Cisc 271 Assignment 3 Selena Sun-10152968

Written Report

To find matrix A and data vector b, I used this formula:

$$\begin{aligned} -2\vec{z}_j^T\vec{g}+\sigma &= -\vec{z}_j^T\vec{z}_j \\ \equiv \quad [-2\vec{z}_j^T \;\; 1]\begin{bmatrix}\vec{g}\\ \sigma\end{bmatrix} &= -\vec{z}_j^T\vec{z}_j \end{aligned}$$

Setting matrix A

```
D = [];
B= xydata.' %Transpose the data
for z=1:size(xydata,1)
    D= [D,-2*B(:,z)];
end
```

To find the matrix A, I transposed the original data and extracted the first column. I then multiply it by -2. By looping through the total number of rows, I got a new matrix that is the first 2 columns of A.

```
C=D.';
A=[C ones(size(xydata,1),1)] %Matrix A
```

After that, I transposed the new matrix again and concatenate a column of 1 to get the final matrix A.

Setting data vector b

```
b = [];
f=[];
c = xydata.' %Transpose the data

%for all column in the dataset
for z=1:size(xydata,1)
    d = c(:,z); %This find the first column
    w = d.';    %This is the transpose of the first column
    f = [f,-1*(w*d)];   %Calculate b
end
b =f.'          %Transpose f to get actually b vector
```

To find the data vector b, I transposed the original data and extracted the first column. I then transposed the first column and used it to multiply by the first column of the negation (-1) of the original data to find b. By looping through the total number of rows, I concatenate all the values of b in a vector and then transposed it to get a vertical vector.

After finding A and b, I can now find vector g and sigma.

$$A\begin{bmatrix}\hat{g}\\\hat{\sigma}\end{bmatrix}=\vec{b}$$

Finding vector g and sigma

Given A and b, I used the economy QR decomposition to find Q and R.

$$\vec{y}=[Q_1]^T\vec{b}$$

I then used the equation above to find the y.

$$R_1\vec{x}=\vec{y}$$

With R and y, we can then find x by back-subsitution.

```
x=zeros(3,1)
Ry=[R y]
```

I created a vector of x with three entries of 0. Then, I concatenate R and y into a single matrix in order to use back-substitution.

```
%Back-substitution
x(3)= Ry(3,end)/Ry(3,3);
x(2)= (Ry(2,end)-Ry(2,2+1:3)* x(2+1:3))/Ry(2,2);
x(1)= (Ry(1,end)-Ry(1,1+1:3)* x(1+1:3))/Ry(1,1);
```

Doing the back-substitution, I first found x3 by dividing the last value of the row 3 by the 3rd value of column 3. Then, I substitute x3 in the 2nd row to find x2. I used the same procedure to find x1.

```
coeff=[x(1) x(2) x(3)]
coeff=coeff.'
```

After finding x1, x2 and x3, I put them the coefficients in the matrix coeff.

```
center = zeros(2, 1);
center(1) = x(1);
center(2) = x(2);
sigma=x(3);
```

The center is the first and second x values. The third x value is the sigma.

Finding the radius

$$\hat{\rho} = \sqrt{\hat{g} \cdot \hat{g} - \hat{\sigma}}$$

Now that I have the coefficents, I can find the radius using the square root of the dot product of the center minus the sigma.

Finding the residual error

$$e_j \stackrel{\text{df}}{=} \|\vec{z}_j - \hat{g}\| - \hat{\rho}$$

To find the residual error vector, I used the norm of the first row of the original data minus the center vector. Then I subtract it by the radius.

```
e=[]
for z=1:size(xydata,1)
    j=c(:,z)-center;
    n=norm(j);
    e=[e, n-radius];
end

e=e';
```

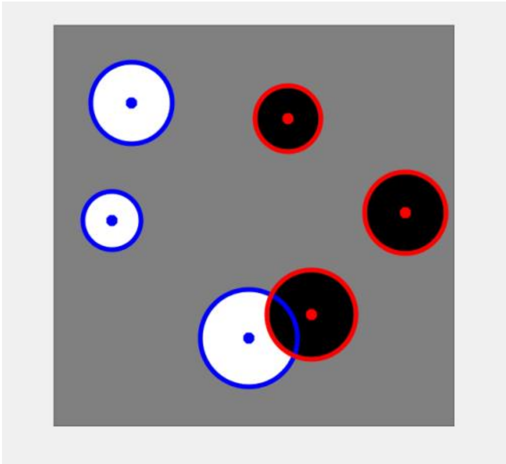
By looping through the rows of the original data, I found the residual error for each row and I combined it in a e vector.

Finding the RMS

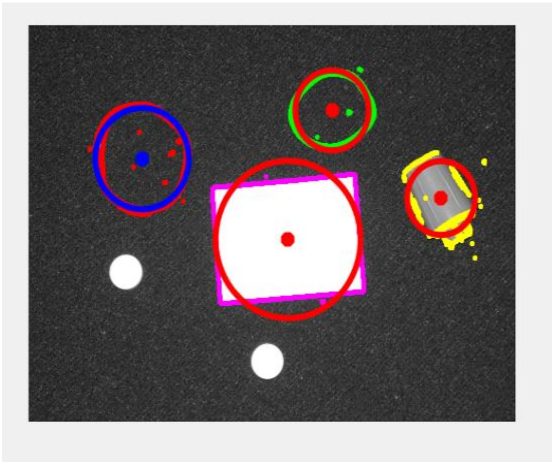
$$\textbf{RMS}(\vec{e}) \stackrel{\text{df}}{=} \sqrt{\|\vec{e}\|^2/m}$$

I found the RMS by finding the norm of the power of 2 of the residual error vector divided by m, then the square root of it all.

Analysis



The image above shows the figure of the disksdemo matlab file. Each circle is an object. As we can see, the outline color lines up perfectly with the shape of the circles. Thus, the RMS for the first 6 circles are extremely small, approximately close to 0. The Least-squares approximation of edges in images are an excellent fit.



This image shows the figure of the pilldemo matlab file. As we can see, the circles in this picture does not fit perfectly with the shape shown in the background. Especially for the pink rectangle in the middle of the image, the Least-squares approximation of edges is a poor fit. This is because the image if a rectangle where we are trying to fit a circle. It will never be as closely fit as a circle shape with another circle shape. The same thing for the small yellow rectangle at the right side of the image. This is a reason why the RMS for this image is bigger than the ones in the previous image.