PROBLEM: Use the result in Practice 3. Assume there is a phase shift of 5° in the receiver carrier. Observe demodulated signals. Compare the result with

$$\begin{cases} \boldsymbol{z_1(t) = m_1(t)\cos(\theta) - m_2(t)\sin(\theta)} \\ \boldsymbol{z_2(t) = m_1(t)\sin(\theta) + m_2(t)\cos(\theta)} \end{cases}$$

I. 推導 Carrier Phase Offset

A. Tx 送出的訊號 $\mathbf{x(t)}$

$$\begin{aligned} \mathrm{x(t)} &= \mathrm{Re}\left\{\mathrm{x(t)} \cdot \sqrt{2}\mathrm{e}^{\mathrm{j}2\pi \mathrm{f_c}t}\right\} \\ &= \sqrt{2}[m_1(t)\cos(2\pi f_c t) - m_2(t)\sin(2\pi f_c t)] \end{aligned}$$

B. Rx 接收的訊號 $\mathbf{y(t)}$

Assume that $\mathrm{y(t) = x(t)}$

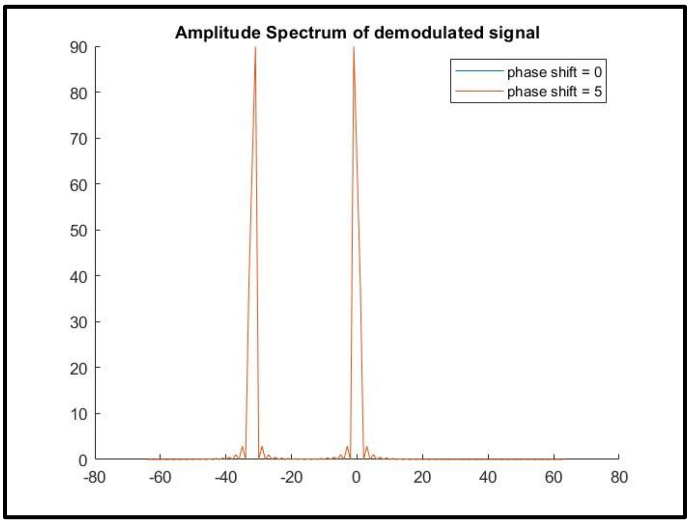
$$\begin{aligned} \mathrm{LPF}\left\{\mathrm{y(t)} \cdot \sqrt{2}\mathrm{e}^{-\mathrm{j}(2\pi \mathrm{f_c}t-\theta)}\right\} &= e^{j\theta} \cdot LPF\left\{\mathrm{y(t)} \cdot \sqrt{2}\mathrm{e}^{-\mathrm{j}2\pi \mathrm{f_c}t}\right\} \\ &= e^{j\theta} \cdot [m_1(t) + jm_2(t)] \\ &= (\cos\theta + \mathrm{j}\sin\theta) \cdot [m_1(t) + jm_2(t)] \\ &= [m_1(t)cos\theta - m_2 sin\theta] + j[m_1(t)\sin(\theta) + m_2(t)\cos(\theta)] \\ &= z_1(t) + jz_2(t) \end{aligned}$$

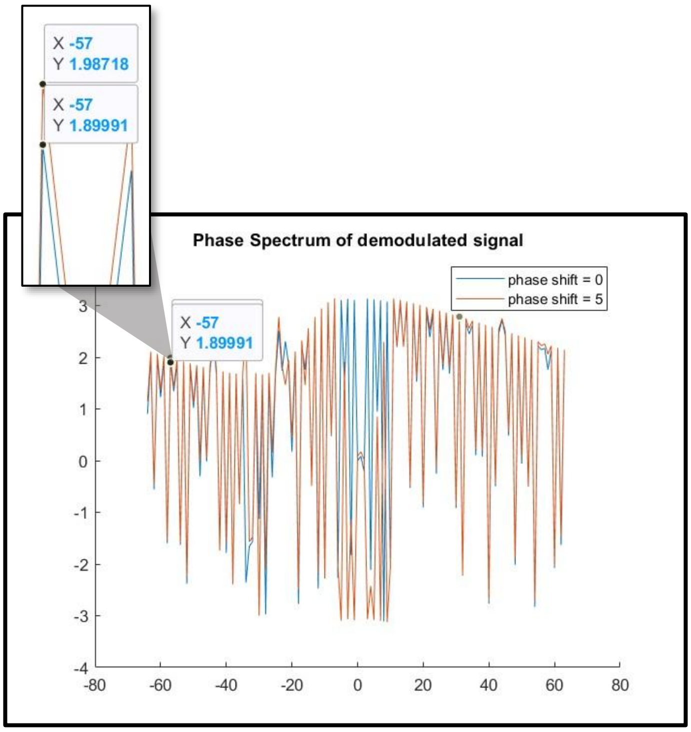
C. 接收端載波存在相位偏移θ，與發射端載波失去正交性，導致解調後的 I、Q 分量互相干擾。

II. MATLAB 模擬

A. Demodulated spectrums

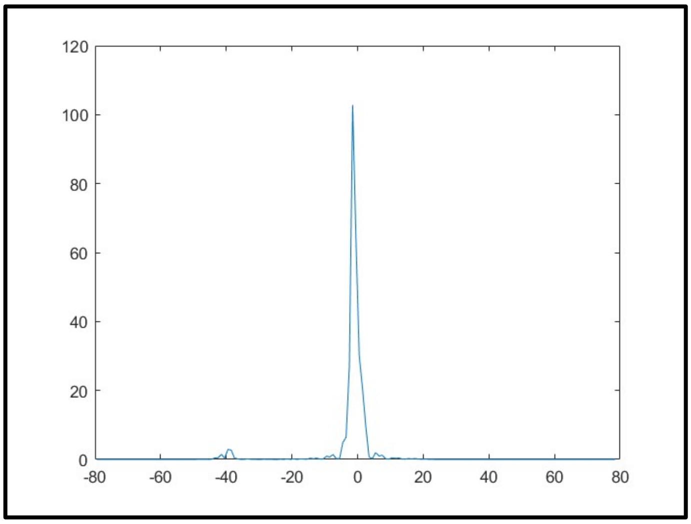
1. 因為接收端載波相位偏移，是乘上一個常數$e^{j\theta}$，大小為 1，所以振幅頻譜大小不變。
2. 但相位頻譜會整體平移θ。這次模擬中，相位偏移$5^\circ \cong 0.0873$ rad，從結果圖看兩者偏移的結果約為$1.98718 - 1.89991 = 0.08727$ rad。





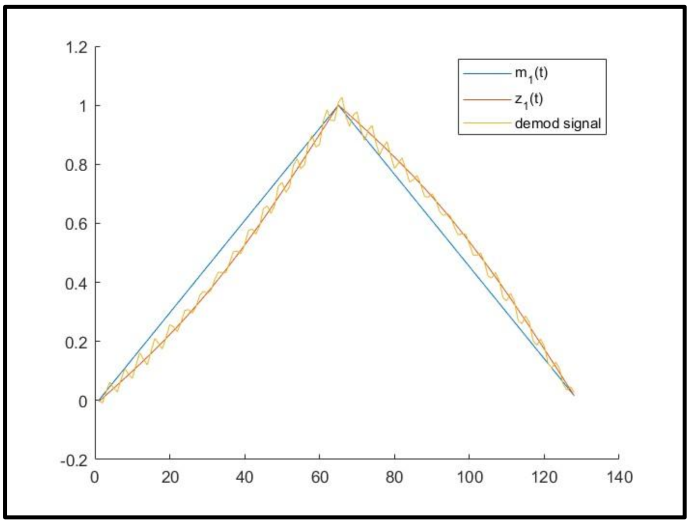
B. Filtered spectrums

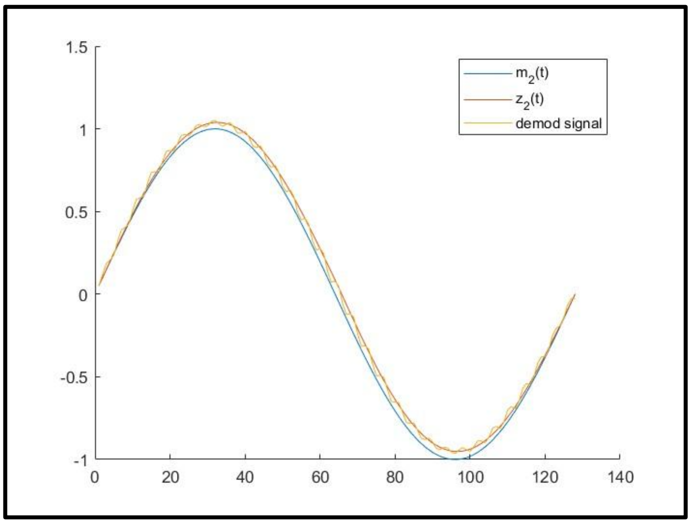
1. 濾掉高頻留下低頻，頻譜剩一根在 DC 位置。



C. Compare the result with $\mathbf{z_1(t), z_2(t)}$

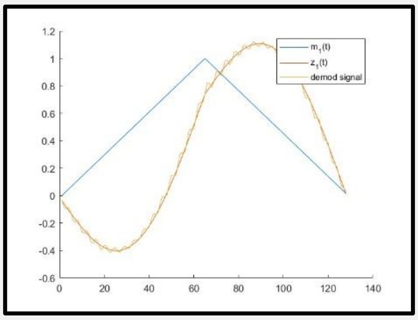
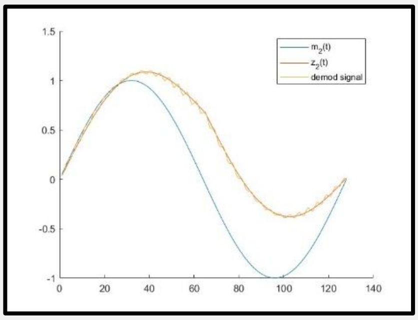
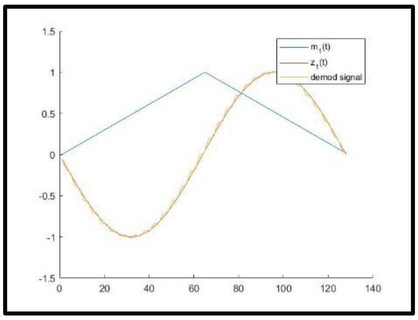
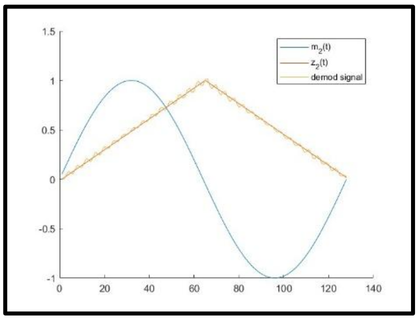
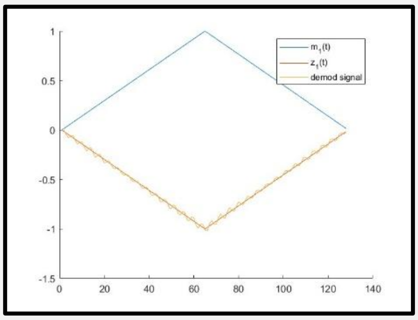
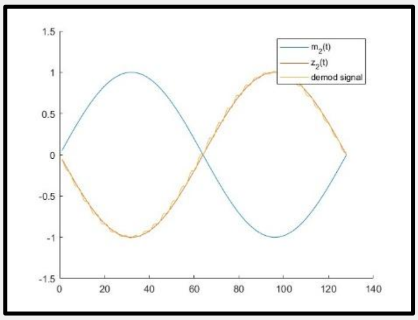
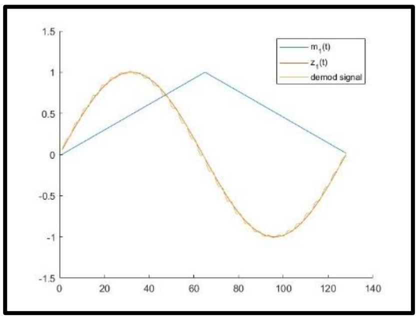
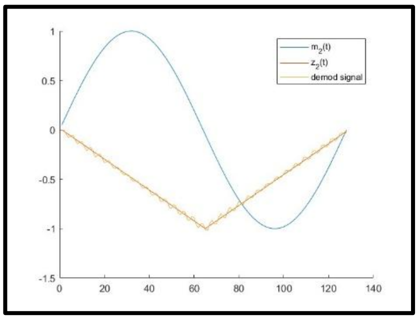
1. 模擬結果和$\mathrm{z}_1(t), z_2(t)$的理論值趨勢相同，但和原本的$\mathrm{m}_1(t), m_2(t)$不再相同，因為接收端相位偏移導致 I、Q 分量互相干擾。





D. Different phase shift θ

1. $\begin{cases} z_1(t) = m_1(t)\cos(\theta) - m_2(t)\sin(\theta) \\ z_2(t) = m_1(t)\sin(\theta) + m_2(t)\cos(\theta) \end{cases}$
2. 相位偏移 45 度時，$z_1(t)$和$z_2(t)$中$m_1(t), m_2(t)$各占比一半。
3. 當相位偏移持續增加，到達±90°時，I、Q 分量完全對調。±的差別為振福相反。
4. 到180°時，回歸$z_1(t)$中僅包含$m_1(t)$，$z_2(t)$中僅包含$m_2(t)$，但震幅相差一個負號。

θ	$z_1(t)$	$z_2(t)$	理論值
45°			$\begin{cases} z_1(t) = \frac{1}{\sqrt{2}} m_1(t) - \frac{1}{\sqrt{2}} m_2(t) \\ z_2(t) = \frac{1}{\sqrt{2}} m_1(t) + \frac{1}{\sqrt{2}} m_2(t) \end{cases}$
90°			$\begin{cases} z_1(t) = -m_2(t) \\ z_2(t) = m_1(t) \end{cases}$
180°			$\begin{cases} z_1(t) = -m_1(t) \\ z_2(t) = -m_2(t) \end{cases}$
−90°			$\begin{cases} z_1(t) = m_2(t) \\ z_2(t) = -m_1(t) \end{cases}$