

DEPARTMENT OF MATHEMATICS & STATISTICS

MT234P

Homework 2

Due by 4 p.m. on March 10, 2023.

1. In each case below, state whether the statement is true or false. Justify your answer in each case.

- (i) There is no subset A in \mathbb{R}^2 such that $\text{Bdy}(A)$ contains exactly four points.
- (ii) There is a subset C in \mathbb{R}^3 such that C is not the empty set and $\text{Int}(C)$ is the empty set.

2. Suppose the directional derivative of $g(x, y)$ at $(1, 2)$ in the direction of $\vec{i} + \vec{j}$ is $2\sqrt{2}$ and the directional derivative of $g(x, y)$ at $(1, 2)$ in the direction of $-2\vec{j}$ is -3 . Find the directional derivative of $g(x, y)$ at $(1, 2)$ in the direction of $-\vec{i} - 2\vec{j}$.

3. (i) Suppose $h(x, y) = \cos(y + x^2) + \sin(x - 2y^2)$. Find $\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial^2 h}{\partial x^2}, \frac{\partial^2 h}{\partial y^2}, \frac{\partial^2 h}{\partial y \partial x}$

(ii) Suppose $z = 2u - 3w^2$, $u = e^{2r+3s}$, $w = s - r^2$. Find, $\frac{\partial z}{\partial r}, \frac{\partial z}{\partial s}$ using the chain rule.

4. Suppose $f(x, y) = y^2 - \cos(y + x)$. Is f differentiable at every point in \mathbb{R}^2 ? Justify your answer.

5. (i) Find the directional derivative of $g(x, y, z) = x^3y^2 - e^z \sin(yx)$ at $(1, \frac{\pi}{2}, 0)$ in the direction of $\vec{i} - 2\vec{j} + 3\vec{k}$.

(ii) Find the directions in which $g(x, y) = 2y^2x - 4e^{yx} \sin x$ increases and decreases most rapidly at $(0, 1)$. Also, at what rate does g change in these directions?