

# Design and Control of the Lift Subsystem of a Two-Wheeled Forklift Robot

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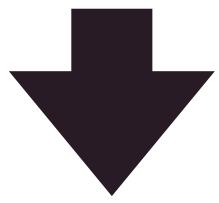
2nd year – Master's degree





# I. Introduction

In modern industrial environments, humans and robots work together.



- Labor shortage in industrialized countries.
- High-risk tasks executed by robots.



[A]

Mobile robots should have a small footprint for safety and space optimization reasons.



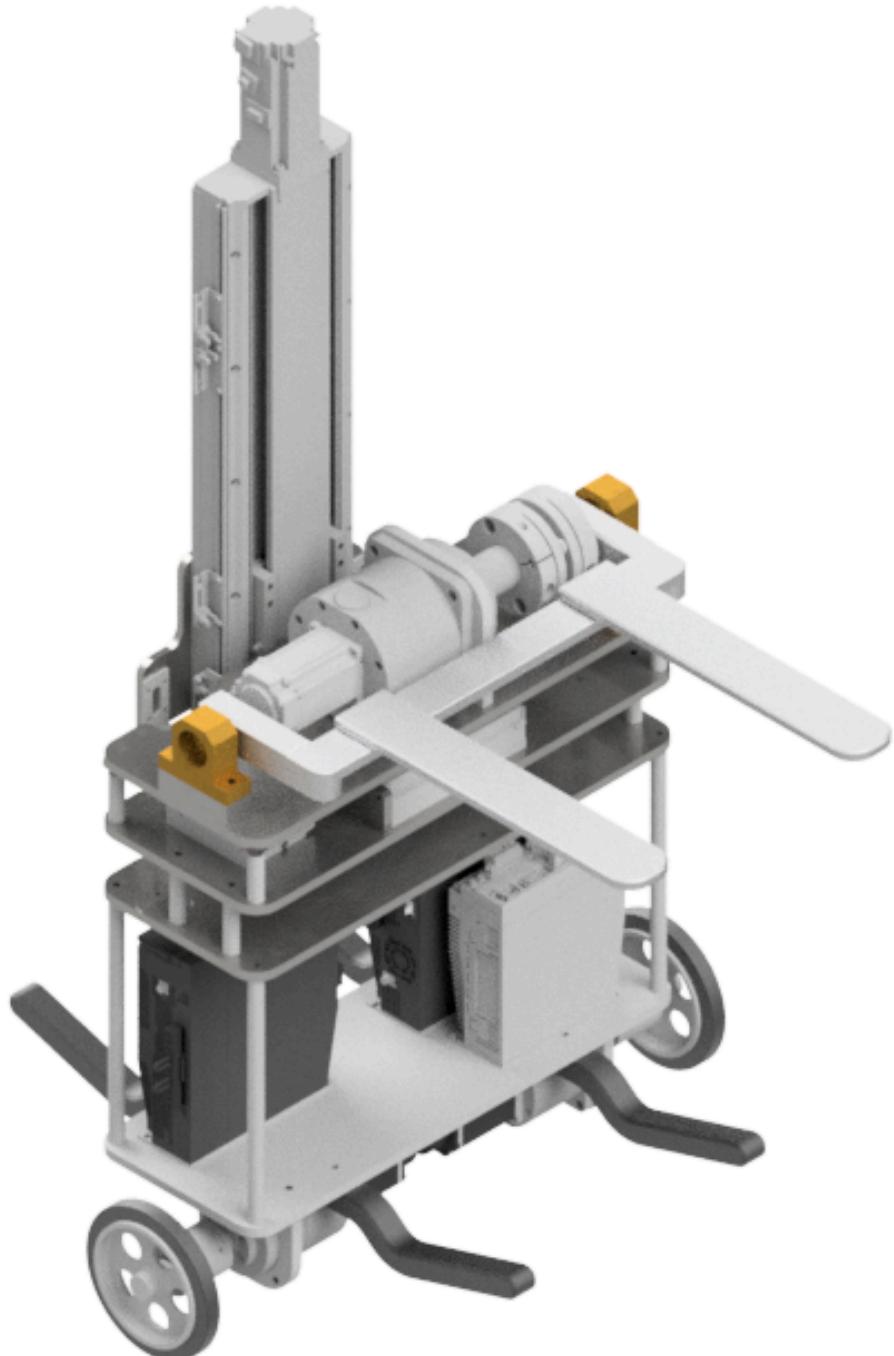
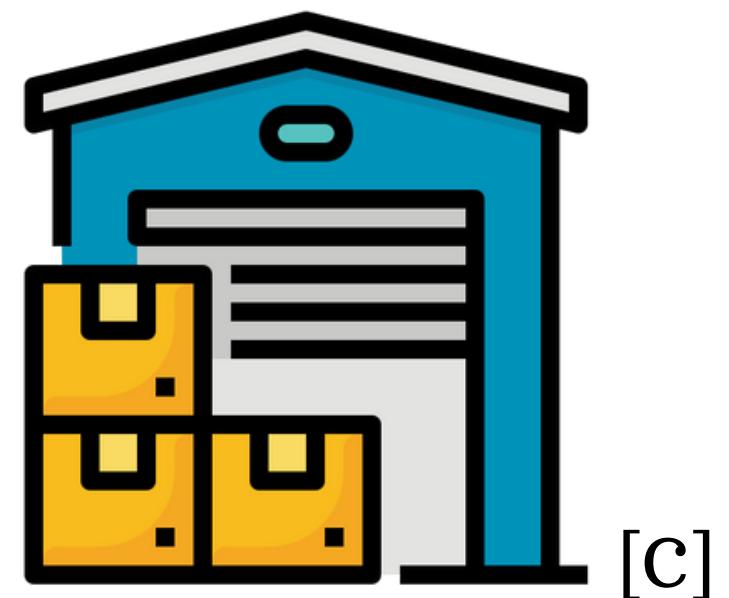
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# I. Introduction

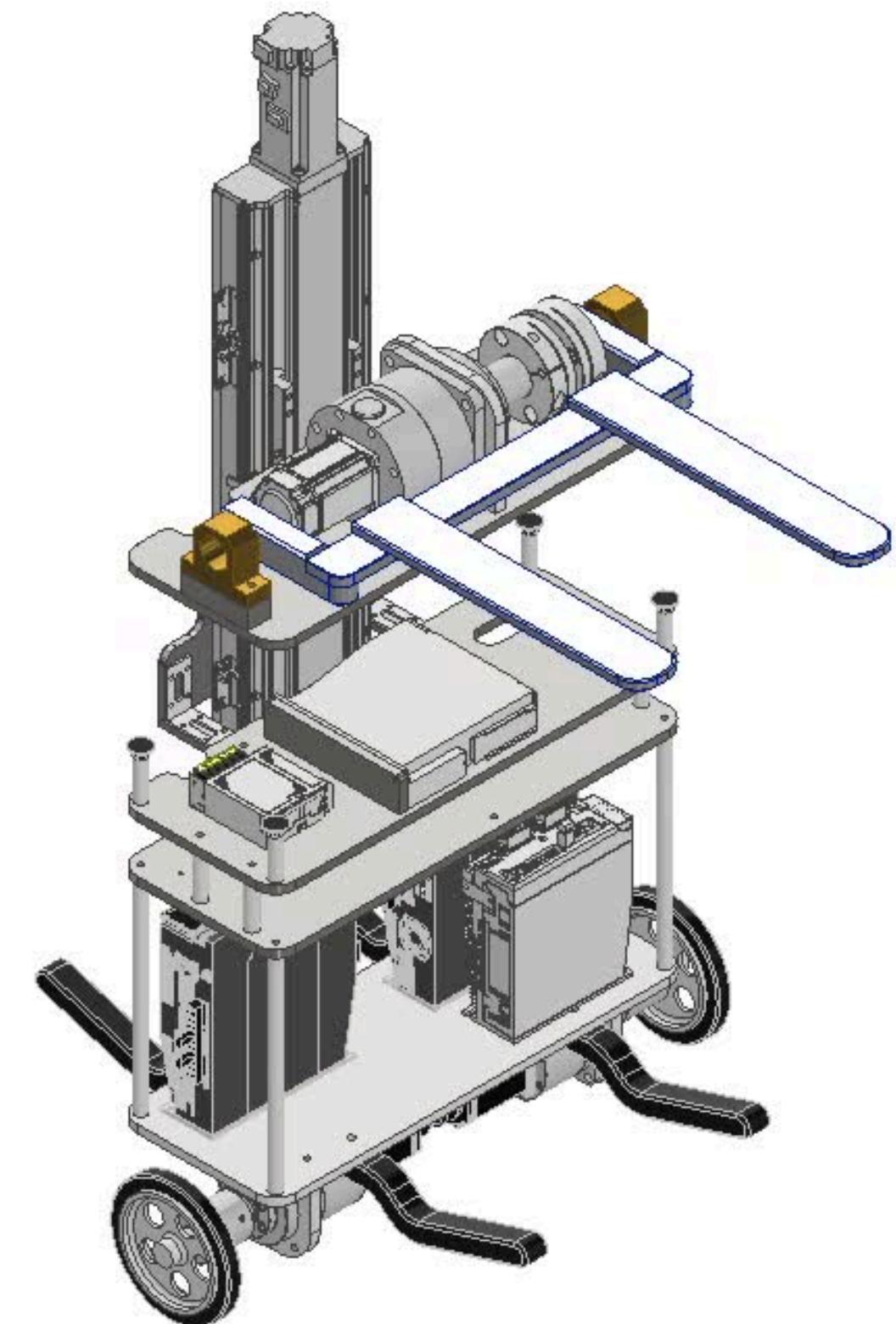
Two-Wheeled Forklift Robot:

- Aimed for load movement in warehouses.
- Self-balanced mobile robot.
- Small footprint.





## II. Conceptual mechanical design





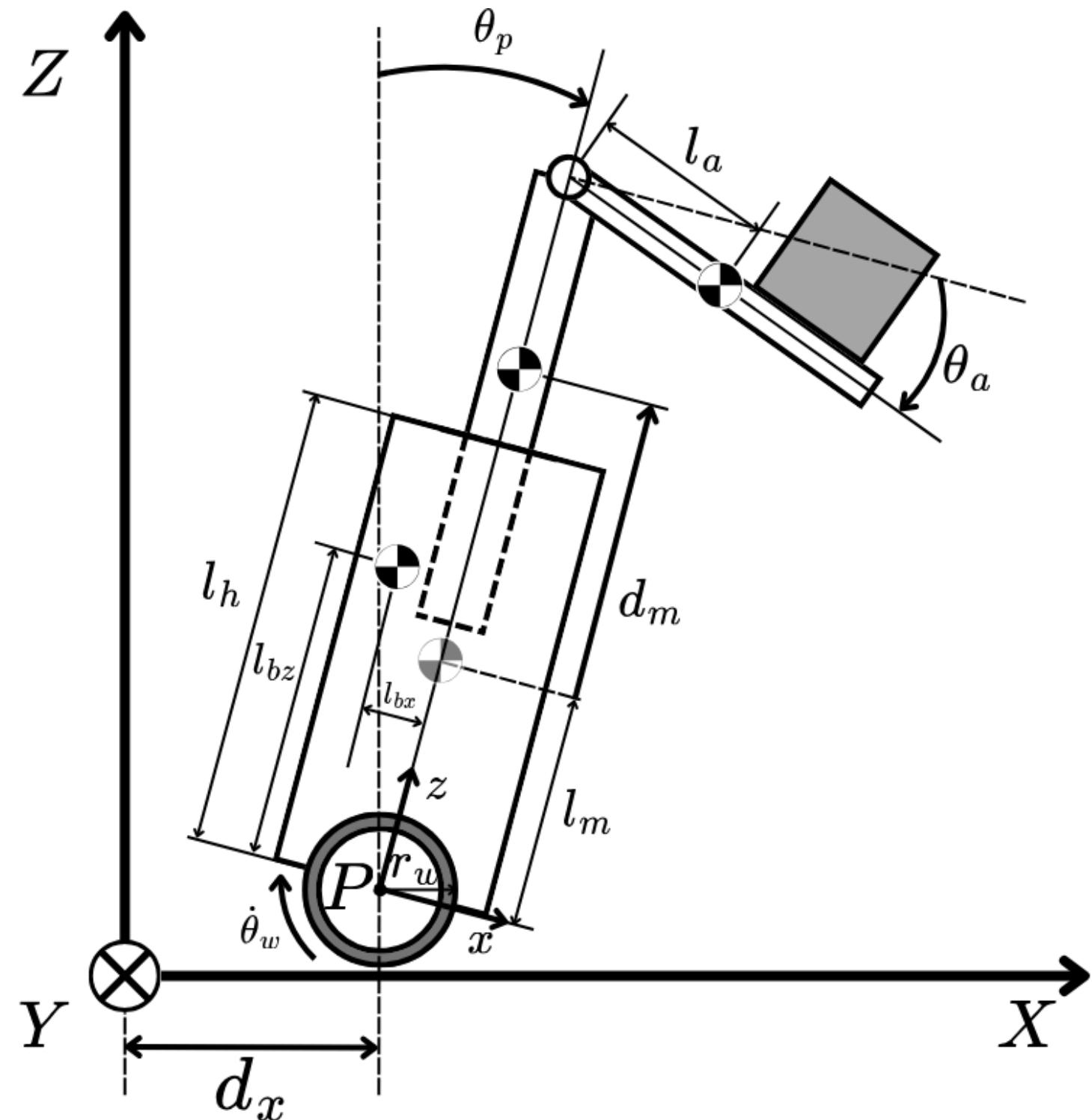
# II. Conceptual mechanical design

## Ball screw linear actuator ETH2-17-L5-350-BC-M40B-E5N5L

Repeatability		mm	( +/-0.005)							
Lead	mm	5	10	20	40					
Maximum Rotating Speed(mm/s)	rpm	3600	3600	3600	3600					
Maximum Linear Speed	mm/s	300	600	1200	2400					
Maximum Payload	Horizontal	kg	120	110	75	16				
	Vertical (For Non-generative power)	kg	40	25	11	9				
	Vertical(external 50W regenerative register)	Kg	50	30	18	15				
Rated Thrust	N	1389	694	347	174					
Stroke Pitch	mm	50-1200mm/50 intervals(50mm pitch)								
Maximum Acceleration	G	0.15	0.31	0.61	1.22					
Ball Screw	Basic Dynamic load rating Ca	N	13428	11223	5667	9656				
	Basic Static load rating Coa	N	29671	24456	11182	20274				
Linear Guide	Dynamic Horizontal	N	7866							
	Static Horizontal	N	78400							
Fixed Bearing	Bsic Dynamic load rating Cr	N	9666							
	Static Load rating Cor	N	6400							
AC Actuator with Motor Output	W	400								
Ball Screw	mm	C7 X ø20								
Linear Guide	mm	W15XH12.5								
Ball Screw	mm	14 X 12								
Home Sensor	Outside	T64N2(NPN)								



### III. Modeling



Kinematics:

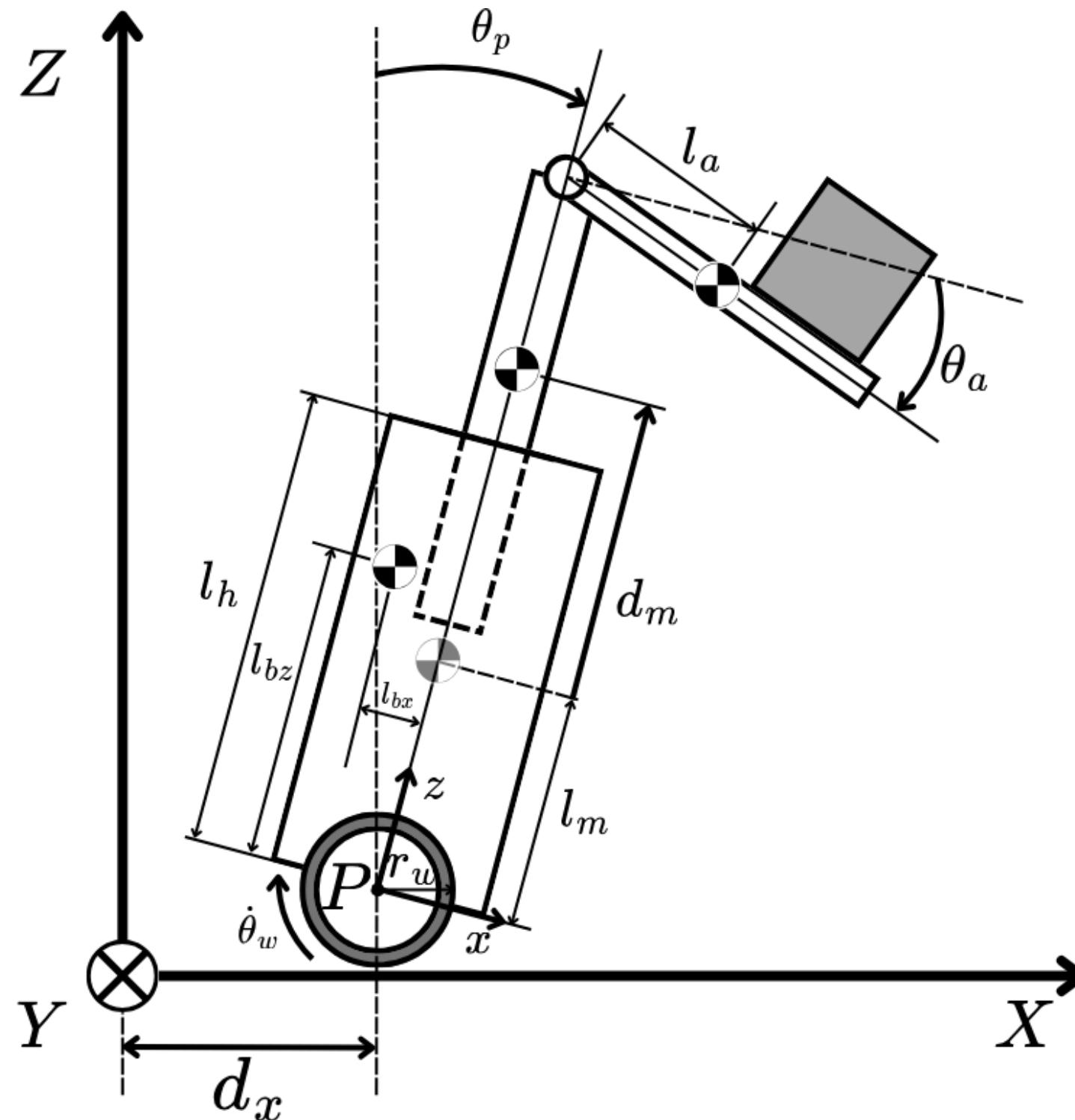
$\theta_w$  : Wheel angle,  $\theta_p$  : Pitch angle,  
 $d_m$  : Lift displacement,  $\theta_a$  : Arm angle

$$\theta_p + \theta_a = 0$$

$$H_T^B = \begin{pmatrix} 1 & 0 & 0 & s_p(l_h + d_m) + l_a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r_w + c_p(l_h + d_m) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



### III. Modeling



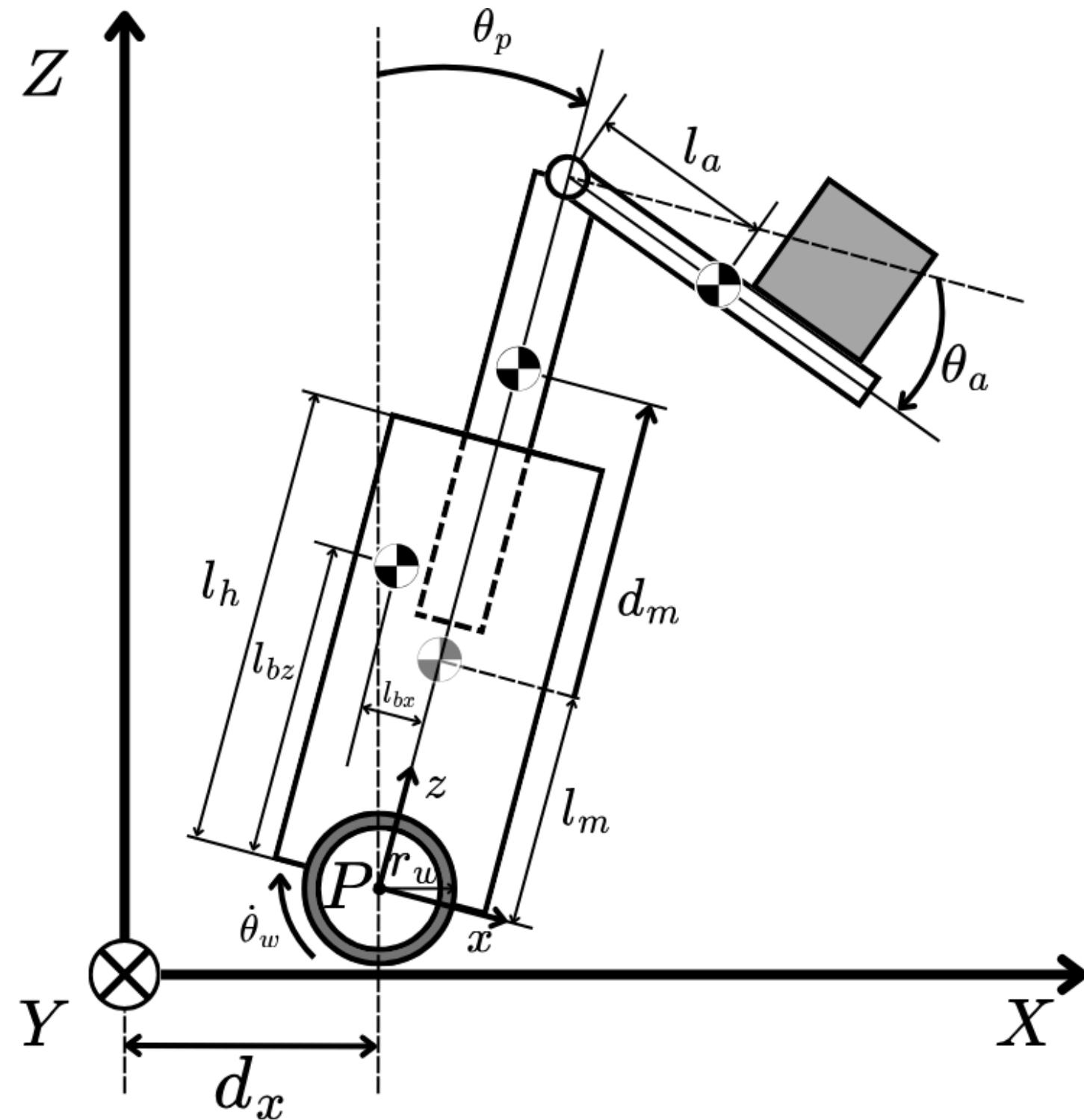
**Kinematics:**

$$CoG = \left( \begin{array}{c} \frac{m_l \left( d_m s_p + l_h s_p + \frac{t_1}{g m_l} \right) + m_a (l_a + d_m s_p + l_h s_p) + m_b (l_{bz} s_p - l_{bx} c_p) + m_m (d_m + l_m) s_p}{m_a + m_b + m_l + m_m} \\ \frac{(m_a + m_l) (d_m c_p + l_h c_p) + m_b (l_{bz} c_p + l_{bx} s_p) + m_m (d_m + l_m) c_p}{m_a + m_b + m_l + m_m} \end{array} \right)$$

$$\begin{cases} r_w + c_p(l_h + d_m) = z \\ c_p \dot{d}_m - s_p(l_h + d_m) \dot{\theta}_p = \dot{z} \\ CoG_x(\theta_p, d_m, m_l, t_1) = \sin(\theta_t) l_t \\ \dot{\theta}_p = 0 \end{cases}$$



### III. Modeling



Dynamics:

$$M(q)\ddot{q} + H(\dot{q}, q) + G(q) = \tau$$

Generalized coordinates:

$$q = [\theta_w \quad \theta_p \quad d_m \quad \theta_a]^T$$

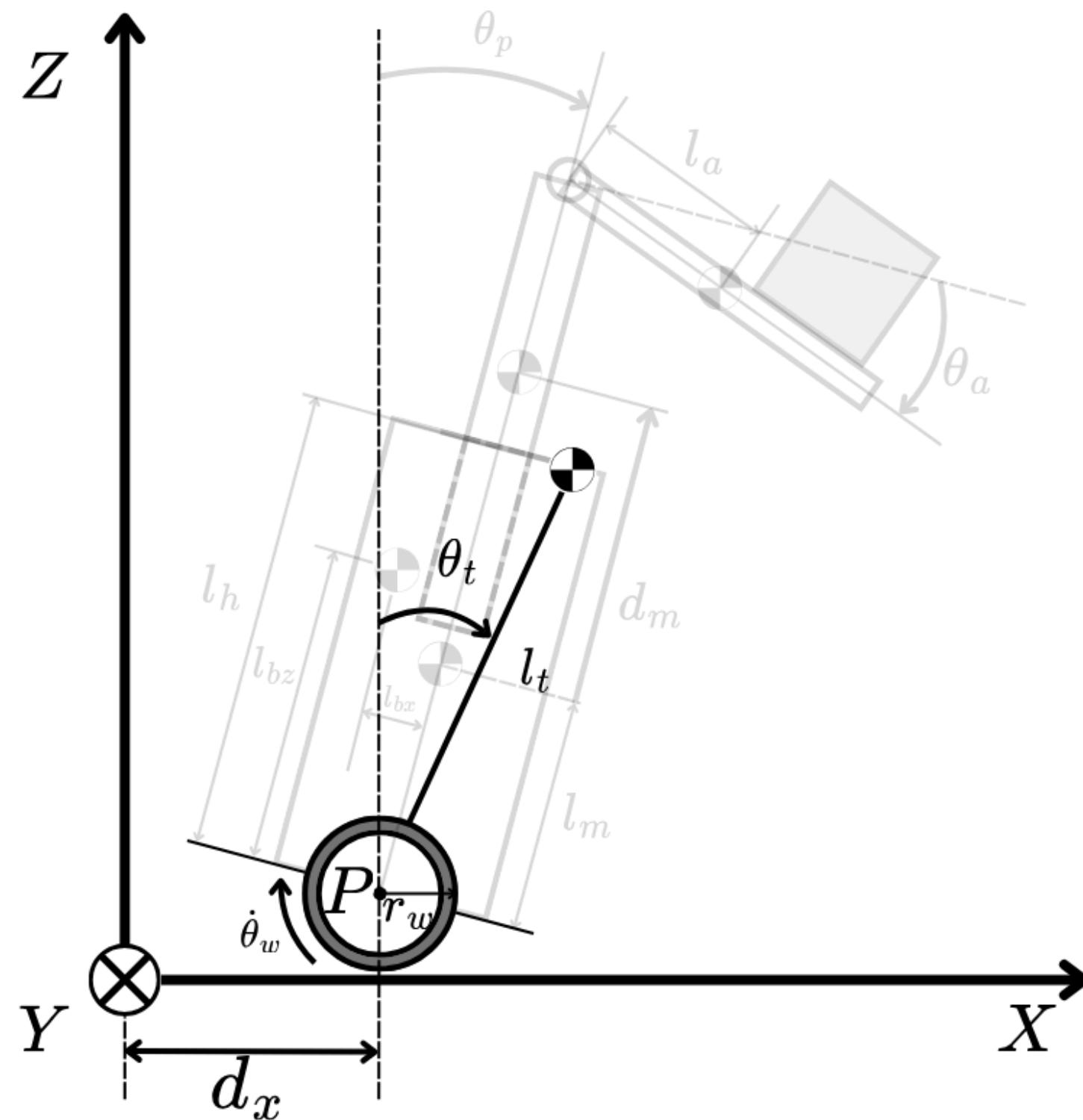
$$\tau = [n_w\tau_w \quad -n_w\tau_w \quad f_m \quad n_a\tau_a]^T$$

$\tau_w$  : Wheel torque,  $f_m$  : Lift force,

$\tau_a$  : Arm torque



### III. Modeling



Simplified dynamics:

$$M^*(q')\ddot{q}' + H^*(\dot{q}', q') + G^*(q') = \tau^*(\ddot{x})$$

Generalized coordinates:

$$q' = [\theta_w \quad \theta_t]^T$$

$$\tau^*(\ddot{x}) = [n_w \tau_w(\ddot{x}) \quad -n_w \tau_w(\ddot{x})]^T$$

$$\theta_t = \text{atan}\left(\frac{CoG_x}{CoG_z}\right), \quad l_t = \|CoG\|$$



## IV. Control

Synthesized Pitch Angle Disturbance Observer (SPADO):

$$m_{11}\ddot{\theta}_w + m_{12}\ddot{\theta}_p + m_{13}\ddot{d}_m + m_{14}\ddot{\theta}_a + h_1 = n_w\tau_w - T_{lw}$$

$$m_{n11}\ddot{\theta}_w^{res} = n_w\tau_w^{ref} - \tilde{\tau}_w^{dist}$$

$$\tilde{\tau}_w^{dist} = (m_{11} - m_{n11})\ddot{\theta}_w + m_{12}\ddot{\theta}_p + m_{13}\ddot{d}_m + m_{14}\ddot{\theta}_a + h_1 + T_{lw}$$

$$m_{21}\ddot{\theta}_w + m_{22}\ddot{\theta}_p + m_{23}\ddot{d}_m + m_{24}\ddot{\theta}_a + h_2 + g_2 = -n_w\tau_w - T_{lp}$$

$$m_{n21}\ddot{\theta}_w^{res} + m_{n22}\ddot{\theta}_p^{res} = -n_w\tau_w^{ref} - \tilde{\tau}_p^{dist}$$

$$\tilde{\tau}_p^{dist} = (m_{21} - m_{n21})\ddot{\theta}_w + (m_{22} - m_{n22})\ddot{\theta}_p + m_{23}\ddot{d}_m + m_{24}\ddot{\theta}_a + h_2 + g_2 + T_{lp}$$



## IV. Control

SPADO:

$$m_{n22}\ddot{\theta}_p^{res} + \frac{m_{n21} + m_{n11}}{m_{n11}}n_w\tau_w^{ref} = \frac{m_{n21}}{m_{n11}}\tilde{\tau}_w^{dis} - \tilde{\tau}_p^{dist}$$

$$\tilde{\tau}_s^{dist} = \tilde{\tau}_p^{dist} - \frac{m_{n21}}{m_{n11}}\tilde{\tau}_w^{dis}$$

$$m_{n22}\ddot{\theta}_p^{res} + \frac{m_{n21} + m_{n11}}{m_{n11}}n_w\tau_w^{ref} = -\tilde{\tau}_s^{dist}$$

$$\hat{\tau}_s^{dist} = \frac{g_s}{s + g_s}(g_s m_{n22}\dot{\theta}_p^{res} - \frac{m_{n21} + m_{n11}}{m_{n11}}n_w\tau_w^{ref}) - g_s m_{n22}\dot{\theta}_p^{res}$$

Pseudo differentiation



## IV. Control

PD controller pitch angle:

$$V = \frac{1}{2}K_1(\theta_p^{cmd} - \theta_p^{res})^2 + \frac{1}{2}K_2(\dot{\theta}_p^{cmd} - \dot{\theta}_p^{res})^2$$

$$\dot{V} = (\dot{\theta}_p^{cmd} - \dot{\theta}_p^{res})(K_1(\theta_p^{cmd} - \theta_p^{res}) + K_2(\ddot{\theta}_p^{cmd} - \boxed{\ddot{\theta}_p^{res}}))$$

$$\dot{V} = -K_3(\dot{\theta}_p^{cmd} - \dot{\theta}_p^{res})^2$$

$$\tau_w^{ref} = -\frac{m_{n11}m_{n22}}{n_w(m_{n21} + m_{n11})} \left( K_{pp}(\boxed{\theta_p^{cmd}} - \theta_p^{res}) + K_{dp}(\boxed{\dot{\theta}_p^{cmd}} - \dot{\theta}_p^{res}) + \boxed{\ddot{\theta}_p^{cmd}} \right) - \frac{m_{n11}}{n_w(m_{n21} + m_{n11})} \boxed{\hat{\tau}_s^{dist}}$$

$$K_{pp} = \frac{K_1}{K_2}, \quad K_{dp} = \frac{K_3}{K_2}$$



## IV. Control

Fork Disturbance Observer (FDOB):

$$m_{41}\ddot{\theta}_w + m_{42}\ddot{\theta}_p + m_{43}\ddot{d}_m + m_{44}\ddot{\theta}_a + h_4 + g_4 = n_a\tau_a - T_{la}$$

$$m_{n44}\ddot{\theta}_a^{res} = n_a\tau_a^{ref} - \tilde{\tau}_a^{dist}$$

$$\tilde{\tau}_a^{dist} = m_{41}\ddot{\theta}_w^{res} + m_{42}\ddot{\theta}_p^{res} + m_{43}\ddot{d}_m^{res} + (m_{44} - m_{n44})\ddot{\theta}_a^{res} + h_4 + g_4 + T_{la}$$

$$\hat{\tau}_a^{dist} = \frac{g_a}{s + g_a}(n_a\tau_a^{ref} + g_a m_{n44}\dot{\theta}_a^{res}) - g_a m_{n44}\dot{\theta}_a^{res} \rightarrow \text{Pseudo differentiation}$$

PD controller fork angle:

$$\tau_a^{ref} = K_{pa}(\theta_a^{cmd} - \theta_a^{res}) + K_{da}(\dot{\theta}_a^{cmd} - \dot{\theta}_a^{res}) + \hat{\tau}_a^{dist}$$



## IV. Control

Fork Reaction Torque Observer (FRTOB):

$$T_{la} = \tilde{\tau}_a^{ext} + \tilde{\tau}_a^{fric}$$

$$\tilde{\tau}_a^{reac} = m_{41}\ddot{\theta}_w^{res} + m_{42}\ddot{\theta}_p^{res} + m_{43}\ddot{d}_m^{res} + (m_{44} - m_{n44})\ddot{\theta}_a^{res} + h_4 + \tilde{\tau}_a^{ext}$$

$$m_{n44}\ddot{\theta}_a^{res} = n_a \tau_a^{ref} - g_4 - \tilde{\tau}_a^{fric} - \tilde{\tau}_a^{reac}$$

$$\hat{\tau}_a^{reac} = \frac{g_r}{s + g_r} (n_a \tau_a^{ref} + g_r m_{n44} \dot{\theta}_a^{res} - g_4 - \tilde{\tau}_a^{fric}) - g_r m_{n44} \dot{\theta}_a^{res}$$

Pseudo differentiation

$$t_1 = -\hat{\tau}_a^{reac}$$



## IV. Control

Lift Disturbance Observer (LDOB):

$$m_{31}\ddot{\theta}_w + m_{32}\ddot{\theta}_p + m_{33}\ddot{d}_m + m_{34}\ddot{\theta}_a + h_3 + g_3 = f_m - F_{lm}$$

$$m_{n33}\ddot{d}_m^{res} = f_m^{ref} - \tilde{f}_m^{dist}$$

$$\tilde{f}_m^{dist} = m_{31}\ddot{\theta}_w^{res} + m_{32}\ddot{\theta}_p^{res} + (m_{33} - m_{n33})\ddot{d}_m^{res} + m_{34}\ddot{\theta}_a^{res} + h_3 + g_3 + F_{lm}$$

$$\hat{f}_m^{dist} = \frac{g_m}{s + g_m}(f_m^{ref} + g_m m_{n33} \dot{d}_m^{res}) - g_m m_{n33} \dot{d}_m^{res} \rightarrow \text{Pseudo differentiation}$$

PD controller lift displacement:

$$f_m^{ref} = K_{pm}(d_m^{cmd} - d_m^{res}) + K_{dm}(\dot{d}_m^{cmd} - \dot{d}_m^{res}) + \hat{f}_m^{dist}$$



## IV. Control

Lift Reaction Force Observer (LRFOB):

$$F_{lm} = \tilde{f}_m^{ext} + \tilde{f}_m^{fric}$$

$$\tilde{f}_m^{reac} = m_{31}\ddot{\theta}_w^{res} + m_{32}\ddot{\theta}_p^{res} + (m_{33} - m_{n33})\ddot{d}_m^{res} + m_{34}\ddot{\theta}_a^{res} + h_3 + \tilde{f}_m^{ext}$$

$$m_{n44}\ddot{d}_m^{res} = f_m^{ref} - g_3 - \tilde{f}_m^{fric} - \tilde{f}_m^{reac}$$

$$\hat{f}_m^{reac} = \frac{g_r}{s + g_r} (f_m^{ref} + g_r m_{n33} \dot{d}_m^{res} - g_3 - \tilde{f}_m^{fric}) - g_r m_{n33} \dot{d}_m^{res}$$

Pseudo differentiation

$$m_l = \frac{\hat{f}_m^{reac}}{g \cos(\theta_p)}$$



## IV. Control

Wheel Position Control:

$$\theta_p^{cmd-PD} = K_{pw}(\theta_w^{cmd} - \theta_w^{res}) + K_{dw}(\dot{\theta}_w^{cmd} - \dot{\theta}_w^{res})$$

Total pitch command:

$$\theta_p^{cmd} = \theta_p^{cmd-IS} + \theta_p^{cmd-PD}$$



# V. Results

<b>Parameter</b>	<b>Explanation</b>	<b>Value</b>
$K_{pp}$	P-gain of pitch angle control	200
$K_{pd}$	D-gain of pitch angle control	50
$K_{pa}$	P-gain of arm position control	200
$K_{da}$	D-gain of arm position control	50
$K_{pm}$	P-gain of lift position control	200
$K_{dm}$	D-gain of lift position control	50
$K_{pw}$	P-gain of wheel position control	0.07
$K_{dw}$	D-gain of wheel position control	0.09
$g_s$ (rad/s)	Cutoff angular frequency of SPADO	$2\pi 6$
$g_a$ (rad/s)	Cutoff angular frequency of arm FDOB	$2\pi 5$
$g_r$ (rad/s)	Cutoff angular frequency of FRTOB, LRFOB	$2\pi 1$
$g_m$ (rad/s)	Cutoff angular frequency of LDOB	$2\pi 4$
$N_w$	Wheel gear ratio	11
$N_a$	Arm gear ratio	33

<b>Parameter</b>	<b>Explanation</b>	<b>Value</b>
$I_{wy}$ ( $\text{kg m}^2$ )	Wheel inertia	0.0052
$I_{by}$ ( $\text{kg m}^2$ )	Body inertia	4.0590
$I_{my}$ ( $\text{kg m}^2$ )	Lift inertia	0.0482
$I_{ay}$ ( $\text{kg m}^2$ )	Arm inertia	0.0890
$l_{bz}$ (m)	Body CoG z coordinate	0.3206
$l_{bx}$ (m)	Body CoG x coordinate	0.0246
$l_m$ (m)	Lift CoG distance	0.3975
$l_a$ (m)	Arm CoG distance	0.2200
$m_w$ (kg)	Wheel mass	1.21
$m_b$ (kg)	Body mass	64.1
$m_m$ (kg)	Lift mass	21.5
$m_a$ (kg)	Arm mass	3.5
$m_l$ (kg)	Load mass	5



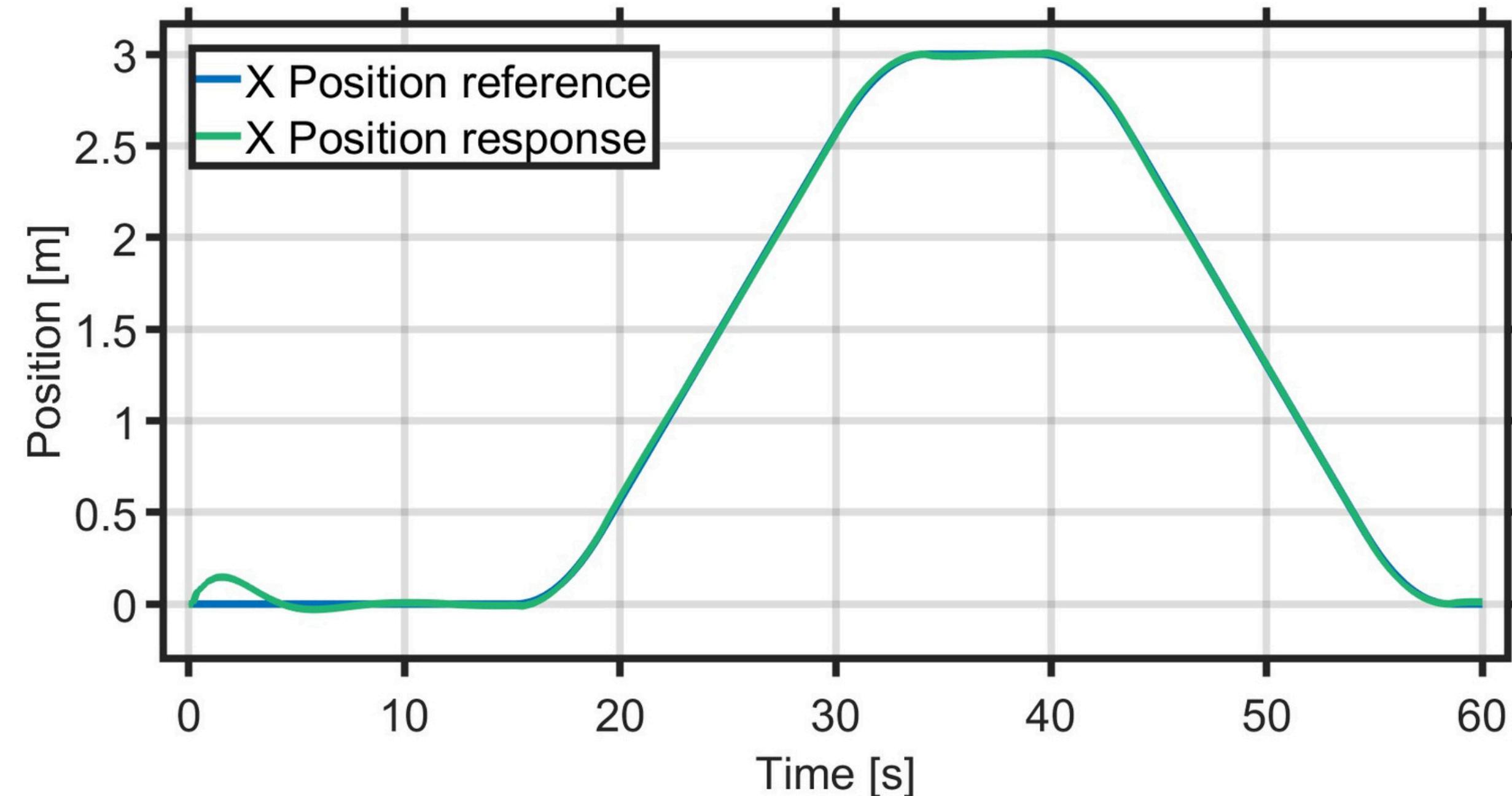
# V. Simulation





## V. Simulation

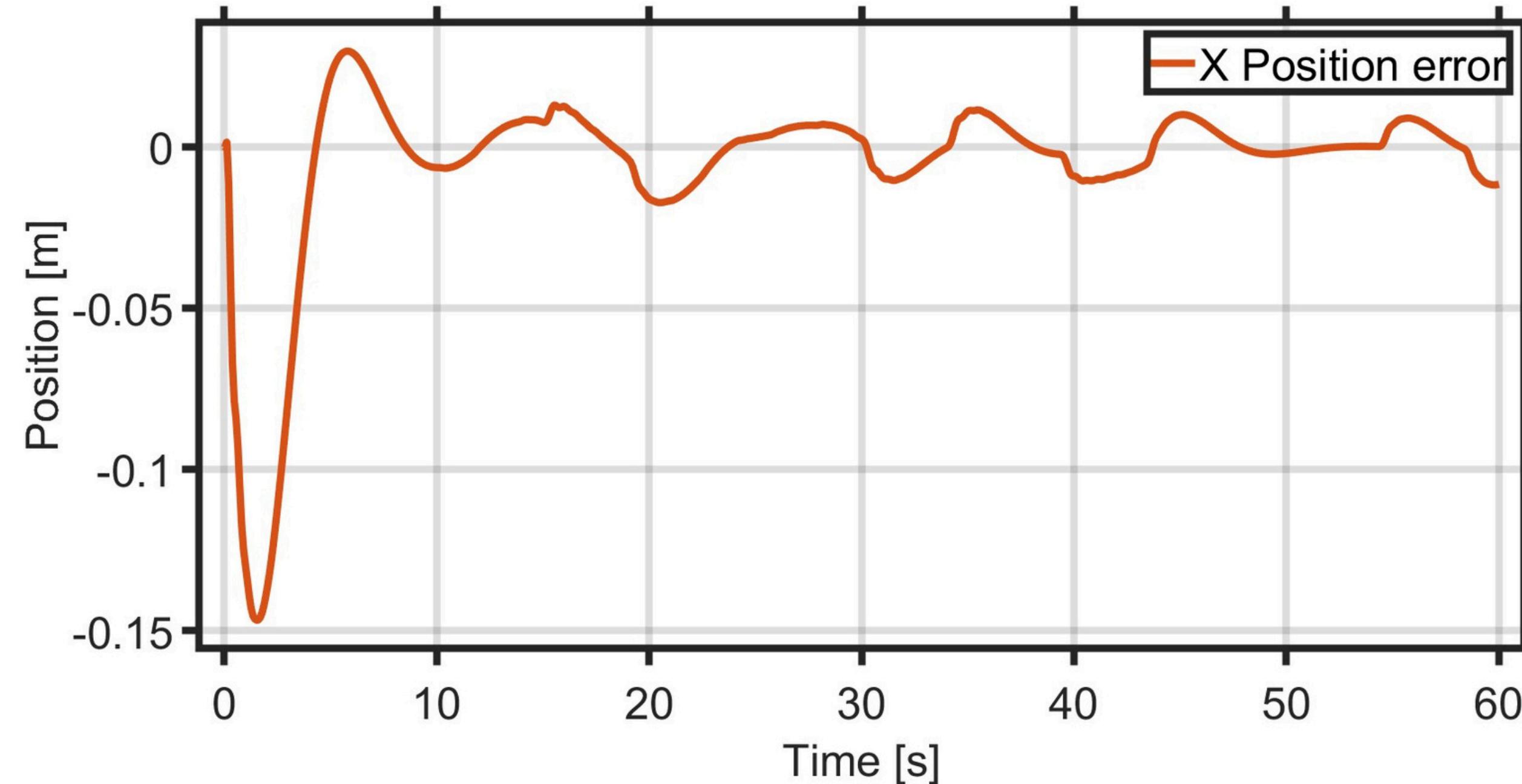
Horizontal displacement of the robot





## V. Simulation

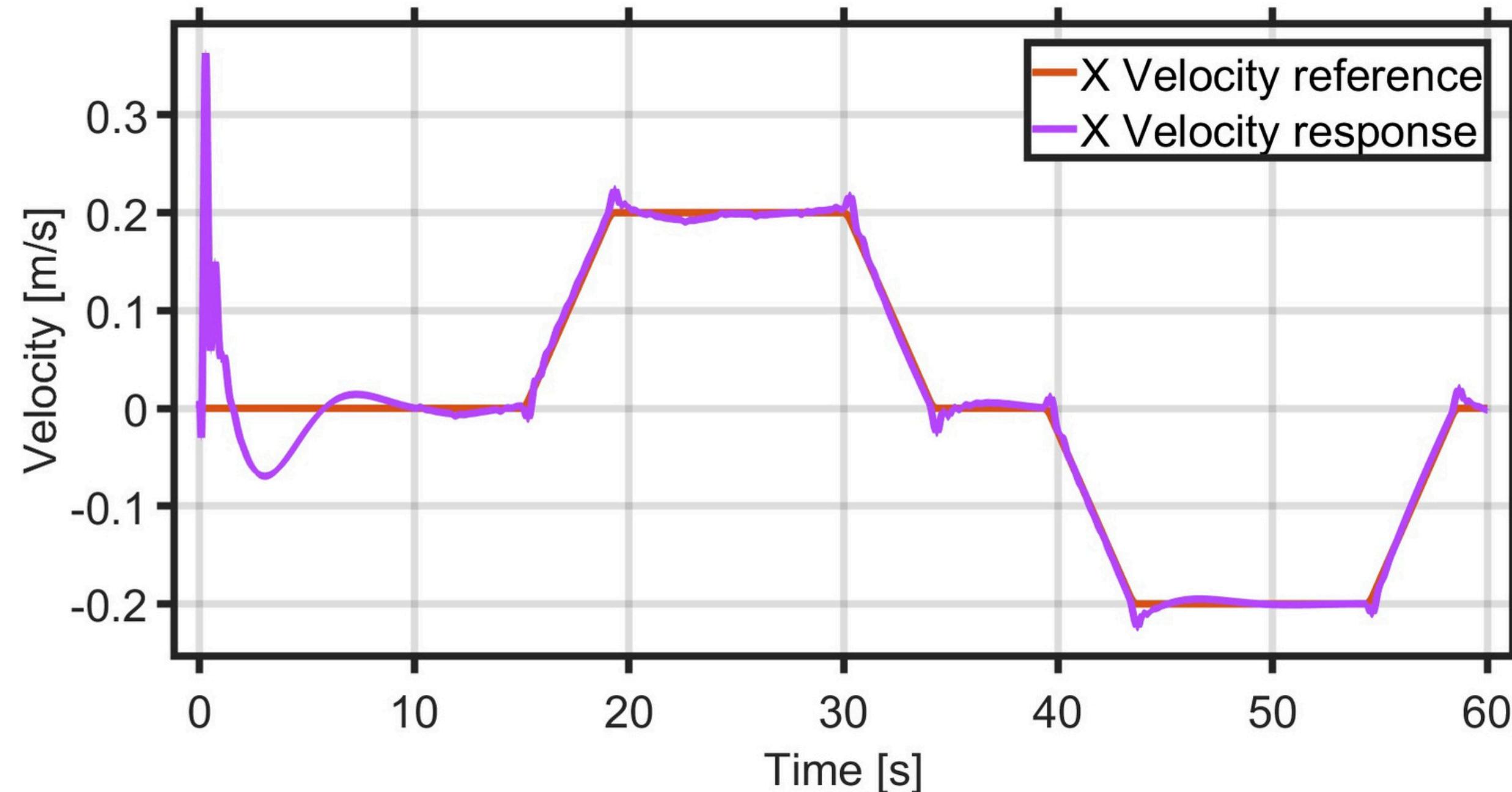
Position error of the robot





# V. Simulation

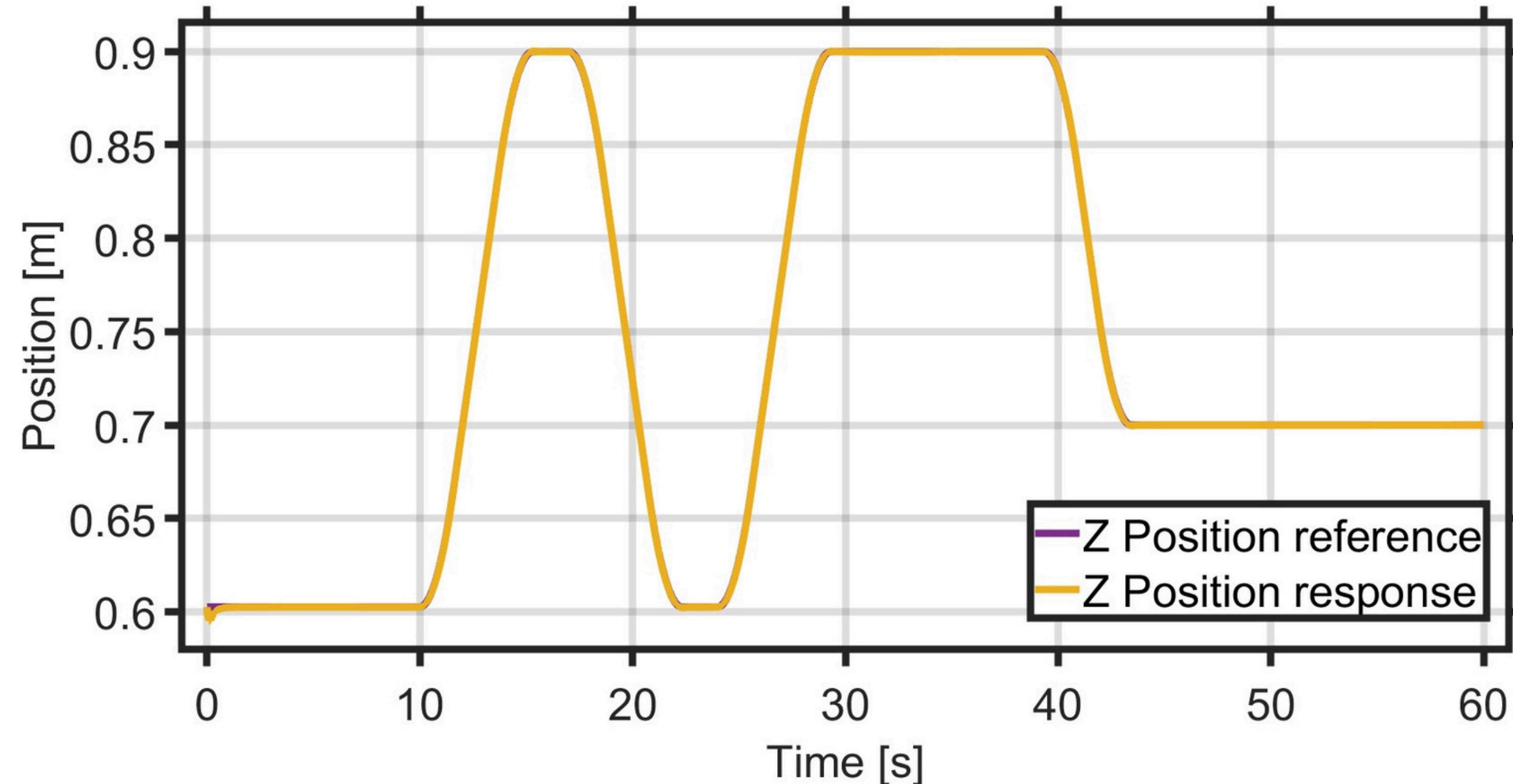
Horizontal velocity of the robot





## V. Simulation

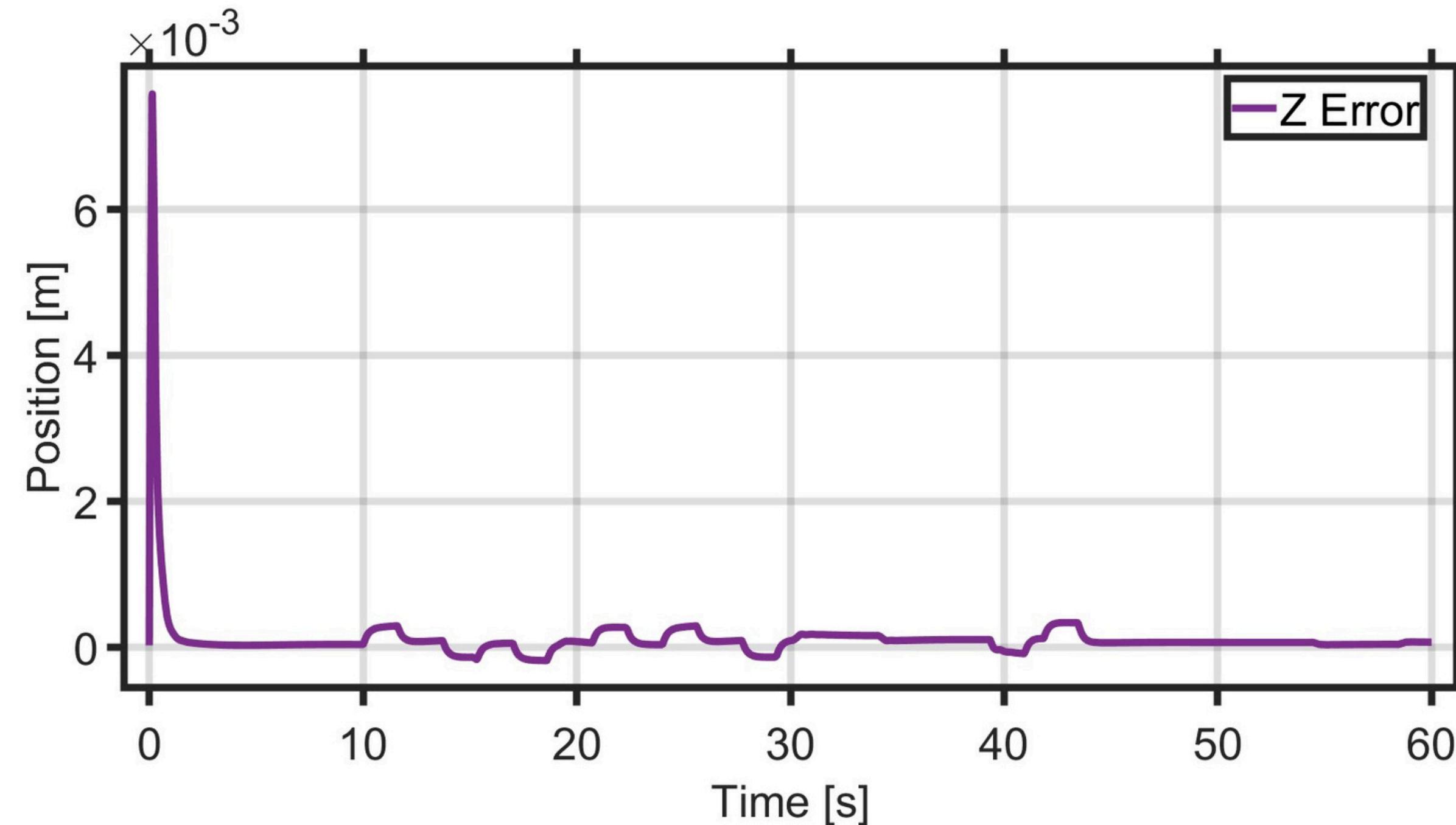
Vertical position of the fork





# V. Simulation

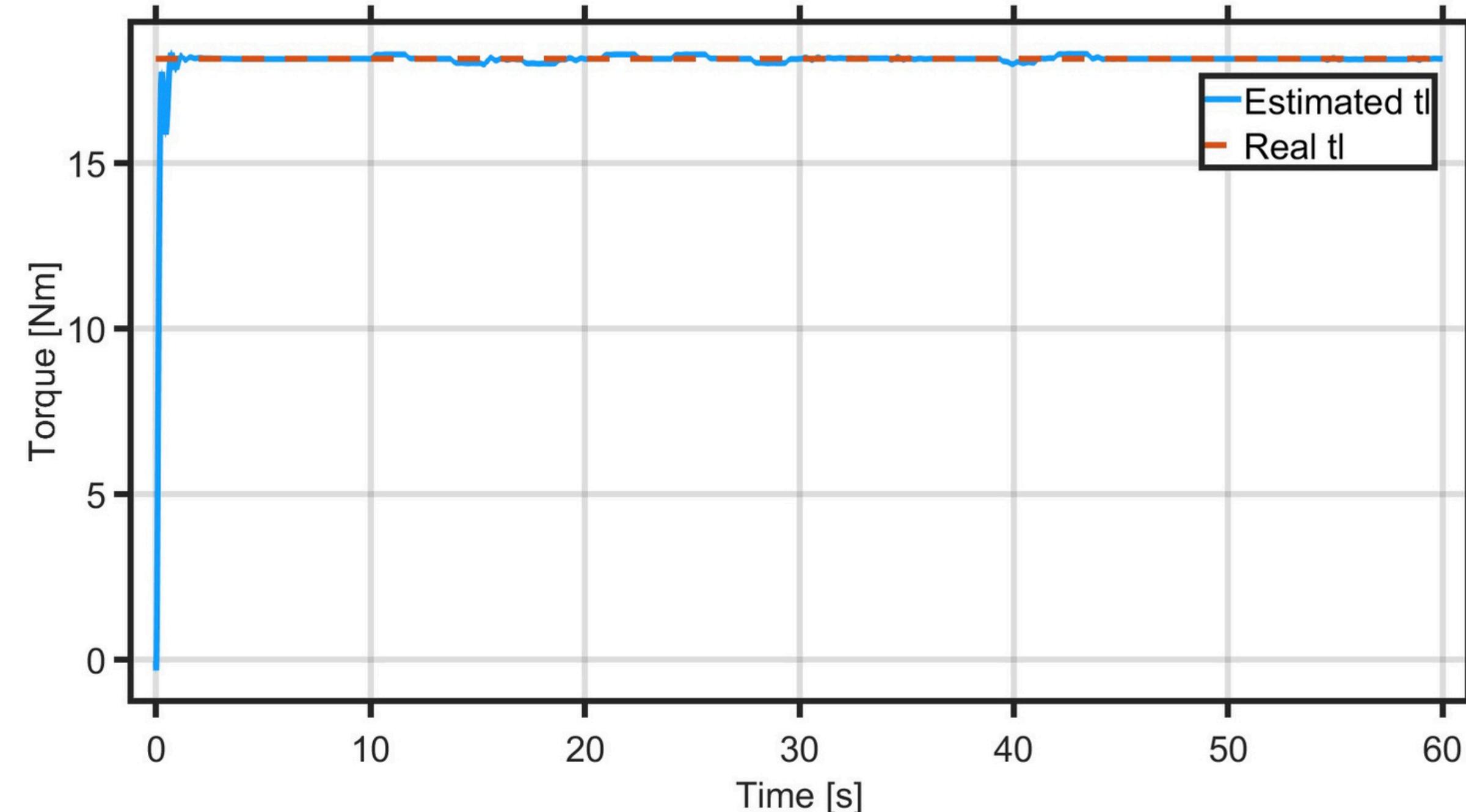
Position error of the fork





## V. Simulation

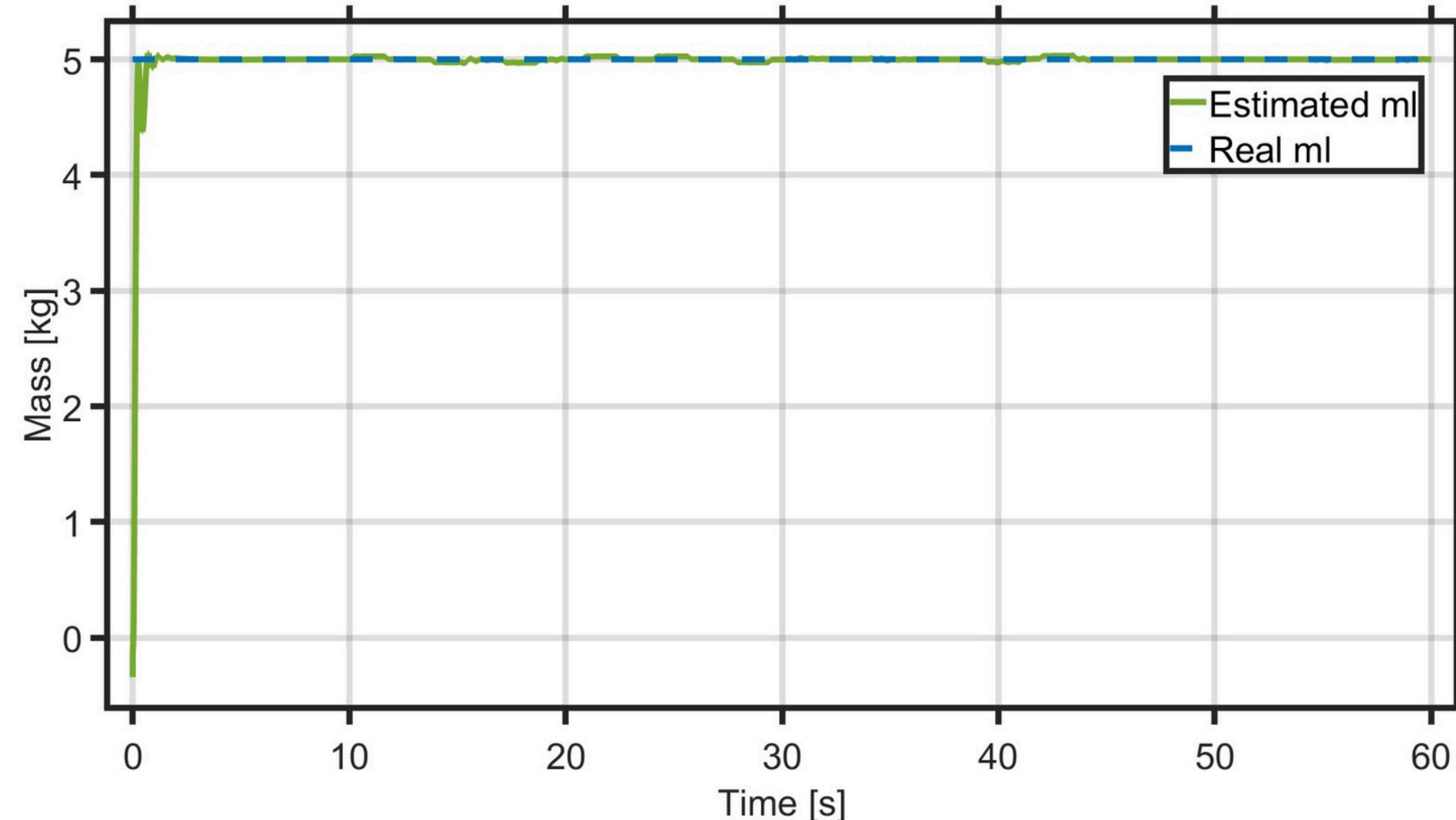
Estimated external torque over the fork (18.142Nm)





## V. Simulation

Estimated mass of the external load (5kg)





# VI. Conclusions and Future work

- The proposed motion planning and control strategies allow the TWFR to execute trajectories in the X and Z coordinate axes simultaneously with an admissible position error.
- The implementation of adaptive IK and ID solutions that depend on the estimation of physical properties of the external load provides great flexibility in industrial environments where it is not possible to control the variables of the load that will be manipulated.
- The next steps for this work are the construction of the lift subsystem and the physical implementation of the presented motion planning and control algorithms.

# References

- [1] H. Kanazawa, K. Ishizaki, Y. Miyata, M. Nawa, N. Kato, and T. Murakami, "Model-Based Pitch Angle Compensation for Center of Gravity Variation in Underactuated System with an Arm," in Proceedings of the 2023 IEEE 32nd International Symposium on Industrial Electronics (ISIE), 2023, pp. 1-6.
- [2] H. Yajima, K. Ishizaki, Y. Miyata, M. Nawa, N. Kato, and T. Murakami, "Posture Stabilization Control Compensating Variation of Body Center of Gravity in Underactuated System," in Proceedings of the 2023 IEEE International Conference on Mechatronics (ICM), 2023, pp. 1-6.
- [3] J. Ito and T. Murakami, "Underactuated Control for Two-Wheeled Mobile Robot with an Arm Using Torque Constraint Conditions and Disturbance Observer," in Proceedings of the 2023 IEEE 32nd International Symposium on Industrial Electronics (ISIE), 2023, pp. 1-6.
- [4] Mezawa, M. (2022). Virtual Suspension Control in Two-Wheeled Transport Robot (Master's thesis, Keio University).
- [A] <https://www.flaticon.com/free-icons/comparison>
- [B] <https://www.flaticon.com/free-icons/standard>
- [C] <https://www.flaticon.com/free-icons/storage>

# Thank you!