Kalman Filter

2019.03.04

차례

1. What is Filter?

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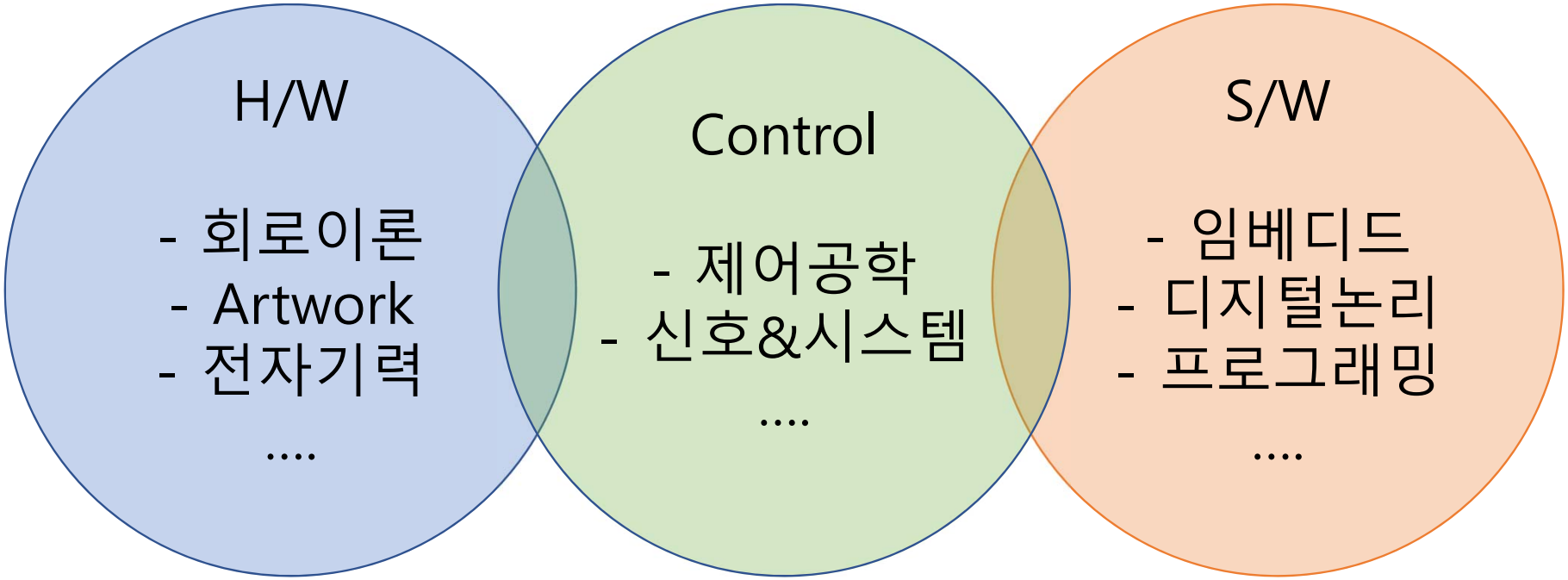

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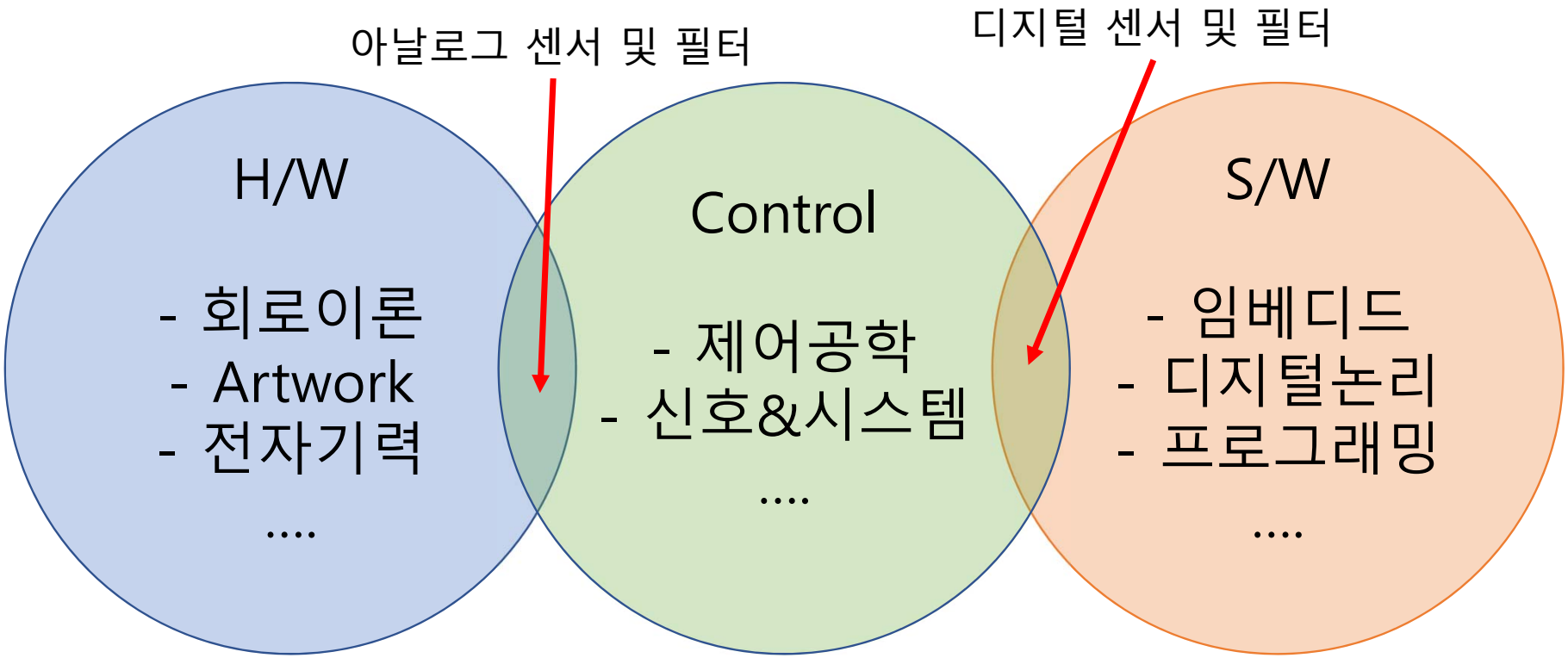

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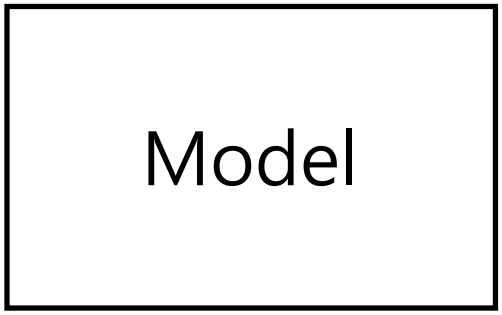

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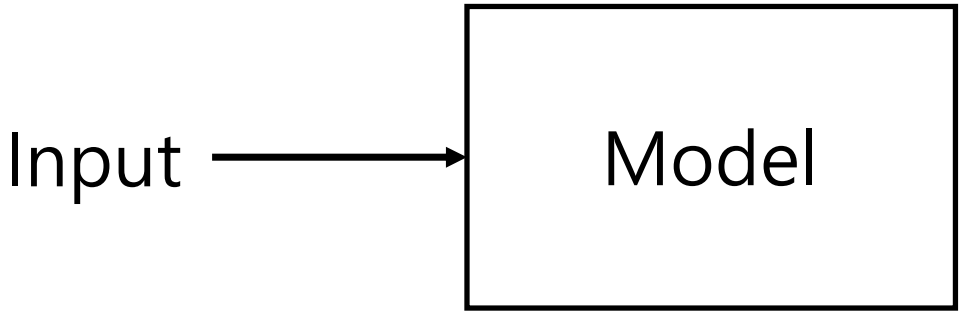

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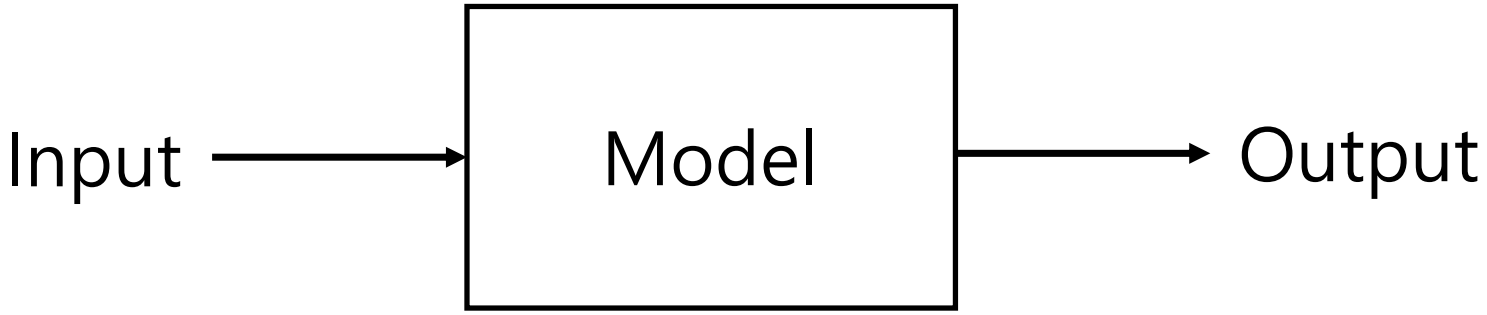

1. What is Filter?

Model

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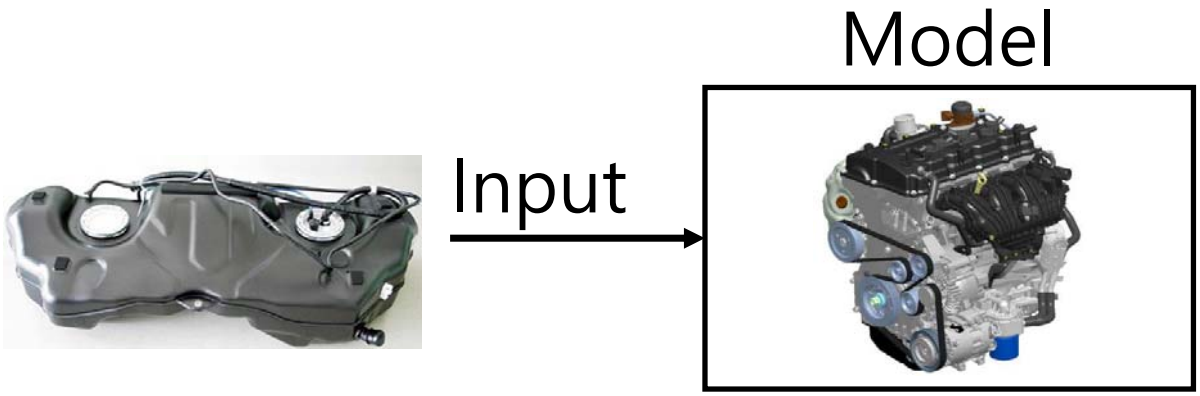

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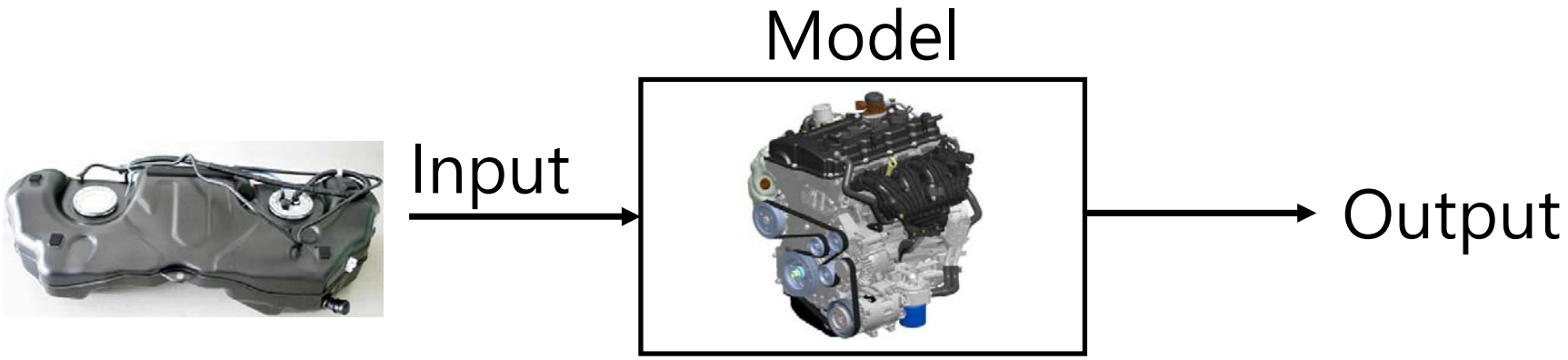

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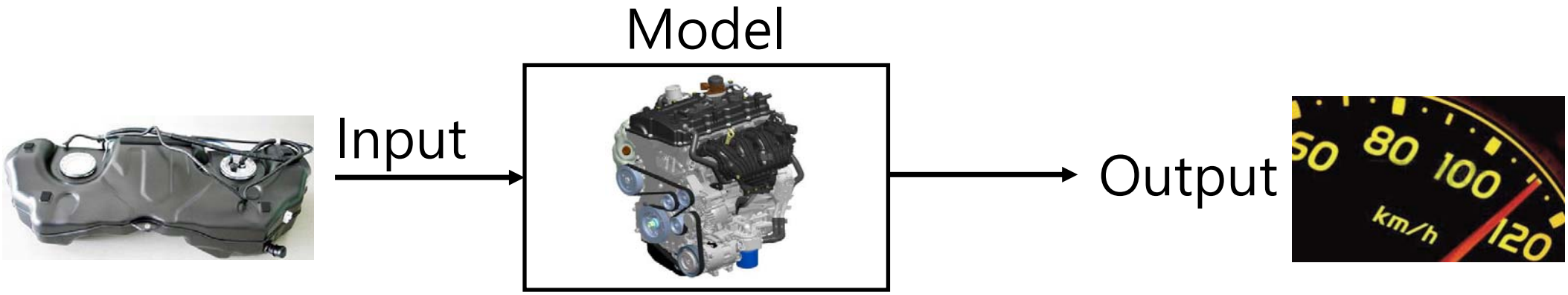

1. What is Filter?

Input

Model

Output

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Input

Model

Output

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Input

Model

Output

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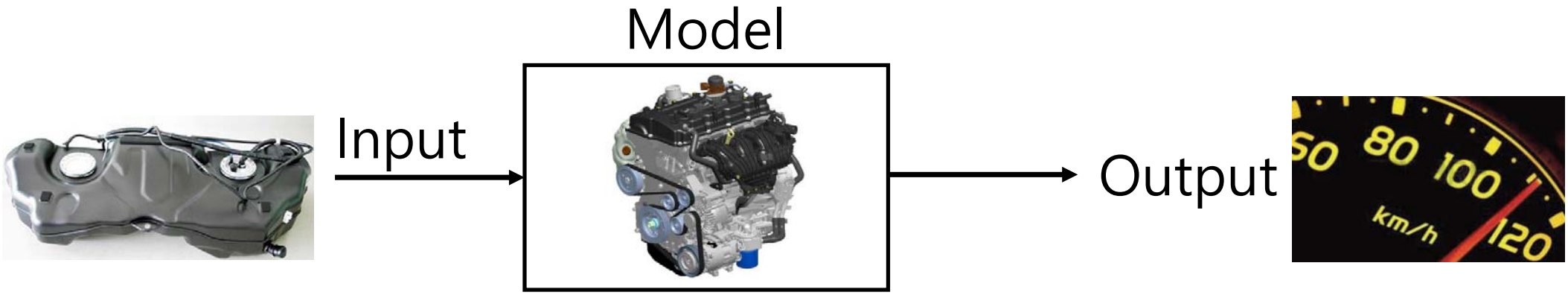

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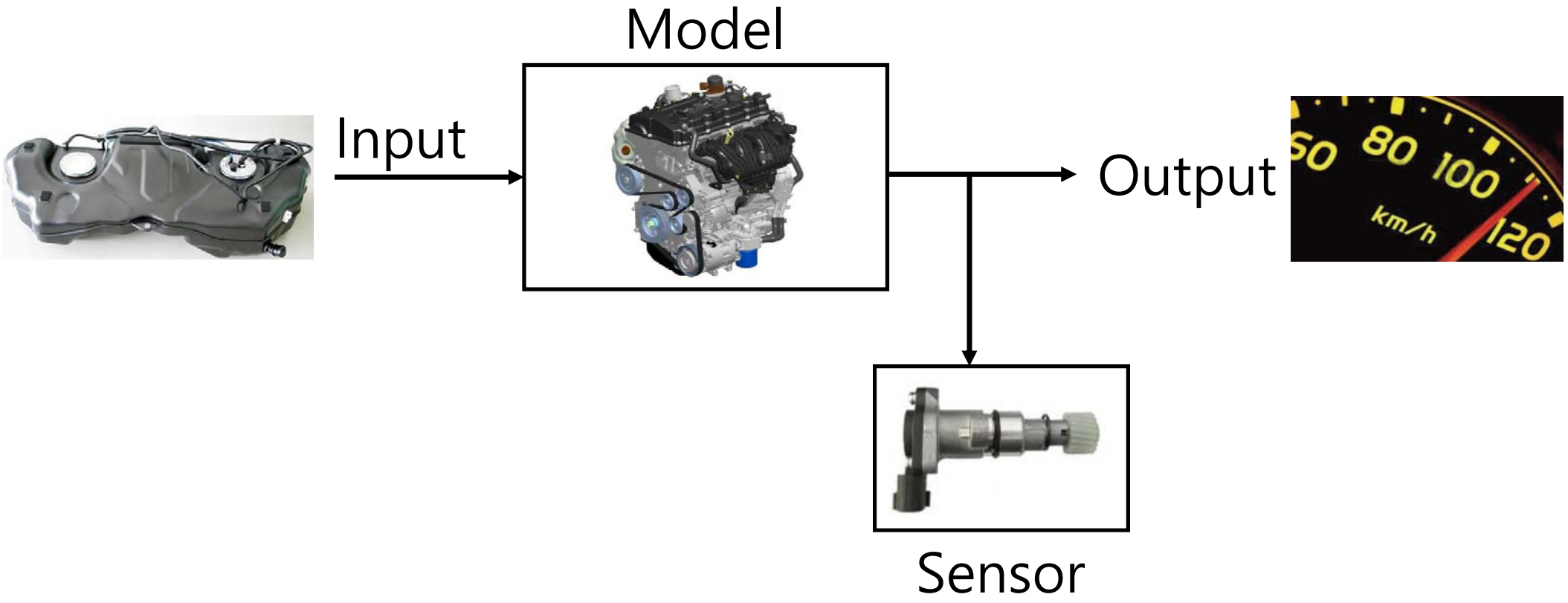

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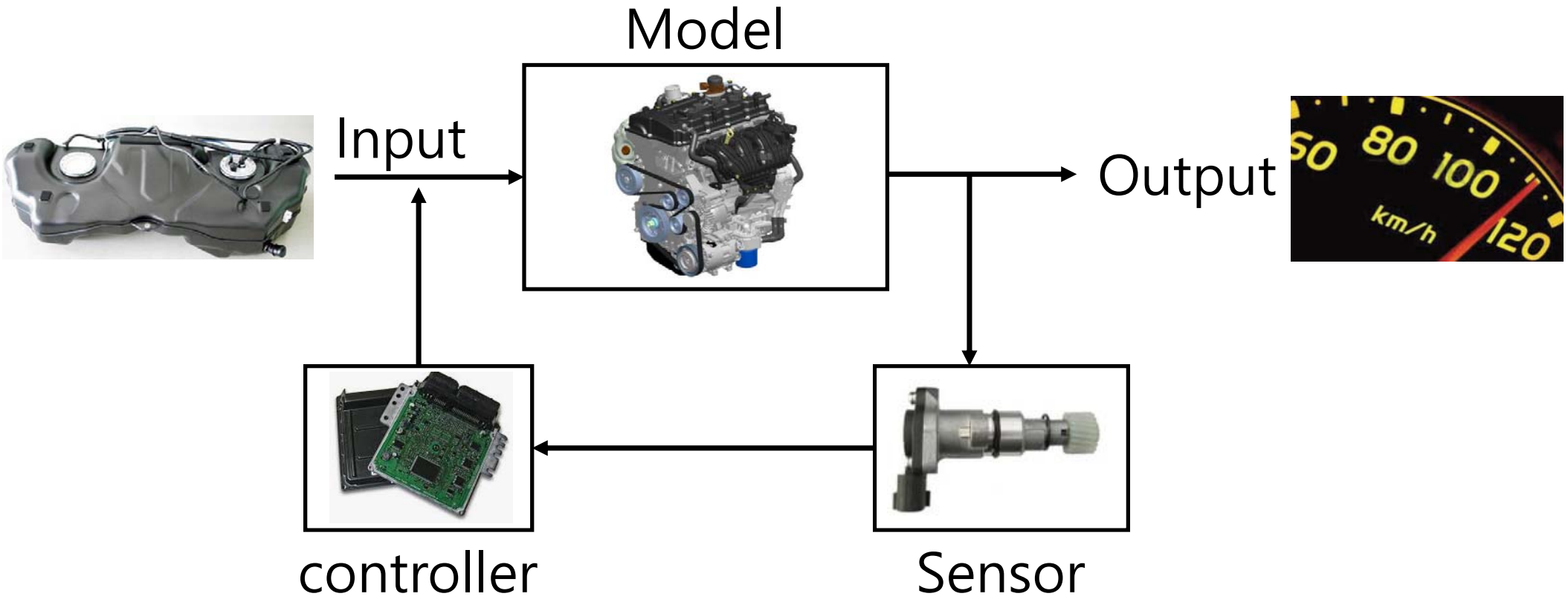

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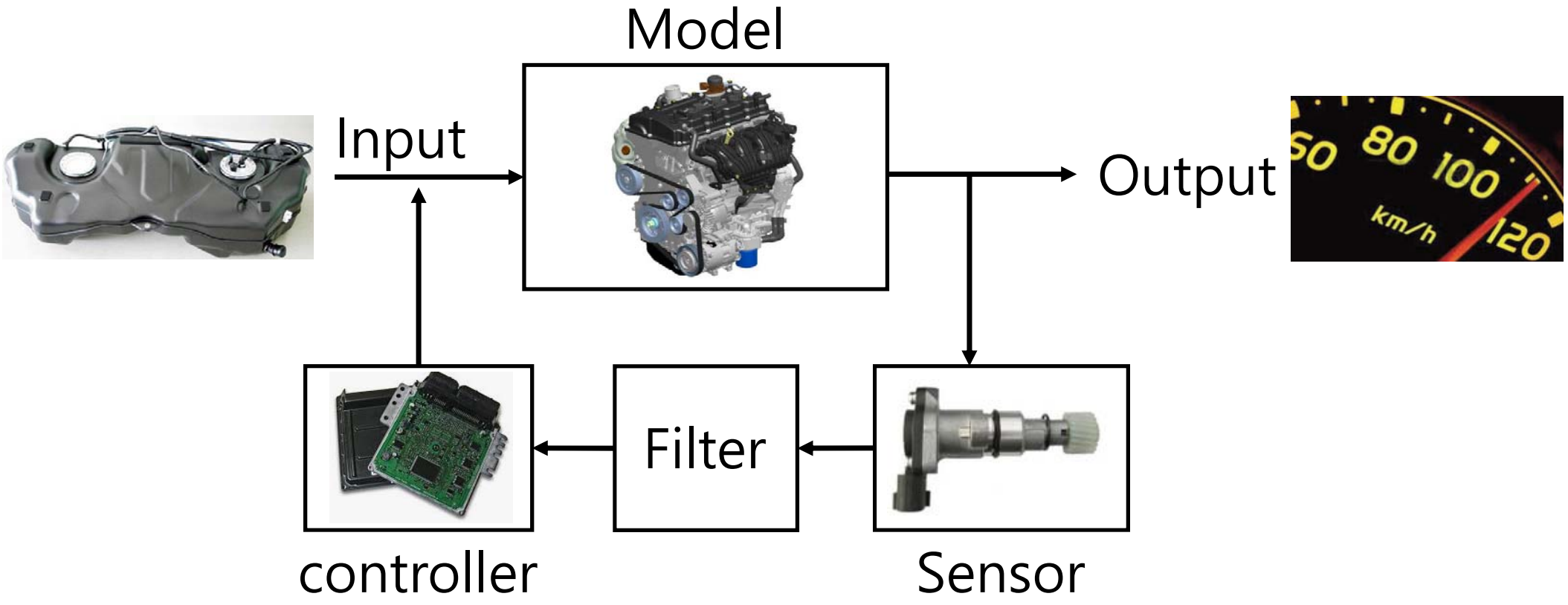

1. What is Filter?

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Signal

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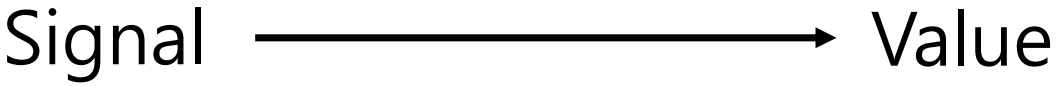

1. What is Filter?

Signal ⟶ Filter ⟶ Value

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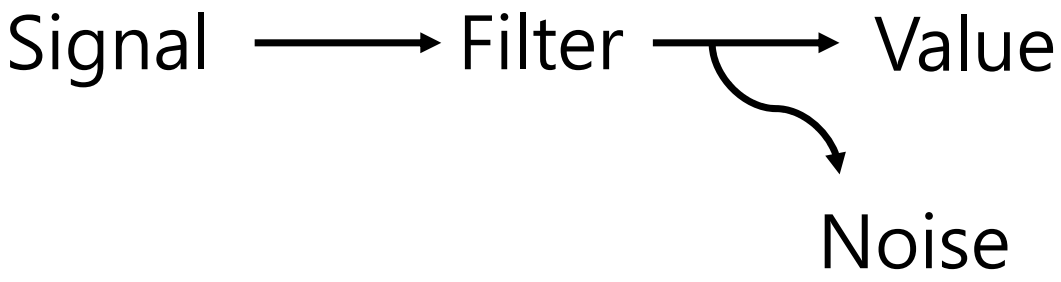

1. What is Filter?

Noise

- High Frequency

1. What is Filter?

Noise

- High Frequency

- Low Frequency

1. What is Filter?

Noise

- High Frequency
- Band Frequency
- Low Frequency

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Noise

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Filter

1. What is Filter?

Noise

- High Frequency

- Band Frequency

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Filter

- Low Pass

1. What is Filter?

Noise

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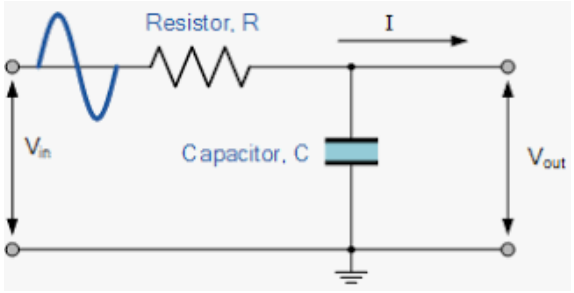
⟶ Filter

- Low Pass

Circuit



1. What is Filter?

Noise

- High Frequency

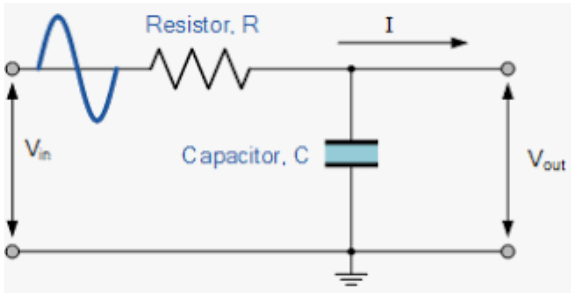
- Band Frequency

- Low Frequency

Filter

- Low Pass

Circuit

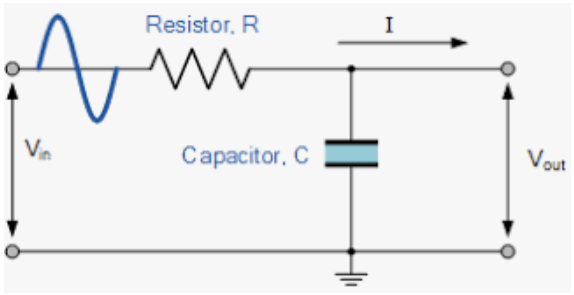


Fig

1. What is Filter?

Noise — Filter — Circuit — Fig

- High Frequency ⟶ - Low Pass



- Band Frequency

- Low Frequency ⟶ - High Pass

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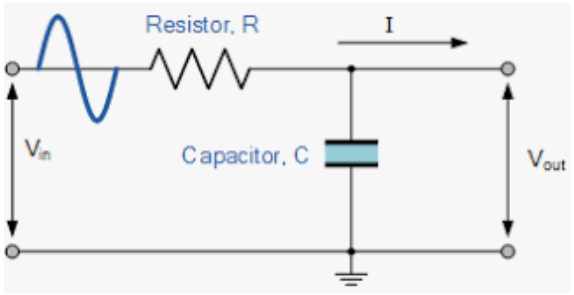
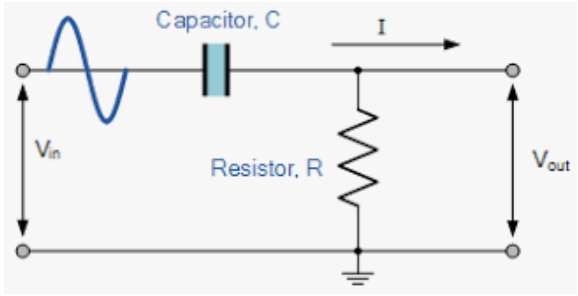
Noise — Filter — Circuit — Fig

- High Frequency ⟶ - Low Pass



- Band Frequency

- Low Frequency ⟶ - High Pass



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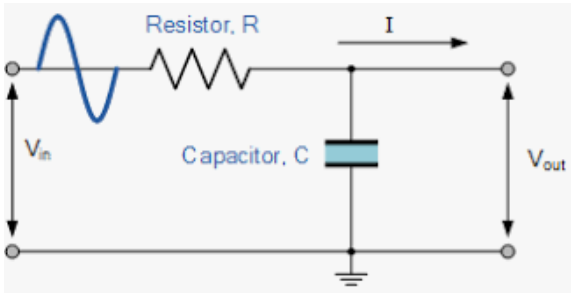
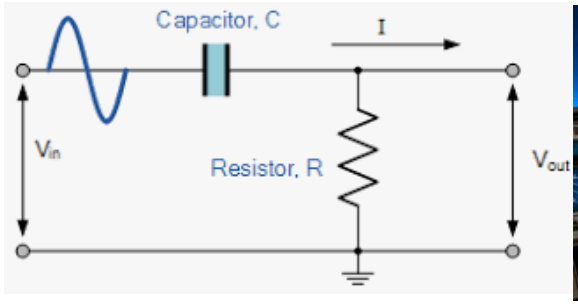
Noise — Filter — Circuit — Fig

- High Frequency ⟶ - Low Pass



- Band Frequency

- Low Frequency ⟶ - High Pass



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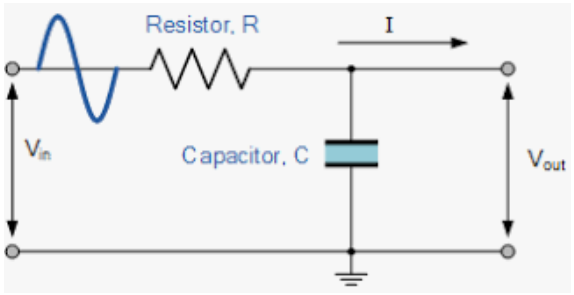
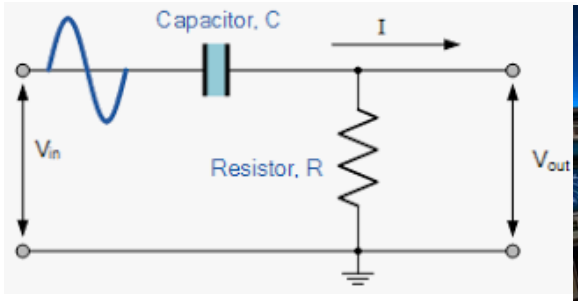
Noise	Filter	Circuit	Fig
- High Frequency ⟶	- Low Pass		
- Band Frequency ⟶	- Band Pass		
- Low Frequency ⟶	- High Pass		

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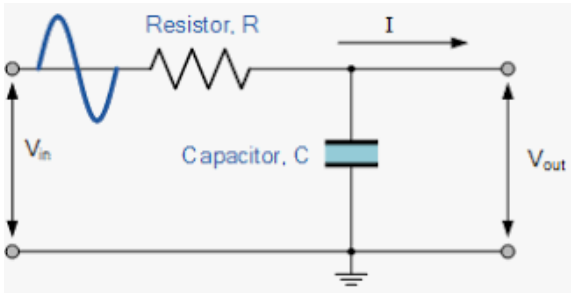
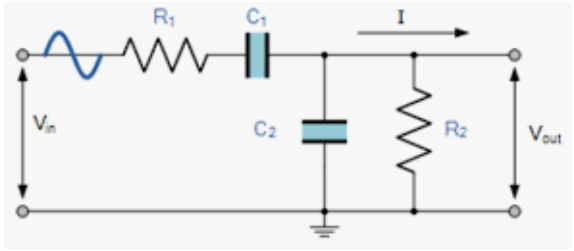
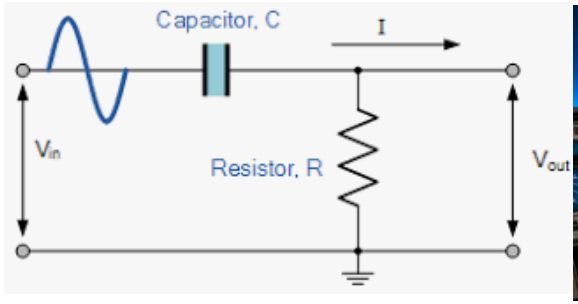
- High Frequency ⟶	- Low Pass		
- Band Frequency ⟶	- Band Pass		
- Low Frequency ⟶	- High Pass		

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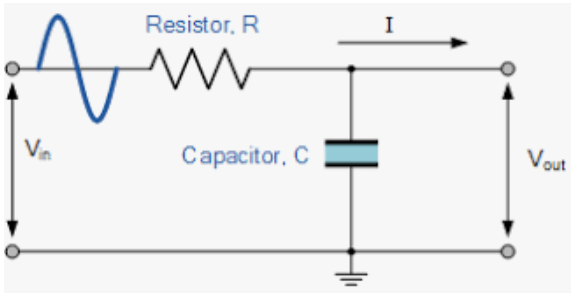
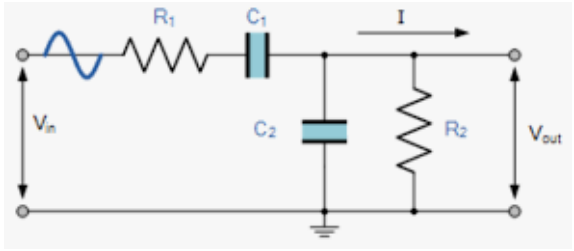
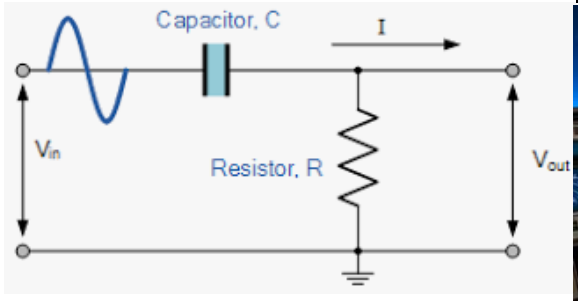
- High Frequency ⟶	- Low Pass		
- Band Frequency ⟶	- Band Pass		
- Low Frequency ⟶	- High Pass		

2. Average Filter?

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$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

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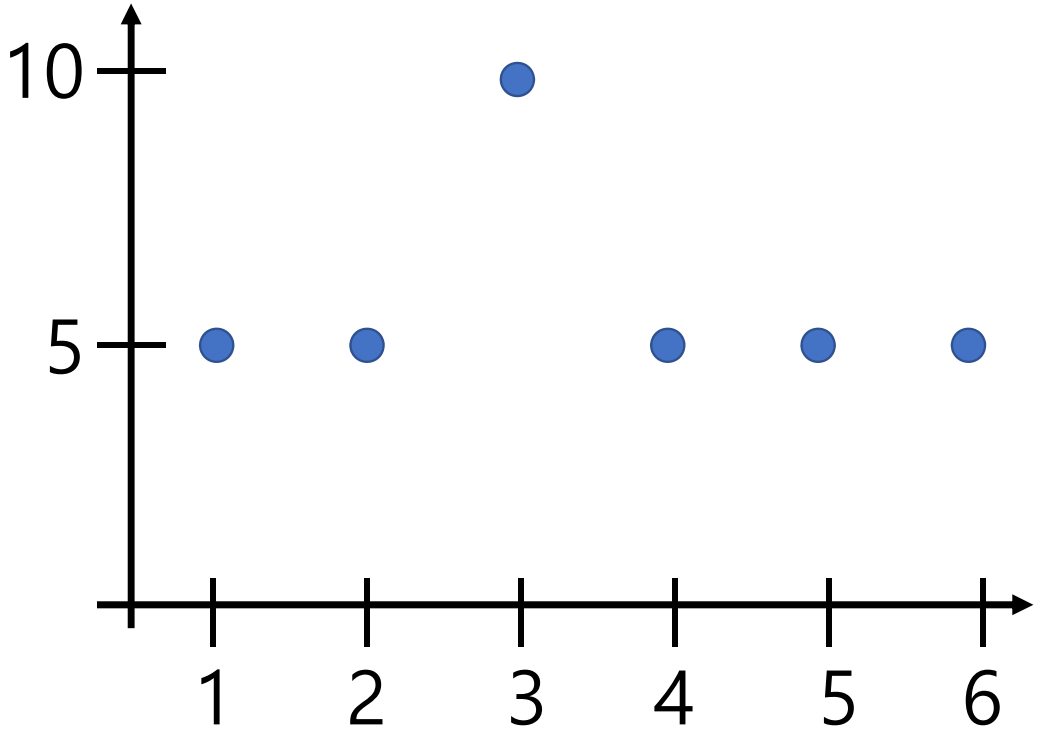

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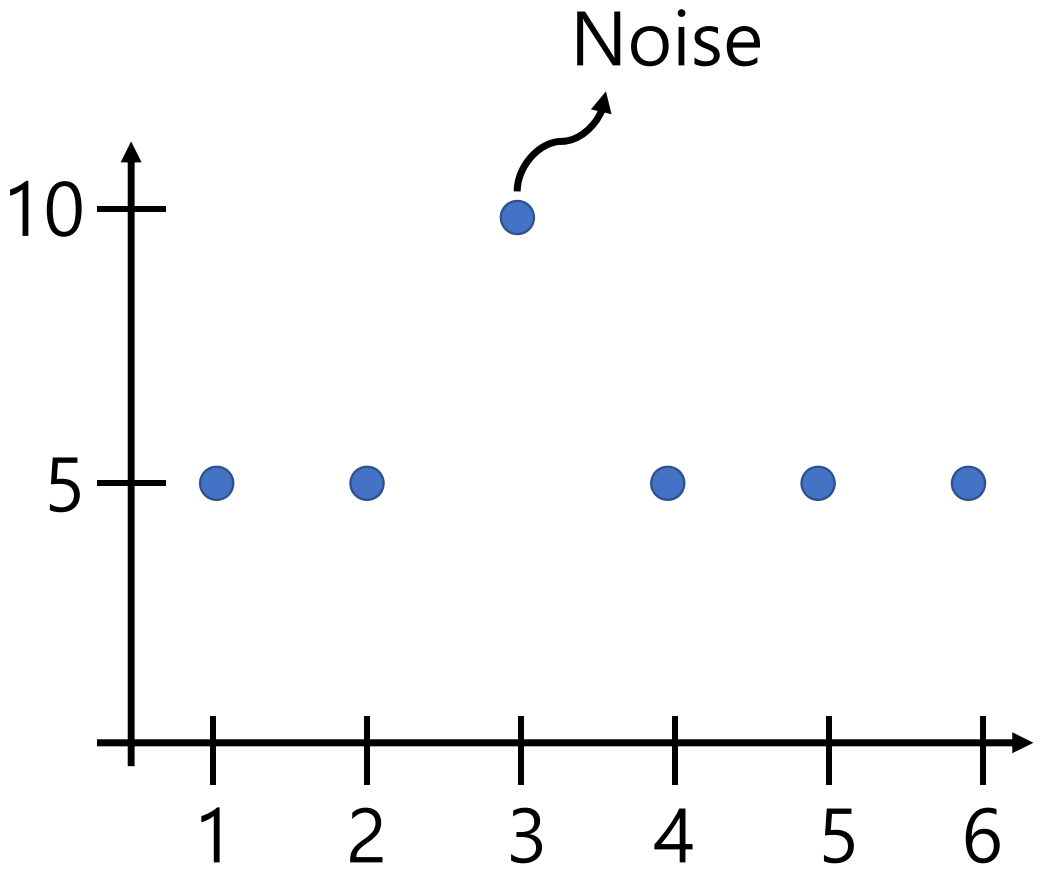
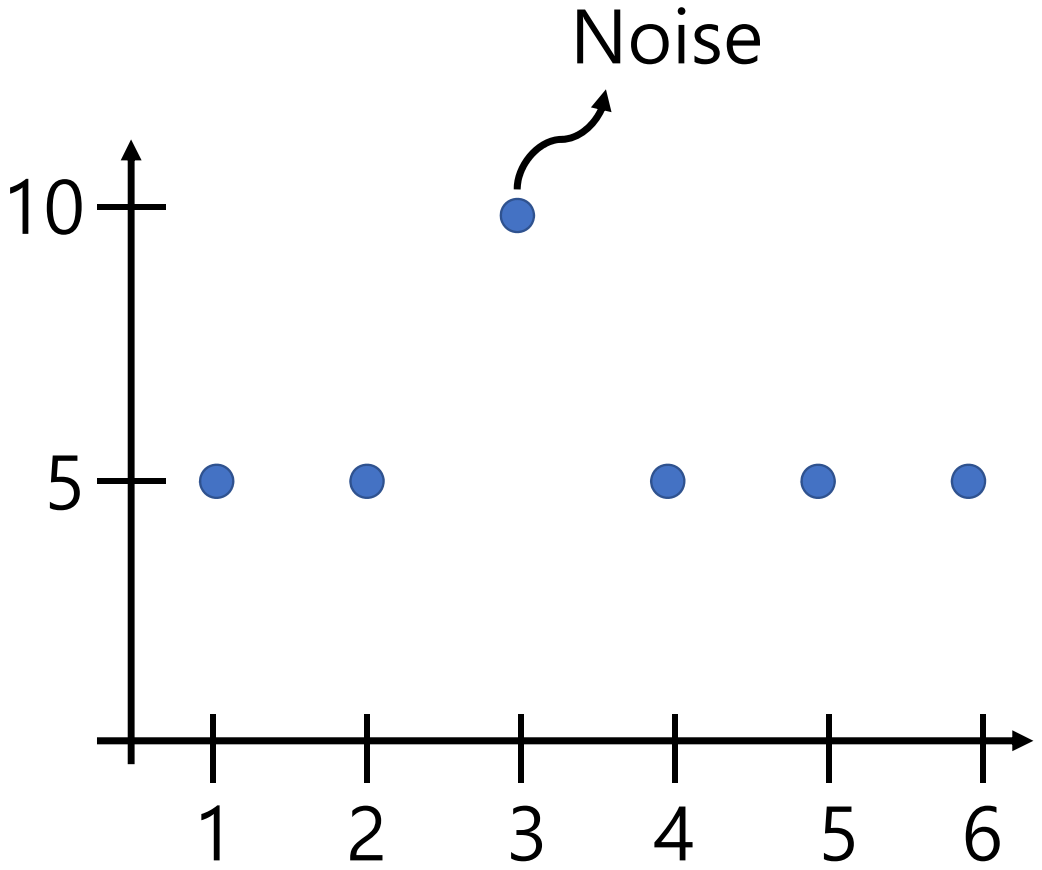
$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$



2. Average Filter?

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$= \frac{5 + 5 + 10 + 5 + 5 + 5}{6}$$



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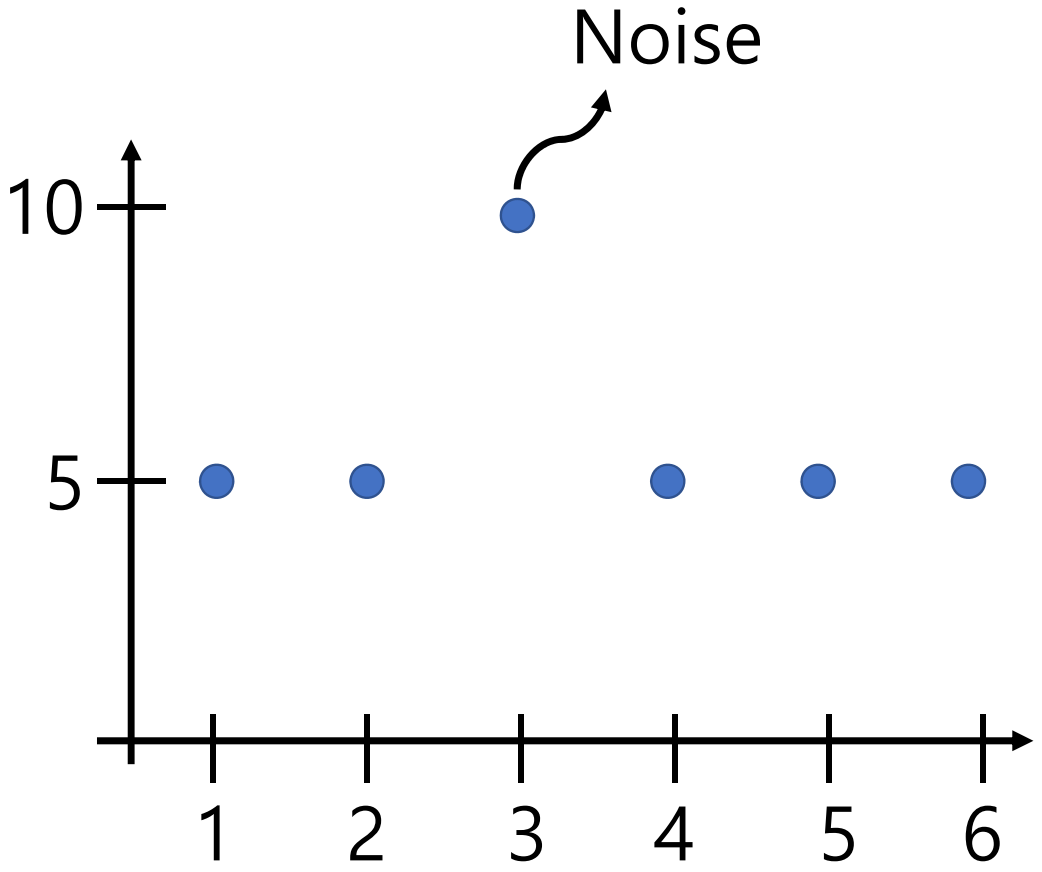
$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$= \frac{5 + 5 + 10 + 5 + 5 + 5}{6}$$

$$= \frac{35}{6} = 5.8333\ldots..$$



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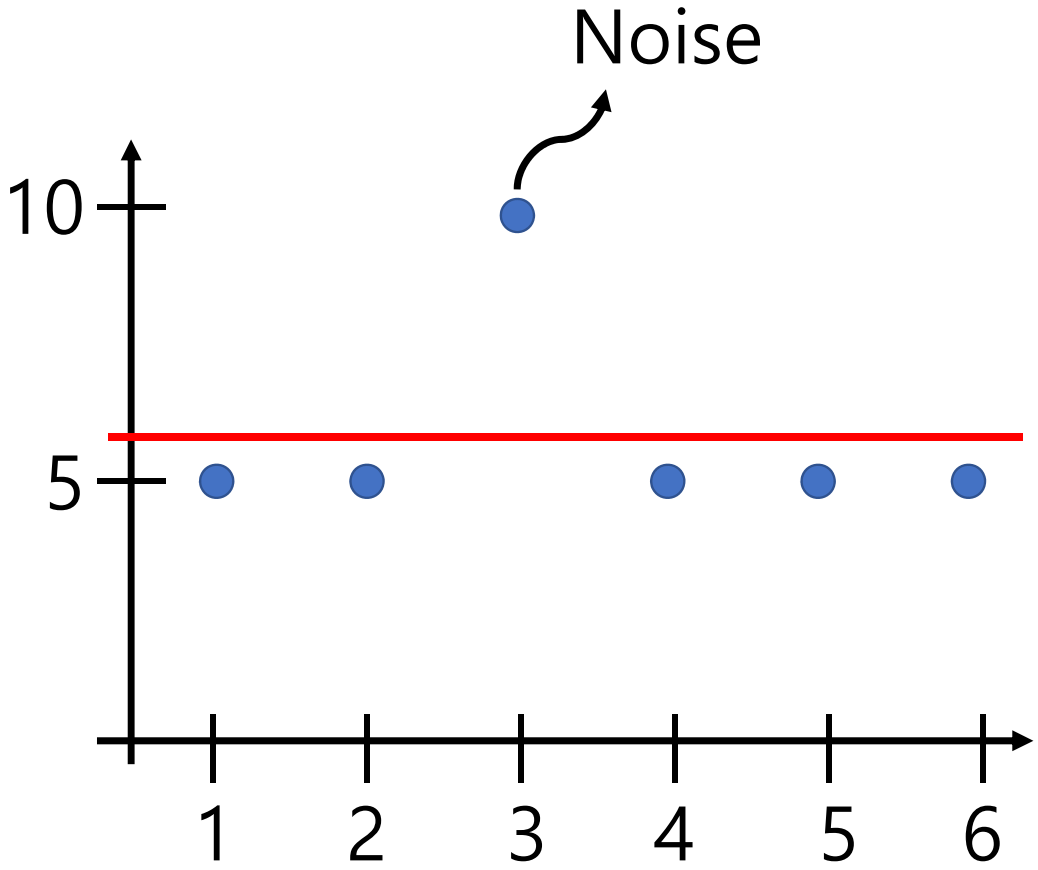

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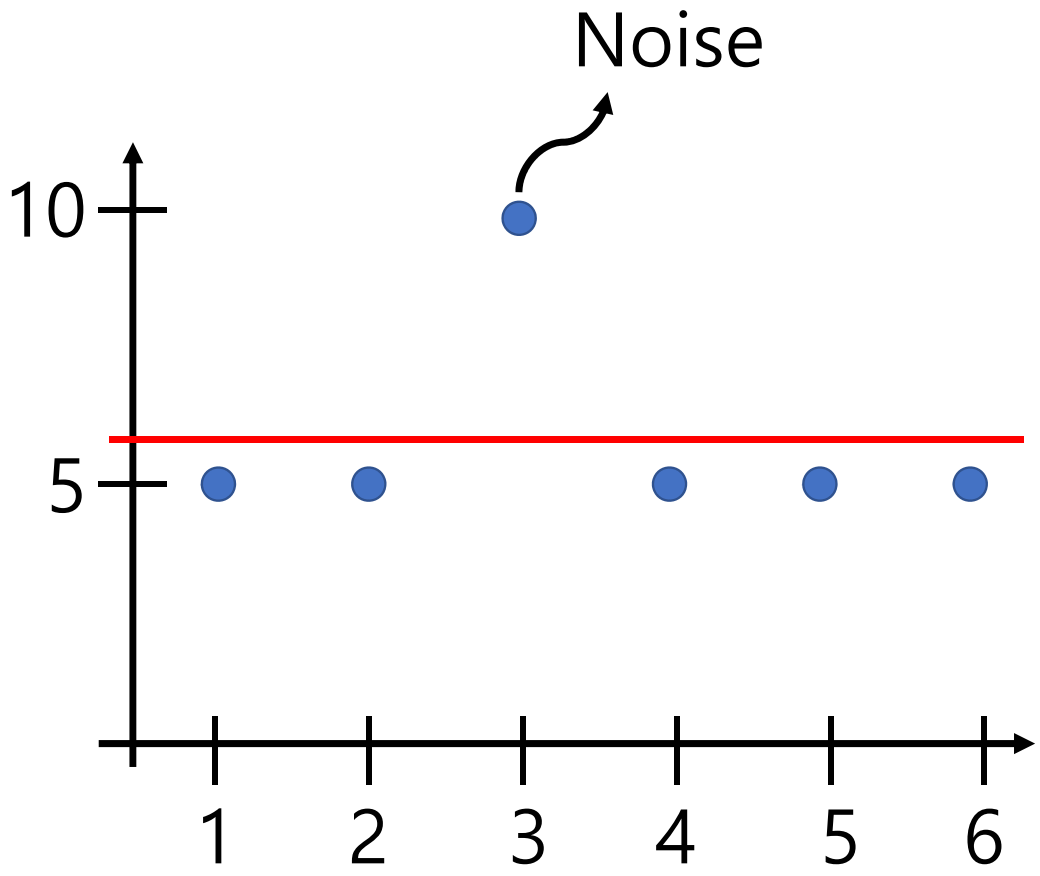
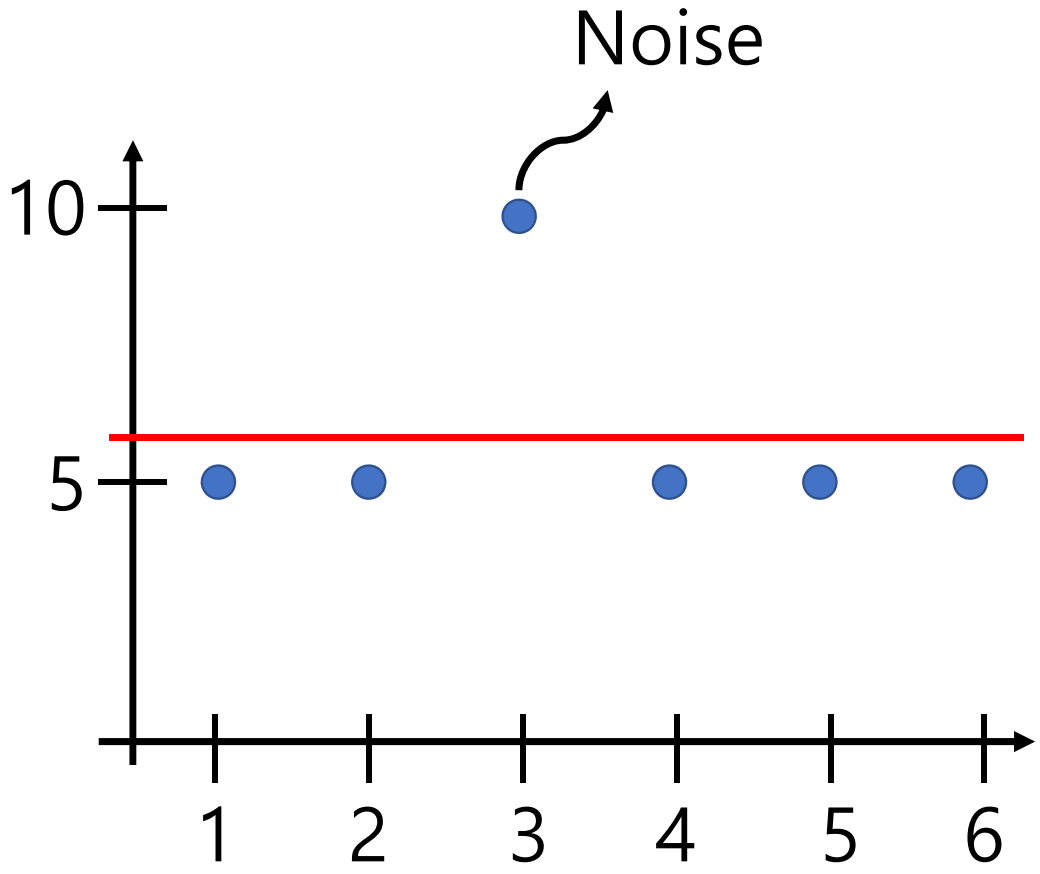


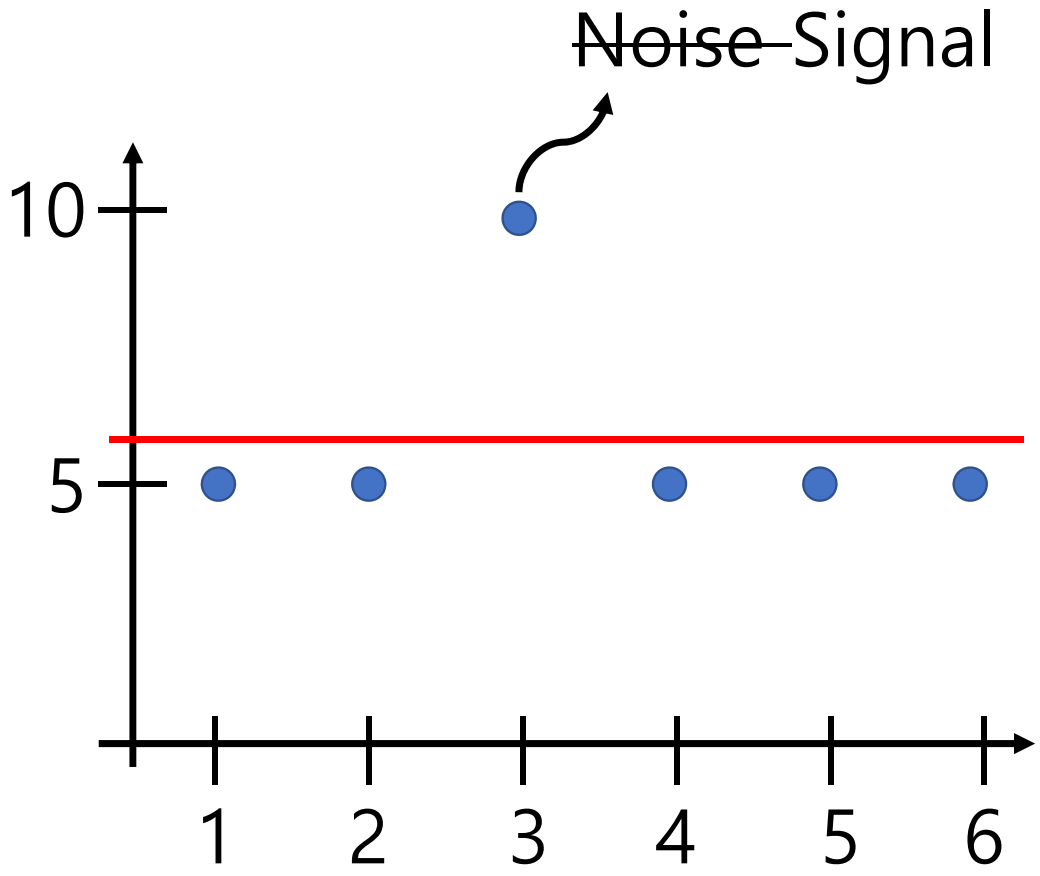
1. Estimation value incases

2. Average Filter?



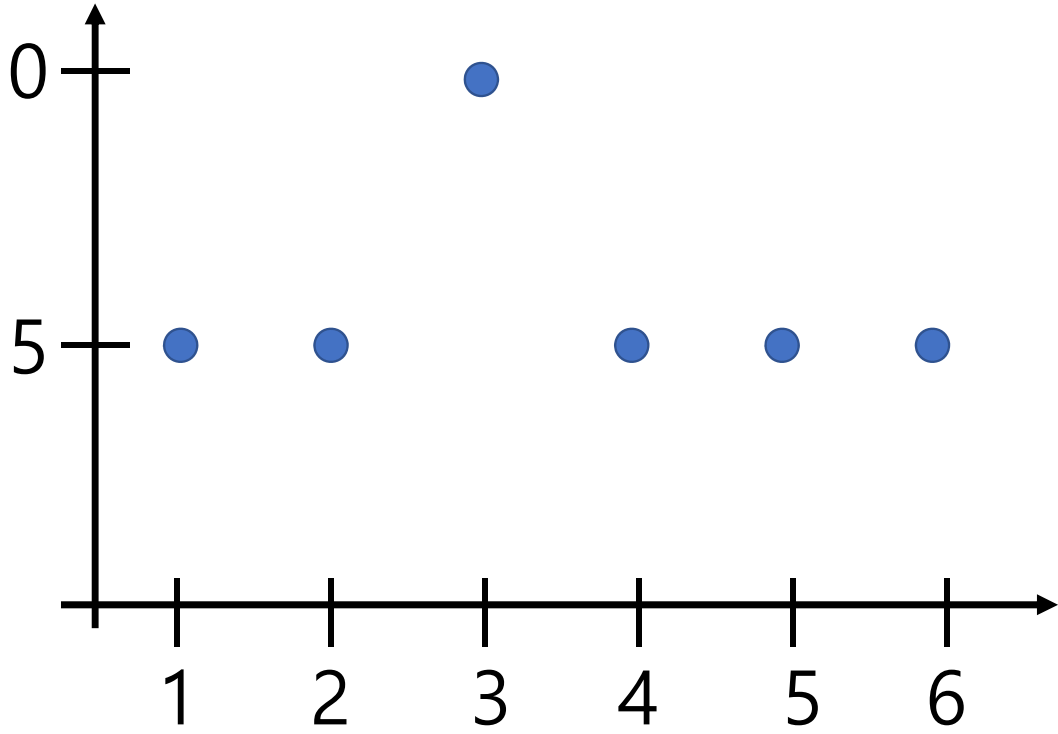
1. Estimation value incases
2. Time delay

2. Average Filter?



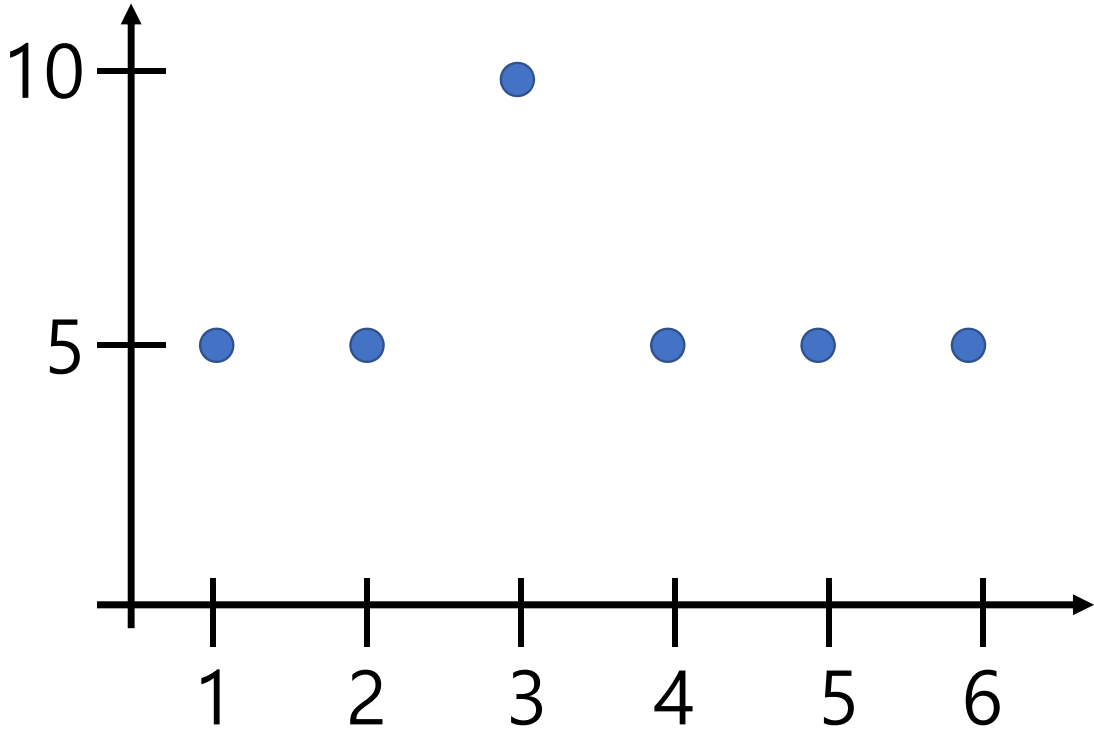
1. Estimation value incases
2. Time delay
3. Missing data

3. Moving Average Filter?



3. Moving Average Filter?

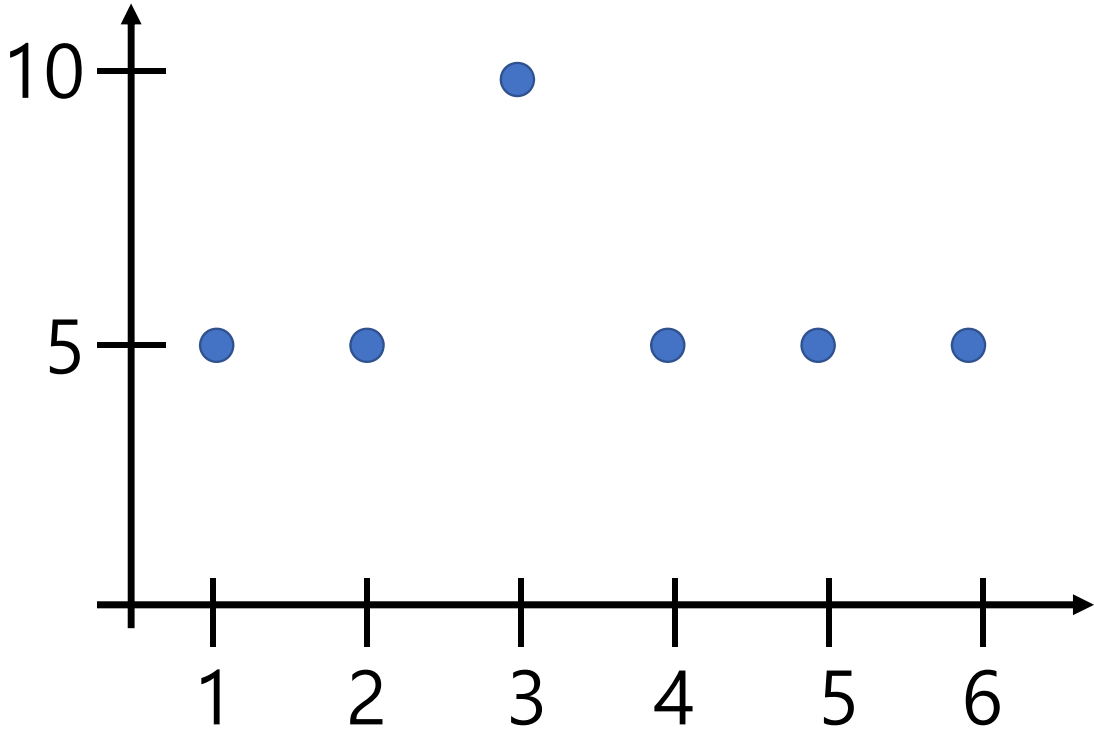
$$Mov.Avg.Filter = \frac{x_1 + \cdots + x_k}{k}$$



3. Moving Average Filter?

$$Mov.Avg.Filter = \frac{x_1 + \cdots + x_k}{k}$$

$$MAF_1 = \frac{5+5}{2} = 5$$



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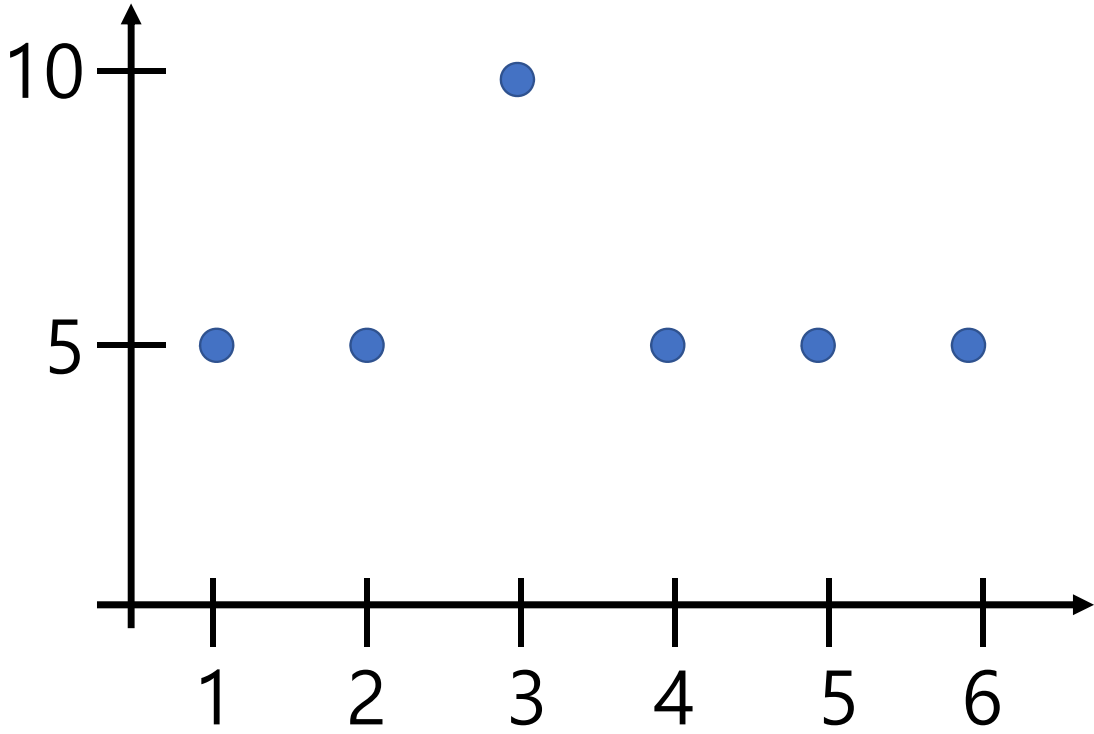
$$Mov.Avg.Filter = \frac{x_1 + \cdots + x_k}{k}$$

$$MAF_1 = \frac{5+5}{2} = 5$$

$$MAF_2 = \frac{10+5}{2} = 7.5$$



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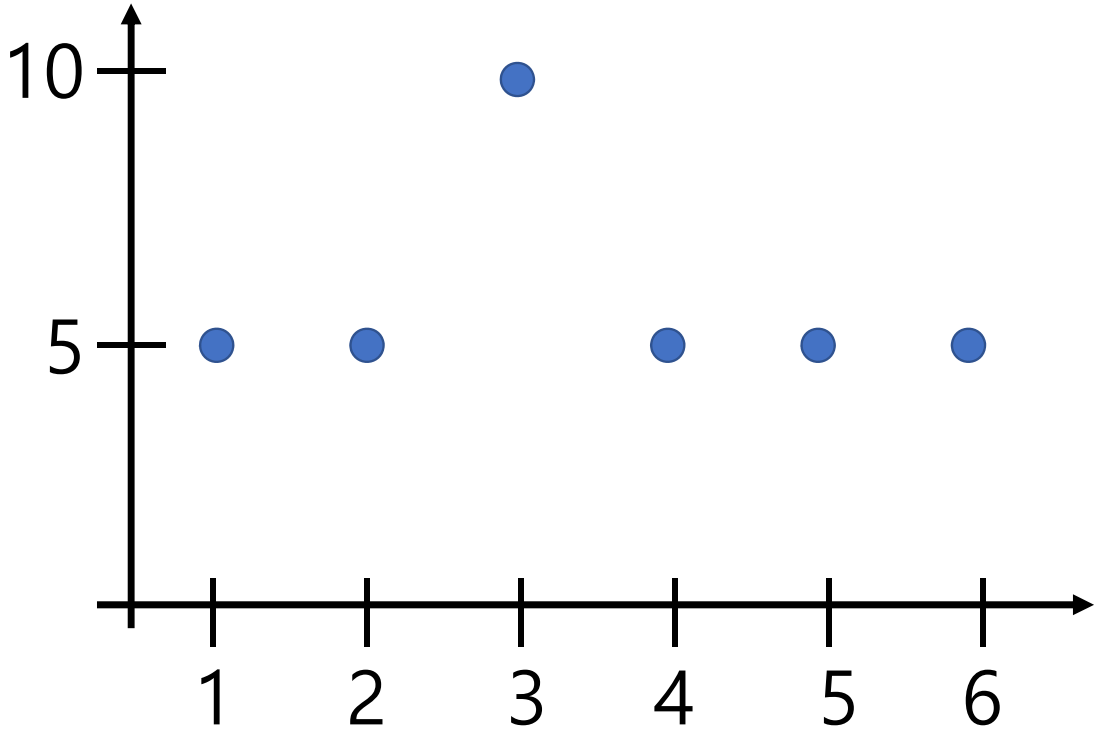
$$MAF_3 = \frac{5+5}{2} = 5$$



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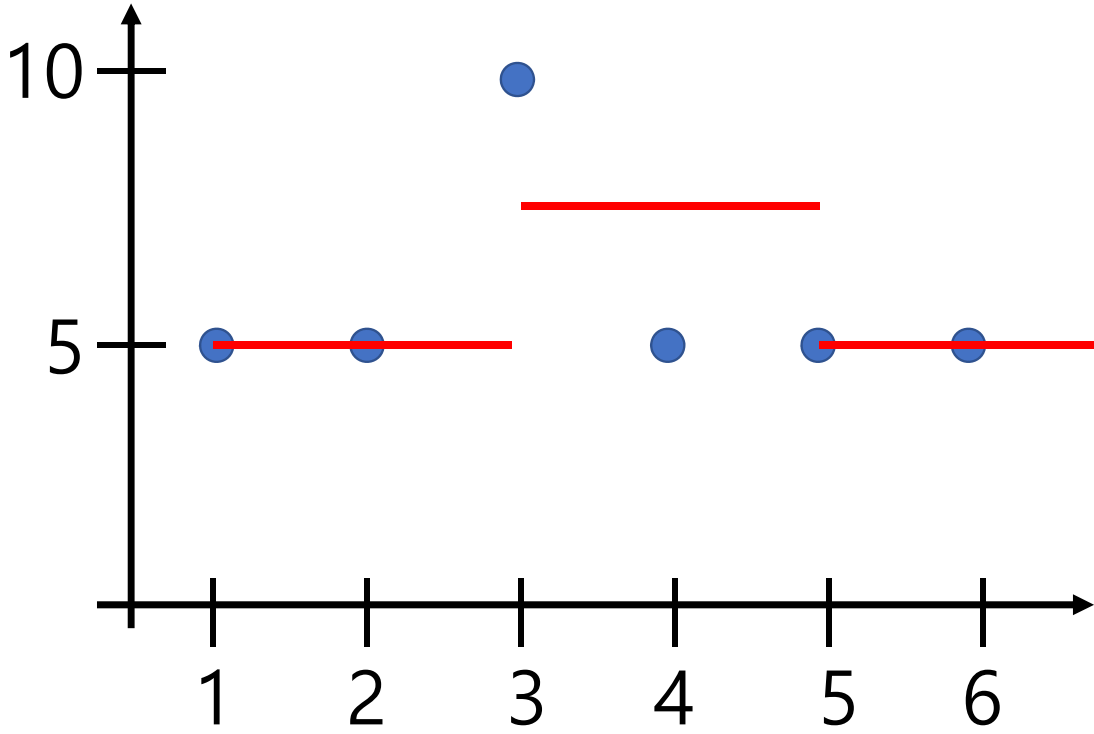
$$MAF_3 = \frac{5+5}{2} = 5$$



4. Recursive

4. Recursive

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

4. Recursive

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

4. Recursive

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

Batch expression

4. Recursive

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

Batch expression

1. All data storage

4. Recursive

$$Avg.Filter = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

Batch expression

1. All data storage
2. Low Efficiency

4. Recursive

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$k\bar{x}_k = x_1 + x_2 + \cdots + x_k$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$k\bar{x}_k = x_1 + x_2 + \cdots + x_k$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_k}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

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$$\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$k\bar{x}_k = x_1 + x_2 + \cdots + x_k$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

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$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

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$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

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$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \bar{x}_{k-1} + \frac{x_k}{k-1}$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \bar{x}_{k-1} + \frac{x_k}{k-1} \rightarrow \times \frac{k-1}{k} \rightarrow$$

4. Recursive

Goal : $\bar{x}_{k-1}$

$$\bar{x}_{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \frac{x_1 + x_2 + \cdots + x_{k-1}}{k-1} + \frac{x_k}{k-1}$$

$$\frac{k\bar{x}_k}{k-1} = \bar{x}_{k-1} + \frac{x_k}{k-1} \rightarrow \times \frac{k-1}{k} \rightarrow \bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k \qquad \alpha \equiv \frac{k-1}{k}$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k \qquad \alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

$$1 - \alpha =$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

$$1 - \alpha = 1 - 1 - \frac{1}{k}$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

$$1 - \alpha = 1 - 1 + \frac{1}{k} = \frac{1}{k}$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

$$1 - \alpha = 1 - 1 - \frac{1}{k} = \frac{1}{k}$$

$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

4. Recursive Goal : $\bar{x}_{k-1}$

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\alpha \equiv \frac{k-1}{k} = 1 - \frac{1}{k}$$

$$1 - \alpha = 1 - 1 - \frac{1}{k} = \frac{1}{k}$$

$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

4. Low Pass Filter

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

4. Low Pass Filter

$$\bar{x}_k = \frac{k-1}{k}\bar{x}_{k-1} + \frac{1}{k}x_k$$

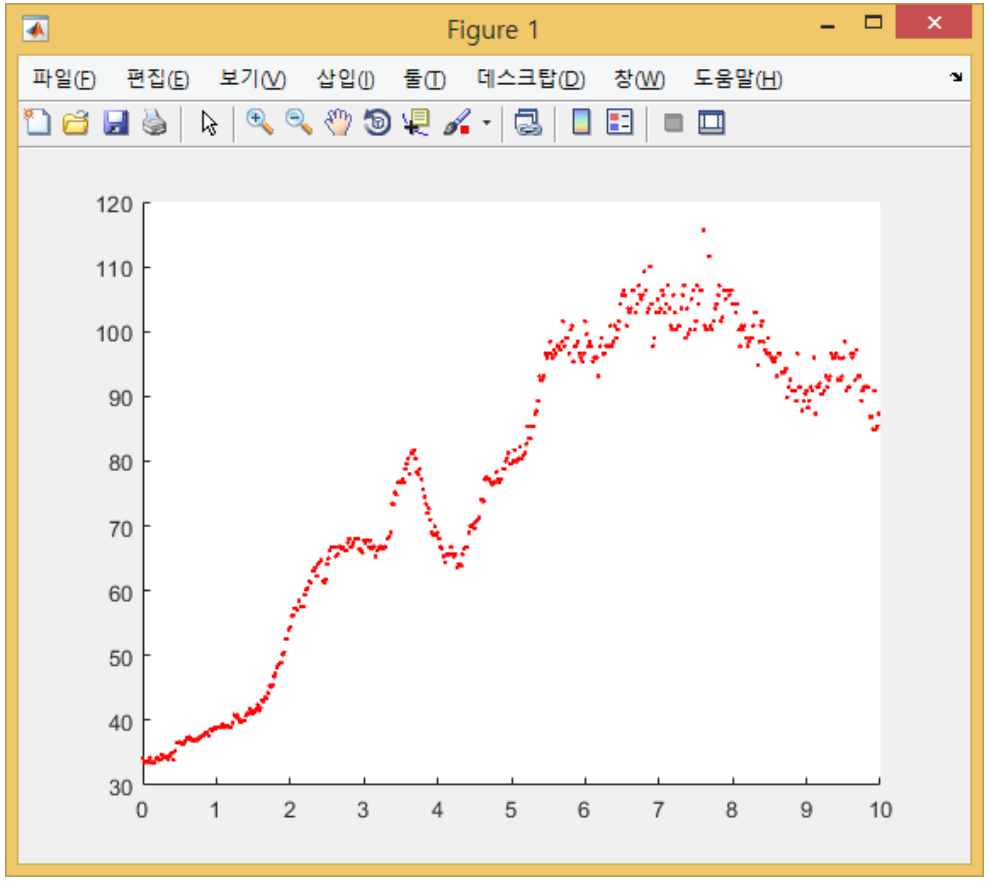
$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

$$0 < \alpha < 1$$

4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

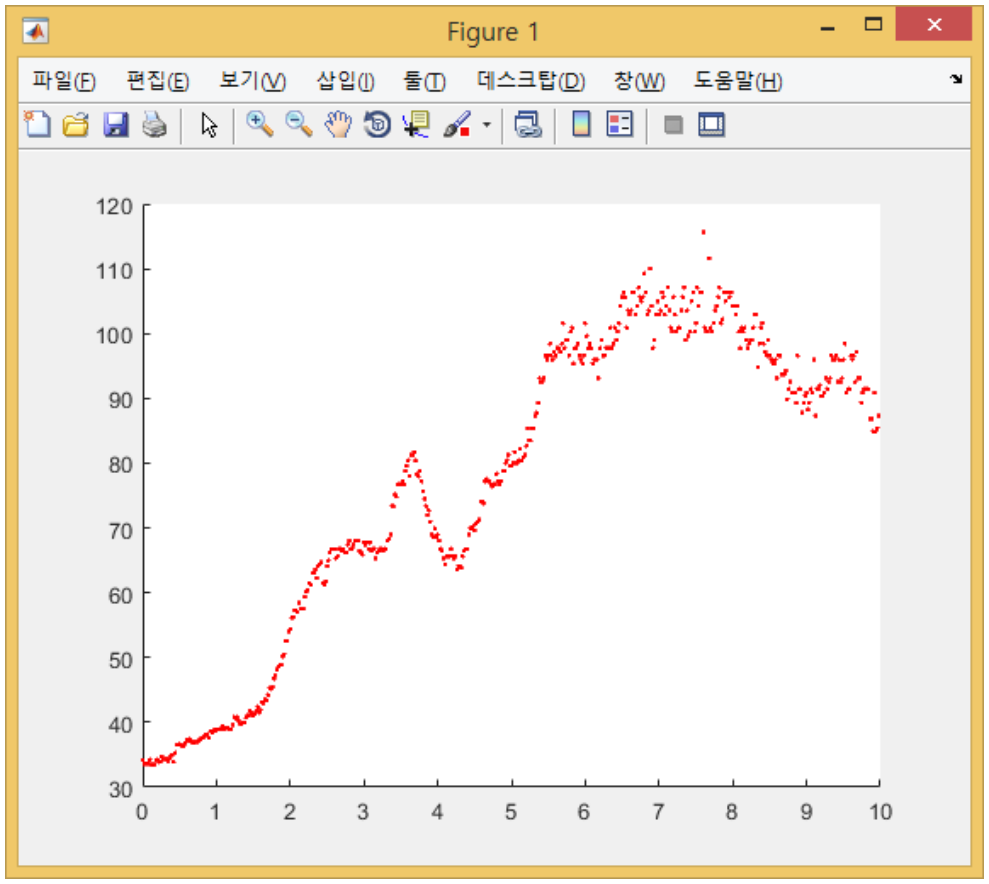
$$\alpha = 0.5 \qquad \bar{x}_k = 0.5 \bar{x}_{k-1} + 0.5 x_k$$

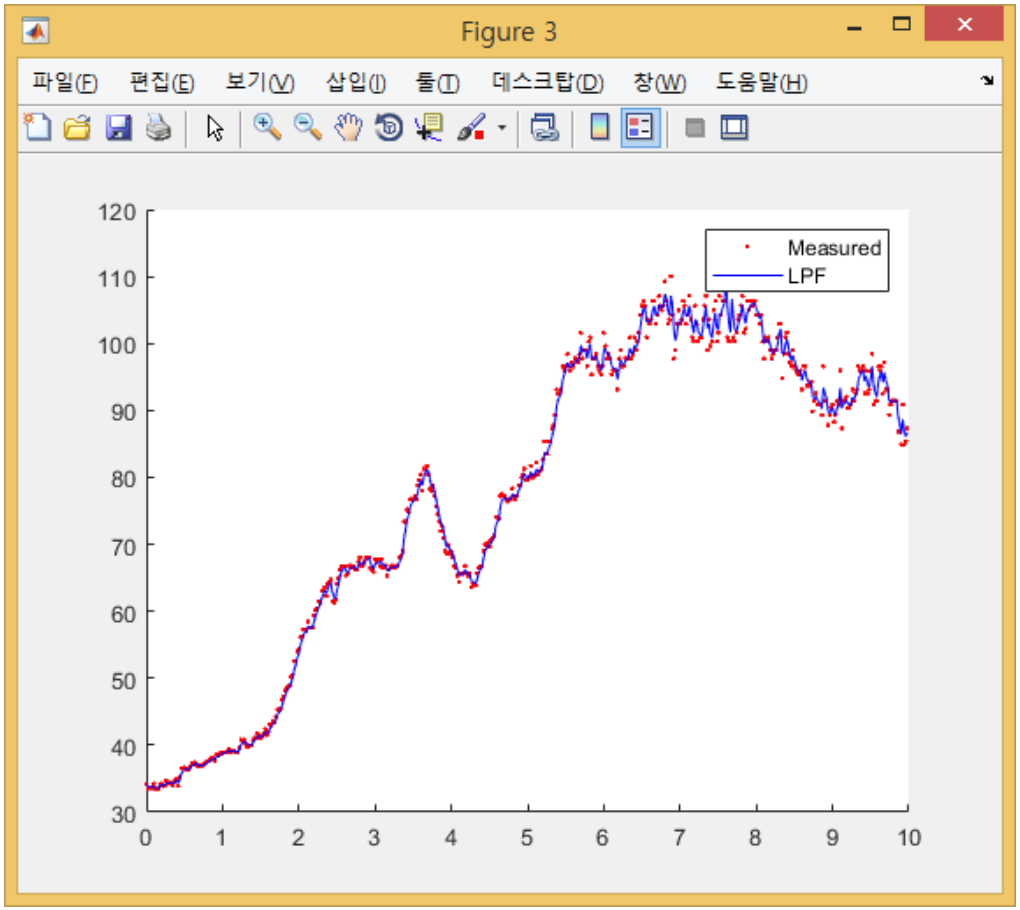


4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

$$\alpha = 0.5 \qquad \bar{x}_k = 0.5\bar{x}_{k-1} + 0.5x_k$$

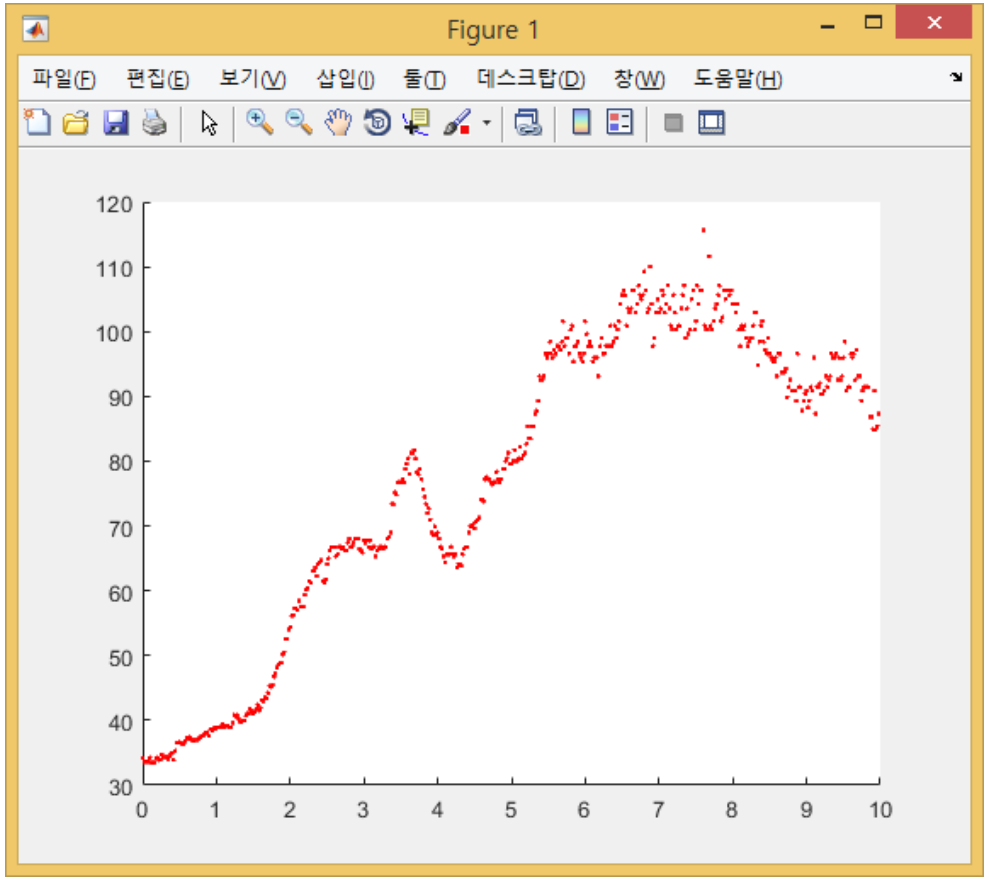




4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

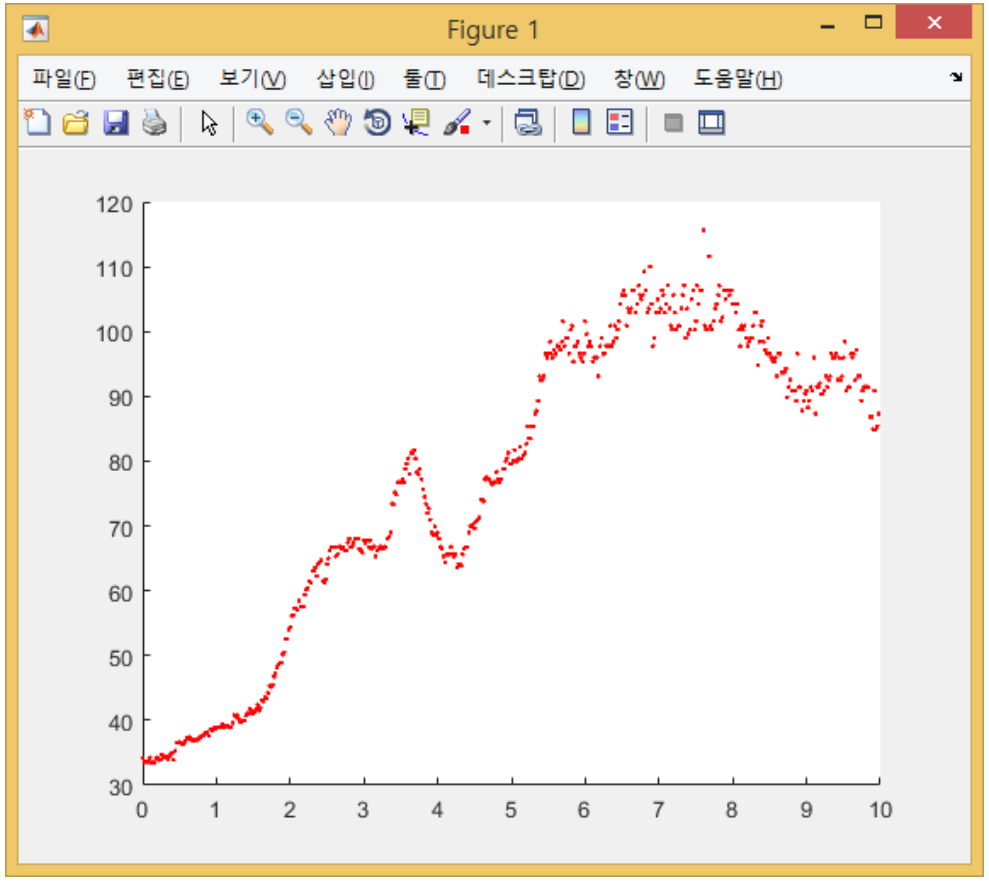
$$\alpha = 0.2 \qquad \bar{x}_k = 0.2\bar{x}_{k-1} + 0.8x_k$$

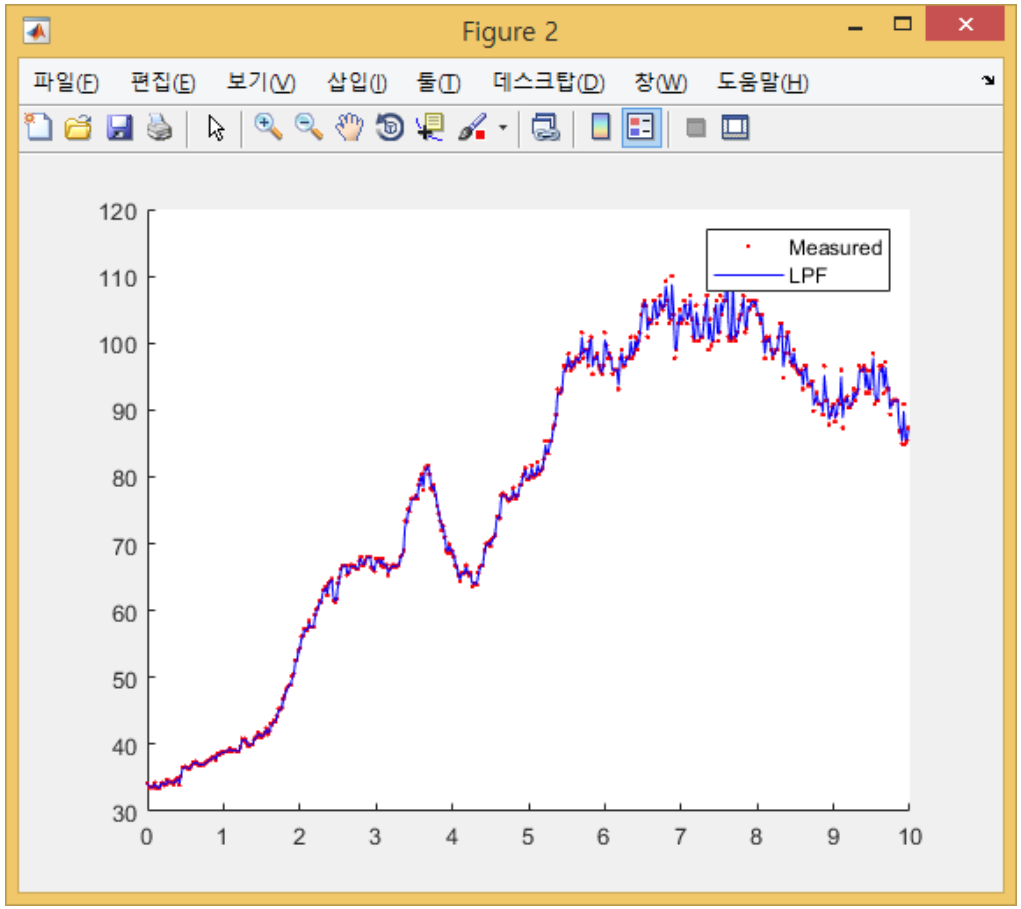


4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

$$\alpha = 0.2 \qquad \bar{x}_k = 0.2\bar{x}_{k-1} + 0.8x_k$$

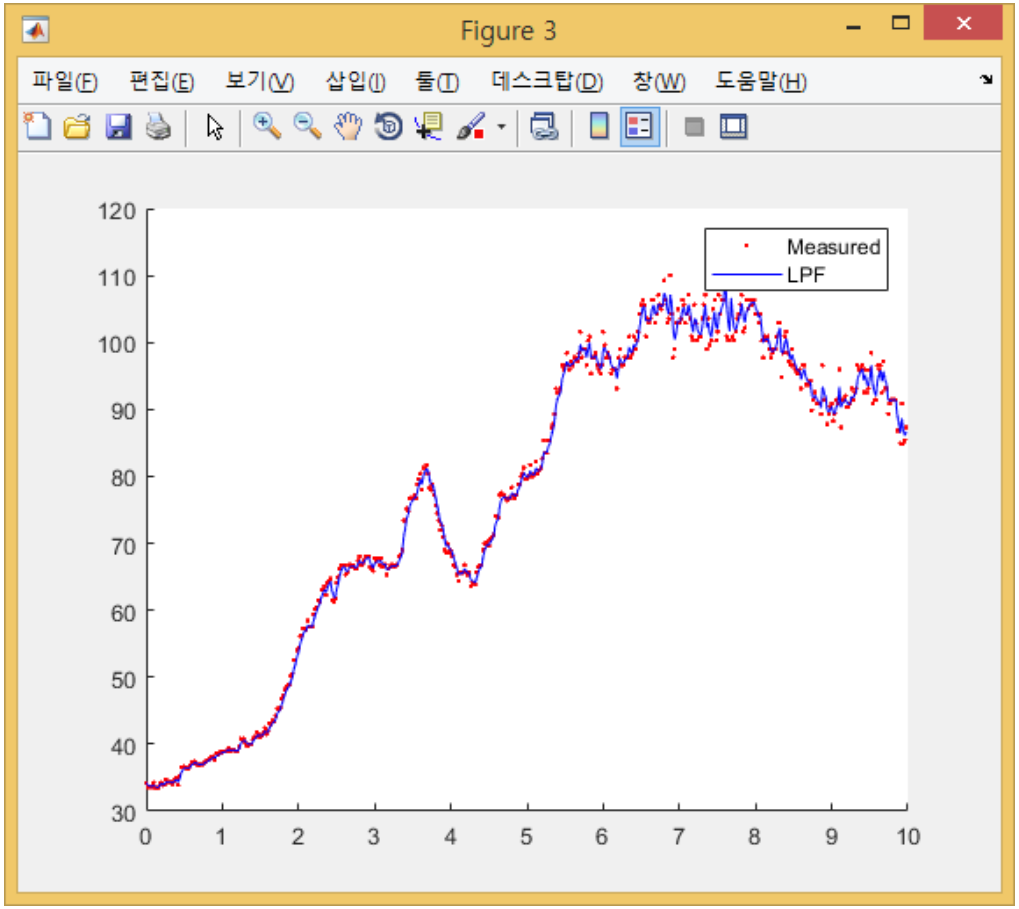


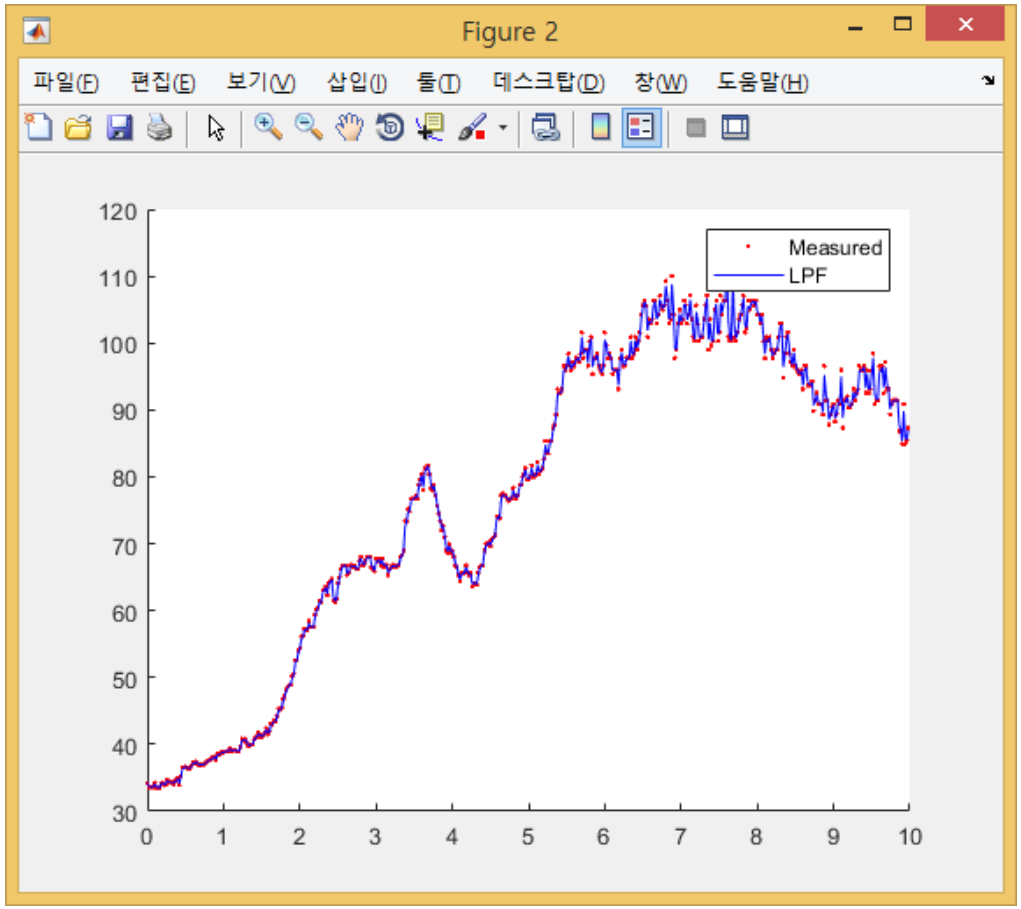


4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

$$\alpha = 0.2 \qquad \bar{x}_k = 0.2\bar{x}_{k-1} + 0.8x_k$$

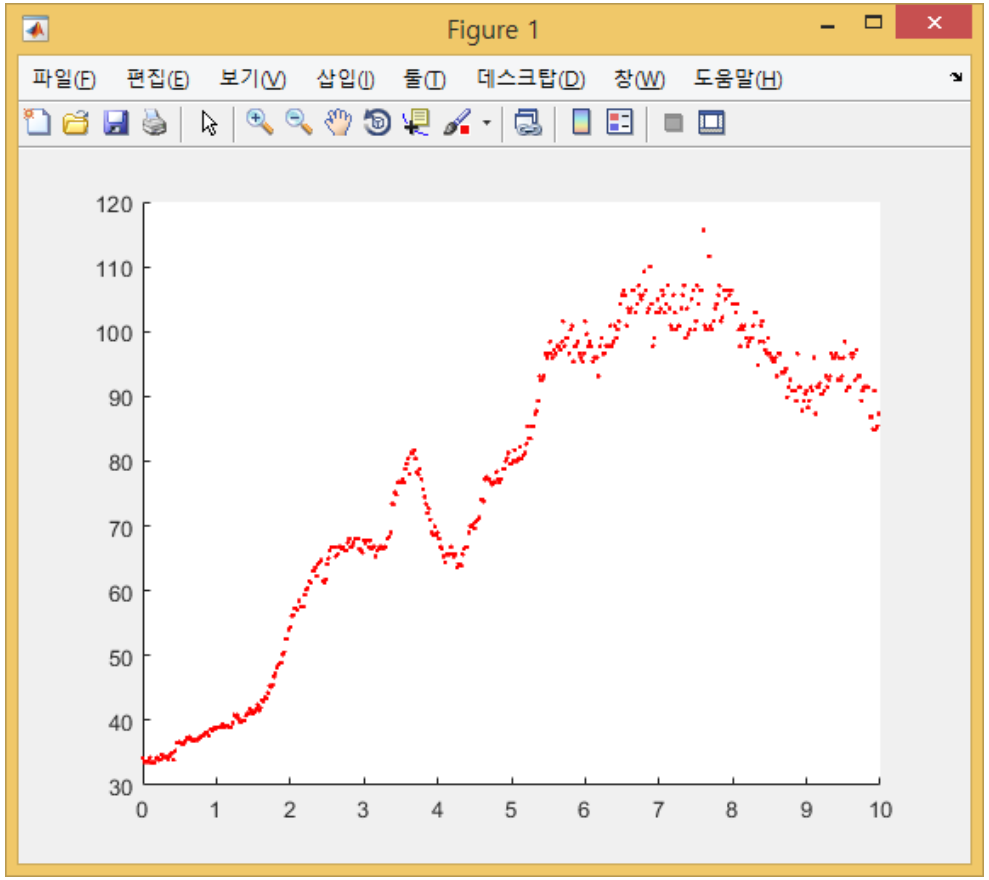




4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

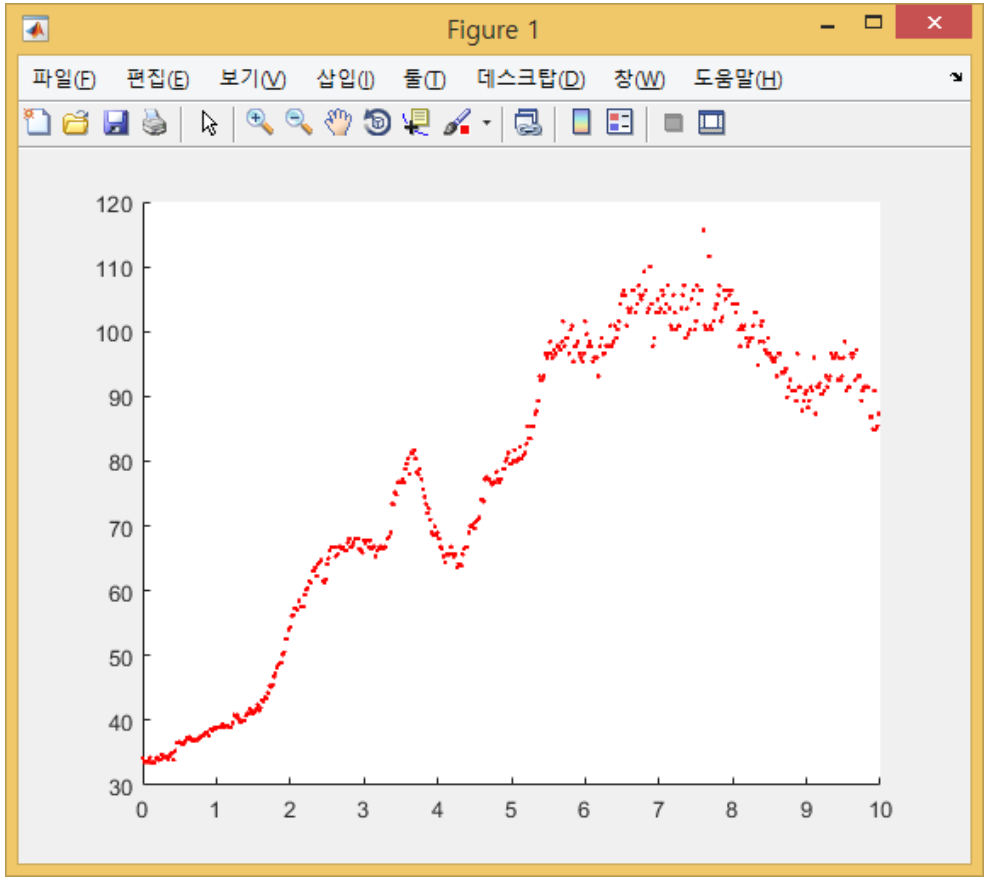
$$\alpha = 0.8 \qquad \bar{x}_k = 0.8 \bar{x}_{k-1} + 0.2 x_k$$

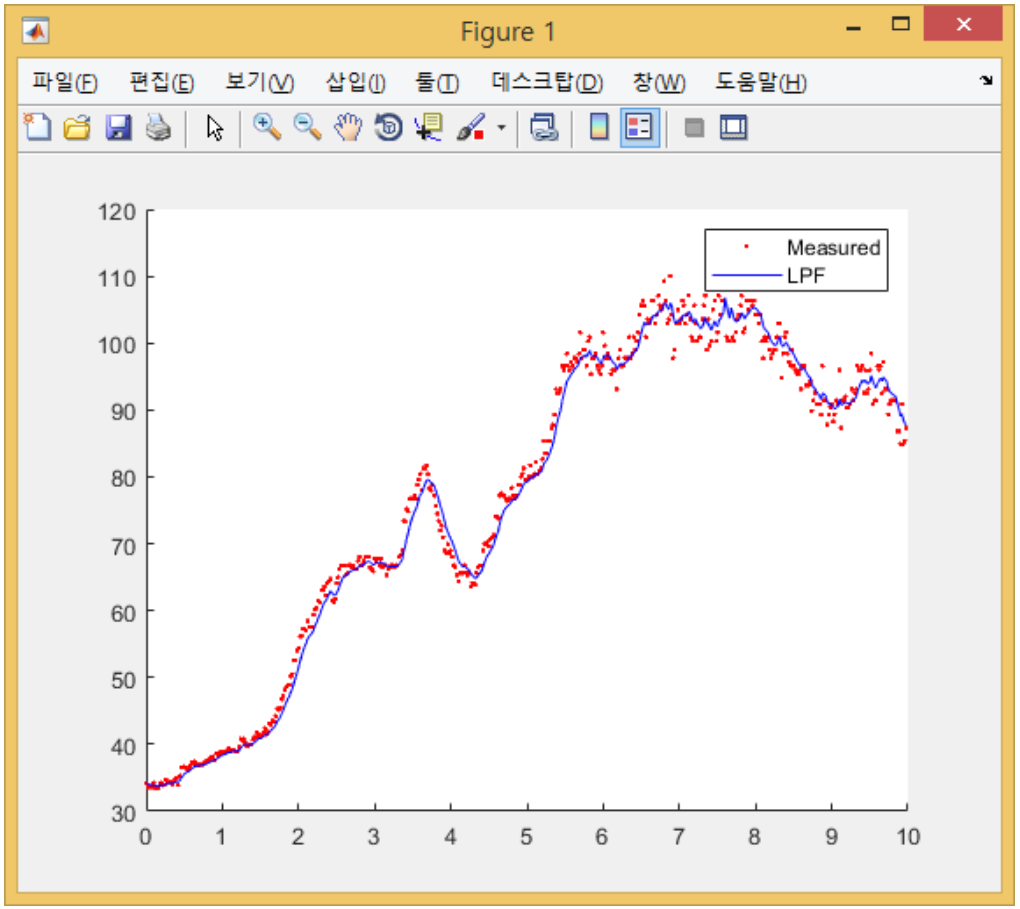


4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \qquad 0 < \alpha < 1$$

$$\alpha = 0.8 \qquad \bar{x}_k = 0.8 \bar{x}_{k-1} + 0.2 x_k$$

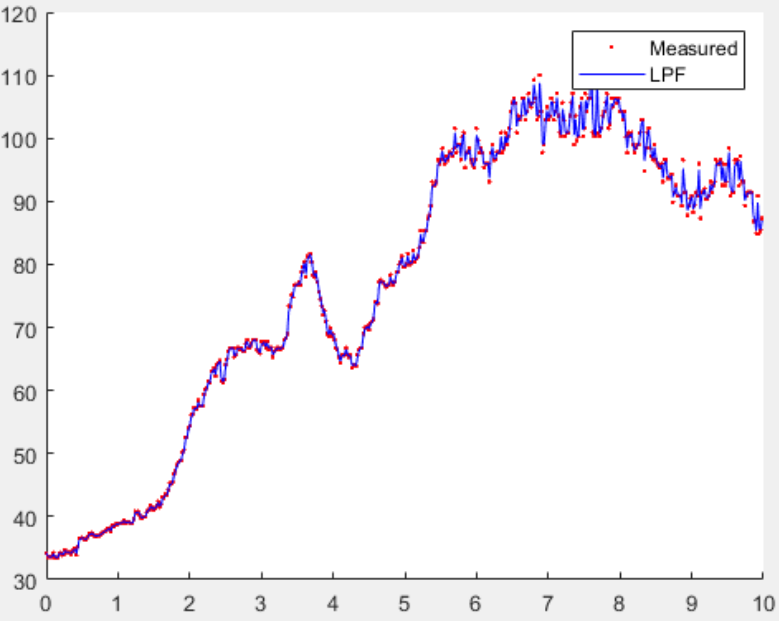


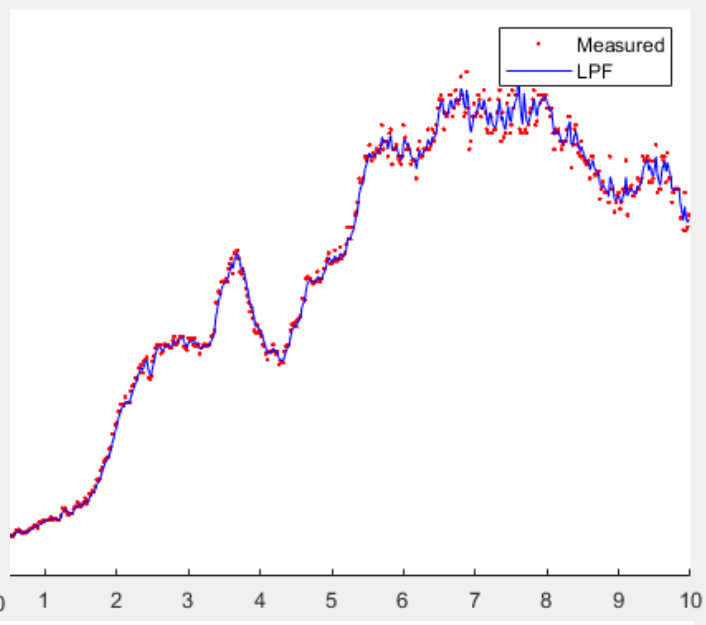


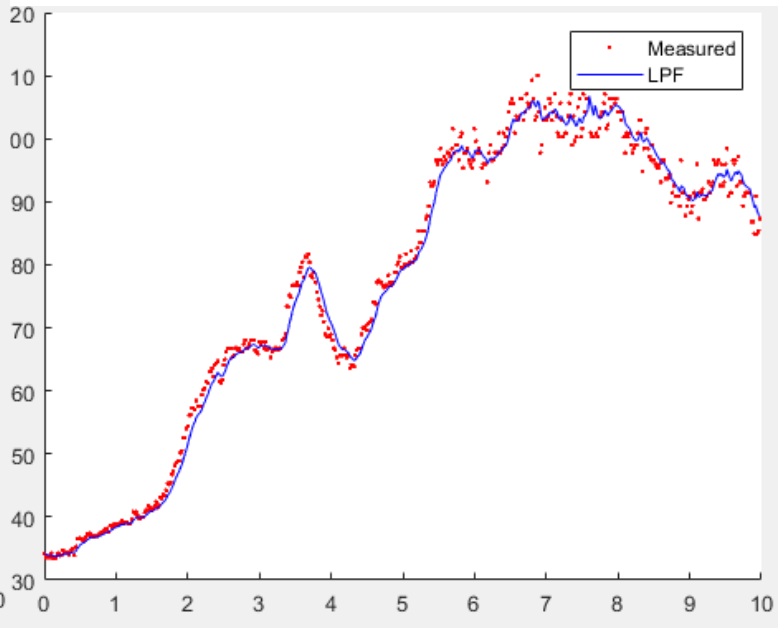
4. Low Pass Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k \quad 0 < \alpha < 1$$

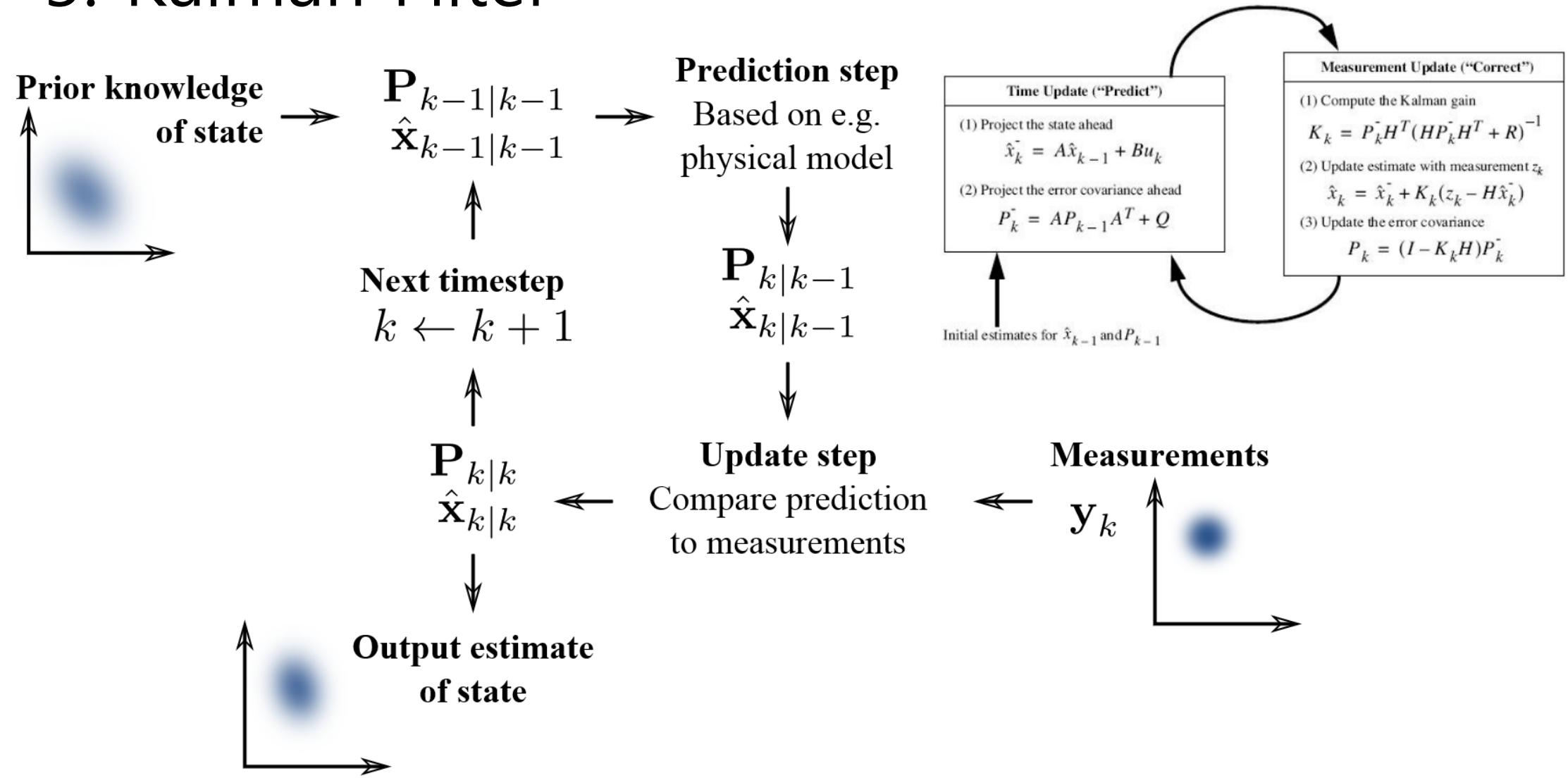
$\alpha = 0.2$ $\alpha = 0.5$ $\alpha = 0.8$



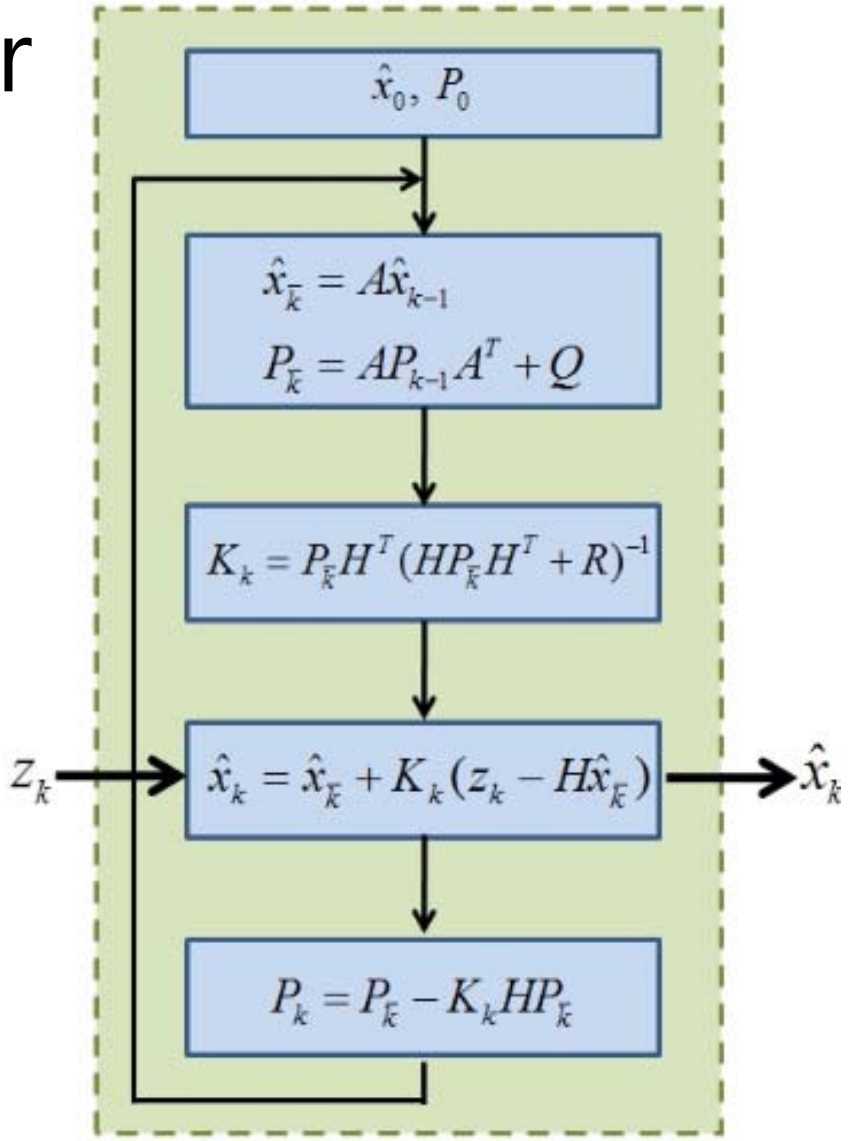




5. Kalman Filter

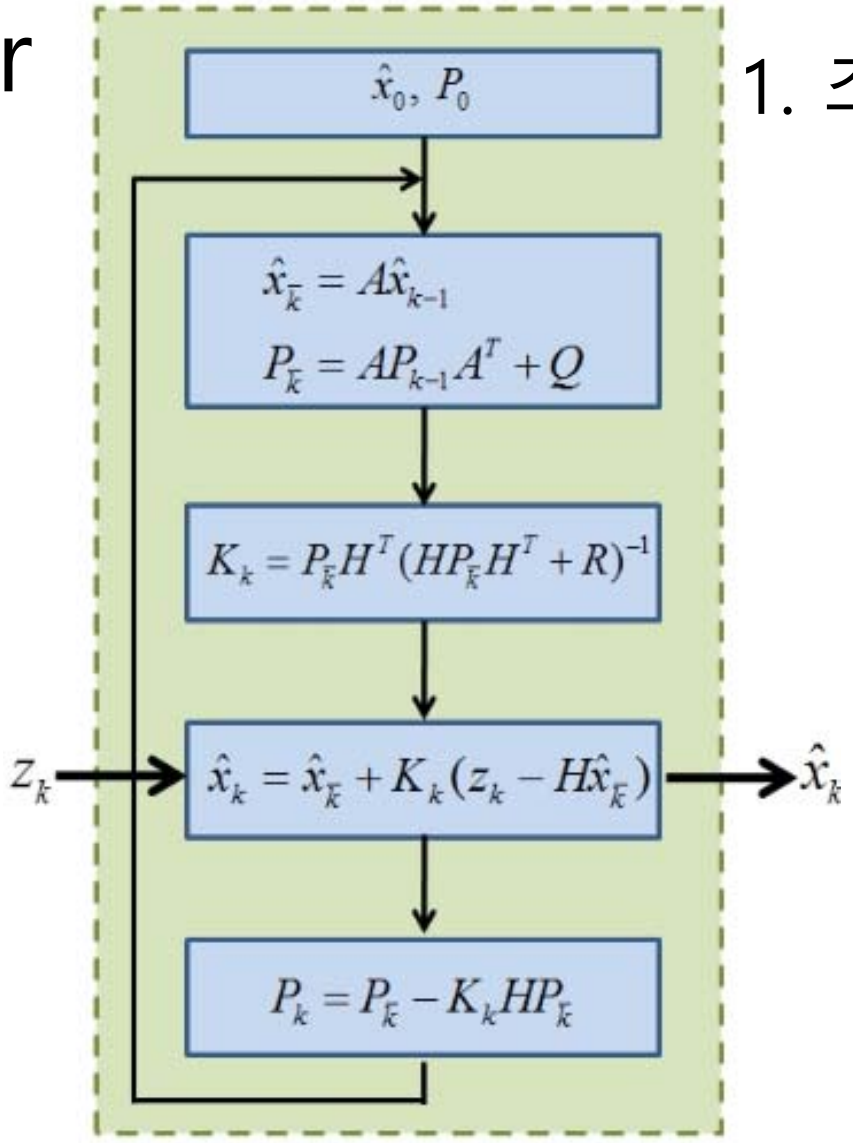


5. Kalman Filter



Kalman Filter algorithm

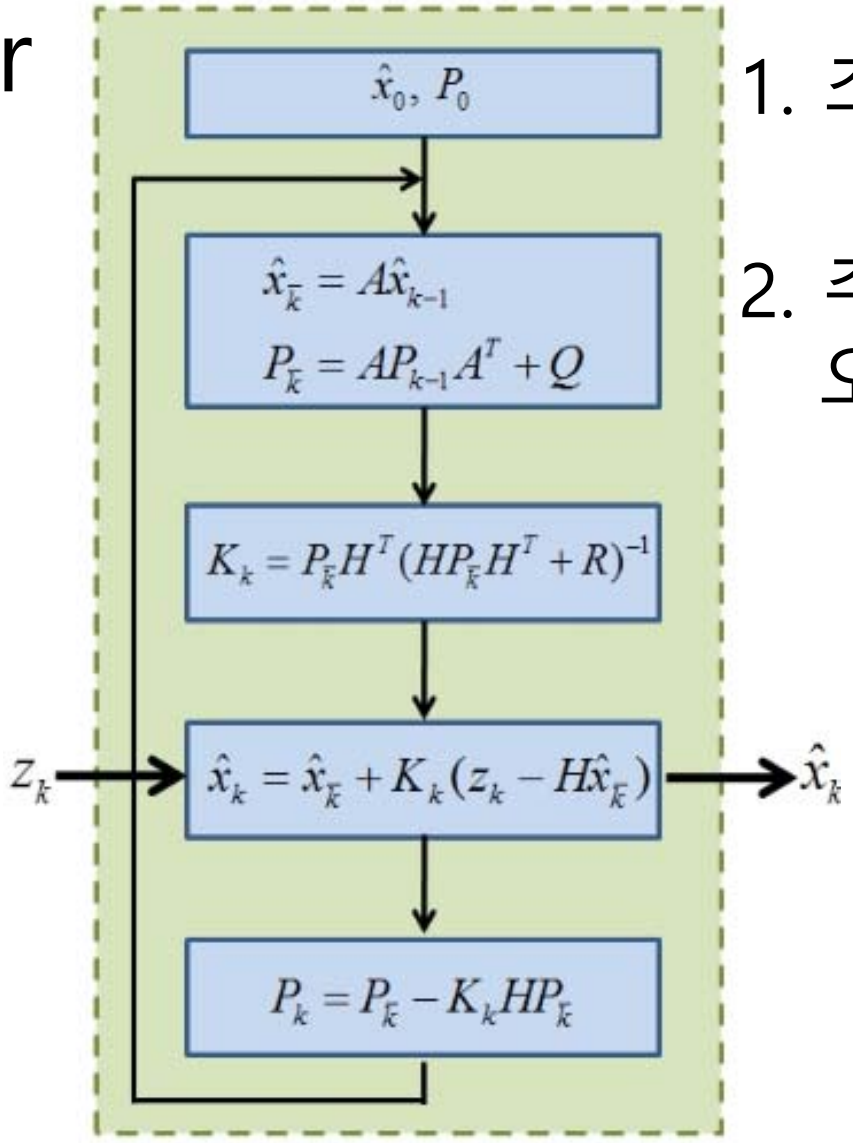
5. Kalman Filter



Kalman Filter algorithm

1. 초기값 선정

5. Kalman Filter

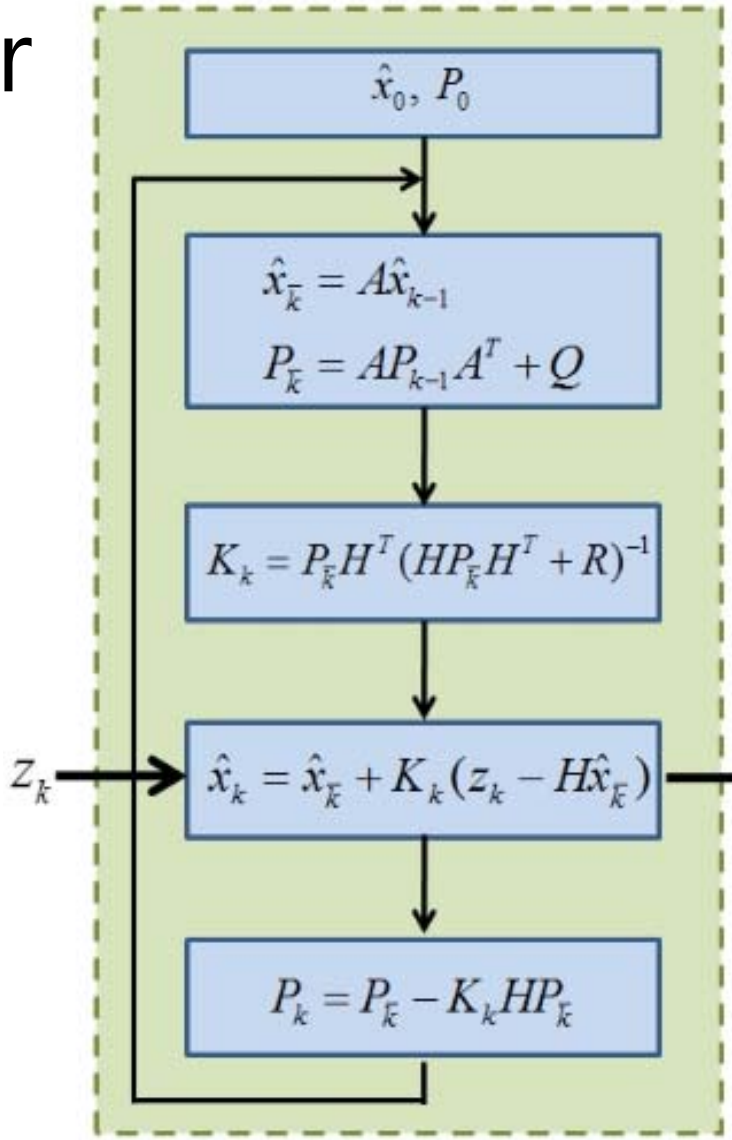


Kalman Filter algorithm

1. 초기값 선정

2. 추정값과
   오차 공분산 예측

5. Kalman Filter



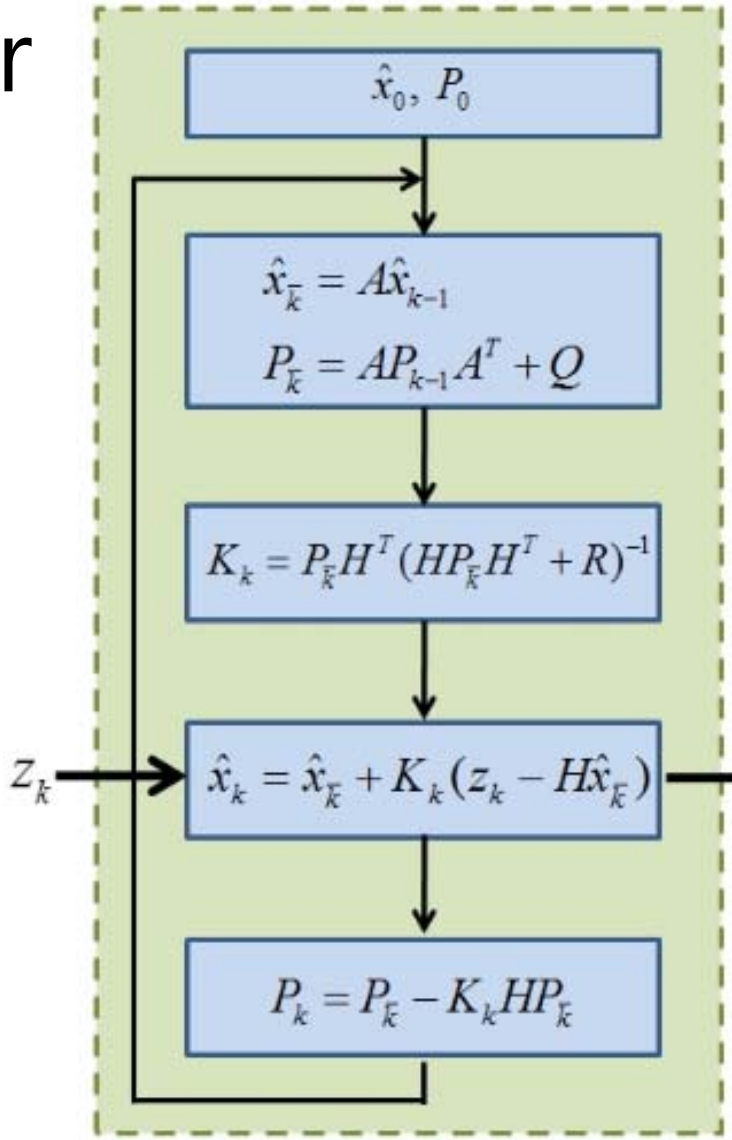
Kalman Filter algorithm

1. 초기값 선정

2. 추정값과
   오차 공분산 예측

3. 칼만 이득 계산

5. Kalman Filter



Kalman Filter algorithm

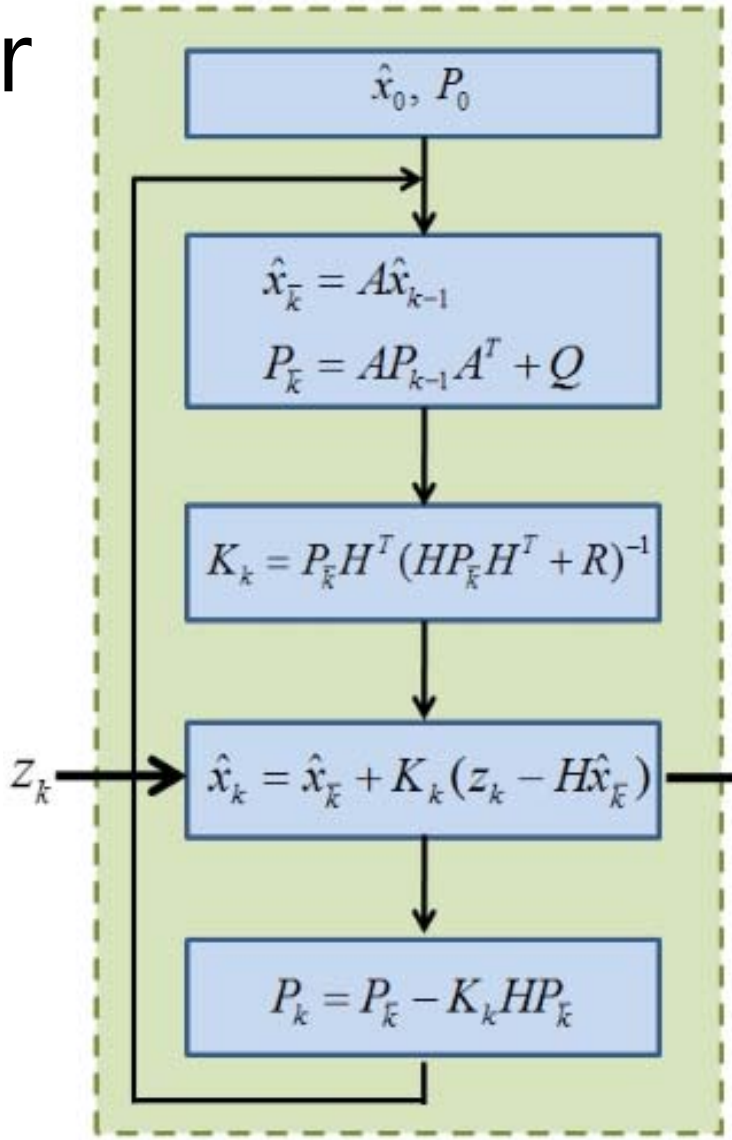
1. 초기값 선정

2. 추정값과
   오차 공분산 예측

3. 칼만 이득 계산

측정값

5. Kalman Filter



Kalman Filter algorithm

1. 초기값 선정

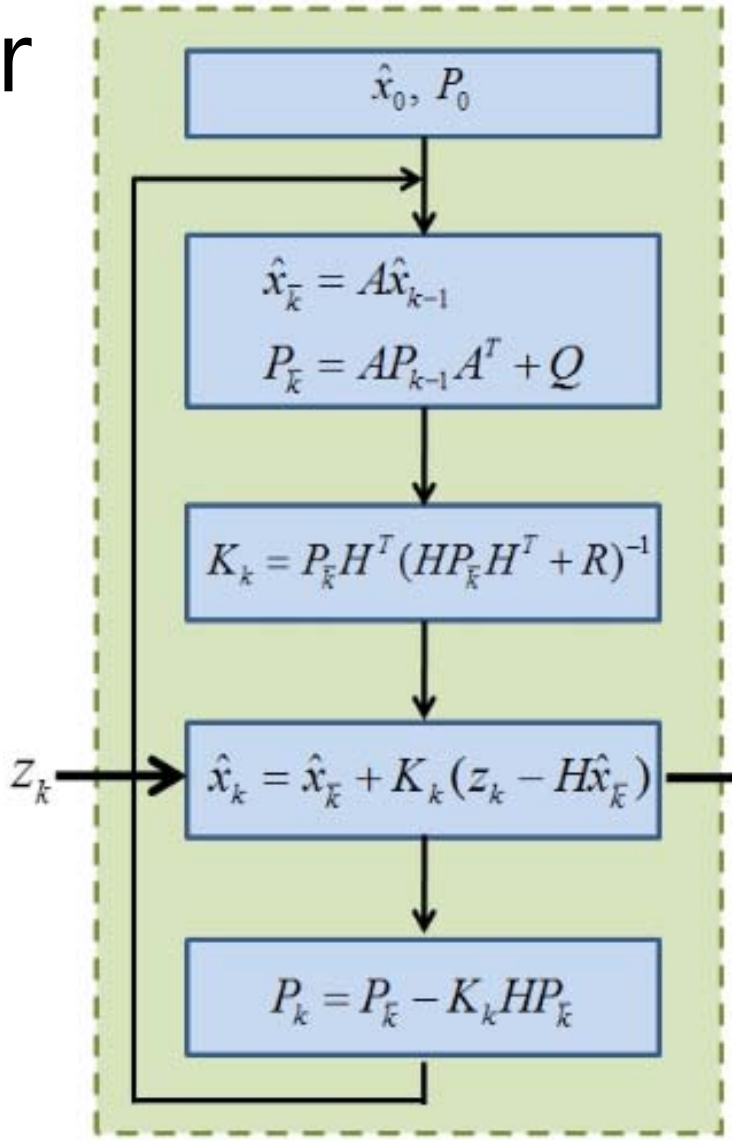
2. 추정값과
오차 공분산 예측

3. 칼만 이득 계산

측정값 z_k

$\hat{x}_k$ 4. 추정값 계산

5. Kalman Filter



Kalman Filter algorithm

1. 초기값 선정

2. 추정값과
오차 공분산 예측

3. 칼만 이득 계산

측정값 z_k

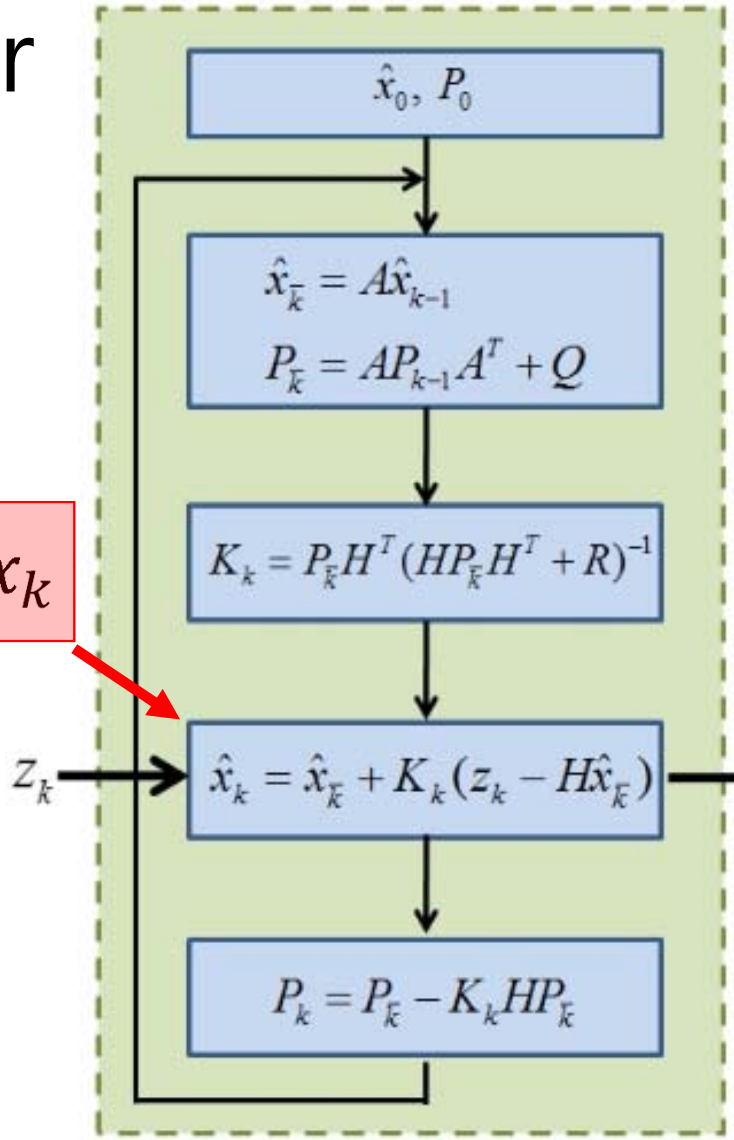
$\hat{x}_k$ 4. 추정값 계산

5. 오차 공분산 계산

5. Kalman Filter

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k$$

측정값



Kalman Filter algorithm

1. 초기값 선정
2. 추정값과 오차 공분산 예측
3. 칼만 이득 계산
4. 추정값 계산
5. 오차 공분산 계산