

Third Assignment, MT231P Integration

Mehmet Yesil

Due: 30-03-2023 10:00am

1. Recall that for $a \in \mathbb{R}$, $\delta > 0$ and a function $f: (a - \delta, a + \delta)$ we write

$$\left. \frac{d}{ds} \right|_{s=a} f(s) = f'(a) = \lim_{s \rightarrow a} \frac{f(s) - f(a)}{s - a}.$$

Find the following

(a) $\left. \frac{d}{dx} \right|_{x=1} \left. \frac{d}{dy} \right|_{y=2} \int_0^{xy} xw + \exp(w^2 x^2) dw,$

(b) $\left. \frac{d}{dx} \right|_{x=2} \int_0^x \left(e^{\sqrt{u}} + \left. \frac{d}{dv} \right|_{v=u} e^{uvx} \right) du.$

2. (a) Compute $\int_{-3}^7 \text{sgn}$, where the sign function $\text{sgn}: \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}.$$

- (b) Compute $\int_0^5 [x] dx$, where we denote by $[x]$ the largest integer not greater than $x \in \mathbb{R}$. Thus, $[x] = k \in \mathbb{Z}$ if and only if $k \leq x < k + 1$.

3. Use substitution method to find the following

(a) $\int_0^2 \frac{e^x}{\sqrt{1 + e^x}} dx,$

(b) $\int_1^2 \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx.$

4. (a) Draw the area of the region bounded between the lines $y = 1$, $y = x$ and the curve $y = \frac{x^2}{4}$, then find the area using definite integral.
- (b) Draw the area of the region bounded by the curves $y = 2x^3 - x^2 - 5x$ and $y = -x^2 + 3x$ between the intersection points, then find the area using definite integral.