Multivariable Calculus

Notes

Angel Cervera Roldan
21319203

Contents

Limits and partial derivatives **3**

Limits and partial derivatives

Definition Open and closed balls in $\mathbb{R}$

An open ball in $\mathbb{R}$ is a set in the form:

$$B = \{(x, y) \in \mathbb{R}^2 | (x - a)^2 + (y - b)^2 < t^2\}$$

A closed ball in $\mathbb{R}$ is a set in the form:

$$B = \{(x, y) \in \mathbb{R}^2 | (x - a)^2 + (y - b)^2 \leq t^2\}$$

A point (x, y) in a subset T of $\mathbb{R}^2$ is called an interior point if (x, y) is the center of an open ball that is a subset of T. The interior of a subset X of $\mathbb{R}^2$ is the set of all interior points of X. We denote this by $Int(X)$.

A point (x, y) is a boundary point of a subset W of $\mathbb{R}^2$ if **every** open ball with center (x, y) contains points that are not in W and also contains points that are in W. The boundary of W is the set of all boundary points of W. We denote this by $Bdy(W)$.

We say that a subset G of $\mathbb{R}^2$ is open iff $Int(G) = G$

We say that a subset Z of $\mathbb{R}^2$ is closed iff $Bdy(Z)$ is a subset of Z. This means that if Z is closed, then every boundary point of Z is an element of Z.

We say that a subset T of $\mathbb{R}^2$ is bounded if it is a subset of an open ball.

Definition Curve and graph of of f

Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a function, then:
The curve of the function is defined as the following set:

$$C_w = \{(x, y) \in \mathbb{R}^2 | f(x, y) = w\}$$

The graph of the function is defined as the following set:

$$G = \{(x, y, z) \in \mathbb{R}^3 | (x, y) \in \text{domain of } f, \text{ and } z = f(x, y)\}$$