CSE12 - Lecture 14 - A00

Wednesday, October 26, 2022 8:00 AM

PA3 released today
PA4 hard deadline → today
PA2 Late / Resubmit → Tuesday @ 11pm

Quicksort: Another magical (recursive) algorithm

https://www.youtube.com/watch?v=ywWBy6J5gz8

14	4	9	12	15	8	19	2

Select a pivot element:

14	4	9	**12**	15	8	19	2

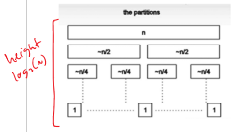
"Partition" the elements in the array (smaller or equal to pivot, larger or equal to pivot)

2	4	9	8	15	**12**	19	14

Magically sort the smaller elements and the larger elements (Quicksort)

2	4	8	9	12	15	19	21

Quick Sort: Using a "good" pivot



How many levels will there be if you choose a pivot that divides the list in half?

A. 1
B. log(n)
C. n
D. n*log(n)
E. n^2

If the time to partition on each level takes N comparisons, how long does Quicksort take with a good partition?

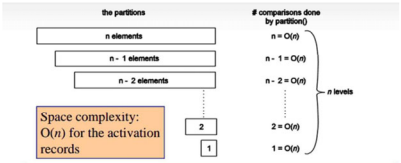
A. O(1)
B. O(log(n))
C. O(n)
D. O(n*log(n))
E. $O(n^2)$

Which of these choices would be the *worst* choice for the pivot?

A. The minimum element in the list
B. The last element in the list
C. The first element in the list
D. A random element in the list

best choice: median value

Quick sort with a bad pivot



If the pivot always produces one empty partition and one with $n-1$ elements, there will be n levels, each of which requires $O(n)$ comparisons: $O(n^2)$ time complexity

Which of these choices is a better choice for the pivot?

A. The first element in the list
B. A random element in the list
C. They are about the same

There are many ways to partition!

Quick sort – Middle Pivot

1. We always pick the middle location as pivot
2. The data we sort is {2, 3, 1, 8, 4, 6, 7}

After the first split, what is the order of elements in the list that was <= pivot?

A. 1 2 3 4
B. 2 3 1 4
C. 4 3 2 1
D. 3 4 1 2
E. None of the above

5 10 5 7 9 2 8 6 4

3 5 2 4 6 10 7 9 8

3 2 4 5 7 10 9 8

2 3 4 8 10 9

10

sort {12, 4, 9, 3, 15, 8, 19, 2}

What is the first pivot? 15

How would you sort the data around the pivot?

lowIndex = low 0
highIndex = high − 1 7
pivotIndex = (high−low)/2 = (8−0)/2 = 4

1/5 X/5 4/5 4/5 4/5

sorted array → 1st element is n^2

(1) 2 3 4 5

O O O O

low = 0 1 2 3 4 5 6 7

high: 7 6

(12) 4 9 3 15 8 19 2

(2) 4

(9) 4 3

(4) 3 15

(3) 15

(8) 15 19

loop

if value[lowIndex] < pivotValue
  lowIndex ++

else
  swap(lowIndex, highIndex)
  highIndex --

if value[highIndex] > pivotValue
  highIndex --

else
  swap(lowIndex, highIndex)
  lowIndex ++

if (low >? high)
  done = true

swap to put pivot in correct place?

```
import java.util.Arrays;
public class Sort {
  static void selectionSort(int[] arr) {
    for(int i = 0; i < arr.length; i += 1) {
      int minIndex = i;
      for(int j = i; j < arr.length; j += 1) {
        if(arr[minIndex] > arr[j]) { minIndex = j; }
      }
      int temp = arr[i];
      arr[i] = arr[minIndex];
      arr[minIndex] = temp;
    }
  }
  static void insertionSort(int[] arr) {
    for(int i = 0; i < arr.length; i += 1) {
      for(int j = i; j > 0; j -= 1) {
        if(arr[j] < arr[j-1]) {
          int temp = arr[j-1];
          arr[j-1] = arr[j];
          arr[j] = temp;
        }
        else { break; } // new! exit inner loop early
      }
    }
  }
}
```

O(1)

1 2 4 5

sorted array

O(N)

5 4 2 1 1 swap

4 5 2 1 2 swap

2 4 5 1 3 swap

```
import java.util.Arrays;
public class SortFaster {

  static int[] combine(int[] p1, int[] p2) {...}
  static int[] mergeSort(int[] arr) {
    int len = arr.length
    if(len <= 1) { return arr; }
    else {
      int[] p1 = Arrays.copyOfRange(arr, 0, len / 2);
      int[] p2= Arrays.copyOfRange(arr, len / 2, len);
      int[] sortedPart1 = mergeSort(p1);
      int[] sortedPart2 = mergeSort(p2);
      int[] sorted = combine(sortedPart1, sortedPart2);
      return sorted;
    }
  }

  static int partition(String[] array, int l, int h) {...}
  static void qsort(String[] array, int low, int high) {
    if(high - low <= 1) { return; }
    int splitAt = partition(array, low, high);
    qsort(array, low, splitAt);
    qsort(array, splitAt + 1, high);
  }
  public static void sort(String[] array) {
    qsort(array, 0, array.length);
  }
}
```

	Insertion	Selection	Merge	Quick
Best case time	Sorted Array $\Theta(N)$	$\Theta(N^2)$	$\Theta(N * \log_2(N))$	Median value $\Theta(N * \log_2(N))$
Worst case time	Reverse Sorted $\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N * \log_2(N))$	$\Theta(N^2)$ Average case: $\Theta(N * \log(N))$
Key operations	swap(a, j, j-1) (until in the right place)	swap(a, i, indexOfMin) (after finding minimum value)	l = copy(a, 0, len/2) r = copy(a, len/2, len) ls = sort(l) rs = sort(r) merge(ls, rs)	p = partition(a, l, h) sort(a, l, p) sort(a, p + 1, h)

Last note about sorting

Not only do we care about runtime, we also care about

- Space: do we need extra storage?
- Stable: if we have duplicates, do we maintain the same ordering?

Algorithm	Space	Stable
Bubble sort	O(1)	Yes
Selection sort	O(1)	No
Insertion sort	O(1)	Yes
Heap sort	O(1)	No
Merge sort	O(n)	Yes
Quick sort	O(logn)	No

key value

{"1", "Greg"}

{"1", "Elsa"}

[{"1", "Greg"}, {"1", "Elsa"}]

[{"1", "Elsa"}, {"1", "Greg"}]

Unstable ordering

Java → for ArrayList

↳ Quicksort → primitives

↳ MergeSort → objects