MT231 - Analysis 1

Notes

Angel Cervera Roldan

Contents

1 Set theory

1.1 Intro to Set Theory

TODO

1.2 Functions

Definition Function

Let A and B bet two sets, then a function $f : A \to B$ is a set of ordered pairs in $A \times B$ with the property that if (a, b) and (a, b') are elements of f, then $b = b'$

With the above definition, the domain of f is the set of all possible values for a. We can also set restrictions on functions. Take $C \subset A$, then we can restrict f to C:

$$f|_C : C \to B \quad f|_C(x) = f(a) \quad \forall a \in C$$

Some functions have no formula, some do. Some functions can even have multiple branches, for example, take:

$$f = \begin{cases} x + 1 & \text{for } x \leq 0 \\ x - 1 & \text{for } x > 0 \end{cases}$$

Fuctions can also be composed, take $f : A \to B$, and $g : C \to D$, then the composition of f and g is written as $(g \circ f)(x) = g(f(x))$ for all x where $f(x) \in C$.

Definition Injections / One-to-One

A function $f : A \to B$ is one to one if for any pair $a, b \in A$ $f(a) = f(b)$ only if $a = b$.
In other words, there are no two **different** inputs that will yield the same output

Definition Surjections / Onto

A function $f : A \to B$ is onto if $\forall b \in B$, $\exists a \in A$ such that $f(a) = b$

Definition Bijections

A function that is both injective and surjective

Theorem

Let $f : A \longrightarrow B$ be a bijection. Then its inverse, $g = f^{-1}$ exists.

We must define $g : B \longrightarrow A$ such that $(g \circ f)(a) = a \quad \forall a \in A$, furthermore, $(f \circ g)(b) = b \quad \forall b \in B$

Let $b \in B$, because f is onto, then $\exists a \in A$ such that $f(a) = b$. Also, f is one to one, so a is a unique elment. We define $g(b) = a$.

Then $g(f(a)) = g(b) = a$ for all $a \in A$. And $f(g(b)) = f(a) = b$ for all $b \in B$. Therefore $g = f^{-1}$

Theorem

Let $f : A \longrightarrow B$ be a bijection, and $g : B \longrightarrow C$ be a bijection. Then $g \circ f : A \longrightarrow C$ is a bijection.

To show that $g \circ f$ is a bijection, we need to show that it is one to one, and that it is onto.

Take $a_1, a_2 \in A$, if $g(f(a_1)) = g(f(a_1))$, then because g is one to one, $f(a_1) = f(a_2)$, because f is also one to one, then $a_1 = a_2$, therefore $g \circ f$ is one to one.

Take $c \in C$, because g is onto, then $\exists b \in B$ such that $g(b) = c$, and because f is also onto, then $\exists a \in A$ such that $f(a) = b$, therefore, $g \circ f$ is onto.

Because we have shown that $g \circ f$ is onto and one to one, we have proven that $g \circ f$ is a bijection.