SVD: $\underbrace{A}_{\text{rectangular}} = \underbrace{U}_{\text{orthogonal}} \underbrace{\Sigma}_{\text{rectangular}} \underbrace{V^T}_{\text{orthogonal}}$

rectangular

orthogonal

Thm: QS factorization (a.k.a polar decomposition)

any $n \times n$ matrix A can be written as $A = QS$

orthogonal

symmetric positive semi-def.

Proof: $A = \underbrace{U \Sigma V^T}_{n \times n} = U \underbrace{V^T V}_{I} \Sigma V^T =$

$= \underbrace{(UV^T)}_{Q} \cdot \underbrace{(V \Sigma V^T)}_{S}$

S is positive semi-def, b/c its eigenvalues, i.e. the diagonal entries of Σ, are ≥ 0

why is $Q = UV^T$ orthogonal?

$Q^T Q = (UV^T)^T UV^T = V^{TT} \underbrace{U^T U}_{= I} V^T = V \cdot V^T = I$

$n = 1$; $a = q \cdot s$

orthogonal means $q^2 = 1 \Rightarrow q = \pm 1$

pos semi-def means $s \geq 0$

Types of factorizations of matrices:

$A = LU$

$A = LDU$

$A = QR$

$A = VDV^{-1}$

$A = U \Sigma V^T$

$A = QS$

4 classes on probability ⇝ likelihood of unknown events

1 class on statistics ⇝ analysis of known events

- experiment with n different outcomes, which happen with probabilities $p_1 \ldots p_n$ (discrete probabilities)

CONSTRAINTS:

$p_1, \ldots, p_n > 0$

$p_1 + \ldots + p_n = 1$

- assume that each outcome comes with a real number value $x_1, \ldots x_n$

(e.g. an amount of money you win if the outcome is $p_1, \ldots p_n$ respective)

example:

coin toss has 2 outcomes

heads: $p_H = \frac{1}{2}$

tails: $p_T = \frac{1}{2}$

<u>Def</u>: (mean = expected value) is the weighted sum of values of outcomes

$$\mu = p_1 x_1 + \ldots + p_n x_n$$

<u>Def</u>: (variance)

$$\Sigma = p_1 (x_1 - \mu)^2 + \ldots + p_n (x_n - \mu)^2$$

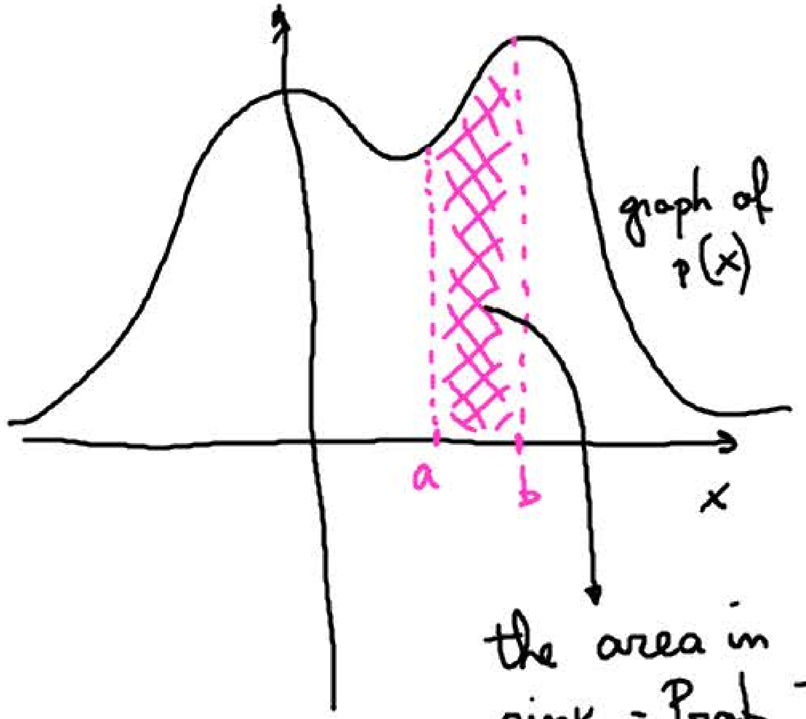
$\sigma = \sqrt{\Sigma}$ is the standard deviation

Continuous probability : any real number can be the value of our experiment, and the probability that this happens is governed by a

probability distribution $p(x) : \mathbb{R} \longrightarrow \boxed{\mathbb{R}_{\geq 0}}$

(intuitively but incorrectly, $p(x)$ = the probability that the outcome is x)

for any interval $[a, b]$, the probability that the outcome lands between a and b is $\underbrace{\int_a^b p(x)\,dx}_{\text{Prob}(x \in [a,b])}$.



the area in pink = Prob that outcome lands between a and b

CONSTRAINT:

$$1 = \text{Prob}\big(X \in (-\infty, \infty)\big) = \int_{-\infty}^{\infty} p(x)\,dx$$

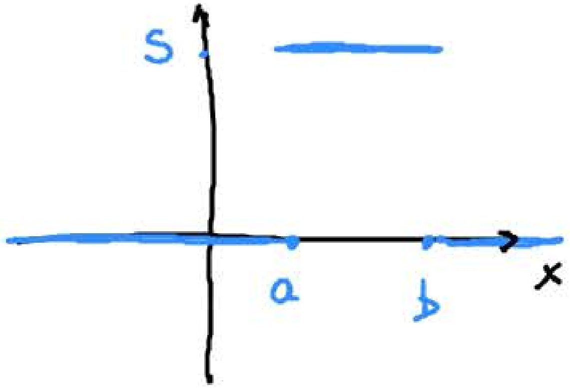
MEAN

$$\mu = \int_{-\infty}^{\infty} p(x) \cdot x\,dx$$

VARIANCE

$$\Sigma = \int_{-\infty}^{\infty} p(x)\,(x-\mu)^2\,dx$$

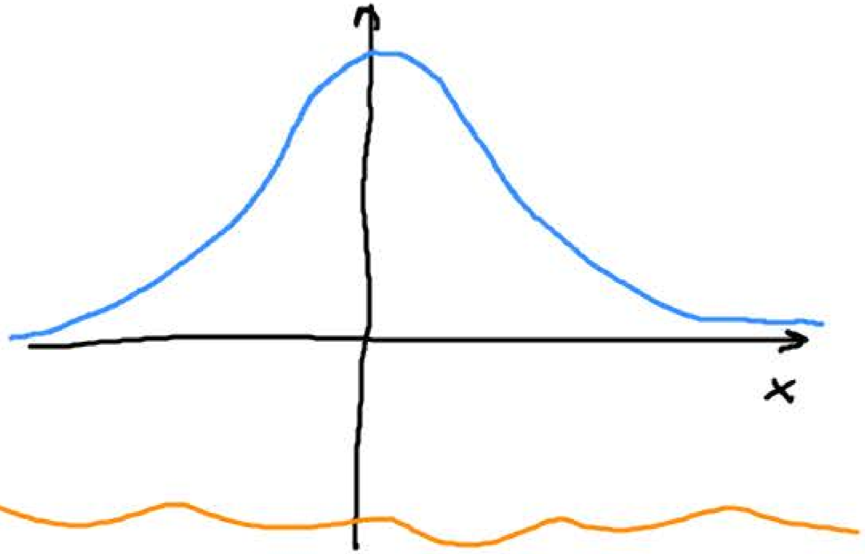
Examples: $p(x) = \begin{cases} s & \text{if } x \in [a,b] \\ 0 & \text{if } x \notin [a,b] \end{cases}$



$$1 = \int_{-\infty}^{\infty} p(x)\,dx = \int_a^b s \cdot dx = s(b-a) \implies s = \frac{1}{b-a}$$

Normal (Gaussian) distribution: $p(x) = \dfrac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$$



$$\mu = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} x\,dx = 0$$

$$\Sigma = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} x^2 dx = 1$$

if $p(x) = \dfrac{e^{-\frac{(x-\mu)^2}{2\Sigma}}}{\sqrt{2\pi\Sigma}}$ then the mean will be μ

the variance will be Σ