Cosmic Acceleration via Modified Gravity

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Presentation Outline

- Background (GR)
- Motivation for modifying gravity.
- *f(R)* theories of gravity.
- Teleparallel gravity and extensions-*f(T)*.
- Results and future work.
- Discussion.

1. General Relativity

- Einstein's General Relativity formulated in 1915.
- It describes the connection between the curvature of space-time and the matter/radiation content of the Universe.
- The theory is encapsulated in the Einstein field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}^{EM}}$$

GEOMETRY

$R_{\mu\nu}$ = the Ricci curvature tensor.
$g_{\mu\nu}$ = the metric tensor.
R = the Ricci scalar.

CONTENT

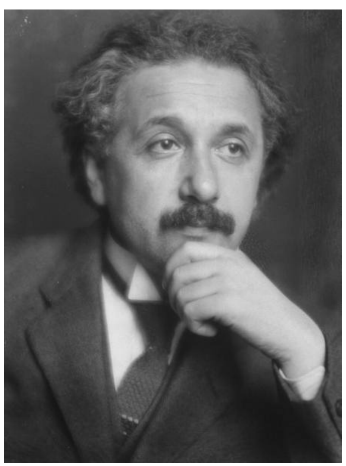
$T_{\mu\nu}^{EM}$ = the energy –momentum tensor.
G = Newton's gravitational constant.

NB - all the equations are in natural units,
Greek indices run from 0-3 over space-time.

2. The Cosmological Constant

- Introduction of the Cosmological Constant Λ to permit static solutions.
- It was included in the geometry side of the equation.

$$\boxed{g_{\mu\nu}\Lambda + R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}^{EM}}$$



Ref: LIFE Photo Archive

- Static solution is unstable to small perturbations.
- BUT, this didn’t matter because the Universe was found to be **EXPANDING.**

3. Accelerated expansion of the Universe

- In 1929, Hubble showed that the Universe is **expanding**.
- **AND,** in 1998, Reiss et al., Perlmutter et al. simultaneously discovered the expansion was **accelerating** (Supernovae Type 1a).
- **Dark energy** causing the acceleration? What is it?

Dark energy modelled as a perfect fluid with an equation of state parameter $w \leq -\frac{1}{3}$.

Equation of state $\longrightarrow$ $\boxed{P=w\rho}$

P is the pressure.
w is the equation of state parameter.
ρ is the density of the material.

$$\boxed{R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu}+T^{DE}{}_{\mu\nu})}$$

Dark energy on the right hand side.

- The cosmological constant -represents DE with *w=-1*, **ΛCDM** model.
- **It is a very good model compared with observations at high and low z.**

4. Motivation for modifying GR

- Two problems with ΛCDM :

1. The cosmological constant problem
The vacuum energy predicted by quantum field theory -about 10^120 times greater than observation..

Observations: $\Lambda \approx 10^{-83}\ GeV^2$

QFT: $\Lambda \approx 10^{38}\ GeV^2$

Durrer et al (2008) arXiv: 0811.4132.

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2. The coincidence problem.
The matter density of the Universe today is of the same order as dark energy density -very unlikely.

Density of matter: $\rho_M \propto \left(\frac{1}{a}\right)^3$

a= scale factor

Density of Λ today: $\rho_\Lambda = \frac{\Lambda}{8\pi G}$

BUT $\rho_\Lambda \approx \rho_M$

POSSIBLE SOLUTION: Avoid using Λ, **modify the geometry side of the Einstein equation.**

5. The action of GR.

- The action *S* describes the trajectory of a particle could take between point A and B.

$$\boxed{S = \int_A^B ds}$$

- The infinitesimal line element in GR is:

$$\boxed{ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}}$$

- In GR, the action is the sum of Einstein-Hilbert Action *SEH* (geometry) and the matter action *SM*.

$$\boxed{S = SEH + SM}$$

- The Einstein-Hilbert Action is a function of the Ricci scalar:

$$\boxed{S_{EH} = \frac{1}{16\pi G} \int \sqrt{-g} R \, d^4 x}$$

R= the Ricci scalar
g = the determinant of the metric

Modifying the geometry part (*SEH*) of the action= modification to gravity .

6. *f(R)* theories of gravity

- The action becomes:

$$\boxed{S_R = \frac{1}{16\pi G}\int \sqrt{-g}\,[\,f(R) + L_M]d^4x}$$

- Allows an extra scalar degree of freedom- dubbed the scalaron (Starobinsky, 1980).
- **Couples to matter – acts like a fifth force!**
- To provide late-time acceleration – mass of scalaron must be small.
- **HOWEVER , no fifth force has been observed in the Solar System.**
- The mass of the scalaron evolves with environment i.e. –high mass and short range in solar system–**CHAMELEON MECHANISM.**

More Constraints…

7. *f(R)* -constraints

Constraint	Region	Reason
		To prevent ghosts.
		To prevent tachyons.
	High density	To adhere to local gravitational constraints.

Constraint	Region	Reason
$\frac{df(R)}{dR} > 0$	$R \geq R_0$	To prevent ghosts.
$\frac{d^2 f(R)}{dR^2} > 0$	$R \geq R_0$	To prevent tachyons.
$f(R)$ $R\text{-}2\Lambda$ (ΛCDM)	High density	To adhere to local gravitational constraints.

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R_0 is the Ricci scalar of the background

- As a result, the model needs to be finely tuned.

Example: $f(R) = R - \mu R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1}$

Hu and Sawicki (2007), arXiv:0705.1158

with $n, \mu, R_0 > 0$

8. Teleparallel gravity

- When the Ricci scalar *R* is replaced with *T*, the torsion scalar- equivalent to GR.

$$S_T = \frac{1}{16\pi G}\int \sqrt{-g}\,[T + L_M]\,d^4x$$

T is the torsion scalar

- **Original motivation-** Einstein proposed teleparallel gravity to unite electro-magnetism and gravity.

- Replaces the Christoffel symbols of GR $\longrightarrow$ Γρμν

  - with the Weitzenbock connections .. Γρμν $\longrightarrow$ w

9. *f(T)* theories of gravity

- Original motivation lost, BUT renewed interest recently to model late-time acceleration.
- Following the process of *f(R)* theories, promote to a function of $T \to f(T)$, giving the action:

$$S_T = \frac{1}{16\pi G} \int \sqrt{-g}\, [\, f(T) + L_M] d^4 x$$

Advantages of *f(T)* over *f(R)*:

- Not so tightly constrained: $f/T \longrightarrow 0$ at early times,
  must have $w=-1$ at late times.

- Equations of motion are *2nd* order compared to *4th* in *f(R)* theory.

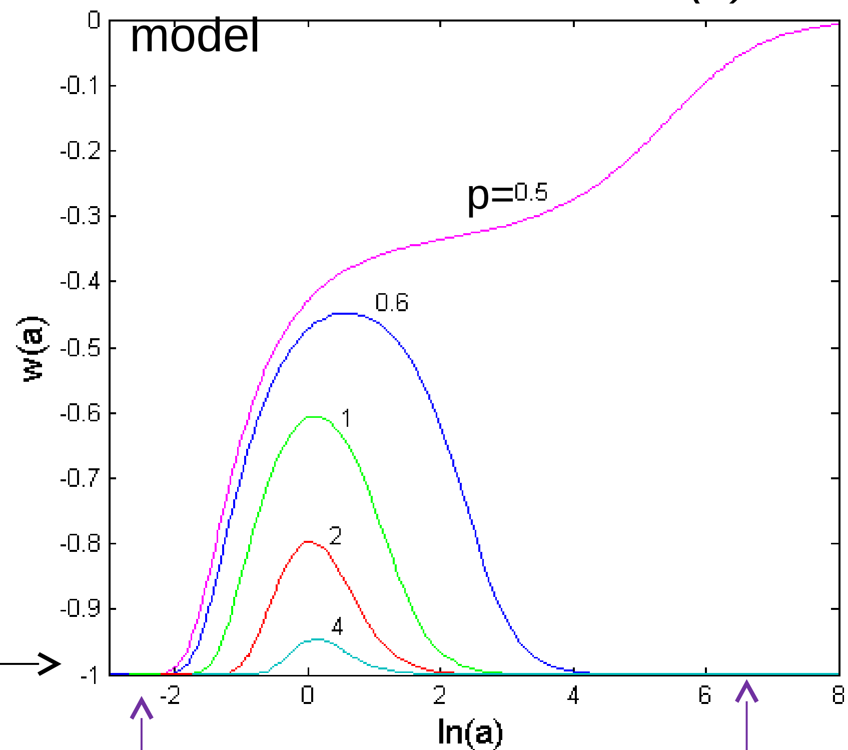
10. Results and future work

$$f(T) = \frac{1-\Omega_M}{1-(1+p)e^{-p}} T_0 \left(1 - e^{-p\sqrt{T/T_0}}\right)$$

Linder (2010), arXiv:1005.3039v2

where: *T0* is the torsion scalar today.
p is an adjustable parameter.
ΩM is the matter density

Evolution of *w* in Linder's *f(T)* model



- **Good model** -when $p > 0.5$ as mimics ΛCDM at high and low z.

z.

- **Future work-** to fit observational data from SN1a to this model.

w=-1, equivalent to ΛCDM

High z

Low z

10. Discussion (and future work)

1. Is it possible to distinguish between dark energy and modified gravity observationally?
   - Not yet!
   - Through the evolution of density perturbations -Dent et al.(2010) arXiv: 1008.1250

Future observations will show preference for either dark energy or modified gravity.

Finally (in the meantime)

2. Are *f(T)* and *f(R)* sensible theories?
   - *f(T)* not Lorentz invariant-Sotirou et al. (2010), arXiv: 1012.4039,
   - *f(R)* too tightly constrained, the need for Lorentz invariance-Durrer et al. (2008), arXiv:0811.413.

$\Gamma\rho\mu\nu = \Gamma\rho\nu\mu$

Christoffel symbols of GR, symmetric under lower index exchange.

$T\rho\nu\mu^{w} = \Gamma\rho\mu\nu^{w} - \Gamma\rho\nu\mu$

Torsion tensor is the difference between Weitzenbock connections under lower index exchange.

- Kunz et al. (2007) arXiv:0612452, say it is **not** possible to see the difference between dark energy and modified gravity in growth of perturbations.
- Dent et al. (2010) arXiv:1010.2215, say that the equivalent *f(T)* theory compared to quintessence will leave a signature in the growth of density perturbations.

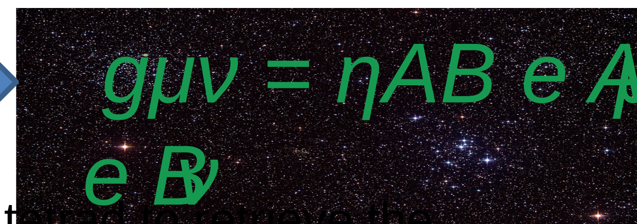
9. *f(T)* continued…

- **Instead of the metric** - the **tetrad**. $e^A_\mu = diag\,(1,a,a,a)$

where *a* is the scale factor of the expansion.

  - the Latin indices represent tangent space and range from *0-3*.
  - the Greek indices represent space-time and run from *0-3*.

- The tetrad are the orthonormal tangent vectors related to the metric by



- The action is varied with respect to the tetrad to retrieve the modified equations of motion.

$$T^\rho{}_{\mu\nu} \equiv e^\rho_A \left(\partial_\mu e^A_\nu - \partial_\nu e^A_\mu\right).$$

$$K^{\mu\nu}{}_\rho \equiv -\frac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right)$$

$$S_\rho{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho T^{\alpha\mu}{}_\alpha\right)$$

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}$$

Field equations

$$e^{-1}\partial_{\mu}\left(e.e^{\rho}_{A}S_{A}{}^{\mu\nu}\right)\left[1+f_{,T}\left(T\right)\right]-e^{\lambda}_{A}T^{\rho}{}_{\mu\lambda}S^{\rho}{}_{\nu\mu}\left[1+f_{,T}\left(T\right)\right]$$
$$+e^{\rho}_{A}S_{\rho}{}^{\mu\nu}\partial_{\mu}\left(T\right)f_{,TT}\left(T\right)-\frac{1}{4}e^{\nu}_{A}\left[T+f\left(T\right)\right]=4\pi G e^{\rho}_{A}T^{\mathrm{EM}}_{\rho}{}^{\nu}.$$

$$H^2=\frac{8\pi G}{3}\rho-\frac{f}{6}-2H^2f_{,T}$$

Modified Friedman equation

$$\left(H^2\right)'=\frac{16\pi Gp+6H^2+f+12H^2f_{,T}}{24H^2f_{,TT}-2-2f_{,T}}$$

Modified acceleration equation

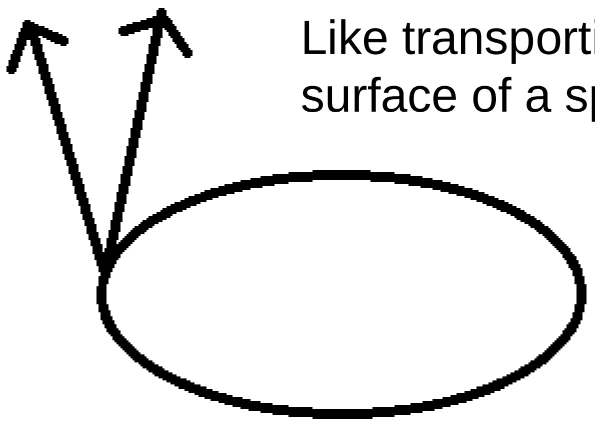
$$\frac{d\ln\left(a\right)}{dT}=\left(-\frac{1}{3}\right)\frac{1+f_{T}+2Tf_{,TT}}{T-f+2Tf_{,T}}$$

$$w=-\frac{\frac{f}{T}-f_{,T}+2Tf_{,TT}}{\left(1+f_{,T}+2Tf_{,TT}\right)\left(\frac{f}{T}-2f_{,T}\right)}$$

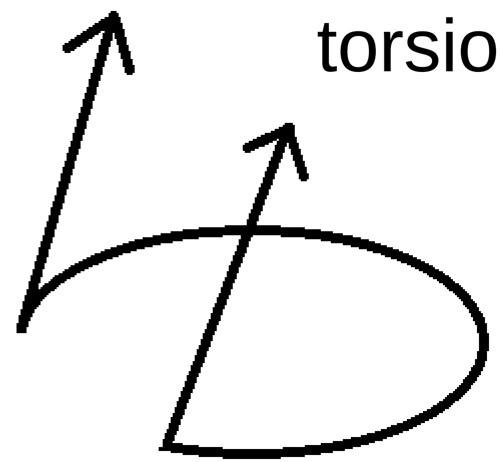
Curvature and torsion

curvature

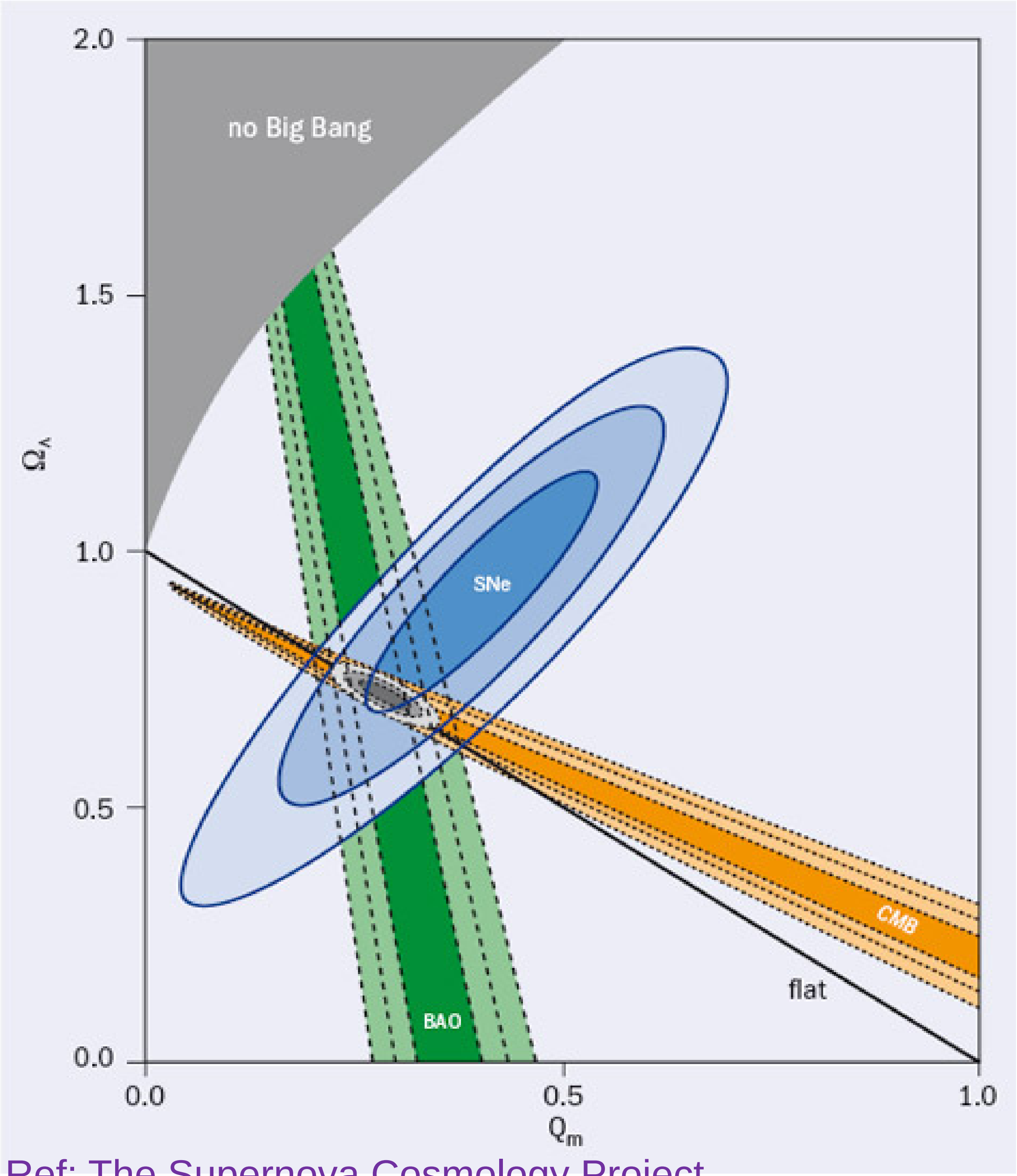
Like transporting a vector over the surface of a sphere



torsion



Torsion proportional to the difference when two vectors are parallel transported along each other.



Ref: The Supernova Cosmology Project

GR tests in the Solar System

- **Precession of Mercury's orbit.**
  - Newton out by approx. 43 arcsecs per century
  - Einstein's GR predicts this very accurately.
- **Bending of light rays close to the Sun**
  - Light from stars that travels very close to the Sun is bent.
- **Radar Echo Delays**
  - Time dilation of radio signals when passing close to a massive body

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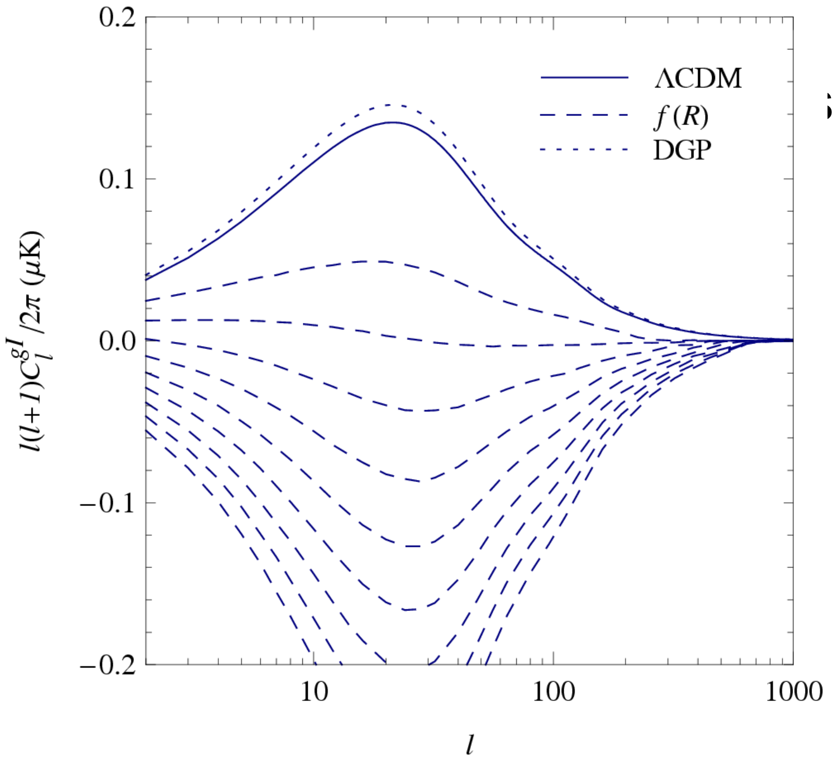
Distinguishing DE from modified gravity



Galaxy-ISW cross correlations
Figure from : Lombriser (2008) arXiv:

How should teleparallel gravity unite EM and gravity?