18.06 – linear algebra

read Course Info

lectures

- please do not use audio
- ask any questions via chat
  Mayowa will answer

Course Admin = Jeremy Hahn

excuses/absences/etc

S^3

recitations + office hours

- must attend synchronously
  ↳ the one you are enrolled
  (Theresa can help with sign-up)
- if you want the in-person recitation, sign-up ASAP, because you must be enrolled in CovidPass

TBA shortly through Canvas

Linear Algebra = a language for dealing with $\begin{cases} \text{coordinates} \\ \text{linear eqns} \\ \text{vectors} \\ \text{geometric transformations} \end{cases}$

machine

how to use it

how/why it works

system of linear eqns:

goal is to solve

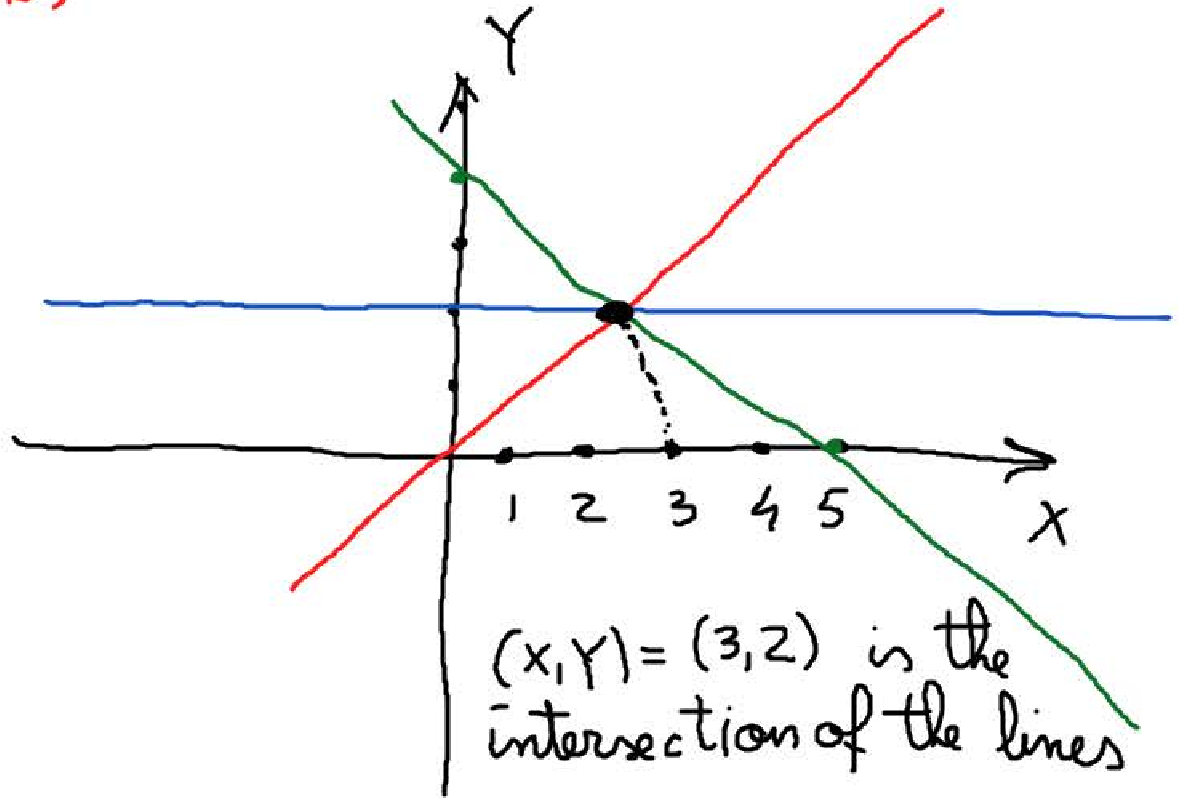
$(x,y) = (3,2)$

$$\begin{cases} 2x - 3y = 0 \\ x + y = 5 \\ 2y = 4 \end{cases} \rightsquigarrow 2\cdot 3 - 3\cdot 2 = 0$$

BACK-SUBSTITUTION

$$\Downarrow \quad y = 2$$

$$\Downarrow \quad x = 5 - y = 5 - 2 = 3$$



$(x,y) = (3,2)$ is the intersection of the lines

$$\begin{cases} 2X - 3Y = 0 \\ X + Y = 5 \\ 2Y = 4 \end{cases} \quad \xrightarrow{\text{MATRIXIFY}} \quad \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$$

3×2 matrix (known)
A

2×1 matrix (unknown)
v

3×1 matrix (known)
b

Matrix multiplication

$A \cdot B$ involves multiplying rows of A with the columns of B

if A is an $m \times n$ matrix → m rows, n columns
and B —"— $n \times p$ matrix
then AB —"— $m \times p$ matrix

$$2 \cdot X + (-3) \cdot Y = 0$$
$$1 \cdot X + 1 \cdot Y = 5$$
$$0 \cdot X + 2 \cdot Y = 4$$

Upshot: the system of eqns we started off with is equivalent with the single eqn

$$\boxed{Av = b}$$

of matrices

Def: an $n \times 1$ matrix is called a (column) vector:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \rightsquigarrow (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$$

n-dimensional space

Adding vectors of the same size

$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$ then $v + w = \begin{bmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{bmatrix}$

Multiplying vectors by scalar

numbers

$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and $c \in \mathbb{R}$ then $cv = \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix}$

Linear combination of vectors V and W : $\alpha V + \beta \cdot W$

where α, β are some numbers

coefficients of the linear combination

$v = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad w = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \in \mathbb{R}^3$

$$\alpha v + \beta w = \begin{bmatrix}\alpha\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\beta\end{bmatrix} = \begin{bmatrix}\alpha\\0\\\beta\end{bmatrix}$$

as α, β run over any real numbers, this traces out the xz-plane in XYZ-space

Fact: given two vectors, the set of their linear combinations traces out a plane

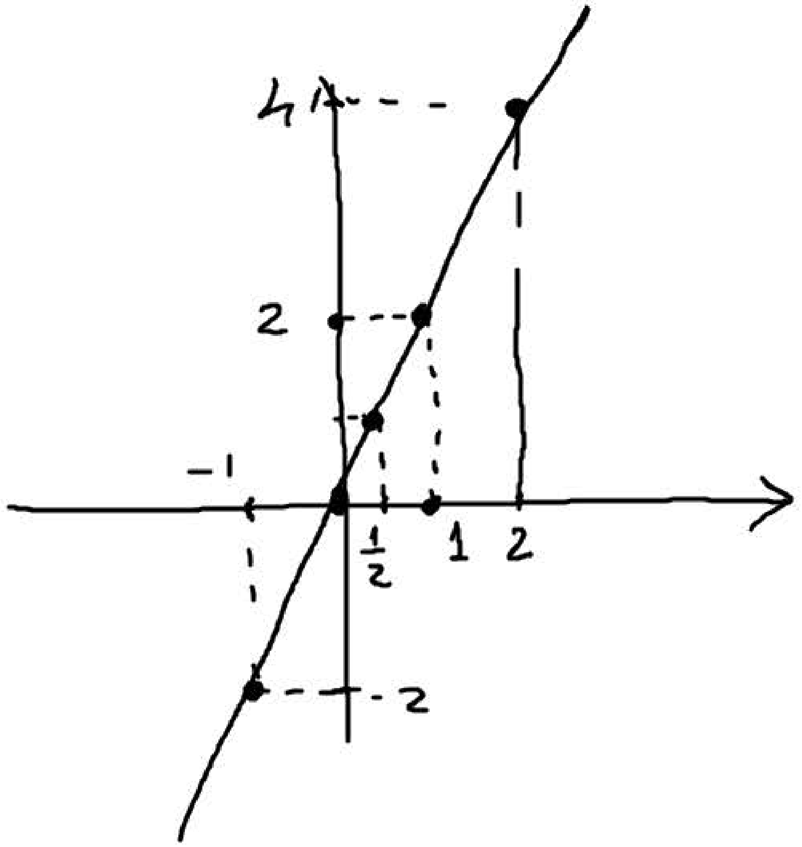
Easier fact: given one vector, the set of its linear combinations is a line

v

$c \cdot v$ for some $c \in \mathbb{R}$

$\alpha v + \beta v = (\alpha + \beta) v$

ex: $v = \begin{bmatrix}1\\2\end{bmatrix}$, $cv = \begin{bmatrix}c\\2c\end{bmatrix}$



$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \left(X \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Y \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) =$$

A

$$= X \cdot \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Y \cdot \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = X \cdot \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} + Y \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{unit vectors of axes of } \mathbb{R}^2}$$

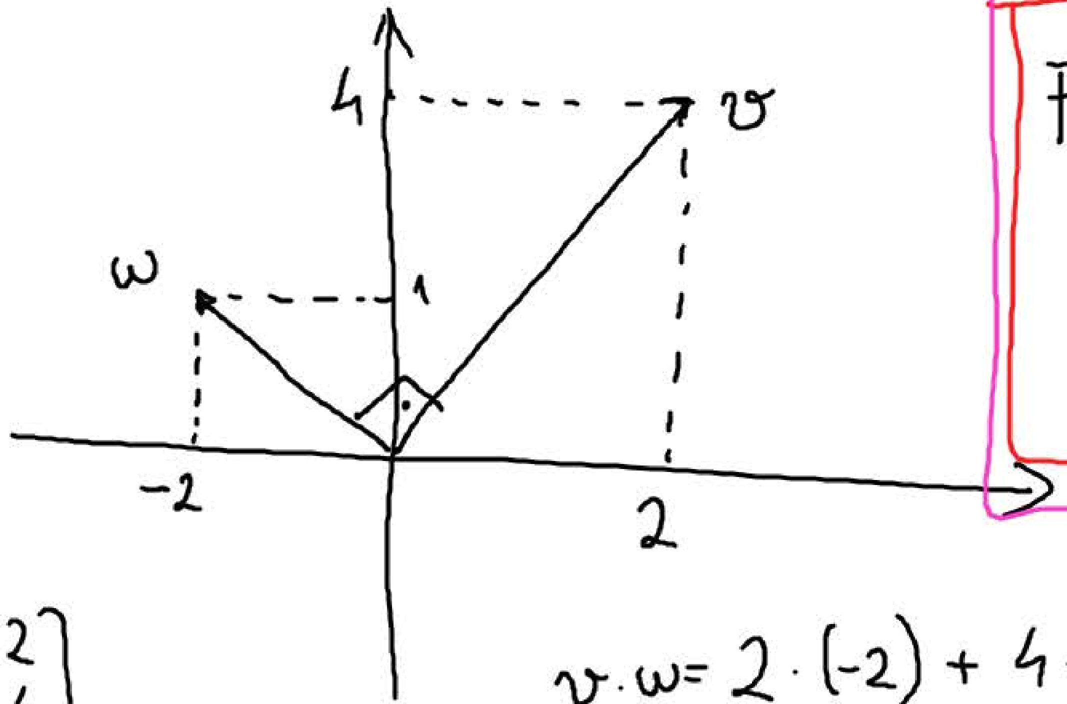
$$= X \cdot \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + Y \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = \text{a linear combination of the columns of } A$$

$Av = b$ $\rightsquigarrow$ which combination of the columns of A gives you the vector b

Dot product : $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ $w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$

the number $\quad v \cdot w = v_1 w_1 + \dots + v_n w_n$

more generally, $v \cdot w$ gives you information about the angle Θ between v and w



Fact : $v \cdot w = 0$

$\Updownarrow$

$v \perp w$

$v \cdot v = v_1^2 + \dots + v_n^2 = \|v\|^2$

length of v

$\|v\| = \sqrt{v \cdot v}$

$v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$w = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$v \cdot w = 2 \cdot (-2) + 4 \cdot 1 = 0$