CSE12 - Lecture 14 - C00

Wednesday, October 26, 2022 11:00 AM

PA5 released today
PA4 hard deadline → today
PA2 Late/Resubmit → Tuesday @10 pm

Quicksort: Another magical (recursive) algorithm

https://www.youtube.com/watch?v=ywWBy6J5gz8

14	4	9	12	15	8	19	2

Select a pivot element:

14	4	9	12	15	8	19	2

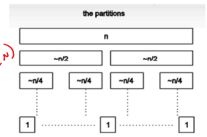
"Partition" the elements in the array (smaller or equal to pivot, larger or equal to pivot)

2	4	9	8	15	12	19	14

Magically sort the smaller elements and the larger elements (Quicksort)

2	4	8	9	12	15	19	21

Quick Sort: Using a "good" pivot



height $\log_2(n)$

How many levels will there be if you choose a pivot that divides the list in half?

A. 1
B. log(n)
C. n
D. n*log(n)
E. n^2

If the time to partition on each level takes N comparisons, how long does Quicksort take with a good partition?

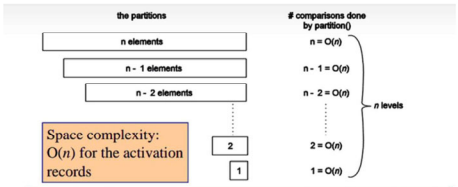
A. O(1)
B. O(log(n))
C. O(n)
D. O(n*log(n))
E. $O(n^2)$

Which of these choices would be the *worst* choice for the pivot?

A. The minimum element in the list
B. The last element in the list
C. The first element in the list
D. A random element in the list

best choice
↳ median value

Quick sort with a bad pivot



Space complexity: O(*n*) for the activation records

If the pivot always produces one empty partition and one with $n-1$ elements, there will be n levels, each of which requires O(n) comparisons: O(n^2) time complexity

Which of these choices is a better choice for the pivot?

A. The first element in the list
B. A random element in the list
C. They are about the same

There are many ways to partition!

Quick sort – Middle Pivot

1. We always pick the middle location as pivot
2. The data we sort is {2, 3, 1, 5, 4, 6, 7}

After the first split, what is the order of elements in the list that was <= pivot?

A. 1 2 3 4
B. 2 3 1 4 ?
C. 4 3 2 1
D. 3 4 1 2
E. None of the above

10 3 5 9 2 7 6 4

sort {12, 4, 9, 3, 15, 8, 19, 2}

What is the first pivot? 15

How would you sort the data around the pivot?

lowIndex = low (0)
highIndex = high - 1 (7)
pivotIndex = (high-low)/2 = (8+0)/2 = 4

low = pt 1 2 3 4 5 6 7
high = 7 6
pi = 4 7
pv = 15

(12) 4 (9) 3 (15) (8) 19 2
(2) (4) (3)
4 5
15
(15) (19)

loop

if value[lowIndex] <=? pivotValue
lowIndex ++

else
swap(lowIndex, highIndex)
highIndex --

if value[highIndex] > pivotValue
highIndex --

else
swap(lowIndex, highIndex)
lowIndex ++

if (low >=? high)

```
import java.util.Arrays;
public class Sort {
  static void selectionSort(int[] arr) {
    for(int i = 0; i < arr.length; i += 1) {
      int minIndex = i;
      for(int j = i; j < arr.length; j += 1) {
        if(arr[minIndex] > arr[j]) { minIndex = j; }
      }
      int temp = arr[i];
      arr[i] = arr[minIndex];
      arr[minIndex] = temp;
    }
  }
  static void insertionSort(int[] arr) {
    for(int i = 0; i < arr.length; i += 1) {
      for(int j = i; j > 0; j -= 1) {
        if(arr[j] < arr[j-1]) {
          int temp = arr[j-1];
          arr[j-1] = arr[j];
          arr[j] = temp;
        }
        else { break; } // new! exit inner loop early
      }
    }
  }
}

import java.util.Arrays;
public class SortFaster {

  static int[] combine(int[] p1, int[] p2) {...}
  static int[] mergeSort(int[] arr) {
    int len = arr.length
    if(len <= 1) { return arr; }
    else {
      int[] p1 = Arrays.copyOfRange(arr, 0, len / 2);
      int[] p2= Arrays.copyOfRange(arr, len / 2, len);
      int[] sortedPart1 = mergeSort(p1);
      int[] sortedPart2 = mergeSort(p2);
```

sorted N [O(1)

sorted array
1 2 3 4 5
O(N)

reverse sorted O(N²)
5 4 3 2 1 1 swap
4 3 5 2 swaps
2 3 4 5 3 swaps

```
static int[] combine(int[] p1, int[] p2) (...)
static int[] mergeSort(int[] arr) {
  int len = arr.length
  if(len <= 1) { return arr; }
  else {
    int[] p1 = Arrays.copyOfRange(arr, 0, len / 2);
    int[] p2= Arrays.copyOfRange(arr, len / 2, len);
    int[] sortedPart1 = mergeSort(p1);
    int[] sortedPart2 = mergeSort(p2);
    int[] sorted = combine(sortedPart1, sortedPart2);
    return sorted;
  }
}

static int partition(String[] array, int l, int h) (...)
static void qsort(String[] array, int low, int high) {
  if(high - low <= 1) { return; }
  int splitAt = partition(array, low, high);
  qsort(array, low, splitAt);
  qsort(array, splitAt + 1, high);
}
public static void sort(String[] array) {
  qsort(array, 0, array.length);
}
}
```

if (low >=? high)
done = true

Swap?
Changed index?

	Insertion	Selection	Merge	Quick
Best case time	sorted array $\Omega(n)$	$\Theta(N^2)$	$\Theta(N * \log_2(N))$	Median value $\Theta(n * \log_2(n))$
Worst case time	reverse sorted $\Omega(N^2)$	$\Theta(N^2)$	$\Theta(N * \log_2(N))$	$\Theta(N^2)$ Average case: $\Theta(N * \log(N))$
Key operations	swap(a, j, j-1) (until in the right place)	swap(a, i, indexOfMin) (after finding minimum value)	l = copy(a, 0, len/2) r = copy(a, len/2, len) ls = sort(l) rs = sort(r) merge(ls, rs)	p = partition(a, l, h) sort(a, l, p) sort(a, p + 1, h)

Last note about sorting

Not only do we care about runtime, we also care about
- Space: do we need extra storage?
- Stable: if we have duplicates, do we maintain the same ordering?

Algorithm	Space	Stable
Bubble sort	O(1)	Yes
Selection sort	O(1)	No
Insertion sort	O(1)	Yes
Heap sort	O(1)	No
Merge sort	O(n)	Yes
Quick sort	O(logn)	No

[{"1", "Gray"}, {"1", "Tina"}]

[{"1", "Gray"}, {"1", "Tina"}]
[{"1", "Tina"}, {"1", "Gray"}]
⇒ unstable ordering

Java ⇒ for ArrayLists
↳ primitives → Quicksort
↳ Objects → Merge sort