

# I. Homework 1

A sinusoidal signal has a frequency of 1MHz.

A. It is sampled with a frequency of 4MHz. Find the discrete frequency of the sampled signal. Plot its spectrum with a DFT of size 256.

$$1. f_d = \frac{f}{f_s} = \frac{1\text{M}}{4\text{M}} = 0.25 \text{ cycles/sample}$$

<NOTE> Sampling:  $\cos(2\pi f n T) = \cos\left(2\pi \frac{f}{f_s} n\right) = \cos(2\pi f_d n)$

$$2. \because f_d = 0.25 = \frac{64}{256} = \frac{i}{N}$$

$$\therefore \text{頻率剛好落在 DFT 格點上，只有兩個非零頻點} \begin{cases} k = i = 64 \\ k = N - i = 192 \end{cases}$$

(圖表為下方左圖)

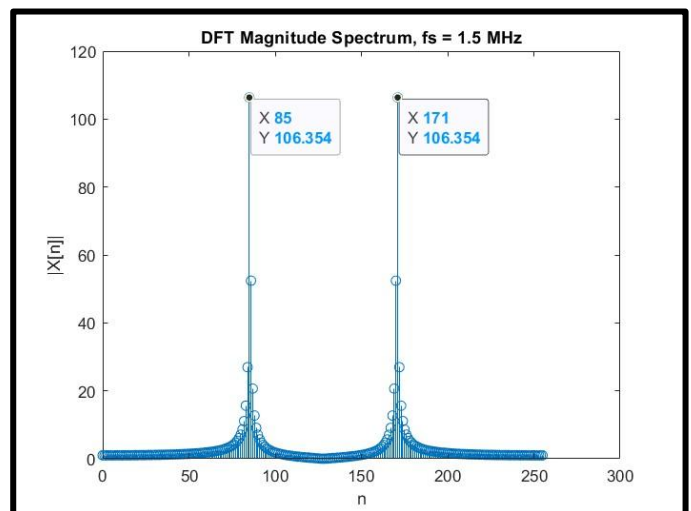
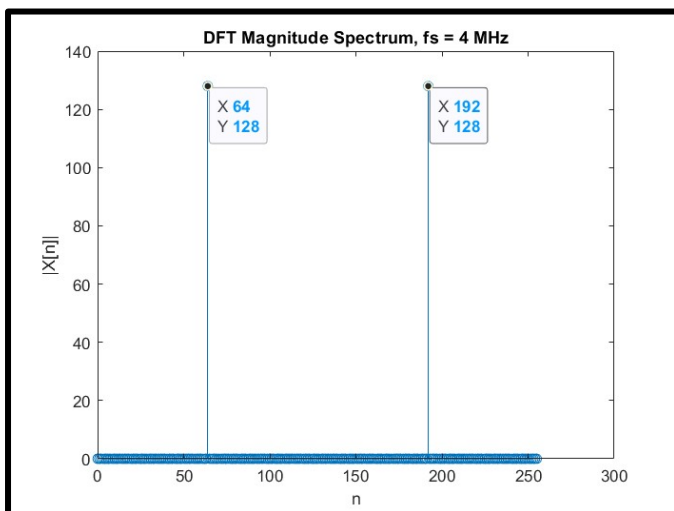
B. If the signal is sampled with 1.5MHz, find the frequency of the sampled signal. Plot its spectrum with a DFT of size 256.

$$1. f_d = \frac{f}{f_s} = \frac{1\text{M}}{1.5\text{M}} = \frac{2}{3} \text{ cycles/sample}$$

$$2. \because f_d = \frac{2}{3} \neq \frac{170.67}{256} \neq \frac{i}{N}$$

$$\therefore \text{頻率不對齊 DFT 格點，能量會分散，但還是集中在 } f_d \text{ 附近} \begin{cases} k = 171 \\ k = 256 - 171 = 85 \end{cases}$$

(圖表為下方右圖)

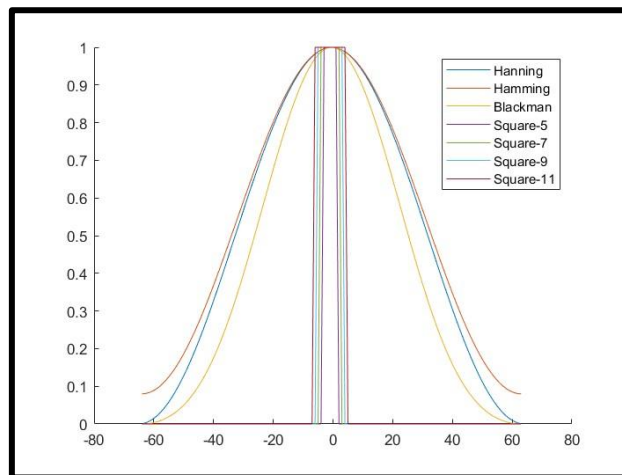


## II. Homework 2

Generate a triangular signal (with length 128, peak 64) and add Gaussian noise to yield an SNR of 15dB. Using the frequency windowing technique to conduct a filtering operation (conducting 128-point DFT). Generate 100 noise sequences and add to the signal. Find an optimum windowing function such that the averaged MSE of the filtered signal is minimized.

### A. 使用的 frequency windowing functions

1. Hanning, Hamming, Blackman
2. 方波，寬度為 5,7,9,11



### B. 直覺推論

1. 三角波的頻譜類似  $(\text{sinc}(f))^2$ ，能量非常集中在中心
2. 高 SNR 時，盡量保留完整的頻譜，能更好還原三角波
3. 低 SNR 時，取中心一部分還原三角波即可，頻譜外圍雜訊干擾過於嚴重

### C. 模擬驗證，計算 averaged MSE

	Lower SNR (for SNR = 1dB)	SNR = 15dB	Higher SNR (for SNR = 30dB)
Hanning	396.2188	16.0046	0.50893
Hamming	431.2509	16.6543	0.53456
Blackman	326.4279	12.3388	<b>0.41717</b>
Square-5	<b>27.6515</b>	5.9179	5.0012
Square-7	47.2029	6.6367	5.0198
Square-9	60.3128	<b>3.4054</b>	0.88017
Square-11	79.2421	3.8135	0.89713

### D. 驗證結果，optimum windowing function with min. MSE

1. 15dB：Square-9
2. Lower SNR：選方波，避免取到外圍被雜訊干擾嚴重的頻譜
3. Higher SNR：選能夠大範圍覆蓋的，因為 noise 小不影響，所以全部取能更好還原