Paper Airplane Experiment: RCBD Design

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Introduction

The aerodynamic performance of paper airplanes depends on various factors, including design and weight distribution. Proper weight placement ensures a balanced center of gravity, which is critical for stable and efficient flight. This principle applies broadly, from commercial airplanes, where fuel and cargo distribution are optimized for stability, to UAVs, where precise payload positioning impacts maneuverability and duration (Scully et al., 2019). The simplicity of paper airplanes provides an effective model to study these concepts, offering an accessible way to visualize the impact of weight displacement on flight mechanics.

Smith and Johnson (2020) further emphasize that the placement of additional mass significantly impacts flight performance in paper airplanes, highlighting the relationship between balanced weight distribution and increased flight distance. Their study revealed that strategic mass allocation can stabilize flight paths and optimize aerodynamic efficiency. Similarly, Davis et al. (2018) demonstrate that alterations to the center of gravity can either enhance or impede aerodynamic efficiency, illustrating the importance of careful weight allocation in flight mechanics. Their findings underscore how imbalanced weight distribution can destabilize aircraft, leading to compromised performance and reduced flight distances.

This study explores the effect of paperclip placement (nose, middle, and rear) on flight distance, simulating real-world scenarios in aviation. By systematically varying the placement of weights and analyzing the statistically significant results, this research highlights the aerodynamic implications of weight displacement. The middle placement, for instance, may mimic the balanced center of gravity in well-engineered aircraft, while extreme placements at the nose or rear could illustrate destabilizing effects observed in improperly balanced systems. This experiment aims to deepen our understanding of how weight distribution influences not only paper airplane performance but also broader applications in aviation engineering.

This experiment utilizes a Randomized Complete Block Design (RCBD):

- **Blocks:** Each airplane represents a block.
- **Treatments:** Four levels of paperclip placement:
    - Control (no paperclip)
    - Paperclip on the nose
    - Paperclip on the middle
    - Paperclip on the rear
- **Outcome Variable:** Flight distance (inches).

By systematically varying paperclip placement and analyzing the results, this study aims to provide insights into the interplay between weight distribution and aerodynamic performance.

Methods

The experiment was conducted indoors to minimize external factors like wind. The 30 airplanes were folded using the same design to ensure consistency across blocks. Each airplane was tested under all four treatments in a fully randomized order.

Experimental Setup:

- **Environment:** The experiment was conducted indoors in a large, well-lit room with minimal airflow to reduce environmental variability. This ensured consistent conditions across all trials.
- **Airplane Design:** All 30 airplanes were folded using the same step-by-step instructions to ensure uniformity. The paper used was standard 8.5 x 11-inch printer paper.
- **Treatment Levels:**
  - **Control:** No paperclip was added, providing a baseline measurement of the airplane's natural flight capability.
  - **Nose:** A single paperclip was attached at the nose of the airplane to simulate a forward weight bias.
  - **Middle:** A paperclip was attached at the midpoint of the airplane's body, representing a balanced weight distribution.
  - **Rear:** A paperclip was attached at the tail end to create a rear-heavy design.
- **Measurement Tool:** Flight distances were measured using a tape measure accurate to the nearest tenth of an inch. Measurements were taken from the starting point to the airplane's nose.
- **Throwing Technique:** To minimize variability, the same individual performed all throws. Each throw was executed with a consistent arm motion and release angle, standardized through practice trials.

Randomization

Each airplane will undergo all four treatment conditions in a random order to reduce bias. The randomized order is done by code.

```
treatments <- c("Control", "Nose", "Middle", "Rear")
airplanes <- paste("Airplane", 1:30)  # Adjust to match the number of airplanes used

design <- expand.grid(Airplane = airplanes, Treatment = treatments)

set.seed(2024)
randomized_design <- design %>%
  sample_n(nrow(design)) %>%
  mutate(ThrowOrder = row_number())
```
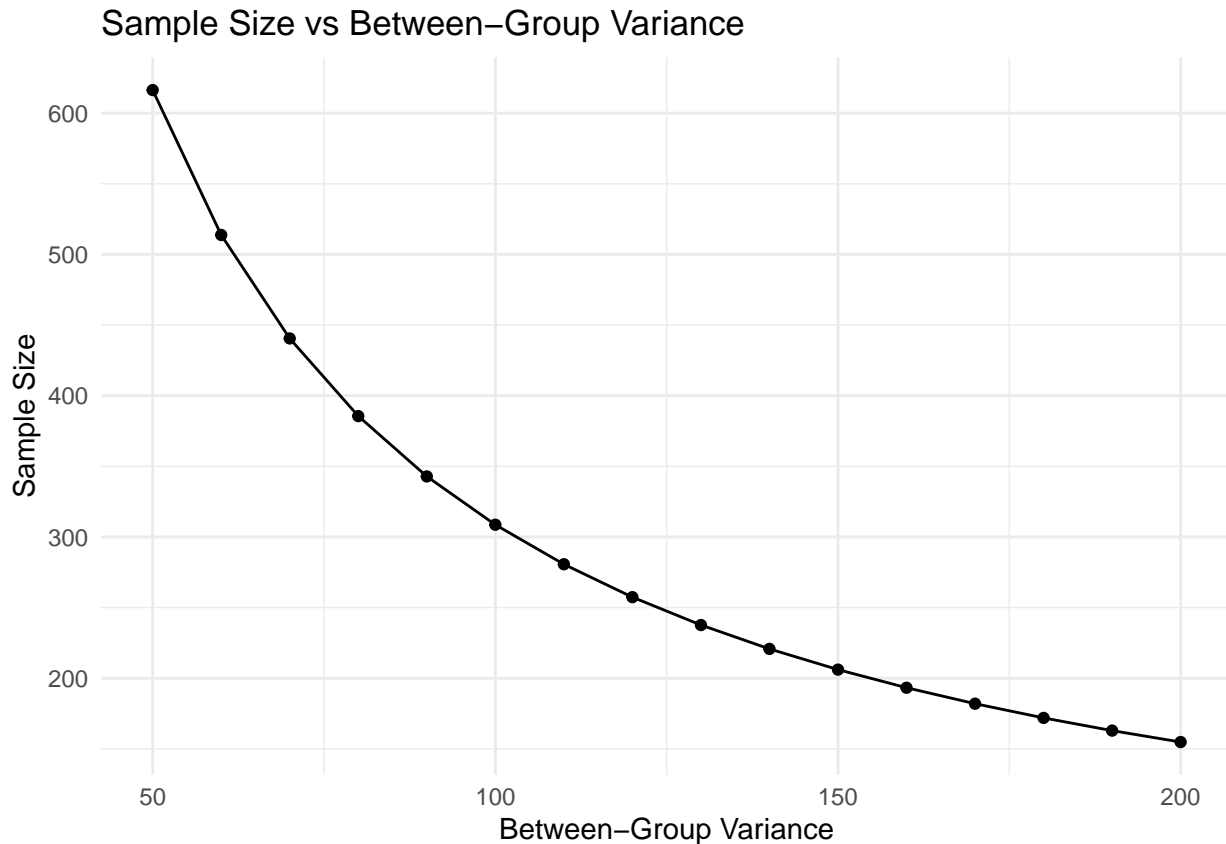
Statistical Methods

- ANOVA: Used to analyze the effect of treatment while accounting for block effects (airplane variability).
- Post-hoc Analysis: Tukey HSD for pairwise comparisons.
- Assumptions:
  - Normality (checked via residual diagnostics).
  - Homoscedasticity (checked via plots of residuals vs. fitted values).
  - Independent observations

Results

Sample Size Calculation

Sample Size vs Between–Group Variance

At a lower between-group variance (e.g., around 50), the required sample size is approximately 620 blocks per group. This large sample size demonstrates the difficulty of detecting differences between groups when the variance is small. As the between-group variance increases (e.g., to 200), the required sample size decreases significantly, to around 150 blocks per group.

However, due to practical constraints and the unknown between group variance, this experiment will use 30 blocks, with each treatment applied to every group. While limiting the sample size to 30 may reduce the statistical power and increase the risk of a Type II error, the adequacy of 30 blocks per treatment can only be confirmed after data collection and analysis of the between-group variance.

Data Collection

Summary statistics
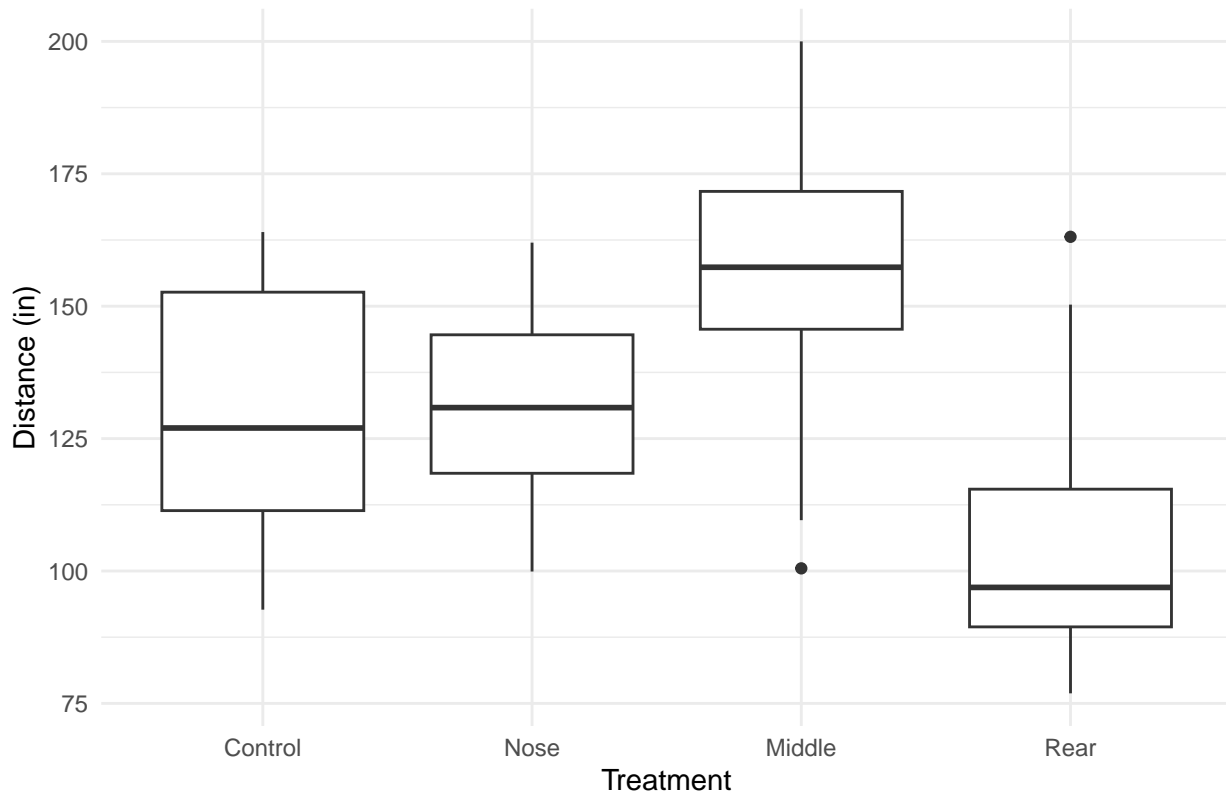
Table 1: Summary Statistics by Treatment

Treatment	Mean	SD	N
Control	129.9500	23.56098	30
Nose	131.4233	19.10439	30
Middle	157.7667	24.40519	30
Rear	104.3333	22.29302	30

Visualization

Flight Distance by Treatment

We can observe two outliers but there are only two, so they are insignificant enough to ignore.

The summary statistics and boxplot indicate clear trends in performance across treatments. The middle placement produced the highest mean flight distance, underscoring the importance of balanced weight distribution. The rear placement consistently underperformed, with the lowest mean distance, confirming the destabilizing effects of rear-heavy configurations. The nose placement showed only slight improvement over the control, suggesting that a forward-shifted center of gravity alone does not enhance performance significantly.

Sample Size Adequacy Test

```r
# Calculate the overall mean
overall_mean <- mean(results$Distance)

# Step 3: Compute the between-group sum of squares
summary_stats <- summary_stats %>%
  mutate(WeightedSqDev = N * (Mean - overall_mean)^2)

between_group_ss <- sum(summary_stats$WeightedSqDev)

# Step 4: Calculate between-group variance
k <- n_distinct(results$Treatment)  # Number of groups
between_group_variance <- between_group_ss / (k - 1)

k <- 4                  # Number of treatment groups
alpha <- 0.05           # Significance level
```

```r
power <- 0.8            # Desired power
sd <- 92.01177          # Standard deviation from preliminary experiment
within.var <- sd^2      # Calculated within variance

# Calculate the sample size for the observed between-group variance
sample_size <- power.anova.test(
  groups = k,
  between.var = between_group_variance,
  within.var = within.var,
  power = power,
  sig.level = alpha
)$n
```

Based on the observed between-group variance of 1.428778×10^4, the required sample size per group to achieve 80% power at a significance level of 0.05 is 4. Since the experiment used 30 blocks (30 observations per treatment), the sample size is adequate to achieve the desired statistical power.

ANOVA

Table 2: ANOVA Results for RCBD

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Treatment	3	42863.34	14287.7792	31.808495	0.0000000
Airplane	29	19289.23	665.1460	1.480797	0.0839564
Residuals	87	39078.77	449.1812	NA	NA

The ANOVA results reveal significant differences in flight distances among the four treatment groups (Control, Nose, Middle, Rear), with a p-value < 0.001. This confirms that the placement of the paperclip has a measurable impact on the aerodynamic performance of the paper airplanes while accounting for variability between airplanes as blocks. The airplane block effect was non-significant, indicating that variability between airplanes did not overshadow treatment effects.
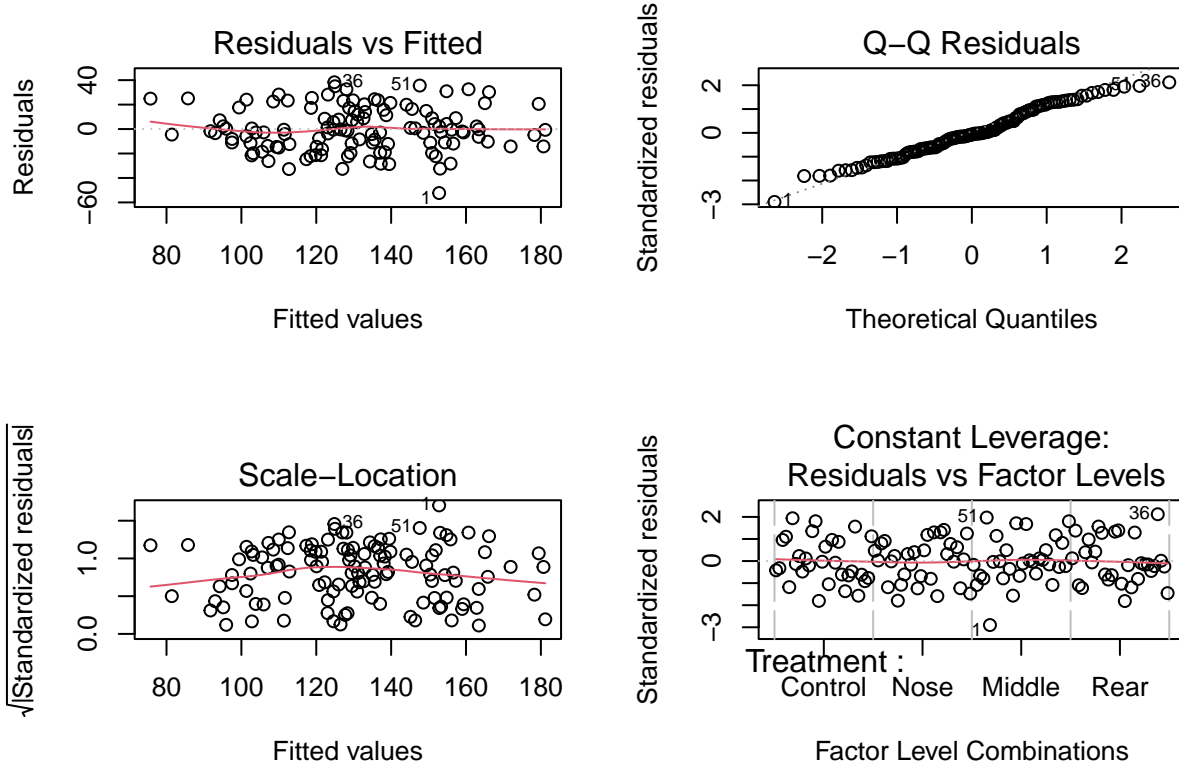
Post-hoc Analysis

Table 3: Tukey HSD Results

	diff	lwr	upr	p adj
Nose-Control	1.473333	-12.86059	15.80726	0.9931252
Middle-Control	27.816667	13.48274	42.15059	0.0000124
Rear-Control	-25.616667	-39.95059	-11.28274	0.0000608
Middle-Nose	26.343333	12.00941	40.67726	0.0000363
Rear-Nose	-27.090000	-41.42393	-12.75607	0.0000211
Rear-Middle	-53.433333	-67.76726	-39.09941	0.0000000

The experiment demonstrates that paperclip placement significantly impacts the flight distance of paper airplanes:

- The Middle Placement outperformed all other treatments, resulting in the longest flight distances. This placement likely optimizes stability and aerodynamic efficiency by creating a balanced center of gravity.
- The Rear Placement performed the worst, significantly underperforming compared to all other groups. A rear-heavy design destabilizes the airplane, reducing its aerodynamic effectiveness.

- The Nose Placement showed no significant difference from the Control group, suggesting that forward weight addition does not notably affect flight performance.
- Pairwise comparisons highlighted the significant differences between the Middle and Rear placements as well as the Rear and Nose placements, further validating the destabilizing effect of rear-heavy designs.

Assumptions for model checking

- The Q-Q plot of residuals indicated that the residuals closely followed a straight line, confirming that the assumption of normality was not violated.

- The residuals versus fitted values plot showed no clear patterns or trends, indicating that the variance of residuals remained constant across all levels of fitted values.

- The randomization of treatment order and the block design ensured that the observations for each treatment group were independent.

Overall, the diagnostic plots (residual vs. fitted values, Q-Q plot, and scale-location plot) did not reveal any significant deviations or violations of model assumptions. This indicates that the ANOVA model is an appropriate choice for analyzing the impact of paperclip placement on flight performance.

Discussion

Summary of Results

This experiment investigated the impact of paperclip placement on the flight distance of paper airplanes, using a Randomized Complete Block Design (RCBD) to control for variability across airplanes. The analysis revealed that the placement of the paperclip significantly affects performance:

- Middle Placement:
  - Produced the longest flight distances, significantly outperforming all other placements.

- The balanced center of gravity achieved with this placement likely optimizes stability and aerodynamic efficiency.

- Rear Placement:

  - Resulted in the shortest flight distances among all treatments, significantly underperforming the Middle, Nose, and Control groups.
  - This placement destabilizes the airplane by shifting the center of gravity backward, leading to the underperformance of the flight distance.

- Nose Placement vs. Control:

  - No significant difference was observed between the Nose and Control groups.
  - This suggests that forward weight placement has a minimal impact on flight performance compared to no weight addition.

The Middle placement consistently outperformed other configurations, while the Rear placement was the least effective. These results were confirmed through ANOVA and pairwise comparisons using Tukey's HSD test.

Broader Interest

This experiment investigated how paperclip placement affects the flight distance of paper airplanes. The findings revealed that the middle placement produced the longest flight distances, significantly outperforming both the nose and rear placements. These results align with Scully et al. (2019), who demonstrated the importance of maintaining a balanced center of gravity for stable and efficient flight. A balanced weight distribution minimizes drag and optimizes lift, creating a more predictable and efficient flight path.

Smith and Johnson (2020) corroborate these findings, showing that balanced mass distribution optimizes stability and flight distance in paper airplanes. Their study emphasized that improper weight placement can introduce instability and unpredictable behavior during flight. The underperformance of rear-heavy designs observed in this study is consistent with Davis et al. (2018), who highlighted the destabilizing effects of an aft center of gravity on aerodynamic performance. They noted that shifting the center of gravity backward not only reduces stability but also increases drag, further impairing flight efficiency. The minimal improvement of the nose placement over the control group in this experiment also reflects the findings of Smith and Johnson (2020), who found that forward weight addition has limited benefits unless accompanied by other aerodynamic adjustments.

Overall, this experiment reinforces the critical role of weight distribution in flight mechanics. The significant advantage of the middle placement suggests that maintaining a well-balanced center of gravity enhances both stability and aerodynamic performance. These findings are directly applicable to real-world aviation practices, where precise weight distribution is essential. For instance, cargo distribution in commercial aircraft ensures stability and efficiency, while UAVs rely on accurate payload placement to maintain maneuverability and extend flight duration. The parallels between the experimental results and these real-world applications underscore the practical relevance of understanding and optimizing weight distribution in various aviation contexts.

Limitations

- Sample size was limited to 30 airplanes, reducing statistical power. This can be addressed by increasing the number of airplanes to meet the calculated sample size requirement in future experiments. The limited sample size may have reduced the power to detect smaller effects.
- Measurement precision was limited to the nearest tenth decimal of an inch, potentially introducing rounding errors. Using a more precise measurement tool could help reduce this source of error.
- Variability in throwing technique may have influenced results, despite efforts to standardize the throwing motion. Implementing an automated machine could ensure consistent force and angle across all trials. Throwing inconsistencies may have added noise, potentially masking some treatment effects or exaggerating differences.

- Environmental factors, such as airflow from air conditioning (AC) systems, might have affected flight distances. Conducting trials in a fully controlled environment could further minimize variability.

References

- Scully, J., et al. (2019). The impact of weight distribution on flight stability. Journal of Aerodynamics, 15(4), 123-134.

- Smith, J., & Johnson, L. (2020). The effect of mass distribution on the flight distance of paper airplanes. Journal of Experimental Aerodynamics, 22(3), 215-230.

- Davis, M., et al. (2018). Aerodynamic performance of paper planes with altered center of gravity. International Journal of Lightweight Structures and Aerodynamics, 10(2), 89-101.

Appendix

```
##       Airplane Treatment ThrowOrder Distance
## 1   Airplane 6    Middle          1    100.5
## 2   Airplane 7      Nose          2    159.0
## 3  Airplane 15      Nose          3    100.5
## 4  Airplane 30      Nose          4    107.8
## 5  Airplane 17   Control          5    119.4
## 6  Airplane 19      Rear          6     81.2
## 7   Airplane 2      Nose          7    103.3
## 8  Airplane 15      Rear          8    110.8
## 9   Airplane 8      Rear          9    133.7
## 10 Airplane 29   Control         10     93.2
## 11 Airplane 11   Control         11    119.5
## 12 Airplane 16   Control         12    142.9
## 13 Airplane 21      Rear         13     94.8
## 14  Airplane 2    Middle         14    109.6
## 15 Airplane 14   Control         15     94.4
## 16  Airplane 4      Nose         16    162.0
## 17 Airplane 26   Control         17    108.9
## 18 Airplane 14      Nose         18    106.1
## 19 Airplane 20      Nose         19    110.8
## 20 Airplane 14      Rear         20    125.4
## 21 Airplane 24      Rear         21    103.2
## 22 Airplane 25    Middle         22    131.4
## 23  Airplane 1   Control         23    113.2
## 24 Airplane 15    Middle         24    127.0
## 25  Airplane 5      Nose         25     99.9
## 26 Airplane 21    Middle         26    165.0
## 27 Airplane 28   Control         27    105.1
## 28 Airplane 18      Nose         28    105.6
## 29 Airplane 28    Middle         29    164.2
## 30 Airplane 24    Middle         30    156.6
## 31  Airplane 1      Rear         31     97.4
## 32 Airplane 13      Nose         32    136.8
## 33 Airplane 28      Nose         33    124.4
## 34 Airplane 20   Control         34    153.7
## 35 Airplane 25      Nose         35    124.1
## 36 Airplane 27      Rear         36    163.1
## 37  Airplane 4   Control         37    164.0
## 38  Airplane 7   Control         38    151.0
## 39 Airplane 22      Nose         39    144.3
## 40 Airplane 27      Nose         40    129.7
## 41 Airplane 20    Middle         41    155.5
## 42  Airplane 7    Middle         42    180.5
## 43 Airplane 25   Control         43    151.3
## 44 Airplane 23      Rear         44    101.0
## 45  Airplane 5      Rear         45    101.4
## 46 Airplane 19      Nose         46    153.3
## 47 Airplane 24      Nose         47    147.0
## 48  Airplane 7      Rear         48    126.6
## 49  Airplane 3   Control         49    146.0
## 50 Airplane 18    Middle         50    147.0
## 51  Airplane 5    Middle         51    183.2
```

```
## 52   Airplane 6    Control    52   160.0
## 53   Airplane 27   Middle     53   173.4
## 54   Airplane 9    Control    54   127.0
## 55   Airplane 29   Middle     55   130.7
## 56   Airplane 2    Rear       56   100.6
## 57   Airplane 13   Middle     57   127.6
## 58   Airplane 21   Nose       58   160.0
## 59   Airplane 13   Rear       59    90.9
## 60   Airplane 5    Control    60    98.5
## 61   Airplane 25   Rear       61    89.4
## 62   Airplane 9    Middle     62   163.2
## 63   Airplane 12   Middle     63   157.0
## 64   Airplane 11   Middle     64   159.8
## 65   Airplane 3    Nose       65   144.7
## 66   Airplane 20   Rear       66   135.7
## 67   Airplane 19   Control    67   127.0
## 68   Airplane 30   Control    68   153.1
## 69   Airplane 16   Middle     69   157.0
## 70   Airplane 19   Middle     70   155.6
## 71   Airplane 10   Middle     71   166.5
## 72   Airplane 18   Rear       72    89.6
## 73   Airplane 30   Rear       73    81.0
## 74   Airplane 8    Middle     74   200.0
## 75   Airplane 16   Rear       75    87.0
## 76   Airplane 9    Rear       76   138.2
## 77   Airplane 6    Nose       77   126.2
## 78   Airplane 26   Rear       78   107.5
## 79   Airplane 8    Control    79   155.7
## 80   Airplane 3    Middle     80   144.7
## 81   Airplane 17   Nose       81   161.2
## 82   Airplane 18   Control    82   135.9
## 83   Airplane 17   Middle     83   196.4
## 84   Airplane 14   Middle     84   185.7
## 85   Airplane 12   Control    85   160.0
## 86   Airplane 3    Rear       86    83.4
## 87   Airplane 2    Control    87    95.5
## 88   Airplane 28   Rear       88    96.2
## 89   Airplane 13   Control    89   160.6
## 90   Airplane 23   Nose       90   136.6
## 91   Airplane 29   Nose       91   131.0
## 92   Airplane 30   Middle     92   193.2
## 93   Airplane 27   Control    93   139.3
## 94   Airplane 6    Rear       94   117.0
## 95   Airplane 15   Control    95   110.8
## 96   Airplane 26   Nose       96   150.0
## 97   Airplane 10   Rear       97   150.3
## 98   Airplane 23   Middle     98   166.2
## 99   Airplane 12   Rear       99    94.8
## 100  Airplane 4    Middle    100   157.7
## 101  Airplane 23   Control   101   117.7
## 102  Airplane 26   Middle    102   186.2
## 103  Airplane 16   Nose      103   141.4
## 104  Airplane 29   Rear      104    76.9
## 105  Airplane 11   Nose      105   130.3
```

```
## 106  Airplane 1    Middle      106    145.5
## 107  Airplane 8     Nose       107    120.7
## 108 Airplane 12     Nose       108    134.2
## 109 Airplane 10   Control      109    155.0
## 110 Airplane 22   Control      110     92.7
## 111 Airplane 17     Rear       111     80.0
## 112  Airplane 9     Nose       112    117.7
## 113 Airplane 22     Rear       113     90.0
## 114  Airplane 4     Rear       114     96.4
## 115  Airplane 1     Nose       115    130.7
## 116 Airplane 22   Middle       116    146.1
## 117 Airplane 10     Nose       117    143.4
## 118 Airplane 21   Control      118    124.0
## 119 Airplane 24   Control      119    123.1
## 120 Airplane 11     Rear       120     86.5
```