MT251 - Euclidean Geometry

Lecture Notes

Angel Cervera Roldan
21319203

24th October 2022

Remakrk 6. Suppose we are in three dimensional space given by $\mathbb{R}^3 = \{(x, y, x) \mid x, y, z \in \mathbb{R}\}$ and suppose that $A, B \in \mathbb{R}^3$
The vector $\vec{u} = \vec{AB}$ can be written as: GIVE A AND B (X, Y, Z) VALUES AND ADD ALGEBRAICALLY

Theorem 2

Suppose $\vec{v} = v_1 i + v_2 j + v_3 k$ and $\vec{w} = w_1 i + w_2 j + w_3 k$ and $t \in \mathbb{R}$.

1. $\vec{v} + \vec{w} = (v_1 + w_1)i + (v_2 + w_2)j + (v_3 + w_3)k$
2. $t\vec{v} = (tv_1 i) + (tv_2 j) + (tv_3)k$
3. $||\vec{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4. $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
5. $\vec{v} \cdot \vec{w} = ||\vec{v}||||\vec{w}|| \cos\theta$, $v, w \neq 0$, where $0 \leq \theta \leq \pi$

Definition 2

Two non 0 vector are said to be perpendicular if the angle between them is $\frac{\pi}{2}$.
The 0 vector is perpendicular to every vector.

Definition 3

Suppose $\vec{u} = u_1 i + u_2 j + u_3 k$ and $\vec{w} = w_1 i + w_2 j + w_3 k$, the cross product of $\vec{u}$ and $\vec{w}$ is denoted by $\vec{u} \times \vec{w}$ and is defined by:

$$\vec{u} \times \vec{w} = (u_2 w_3 - u_3 w_2)i + (u_3 w_1 - u_1 w_3)j + (u_1 w_2 - u_2 w_1)k$$

Remark 8

If $\vec{u}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$, then $u \times w$ gives a vector which is perpendicular to both $\vec{u}$ and $\vec{w}$.

Example 6

Consider the vectors $\vec{u} = i + 2j + k$ and $\vec{w} = 3i + j + 2k$. Find the angle θ between $\vec{u}$ and $\vec{w}$.

$$
\begin{aligned}
\vec{u} \cdot \vec{w} &= 7 \\
||\vec{u}|| &= \sqrt{6} \\
||\vec{w}|| &= \sqrt{14} \\
7 &= \sqrt{6}\sqrt{14}\cos\theta \\
\cos\theta &= \frac{7}{\sqrt{84}} \\
\theta &= \arccos\frac{7}{\sqrt{84}}
\end{aligned}
$$

Vectors in $\mathbb{R}^n$

Definition 4

For $n \geq 1$ we define

$$\mathbb{R}^n = \{(x_1, x_2, ...) \mid x_i \in \mathbb{R} \text{ for } 1 \leq i \leq n\}$$

Theorem 3

Suppose $(x_1, x_2, ..., x_n)$ and $(y_1, y_2, ..., y_n) \in \mathbb{R}^n$.

1. $(x_1, x_2, ..., x_n) + (y_1, y_2, ..., y_n) = (x_1 + y_1, x_2 + y_2, ..., x_n + y_n)$
2. $t(x_1, x_2, ..., x_n) = (tx_1, tx_2, ..., tx_n)$
3. $||(x_1, x_2, ..., x_n)|| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$
4. $\vec{x} \cdot \vec{y} = (x_1y_1, x_2y_2, ..., x_ny_n)$
5. $\vec{v} \cdot \vec{w} = ||\vec{v}||||\vec{w}|| \cos\theta$, $v, w \neq (0, 0, ..., 0)$, where $0 \leq \theta \leq \pi$