their relationship to be:

$$\begin{cases} 1.5 * \text{nearly empty level} = \text{some seats available level} \\ \\ 1.5 * \text{some seats available level} = \text{standing room only level} \end{cases}$$

We observed that the estimated occupancy level from Mondays to Thursdays exhibits the same pattern. Thus, we grouped these four days into one category and named it "weekday". We then calculated the hourly demand for weekdays, Friday, Saturday, and Sunday based on our weekly demand and assumption of occupancy levels' relationships.

3.1.3 Capacity

According to Montreal Metro, the Montreal Metro has two models of subways that are currently in use. The first model, MR-73, was delivered starting in the 1980s. It is currently served green, blue, and yellow lines among all four lines.[7] There are in total 29 MR-73 trains in service. Another model, MPM-10, began to deliver more recently, which was around 2015. It serves orange and green lines, and there are 639 cars nowadays.[8]

Among the two models of subways, each line possesses a different number of trains and cars.

Firstly, the orange line only operates the model of MPM-10. It has 45 trains, and each train has 9 cars, which is in total 405 cars. Then, the green line is the only line that uses both the MPM-10 and MR-73 models. It has 36 MPM-10 trains, which are 324 cars, and 6 MP-73 trains, which are 54 cars. Next, the blue line uses MR-73 only. There are 18 trains, which are 108 cars, operated by the blue line. Lastly, there are only 5 MR-73 trains operated by the yellow line, as it is the shortest line among all four lines. It has 45 cars. It is worth noticing that only the blue line has 6-car set trains, others all have 9-car set trains[7].

In terms of the train capacity, each model has different capacities. For the MR-73, the capacity is 160 passengers per car, of which 40 of them are seated. Hence, a complete 9-car set train can hold 1440 passengers, while the 6-car set blue line train can hold 960 passengers. Then, for the MPM-10, the capacity is approximately 172 passengers per car, 32 of which are seated[7]. In total, the capacity is 1548 passengers for the 9-car set train. In addition, due to the COVID-19 pandemic situation, social distancing is necessary for passengers in the trains. As a result, in our model, the capacity for each train is half of the amount before the pandemic.

3.1.4 Average Waiting Time

For the average waiting time limit data, each line has a different limit during peak, non-peak, and weekend. The peak hours are the two specific time periods of 7 a.m. to 9 a.m. and 4 p.m. to 6 p.m. The rest hours are non-peak hours. And weekends are anytime on Saturday and Sunday. For green and orange lines, the maximum waiting time limit is 5 minutes during peak hours, 10 minutes during non-peak hours, and 12 minutes during weekends. For the blue line, the maximum waiting time limit is 5 minutes during peak hours, 10 minutes during non-peak hours, and 11 minutes during weekends. Similar to the yellow line, it is 5 minutes during peak hours, 10 minutes during non-peak hours, and 10 minutes during weekends[9].

3.2 Model results and interpretation

We constructed our model by using Gurobi - a powerful mathematical optimization solver. We first imported all the required packages. Then, we imported the pre-processed data from Excel spreadsheets, defined decision variables, set the objective function, and added relevant constraints. After finishing constructing the model, we solved the problem by using the optimization function and found the objective value to be around $6.348 million which represents the minimum weekly operation costs of STM's metro lines. This amount is approximately $5.5 million less than the originally budgeted amount planned by STM. The following is a sample of optimal solutions for the 320 decision variables:

Example: Orange Line		
	Current	**Optimal**
Mon-Thu 8-9AM (Peak)	20	24
Mon-Thu 14-15PM (Non-Peak)	15	9
Sat, Sun 9-10AM	10	14

The data for STM's current metro schedule is obtained from Google Map. As we can see, from Monday to Thursday and from 8 a.m. to 9 a.m., which is during the peak hours, there are currently 20 departures of trains for the orange line, however, our optimal solutions suggest that STM should consider increasing the number of trains to 24 to meet the demand while keeping social distancing into consideration. From Monday to Thursday and from 2 p.m. to 3 p.m., which is during the non-peak hours, there are 15 trains departing in an hour, however, STM can consider decreasing the number to 9 because there is not that much demand that needs to be satisfied. Likewise, over the weekends from Saturday to Sunday, STM should consider increasing the number from 10 to 14.

As one can see, by planning the metro's schedule more wisely, STM can increase its operation efficiency by better matching supply and demand while saving a part of the budget for other construction projects. Since STM has prepared plenty of strategic plans, they can consider allocating the surplus to other projects that might benefit the company as well as the residents living in the Great Montreal Area.

4 Problem extensions

4.1 Yellow Orange Blue Line Extension Projects

We utilized online resources and gathered data related to yellow, orange, and blue metro line extension projects: after the implementation of the line extension project, the estimated service interval between two trains during peak hours should decrease from 5 minutes to 4 minutes for yellow line and should decrease from 2.5 minutes to 2 minutes for orange line[**10**]. The passenger demand for the yellow line will increase by 12%. There is no data found on the demand increase of the orange line[**11**], thus we estimated it using the yellow line's data. However, for the blue line extension project, it was supposed to cost $4.5 billion, but according to documents obtained by Radio-Canada, the updated price tag is estimated at around $6 million as sky-high expropriation costs put the future of the project in question[10]. Therefore, we did not consider the blue line extension project here because its cost is way higher than the budget, which would certainly give an infeasible model.

After feeding the model with the new data and estimating for yellow and orange lines, we found that the total cost and number of departures increase for the yellow and orange lines after the metro line extension project. The optimal operation cost for the yellow line after an extension is around $0.183 million per week, and the optimal operation cost for the orange line is approximately $4.88 million per week. In comparison, the optimal operation cost for the orange line without extension is $2.24 million per week, and the operation cost for the yellow line is $0.173 million per week. It is worth noticing that there is a minor increase for the yellow line, but a drastic increase for the orange line. This could be due to much greater demand and usage for the orange line, as it will be the longest line in Montreal.

4.2 Model Sensitivity Analysis

We looked further into the aspect of reduced cost and shadow price of the basic model. Since our model is a mixed-integer optimization problem, we could not print sensitivity analysis related attributes directly. Instead, we used trial and error to test different maximum average waiting times, as they can be manipulated easily.

To evaluate the sensitivity analysis of the maximum waiting time constraint, we used a for loop to evaluate the different changes in waiting time. We examined four additional cases, where the average waiting time for all time slots and for all lines will increase by the same certain amount of time. The four additional cases are: average waiting time increase by 0.5 minutes, increase by 1 minute, increase by 1.5 minutes, and increase by 2 minutes. For instance, the maximum average waiting time will increase by 1 minute compared to their original waiting time for all peak, non-peak, and weekend trains for all four lines in the first case. The rest of the model remains the same. We then solved the model and moved on to the next case.

By running the for loop on these four cases, we can conclude that the minimum optimal solution happens when the average waiting time is higher. The optimal cost for the basic model is $6.35 million, but it is $5.70 million when the average waiting time increases by 2 minutes. To learn from this result, STM could conduct marketing research on passenger flow scheduling and customer segmentation at a specific time period so that they could fluctuate the maximum waiting time a bit. More will be discussed in the recommendation section.

4.3 Application with Queuing Theory

After the data collection steps are done, our team realized that the demand is much smaller than the capacity of the Montreal metros. Consequently, the City of Montreal will not face the overcrowding issue in subway stations. However, as we aim to generalize our model and bring it internationally, we need to find an optimization solution for solving the overcrowding issues in cities with a higher population such as Hong Kong.

Our team is trying to use the queuing theory to solve this problem. Suppose we have the data about how many people will arrive at the station every minute, the capacity of every station (maximum people available for each station), then along with our decision variable, we can add on a new constraint to our model to solve the problem:

$$60 * \Delta_{ijm} \leq C_m * X_{ijk}$$

$$\frac{60}{X_{ijk}} * \Delta - ijm \leq C_m$$

Where C_m is capacity of station m, λ_{ijm} is arrival rate at station m on day j during timeslot i, μ_{ijm} is departure rate at station m on day j during timeslot i and $\Delta_{ijm} = \lambda_{ijm} - \mu_{ijm}$.

This constraint requires that the number of people piling up in a subway station during the time between each train departure is less than the capacity of the subway station. By adding this new constraint, the overcrowding issue can be resolved, and our model will be more robust and can be applied worldwide.

5 Recommendations and Conclusions

As one of the leading public transportation enterprises in Montreal, STM must satisfy the local demand for public transit services. However, in order to sustain in the long run, STM should also consider minimizing its operating costs. The solutions given by our model have well balanced the two factors. Nevertheless, the strategy that we provide is based on the fact that there is a drastic decrease in demand after the pandemic. As a result, we suggest STM conduct market research to find out the root cause that people no longer want to take public transportation and set relevant strategies to solve the issue. To further investigate the possibility of reducing the operation cost, we also recommend STM conduct a survey on the maximum acceptable waiting time for passengers. If passengers are willing to wait for a longer time, STM can further reduce their operational costs by decreasing the number of departure times within an hour. This is feasible because compared to other major cities, Montreal metro system does not have huge demand that requires an extremely high-level service frequency. Therefore, adjusting the frequency might be an economical solution to optimize the problem.

The metro problem is extremely complex and requires a large amount of trivial data that is hard to obtain. Therefore, we need to use accessible data to make educated estimates. Due to the limited source and time that we have, we processed the problem by building a simplified model and consequently did not consider certain situations. For instance, the arrival rate of the passenger may accumulate at the station and exceed the station capacity. Even the situation is not likely to occur in Montreal, some other cities with more severe metro crowding problems such as Beijing may encounter the problem. Therefore, the model we proposed might not apply to other cities. In order to apply the model to other scenarios, the application of queuing theory, as well as the consideration of the number of passengers leaving the train at each station, could be added to our main model as a constraint to improve the breadth of the applicability of our strategy.

From this project, we learned the importance of considering the feasibility of a project before starting to formulate the problem and construct the model. We have spent the majority of our time doing research on the background of the project regarding data collection, data preprocessing, etc. To illustrate, we first need to look for available data, then, we need to examine the reliability and validity of the existing data. After that, we need to preprocess the data into a format that can fit into our model. Apart from data, we also spent plenty of time evaluating the reasonability of our model in terms of whether our model has captured all the necessary constraints that might impact our optimal solution.

6 Appendix

Optimal Solution for Main Problem

	Mon_Thu	Fri	Sat	Sun
5-6am	6	6	6	6
6-7am	14	14	14	14
7-8am	24	24	24	24
8-9am	24	24	24	24
9-10am	9	9	9	9
10-11am	9	9	9	9
11am-12p	9	9	9	9
12-13pm	9	9	9	9
13-14pm	9	9	9	9
14-15pm	9	9	9	9
15-16pm	9	9	9	9
16-17pm	24	24	24	24
17-18pm	24	24	24	24
18-19pm	8	8	8	8
19-20pm	6	6	6	6
20-21pm	6	6	6	6
21-22pm	6	6	6	6
22-23pm	6	6	6	6
23pm-12a	6	6	6	6
12am-1am	0	0	6	0

Table 1: Optimal Schedule for Orange Line

	Mon_Thu	Fri	Sat	Sun
5-6am	6	6	6	6
6-7am	6	6	6	6
7-8am	12	12	12	12
8-9am	12	12	12	12
9-10am	6	6	6	6
10-11am	6	6	6	6
11am-12p	6	6	6	6
12-13pm	6	6	6	6
13-14pm	6	6	6	6
14-15pm	6	6	6	6
15-16pm	6	6	6	6
16-17pm	12	12	12	12
17-18pm	12	12	12	12
18-19pm	6	6	6	6
19-20pm	6	6	6	6
20-21pm	6	6	6	6
21-22pm	6	6	6	6
22-23pm	6	6	6	6
23pm-12a	6	6	6	6
12am-1am	0	0	6	0

Table 2: Optimal Schedule for Yellow Line

	Mon_Thu	Fri	Sat	Sun
5-6am	6	6	6	6
6-7am	10	10	6	6
7-8am	12	12	7	7
8-9am	12	12	8	8
9-10am	7	7	10	10
10-11am	7	7	7	7
11am-12p	7	7	7	7
12-13pm	7	7	7	7
13-14pm	7	7	7	7
14-15pm	7	7	7	7
15-16pm	7	10	7	7
16-17pm	12	12	8	8
17-18pm	12	12	8	8
18-19pm	6	10	8	7
19-20pm	6	6	7	6
20-21pm	6	6	6	6
21-22pm	6	6	6	6
22-23pm	6	6	6	6
23pm-12a	6	6	6	6
12am-1am	0	0	6	0

Table 3: Optimal Schedule for Blue Line

	Mon_Thu	Fri	Sat	Sun
5-6am	6	6	6	6
6-7am	7	7	7	7
7-8am	12	12	12	12
8-9am	12	12	12	12
9-10am	6	6	6	6
10-11am	6	6	6	6
11am-12p	6	6	6	6
12-13pm	6	6	6	6
13-14pm	6	6	6	6
14-15pm	6	6	6	6
15-16pm	6	6	6	6
16-17pm	12	12	12	12
17-18pm	12	12	12	12
18-19pm	6	6	6	6
19-20pm	6	6	6	6
20-21pm	6	6	6	6
21-22pm	6	6	6	6
22-23pm	6	6	6	6
23pm-12a	6	6	6	6
12am-1am	0	0	6	0

Table 4: Optimal Schedule for Green Line

Current Operating Schedule

Current Operation Schedule					
		Blue	Orange	Green	Yellow
Mon - Thu, Fri	Peak Hours	15	20	15	12
	Non-Peak Hours	12	15	10	12
Sat, Sun		7	10	10	6

Table 5: Current Metro Operating Schedule

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