Linear Algebra - Tutorial 3

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Question 1

Let $T \in L\left(\mathbb{R}^3\right)$ be defined by $T(x, y, z) := (x - 2y + z, y - z, x - z)$.

2) Find matrix B of T with $B^{-1} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

3) Find P such that $B = P^{-1}AP$

4) Find a basis for $\ker T$

5) Find a basis for $T(V)$

Part 1

Find a matrix A of T

Given the following:

$$A := \begin{pmatrix} -3 & 1 \\ 3 & -2 \end{pmatrix}$$

$$T \in L(\mathrm{Sym}_2\mathbb{R}) \to T(M) = A^\top M + MA$$

Part 1

Find the matrix H of T

$$A^\top = \begin{pmatrix} -3 & 3 \\ 1 & -2 \end{pmatrix}$$

$$A^\top \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -3a + 3c & b - 2d \\ -3b + 3d & a - 2c \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} A = \begin{pmatrix} -3a + 3c & a - 2c \\ -3b + 3d & b - 2d \end{pmatrix}$$

$$T(e_1) = T\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 6 & -4 \\ -4 & 2 \end{pmatrix}$$

$$T(e_2) = T\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) = \begin{pmatrix} 6 & -4 \\ -4 & 2 \end{pmatrix}$$