First Order Jet Bundles.

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[1] Manuel de León et al. "Hamilton-Jacobi theory in multisymplectic classical field theories". In: *Journal of Mathematical Physics* 58.9 (). DOI: 10.1063/1.5004260. URL: http://arxiv.org/abs/1504.02020.

[2] D. J. Saunders. *The Geometry of Jet Bundles*. London Mathematical Society Lecture Note Series. Cambridge: Cambridge University Press, 1989. DOI: 10.1017/CBO9780511526411.

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For this section and the following, please refer to [2] for further details.

Let (E, π, M) be a bundle, and let $p \in M$.

Definition

Define the local sections ϕ, $\psi \in \Gamma_p(\pi)$ to be 1-equivalent at p if $\phi(p) = \psi(p)$ and if, in some adapted coordinate systems (x^i, u^α) around $\phi(p)$,

$$\frac{\partial \phi^\alpha}{\partial x^i}|_p = \frac{\partial \psi^\alpha}{\partial x^i}|_p$$

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Definition

The equivalence class containing ϕ is called *1-jet of ϕ at p* and is denoted $j^1_p \phi$.

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Definition

The *first jet manifold of* π is the set:

$$\{\, j^1_p\phi \;:\; p \in M,\, \phi \in \Gamma_p\pi \,\}$$

and is denoted $J^1\pi$

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$$\pi_1 : J^1\pi \longrightarrow M$$

$$j^1_p\phi \longmapsto p$$

$$\pi_{1,0} : J^1\pi \longrightarrow E$$

$$j^1_p\phi \longmapsto \phi(p)$$

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Let (U, u) be an adapted coordinate system on E, where $u = (x^i, u^\alpha)$.

Definition

The *induced coordinate system* (U^1, u^1) on $J^1\pi$ is defined by

$$U^1 = \{\, j^1_p\phi \;:\; \phi(p) \in U \,\}$$

$$u^1 = (x^i, u^\alpha, u^\alpha_i)$$

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$$x^i(j^1_p\phi) = x^i(p)$$

$$u^\alpha(j^1_p\phi) = u^\alpha(\phi(p))$$

$$u^\alpha_i(j^1_p\phi) = \frac{\partial \phi^\alpha}{\partial x^i}|_p$$

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Prolongations of Morphisms.

In [1] is defined:

Definition

A section $\psi \in \Gamma(\pi_1)$ is *holonomic* if $j^1(\pi_{1,0} \circ \psi) = \psi$, that is, if there exist a section $\phi = \pi_{1,0} \circ \psi \in \Gamma(\pi)$ such that ψ is the prolongation of ϕ to $J^1\pi$.

END.