DEPARTMENT OF MATHEMATICS & STATISTICS

MT234P

Homework 1

Due by 4 p.m. on February 24, 2023.

1. Suppose $f(x,y) = 3 - \dfrac{10}{\sqrt{3-x+2y}}$

(i) What is the domain and range of f? Justify your answers.

(ii) Find the boundary of the domain of f and find the interior of the domain of f.

(iii) Is the domain of f open, closed, bounded, unbounded? Justify your answer in each case.

2. Find $\lim\limits_{(x,y)\to(0,-1)} \dfrac{x^3-y^2}{2x^5+2y^3-5}$

3. In each case below, state whether the statement is true or false. Justify your answer in each case.

(i) If the number w satisfies $0 \leq 4w < \dfrac{r^3}{4}$, for all $r > 0$, then $w = 0$.

(ii) $\mathrm{Bdy}(A \cup B) = \mathrm{Bdy}(A) \cup \mathrm{Bdy}(B)$, for all subsets A, B in $\mathbb{R}^2$.

4. Does $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^4-x^8}{x^8+y^4}$ exist? Justify your answer.

5. Prove that $\lim\limits_{(x,y)\to(0,0)} \dfrac{3y^2x^2}{5y^2+x^4} = 0$